THIN LAYERS FOR THE LANDAU-LIFSCHITZ EQUATION

David Sanchez

Proceedings of Dynamic Systems and Applications 4 (2004) 239- 244

THIN LAYERS FOR THE LANDAU-LIFSCHITZ EQUATION

DAVID SANCHEZ

Mathématiques appliquées de Bordeaux, UMR 5466, Université Bordeaux 1,
351 Cours de la Libration, 33405 Talence cedex, France.

ABSTRACT. In this paper we study the behaviour of the magnetization in a thin layer of ferromagnetic material when the exchange coefficient is small. We explain the interaction between the boundary layer phenomenon and the thin layer phenomenon.

AMS (MOS) Subject Classification. 35Q60, 78M35, 34E20.

1. INTRODUCTION

We consider here ferromagnetic media which are spontaneously magnetized. They are used for example in computers and in aeronautics for the skins of the planes. In the quasi-stationary case, the behaviour of the magnetization, denoted by u, is described by the Landau-Lifschitz equation:

$$\begin{cases} \dfrac{\partial u}{\partial t} = u \wedge H_e - u \wedge (u \wedge H_e) \qquad \text{in} \quad \mathbb{R}_t^+ \times \Omega, \\ \dfrac{\partial u}{\partial n} = 0 \qquad \text{on} \quad \partial\Omega, \\ u(0,\cdot) = u_0 \qquad \text{in} \quad \Omega, \end{cases} \tag{1}$$

where Ω is a domain filled with ferromagnetic material and immersed in the vacuum, n is the outward unitary normal to $\partial\Omega$ and H_e is the effective magnetic field given by:

$$H_e = \varepsilon^2 \Delta u + \mathcal{H}(u). \tag{2}$$

Here, ε^2 is a small parameter called the exchange coefficient, $\mathcal{H}(u)$ is called the demagnetizing field given by the magnetostatic equations (we assume that we always are at the electromagnetic equilibrium):

$$\begin{cases} \mathcal{H}(u) \in \mathbb{L}^2(\mathbb{R}^3), \\ \text{div } (\mathcal{H}(u) + \overline{u}) = 0 \text{ in } \mathcal{D}'(\mathbb{R}^3), \\ \text{curl } \mathcal{H}(u) = 0 \text{ in } \mathcal{D}'(\mathbb{R}^3), \end{cases} \tag{3}$$

where $\overline{u}$ is the extension of u by 0 outside Ω.
When the exchange coefficient ε is fixed, G. Carbou and P. Fabrie have proved in [6] the existence and uniqueness of local regular solutions for the equations (1)-(3). When the exchange coefficient ε goes to zero, G. Carbou, P. Fabrie and O. Guès have proved in [8] the following theorem on u^ε, solution of the previous equations:

Theorem 1.1 (G. Carbou, P. Fabrie, O. Guès)**.** *Let $u_0 \in \mathbb{H}^5(\Omega)$, $|u_0| = 1$, $\left(\dfrac{\partial u_0}{\partial n}\right)_{|\partial\Omega} = 0$, there exists a function $U^0 \in \mathcal{C}\left([0,+\infty[;\mathbb{H}^5(\Omega)\right)$ solution of*

$$(4) \qquad \begin{cases} \dfrac{\partial U^0}{\partial t} = U^0 \wedge \mathcal{H}(U^0) - U^0 \wedge \left(U^0 \wedge \mathcal{H}(U^0)\right) & \text{in } [0,+\infty[\times\Omega, \\ U^0(0,x) = u_0(x) \quad \text{on } \Omega, \end{cases}$$

which, moreover, fulfills $|U^0(t,x)| = 1$ for all $(t,x) \in [0,+\infty[\times\Omega$. There also exists a function $\widetilde{U}^1 \in \mathbb{L}^\infty_{loc}\left(\mathbb{R}^+_t;\mathbb{H}^4(\Omega)\otimes\mathbb{H}^4(\mathbb{R}^+)\right)$ and a time $T^\varepsilon > 0$ such that $\lim\limits_{\varepsilon\to 0} T^\varepsilon = +\infty$, so that the solution u^ε of the equations (1)-(3) can be written in the form

$$u^\varepsilon(t,x) = U^0(t,x) + \varepsilon\widetilde{U}^1\left(t,x,\frac{\varphi(x)}{\varepsilon}\right) + \varepsilon v^\varepsilon(t,x), \quad t \in [0,T],$$

$\forall\ T < T^\varepsilon$ and with $\varphi(x) = \mathrm{dist}(x,\partial\Omega)$. Moreover v^ε is bounded in $\mathbb{L}^\infty(0,T;\mathbb{H}^1(\Omega))$, and $\varepsilon v^\varepsilon$ is bounded in $\mathbb{L}^\infty(0,T;\mathbb{H}^2(\Omega)) \cap \mathbb{L}^2(0,T;\mathbb{H}^3(\Omega))$.

So the solution u^ε of the equations (1)-(3) admits an asymptotic expansion when ε goes to zero on a time interval $[0,T^\varepsilon[$ with $T^\varepsilon \to +\infty$, whose first order term is the solution of the Landau-Lifschitz equation with $\varepsilon = 0$ and whose second order term is a perturbation which lives in a neighbourhood of the boundary $\partial\Omega$ and whose characteristic thickness is ε.
In [16] we study the case where the demagnetizing field is replaced by an anisotropic field $\Psi(x,u)$ in H_e.

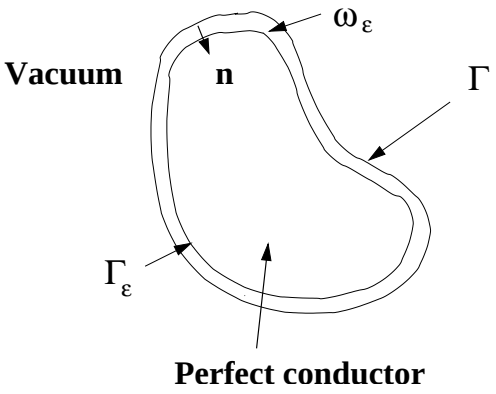
Remark 1.2. *We can study these equations with both the demagnetizing field and the anisotropic field but we ignore the latter for simplification.*

According to the previous result, the perturbation occurs in a ε-thick neighbourhood of $\partial\Omega$. If we now consider a domain with thickness ε, there will be a competition between the behaviours due to the small thickness and the boundary layer that we will describe in the following: in a previous work (see [17]) we considered the case of a ε-thick periodic sheet of ferromagnetic material immersed in the vacuum. We will present here the case of a thin layer of ferromagnetic material spread on a non-flat perfect conductor. We then take into account the geometry of the domain (see [18] for the whole proof).

2. THIN LAYERS

Let us precise the notations. Let Ω be a regular bounded domain of $\mathbb{R}^3$. We denote $\Gamma = \partial\Omega$. For $x \in \Omega$ we introduce $\varphi(x) = \mathrm{dist}(x, \Gamma)$ and $P_\Gamma(x)$ the orthogonal projection of x onto Γ. We remark that since Γ is a regular surface of $\mathbb{R}^3$, φ and P_Γ are regular in a neighbourhood of Γ.
We set

$$\omega_\varepsilon = \{x \in \Omega,\ 0 < \varphi(x) < \varepsilon\}.$$



We denote $\mathcal{U}^\varepsilon = \Omega \setminus \omega_\varepsilon$ the domain where lies the perfect conductor.
The equations we will consider are:

$$(5)\quad \begin{cases} \dfrac{\partial u^\varepsilon}{\partial t} = u^\varepsilon \wedge \left(\varepsilon^2 \Delta u^\varepsilon + \mathcal{H}(u^\varepsilon)\right) - u^\varepsilon \wedge \left(u^\varepsilon \wedge (\varepsilon^2 \Delta u^\varepsilon + \mathcal{H}(u^\varepsilon))\right) \quad \text{in } \omega_\varepsilon, \\ \dfrac{\partial u^\varepsilon}{\partial n} = 0 \ \text{ on } \partial\omega_\varepsilon, \\ u^\varepsilon(0,.) = u_0 \ \text{ in } \omega_\varepsilon, \end{cases}$$

where the operator $\mathcal{H}$ fulfills now

$$(6)\quad \begin{cases} \mathcal{H}(u^\varepsilon) \in \mathbb{L}^2\left(\mathbb{R}^3 \setminus \mathcal{U}^\varepsilon\right), \\ \mathrm{curl}\ \mathcal{H}(u^\varepsilon) = 0 \ \text{ in } \mathbb{R}^3 \setminus \mathcal{U}^\varepsilon, \\ \mathrm{div}\ (\mathcal{H}(u^\varepsilon) + \overline{u^\varepsilon}) = 0 \ \text{ in } \mathbb{R}^3 \setminus \mathcal{U}^\varepsilon, \\ [\mathcal{H}(u^\varepsilon) \wedge n] = 0 \text{ on } \Gamma, \\ \mathcal{H}(u^\varepsilon) \wedge n = 0 \text{ on } \Gamma_\varepsilon, \ \text{(Boundary condition for a perfect conductor)} \\ [(\mathcal{H}(u^\varepsilon) + \overline{u^\varepsilon}) \cdot n] = 0 \text{ on } \Gamma, \\ (\mathcal{H}(u^\varepsilon) + \overline{u^\varepsilon}) \cdot n = 0 \text{ on } \Gamma_\varepsilon, \ \text{(Boundary condition for a perfect conductor)} \end{cases}$$

where n denotes the outward unitary normal to $\partial\omega_\varepsilon$ and $[v]$ denotes the jump of v at the interface $\partial\omega_\varepsilon$.
In order to perform an asymptotic expansion and get the profiles we deal with the rescaled equation. We assume the following conditions are fulfilled:

(H1) The initial data u_0^ε satisfies $\dfrac{\partial u_0^\varepsilon}{\partial n} = 0$ on $\partial\omega_\varepsilon$ and can be written in the form:

$$u_0^\varepsilon(x) = U_0^0(P(x)) \quad +\varepsilon U_0^1\left(P(x), \frac{\varphi(x)}{\varepsilon}\right) + \varepsilon^2 U_0^2\left(P(x), \frac{\varphi(x)}{\varepsilon}\right) \\ +\varepsilon^3 U_0^3\left(P(x), \frac{\varphi(x)}{\varepsilon}\right) + \varepsilon^3 r_0^\varepsilon(x), \quad \forall x \in \omega_\varepsilon,$$

(H2) $|u_0^\varepsilon(x)| = 1$ for all $x \in \omega_\varepsilon$,
(H3) $U_0^0 \in \mathbb{W}^{7,\infty}(\Gamma)$ such that $|U_0^0| \equiv 1$,
(H4) $U_0^1 \in \mathbb{H}^6(\Gamma) \otimes \mathbb{H}^5(0,1)$,
(H5) $U_0^2 \in \mathbb{H}^5(\Gamma) \otimes \mathbb{H}^5(0,1)$,
(H6) $U_0^3 \in \mathbb{H}^4(\Gamma) \otimes \mathbb{H}^5(0,1)$,
(H7) $r_0^\varepsilon \in \mathbb{H}^2(\omega_\varepsilon)$ is the initial data for the remainder term r^ε and fulfills the additional condition:

$$\|r_0^\varepsilon\|^2_{\mathbb{L}^2(\omega_\varepsilon)} + \|\nabla_{\Gamma_z} r_0^\varepsilon\|^2_{\mathbb{L}^2(\omega_\varepsilon)} + \|\varepsilon\partial_z r^\varepsilon\|^2_{\mathbb{L}^2(\omega_\varepsilon)} + \varepsilon^2\left(\|\Delta_{\Gamma_z} r_0^\varepsilon\|^2_{\mathbb{L}^2(\omega_\varepsilon)} + \|\partial_z^2 r_0^\varepsilon\|^2_{\mathbb{L}^2(\omega_\varepsilon)}\right) \le C,$$

where C is an ε-independent constant, ∇_{Γ_z} and Δ_{Γ_z} are the tangential parts of the gradient and Laplace operators.

Remark 2.1. *We work with Sobolev spaces adapted to the anisotropy of the problem:*

$$\mathbb{L}^p(\omega_\varepsilon) = \left\{u \in \mathcal{D}(\omega_\varepsilon),\ \frac{1}{|\omega_\varepsilon|}\int_{\omega_\varepsilon} |u|^p\, dx\right\}, \qquad \mathbb{H}^0(\omega_\varepsilon) = \mathbb{L}^2(\omega_\varepsilon),$$
$$\mathbb{H}^{p+1}(\omega_\varepsilon) = \left\{u \in \mathbb{L}^2(\omega_\varepsilon),\ \nabla_{\Gamma_z} u \in \mathbb{H}^p(\omega_\varepsilon) \ \text{ and } \varepsilon\partial_z u \in \mathbb{H}^p(\omega_\varepsilon)\right\},$$

and with the natural norms on these spaces.

Our main result is the following:

Theorem 2.2. *Under the assumptions (H1)-(H7), if T^ε is the maximum time of existence of the regular solution u^ε of the equation (5) then $\lim\limits_{\varepsilon\to 0} T^\varepsilon = +\infty$ and there exists some profiles U^0, U^1, U^2, U^3 defined on $\mathbb{R}_t^+ \times \Gamma \times (0,1)$ such that for all $T < T^\varepsilon$, $x \in \omega_\varepsilon$ and $t < T$,*

$$u^\varepsilon(t,x) = U^0(t,P(x)) + \varepsilon U^1\left(t, P(x), \frac{\varphi(x)}{\varepsilon}\right) \quad +\varepsilon^2 U^2\left(t, P(x), \frac{\varphi(x)}{\varepsilon}\right) \\ +\varepsilon^3 U^3\left(t, P(x), \frac{\varphi(x)}{\varepsilon}\right) + \varepsilon^3 r^\varepsilon(t,x),$$

where for all $T > 0$,

- $U^0 \in \mathcal{C}^\infty\left(\mathbb{R}_t^+; \mathbb{W}^{7,\infty}(\Gamma)\right)$ *is solution of the equation (5) with $\varepsilon = 0$, i.e.*

$$\begin{cases} \dfrac{\partial U^0}{\partial t} = U^0 \wedge H^0 - U^0 \wedge \left(U^0 \wedge H^0\right) \ \text{ in } \Gamma, \\ U^0(0,.) = U_0^0 \ \text{ in } \Gamma, \end{cases}$$

where $H^0 = -U_3^0 e^3$ becomes a local operator,

- $U^1 \in \bigcap_{k=0}^{2} \left[\mathbb{W}^{k,\infty}\left(0,T;\mathbb{H}^6(\Gamma)\otimes\mathbb{H}^{5-2k}(0,1)\right)\cap\mathbb{H}^k\left(0,T;\mathbb{H}^6(\Gamma)\otimes\mathbb{H}^{6-2k}(0,1)\right)\right]$,
- $U^2 \in \bigcap_{k=0}^{2} \left[\mathbb{W}^{k,\infty}\left(0,T;\mathbb{H}^5(\Gamma)\otimes\mathbb{H}^{5-2k}(0,1)\right)\cap\mathbb{H}^k\left(0,T;\mathbb{H}^5(\Gamma)\otimes\mathbb{H}^{6-2k}(0,1)\right)\right]$,
- $U^3 \in \bigcap_{k=0}^{2} \left[\mathbb{W}^{k,\infty}\left(0,T;\mathbb{H}^4(\Gamma)\otimes\mathbb{H}^{5-2k}(0,1)\right)\cap\mathbb{H}^k\left(0,T;\mathbb{H}^4(\Gamma)\otimes\mathbb{H}^{6-2k}(0,1)\right)\right]$,*item*

  for all $T < T^\varepsilon$ *the function* r^ε *is bounded in* $\mathbb{L}^\infty(0,T;\mathbb{H}^1(\omega_\varepsilon))$ *and the function* $\varepsilon r^\varepsilon$ *is bounded in* $\mathbb{L}^\infty(0,T;\mathbb{H}^2(\omega_\varepsilon))$.

We have then obtained a first order asymptotic expansion valid on the time interval $(0,T^\varepsilon)$ with $T^\varepsilon \to +\infty$ as $\varepsilon \to 0$ and whose first order term is solution of the previous equations taken with $\varepsilon = 0$, *i.e.* solution of the $2D$ Landau-Lifschitz equation with a null exchange coefficient. Also the magnetic field in the $2D$ Landau-Lifschitz equation is no longer a solution of equation (6), it is now a local operator orthogonal to the surface Γ (in accordance with the physics). Morevover the second order perturbation lies in the whole domain contrary to the boundary layer case.

Sketch of the proof.
- Since we can prove that the norm of u is conserved, the Landau-Lifschitz equation is equivalent to the following equation (see [6]):

$$(7)\qquad \begin{cases} \dfrac{\partial u^\varepsilon}{\partial t} - \varepsilon^2\Delta u^\varepsilon = \ \varepsilon^2|\nabla u^\varepsilon|^2 u^\varepsilon + \varepsilon^2 u^\varepsilon \wedge \Delta u^\varepsilon + u^\varepsilon \wedge \mathcal{H}(u^\varepsilon) \\ \qquad\qquad\qquad\qquad -u^\varepsilon \wedge (u^\varepsilon \wedge \mathcal{H}(u^\varepsilon)) \quad \text{in } \mathbb{R}_t^+ \times \omega_\varepsilon, \\ \dfrac{\partial u^\varepsilon}{\partial n} = 0 \quad \text{on } \mathbb{R}_t^+ \times (\Gamma \cup \Gamma_\varepsilon), \\ u^\varepsilon(0,\cdot) = u_0^\varepsilon \quad \text{in } \omega_\varepsilon, \end{cases}$$

since $u \cdot \Delta u = -|\nabla u|^2$ when $|u| \equiv 1$.
- We look for solutions in the form of an asymptotic expansion in ε where the profiles are defined on $\mathbb{R}_t^+$ and on the new coordinates $\Gamma \times (0,1)$,

$$u^\varepsilon(t,x) = U^0\left(t, P_\Gamma(x), \frac{\varphi(x)}{\varepsilon}\right) + \varepsilon U^1\left(t, P_\Gamma(x), \frac{\varphi(x)}{\varepsilon}\right) + \dots$$
$$H^\varepsilon(t,x) = H^0\left(t, P_\Gamma(x), \frac{\varphi(x)}{\varepsilon}\right) + \varepsilon H^1\left(t, P_\Gamma(x), \frac{\varphi(x)}{\varepsilon}\right) + \dots$$

where $P_\Gamma(x)$ is the orthogonal projection of x onto Γ and $\varphi(x) = \mathrm{dist}(x,\Gamma)$.
In the vacuum outside ω we can prove that there is no boundary layer, so for all $x \in \mathbb{R}^3 \setminus \omega$,

$$H^\varepsilon(t,x) = H^0(t,x) + \varepsilon H^1(t,x) + \dots$$

We substitute to u and H their asymptotic expansion in (6)-(7) and we obtain
- transmission conditions for the profiles H^i deduced from the transmission condition written in equation (6),

- the Neumann conditions written in the new coordinates for the profiles,
- the equations fulfilled by the magnetic field outside ω_ε and which can be solved by Lax-Milgram theorem,
- the equations fulfilled by the profiles of u and H in the thin layer. Thanks to the introduction of the new coordinates, the classical differential operators split into two parts, a tangential operator and a normal operator (see [5] and [18]) which allows us to perform the calculus in a simpler way. For example:

If $u : \omega_\varepsilon \longrightarrow \mathbb{R}$ or $\mathbb{R}^3$, we have, setting $\tilde{u}(\sigma, z) = u(x)$ with $\sigma = P_\Gamma(x),\ z = \varphi(x)$:

$$\nabla u(x) = \frac{\partial \tilde{u}}{\partial z}(\sigma, z) n(\sigma) + \left(\nabla_{\Gamma_z} \tilde{u}\right)(\sigma, z),$$

where $\Gamma_z = \{x \in \Omega,\ \mathrm{dist}(x, \Gamma) = z\}$, ∇_{Γ_z} is the tangential gradient on Γ_z when Γ_z is parameterized by Γ.

With this formalism we obtain the equations fulfilled by H^0:

$$\begin{cases} \dfrac{\partial}{\partial z}\left[\left(H^0 + U^0\right) \cdot n\right] = 0, \\ n \wedge \dfrac{\partial}{\partial z} H^0 = 0. \end{cases}$$

We can solve them thanks to the transmission conditions and the solution we obtained outside ω_ε. We obtain $H^0 = -(U^0 \cdot n)n$, *i.e.* $\mathcal{H}$ behaves itsef as a local operator at order ε^0.
Next U^0 fulfilled the following equation:

$$\frac{\partial U^0}{\partial t} - U^0_{zz} = |U^0_z|^2 U^0 + U^0 \wedge U^0_{zz} + U^0 \wedge H^0 - U^0 \wedge \left(U^0 \wedge H^0\right) \qquad \text{in} \quad \Gamma \times (0,1).$$

According to the assumption (H3) we can prove that U^0 does not depend on z, so U^0 fulfills

$$\frac{\partial U^0}{\partial t} = U^0 \wedge H^0 - U^0 \wedge \left(U^0 \wedge H^0\right) \qquad \text{in} \quad \Gamma \times (0,1).$$

By the same way we get the equations fulfilled by H^i, U^i. We obtain for $i = 1, 2, 3$,

$$H^i = -(U^i \cdot n)n + a(U^1, \ldots, U^{i-1}),$$

where a is a linear operator, and U^i fulfills

$$\frac{\partial U^i}{\partial t} - U^i_{zz} = U^0 \wedge U^1_{zz} + F_i(U^0, \ldots, U^{i-1}, U^i) + G_i(U^0, \ldots, U^{i-1}),$$

where F_i is a linear operator in its last variable U^i. We can then obtain the existence of the profiles and the properties we announced above.
- the main difficulty lies in the estimates on the remainder term. Except the technical part of these estimates, there are two important points: we first need to estimate the

remainder term Q^ε of H^ε with the one of u^ε (r^ε) in the same spaces: Q^ε solves

$$\begin{cases} Q^\varepsilon \in \mathbb{L}^2(\omega_\varepsilon), \\ \text{curl } Q^\varepsilon + A(H^0, \overline{U}^0, H^1, \overline{U}^1, H^2, \overline{U}^2, H^3, \overline{U}^3) = 0 \text{ in } \omega_\varepsilon, \\ \text{div } (Q^\varepsilon + \overline{r^\varepsilon}) + B(H^0, \overline{U}^0, H^1, \overline{U}^1, H^2, \overline{U^2}, H^3, \overline{U}^3) = 0 \text{ in } \omega_\varepsilon, \end{cases}$$

where A and B are linear functions of their arguments, with the following transmission conditions

$$\begin{cases} [Q^\varepsilon \wedge n] = 0 \text{ on } \Gamma, \\ [(Q^\varepsilon + \overline{r^\varepsilon}) \cdot n] = 0 \text{ on } \Gamma, \\ Q^\varepsilon \wedge n = 0 \text{ on } \Gamma_\varepsilon, \\ (Q^\varepsilon + \overline{r^\varepsilon}) \cdot n = 0 \text{ on } \Gamma_\varepsilon. \end{cases}$$

Lemma 2.3. *If $r^\varepsilon \in \mathbb{H}^p$, there exists an unique $Q^\varepsilon = R^\varepsilon + S^\varepsilon$ and a constant c_0 such that S^ε is regular, R^ε is linear in r^ε and :*

$$\|R^\varepsilon\|_{\mathbb{H}^p(\omega_\varepsilon)} \leq c_0 \|r^\varepsilon\|_{\mathbb{H}^p(\omega_\varepsilon)}.$$

Secondly we need ε-independent estimates on the remainder term of u. As we use Sobolev embeddings we need to precise the dependence on ε of the Sobolev constants. Following the work of R. Temam and M. Ziane [19], we obtain some other estimates (See [17], [18]). For example we have in the anisotropic spaces (see Remark 2.1):

Lemma 2.4. *There exists c_0 independent of ε such that $\forall u \in \mathbb{H}^1(\omega_\varepsilon)$ and $\forall\ 2 \leq p \leq 6$,*

$$\|u\|_{\mathbb{L}^p(\omega_\varepsilon)} \leq c_0 \|u\|^{\frac{3}{p}-\frac{1}{2}}_{\mathbb{L}^2(\omega_\varepsilon)} \|u\|^{\frac{3}{2}-\frac{3}{p}}_{\mathbb{H}^1(\omega_\varepsilon)}$$

and $\forall\ u \in \mathbb{H}^2(\omega_\varepsilon)$, $\forall\ 2 \leq p \leq +\infty$,

$$\|u\|_{\mathbb{L}^p(\omega_\varepsilon)} \leq c_0 \|u\|^{\frac{1}{4}+\frac{3}{2p}}_{\mathbb{L}^2(\omega_\varepsilon)} \|u\|^{\frac{3}{4}-\frac{3}{2p}}_{\mathbb{H}^2(\omega_\varepsilon)}.$$

□

Remark 2.5. *We have studied here the behaviour of Landau-Lifschitz equations with an exchange coefficient ε^2 in a thin layer of thickness ε. That is the case where the thickness of the boundary layer (if there were any, cf* [8]*) and the domain's thickness are of the same size. If we instead consider a thin layer with a thickness ε^α ($\alpha > 0$, $\alpha = \dfrac{p}{q} \in \mathbb{Q}$), we perform now an asymptotic expansion at the scale $\varepsilon^{\frac{1}{q}}$ and we see two different behaviours. If $\alpha > 1$ we get the same type of behaviour than that of the case $\alpha = 1$, i.e. the behaviour of a thin layer and the localization of H at the low orders. If $\alpha < 1$, the layer's thickness is bigger than the characteristic thickness of the boundary layer. We then obtain a boundary layer in the thin layer but with a smaller thickness ($\varepsilon^{1-\alpha}$) and we also get at the low orders the localization of H.*

Acknowledgments

The author wishes to warmly thank Gilles Carbou and Pierre Fabrie for their help and their advice.

REFERENCES

[1] R. Abraham, J.E. Marsden, T. Ratiu, *Manifolds, tensor analysis and applications*, Applied Mathematical Sciences 75, *Springer Verlag*, 1988.

[2] R.A. Adams, *Sobolev Spaces, Pure and Applied Math.*, Vol 65, *Academic press*, 1975.

[3] H. Brezis, *Analyse fonctionnelle*, *Masson*, 1993.

[4] G. Carbou, Thin layers in micromagnetism, *M3AS*, Vol. 11, No. 9 (2001) 1529-1546.

[5] G. Carbou, Penalization method for viscous incompressible flow around a porous thin layer, preprint MAB.

[6] G. Carbou, P. Fabrie, Regular Solutions for Landau-Lifschitz Equation in a Bounded domain, *Differential and Integral Equations*, 14 (2001), 213-229.

[7] G. Carbou, P. Fabrie, Time average in Micromagnetism, *Journal of Differential Equations* 147 (1998), 383-409.

[8] G. Carbou, P. Fabrie, O. Guès, Couche limite dans un modèle de ferromagnétisme, *Commun. Partial Differ. Equations*, Vol. 27, No. 7-8 (2002), 1467-1495.

[9] R. Dautray et J.L. Lions, *Analyse mathématique et calcul numérique pour les sciences et les techniques*, Tomes 1, 2 et 3, *Coll. du C.E.A.*, *Masson*.

[10] C. Foias, R. Temam, Remarque sur les équations de Navier-Stokes stationnaires et les phénomènes successifs de bifurcation, *Ann. Sc. Norm. Super. Pisa*, Cl. Sci., IV. Ser. 5 (1978), 29-63.

[11] C. Godbillon, *Eléments de topologie algébrique*, *Hermann*, 1971.

[12] H. Haddar, *Modèle asymptotique en ferromagnétisme: Couches minces et homogénéisation*, Thesis (2000), ENPC.

[13] J.L. Lions, *Quelques méthodes de résolution des problèmes aux limites non linéaires*, *Gauthier-Villars*, 1969.

[14] J. Neçàs, *Les méthodes directes en théorie des équations elliptiques*, *Masson*, 1967.

[15] D. Sanchez, Boundary layer on a high-conductivity domain, *CPAA*, Vol. 1, No. 4 (2002), 547-564.

[16] D. Sanchez, Phénomène de couche limite dans un modèle de ferromagnétisme, *Ann. Fac. Sci. Toulouse VI. Ser. Math.*, Vol XI-2, (2002).

[17] D. Sanchez, Behaviour of Landau-Lifschitz Equation in a periodic thin layer, to appear in *Asymptotic Analysis*.

[18] D. Sanchez, Thin layer for Landau-Lifschitz equation, preprint MAB, 2003

[19] R. Temam, M. Ziane, Navier-Stokes equations in three-dimensional thin domains with various boundary conditions, *Adv. Differential Equations*, 1(4), 499-546.

[20] T.J. Willmore, *Riemannian Geometry*, *Oxford Science Publication*, 1993.