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A mixed POD-PGD approach to parametric thermal urban soil modeling: Application to canyon streets

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Abstract

Numerical simulation is a powerful tool for assessing the causes of an Urban Heat Island (UHI) effect or quantifying the impact of mitigation solutions on local climatic conditions. However, the numerical cost associated with such a tool, which may seem low for a section of mesh within the district geometric model, is quite significant at the scale of an entire district. Today, the main challenge consists of achieving both a proper representation of the physical phenomena and a critical reduction in the numerical costs of running simulations. This paper presents a combined parametric urban soil model that accurately reproduces thermal heat flux exchanges between the soil and the urban environment with a reduced computation time. For this purpose, the use of a combination of two reduced-order methods is proposed herein: the Proper Orthogonal Decomposition (POD) method, and the Proper Generalized Decomposition (PGD) method. The developed model is applied to two case studies in order to establish a practical evaluation: an open area independent of the influences of the surrounding surface, namely a parking lot, and a theoretical urban scene with two canyon streets. The mean surface temperature reduction error remains below 0.52°C for a cut computational cost of 80%.

Keyword

POD PGD parametric model Heat transfer Urban soil model SOLENE-microclimat Urban Heat Island Model Order Reduction

Highlights

- We propose a parametric model dedicated to urban soil thermal modeling.
- A combination of two reduced-order methods, i.e. POD and PGD, is presented.
- Calculated temperatures are evaluated with respect to *in situ* measurements.
- The parametric soil model is coupled with the SOLENE-microclimat tool.

- Its accuracy and computational cost are evaluated in an urban setting.

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Nomenclature

α	Thermal diffusivity of the soil [m^2s^{-1}]	k	Thermal conductivity [$W.m^{-1}.K^{-1}$]
γ	Auto-correlation matrix	Q	<i>Snapshots</i> matrix
λ_i	Eigenvalue i	q_ℓ	Latent heat flux [$W.m^{-2}$]
λ	Eigenvalues diagonal matrix	q_c	Sensible heat flux [$W.m^{-2}$]
\mathcal{N}	Reduction order of the POD basis	q_{net}	Net radiative heat flux [$W.m^{-2}$]
Φ	POD Reduced order basis	q_w	Runoff convective heat flux [$W.m^{-2}$]
$\tilde{T}(x, t)$	Approximation of the temperature profile	$T(x, t)$	Temperature profile
ζ	Unknown of the POD reduced model	T_∞	Temperature of the soil below a meter [K]
c	Specific Heat Capacity [$J.m^{-3}.K^{-1}$]	$T_a(t)$	Air temperature [K]
h	Convective heat transfer coefficient [$W.m^{-2}.K^{-1}$]	L	width of the soil layer

1. Introduction

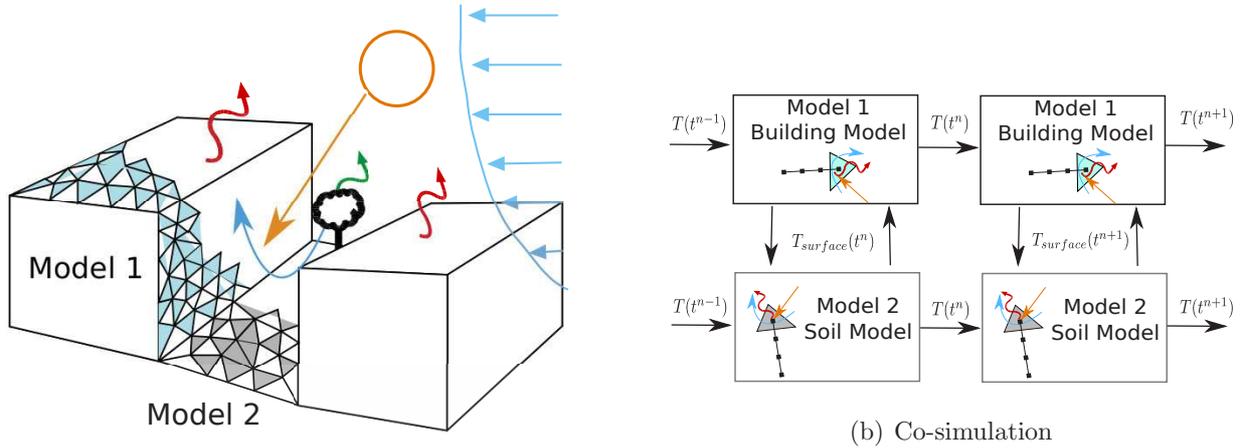
Curbing urban sprawl is a key current objective for urban planners. In the context of global warming, urban densification has negative consequences on local city climate; specifically, it leads to intensification of the urban heat island (UHI) phenomenon [1–3]. For this reason, cities must now be developed in considering UHI as an environmental challenge since it has several consequences on both outdoor comfort and building energy consumption, which can lead to serious health issues.

The storage of solar heat flux by urban materials during the day and its release at night is a main cause of UHI. This heat flux is greater in urban areas than in rural parts due to the higher inertia of construction materials. Surface temperatures drive the various heat fluxes (convection, conduction and long-wave radiative exchanges) at the urban surface. As such, it is of major importance to assess these variables of the urban micro-climate.

In order to mitigate the UHI, its causes must be fully understood and the impact of mitigation solutions on the local micro-climate needs to be quantified. For this purpose, numerical simulation proves to be a powerful tool. Several models have been developed to simulate UHI and its consequences; these include TEB [4] or ARPS-VUC [5], at the city scale,

17 or SOLENE-microclimat [6, 7], Envi-met [8], and EnviBatE [9] at the district scale.

18 Most of these micro-climate tools cited combine several one-dimensional models (soil,
 19 buildings, etc.) using a co-simulation approach. The urban scene is meshed as presented
 20 in Figure 1; moreover, for each piece of mesh, the thermo-radiative balance of an urban
 21 scene is derived. This energy balance is composed of both the short- and long-wave radiative
 22 heat fluxes, as well as the sensible and latent heat fluxes. At each time step t^n , all models
 23 exchange parameters like surface temperature and net radiative heat flux balance with the
 24 other models. During the interval $t \in [t^n, t^{n+1}]$, each model computes the field of interest,
 25 which consists of the temperatures of each surface. Figure 1 displays the co-simulation
 26 process.



(a) Thermo-radiative balance and mesh representation

Figure 1: Thermo-radiative balance of the urban scene with co-simulation

27 In this context, an initial study has been conducted to improve SOLENE-microclimat
 28 soil model performance [10]. A previous study had focused on the ability of the model to
 29 reproduce the particular physical phenomenon. The influence of a large number of parameters
 30 has been assessed in order to improve the model; these include: material properties, layer size,
 31 depth boundary conditions, convective heat transfer coefficient, and discretization. These
 32 improvements however have increased the model computation time. Indeed, at each time
 33 step, the heat transfer equation in the soil must be solved for each spatial mesh of the urban
 34 scene. All told, at each time step, thousands of model computations are processed, thus

35 significantly increasing the computation time. The numerical cost, which may seem small
36 for a portion of the mesh, is considerable at the scale of thousands of such portions. Since
37 our requirements have been raised to an extent commensurate with improved computational
38 resources, the main challenge today remains combining the best representation of the physical
39 phenomena with a critical reduction in associated numerical costs.

40 In this same aim, Gasparin et al. (2017, [11]) proposed combining an analytical approach
41 with a finite difference approach in order to compute the temperature and moisture fields
42 in porous soils. The analytical solution is implemented in the deep soil while the numerical
43 scheme is used to solve the upper layer, where the variations are more pronounced. The
44 two methods are linked through an iterative procedure. For our specific case [11], they were
45 able to reduce computational cost by 16% to 12%, respectively. This method however is
46 counterproductive since it requires an iterative procedure at each time step.

47 Our objective herein is to propose a reduced-order model (ROM) for the soil heat transfer
48 equation. This model needs to accurately reproduce the heat flux being exchanged between
49 the soil and its environment for several weeks in offering reduced computation times.

50 As described above, the thermal modeling of the specific urban soil depends on several
51 parameters, i.e. the thermal characteristics of the soil, the initial temperature profile, the
52 surface energy balance, and the temperature deep in the soil. The thermal characteristics
53 could be considered as a constant over time, though the other parameters evolve at each time
54 step. The transient heat equation thus needs to be solved at each time step. In order to
55 reduce computation time, a parametric model is being proposed. For this purpose, the Proper
56 Generalized Decomposition (PGD) method has been selected. The PGD is an *a priori* model
57 order-reduction method based on distinct variables representing multidimensional problems.
58 This property allows us to circumvent the "curse of dimensionality". Any variable (i.e. initial
59 or boundary condition) can then be defined as an extra-parameter of a model. Hence, the
60 model solution is given as a direct function of x and t coordinates, along with the extra-
61 parameter of the problem. Various examples using the PGD parametric model can be found
62 in applications to heat transfer [12–14] with an emphasis on building energy simulation [14–
63 17].

64 As presented in several references [18, 19] devoted to parametric PGD models, when a

65 dynamic problem needs to be parameterized, the initial conditions must be taken as a prob-
66 lem parameter; this step increases the number of required model parameters. In order to
67 reduce this overall number of parameters, the initial condition should be parameterized. As
68 proposed by Gonzalez et al. (2014, [20]) and Cueto et al. (2016, [18]), the Proper Orthog-
69 onal Decomposition (POD) method is used to parameterize the initial condition, making it
70 an *a posteriori* model order-reduction method. Consequently, a mixed reduction method ap-
71 proach, based on both POD and PGD, is being proposed to generate a combined parametric
72 soil model in the context of a micro-climate simulation.

73 The remainder of this paper is organized as follows. The mixed approach is presented
74 step-by-step in Section 2 and then validated using an analytical solution in Section 3.1. The
75 combined parametric soil model is applied to a case study involving a parking lot in Section
76 3.2. Next, the thermal response of this soil model is compared to *in situ* measurements in
77 order to evaluate its performance. Once validated under actual boundary conditions, the
78 combined parametric model is applied to an urban scene in Section 3.3. For this purpose, it
79 is coupled with the SOLENE-microclimat tool. Lastly, since the main goal of our work is to
80 reduce computation time, the CPU times of the combined POD+PGD and finite difference
81 models, in conjunction with the SOLENE-microclimat tool, are compared.

82 2. Methods

83 2.1. Physical problem and Large Original Model (LOM)

84 This model is being dedicated to urban pavement surfaces, such as parking lots and
85 sidewalks. Since such surfaces are considered to be impervious, only heat transfer will be
86 taken into account (i.e. water transfer will be neglected). The physical problem thus involves
87 transient one-dimensional heat conduction through a soil column (Eq. 1) for time interval
88 $\Omega_t = [0, \tau]$ and space interval $\Omega_x = [0, L]$:

$$c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right), \quad (1)$$

89
90 where k is the thermal conductivity and c the specific heat capacity. Both these variables

91 depend on the space variable x since the domain, i.e. the soil, is composed of several layers.
 92 These thermal properties are assumed to remain constant over time. At the surface $x = 0$,
 93 a ROBIN boundary condition is assumed:

$$-k \frac{\partial T}{\partial x} = q_{net} - q_{\ell} - q_w - h \left(T(x=0) - T_a \right),$$

94 where q_{net} is the net radiative heat flux, q_{ℓ} the latent heat flux and q_w the runoff convective
 95 heat flux. The two last heat fluxes are only computed under rainfall or watering events.
 96 The sensible heat flux is calculated from air temperature T_a varying over time and from a
 97 convective heat transfer coefficient h . At the bottom $x = L$, the soil temperature is imposed
 98 through DIRICHLET boundary conditions:

$$T = T_{\infty},$$

99 where T_{∞} is a constant daily temperature [10]. At $t = 0$, the initial temperature is set
 100 using a space temperature profile:

$$T = T_0(x)$$

101 The problem in Equation (1) can be solved using any classical numerical method. The
 102 so-called Large Original Model (LOM) is defined as the solution to the previous problem with
 103 the finite difference method. The spatial domain Ω_x is discretized into a grid composed of
 104 N_x nodes, while the time domain Ω_t is discretized into N_t time steps. An implicit scheme is
 105 employed, with backward first-order derivatives for the time derivation and a center second-
 106 order derivative scheme for the spatial derivation. In the following sections, the construction
 107 of this combined parametric reduced-order models (ROM) for Eq. (1) will be detailed.

108 *2.2. Building the combined parametric model*

109 The PGD method is used in order to propose an accurate parametric solution of the
 110 formulated soil problem with a shorter computation time, in anticipation of coupling with a
 111 micro-climate tool.

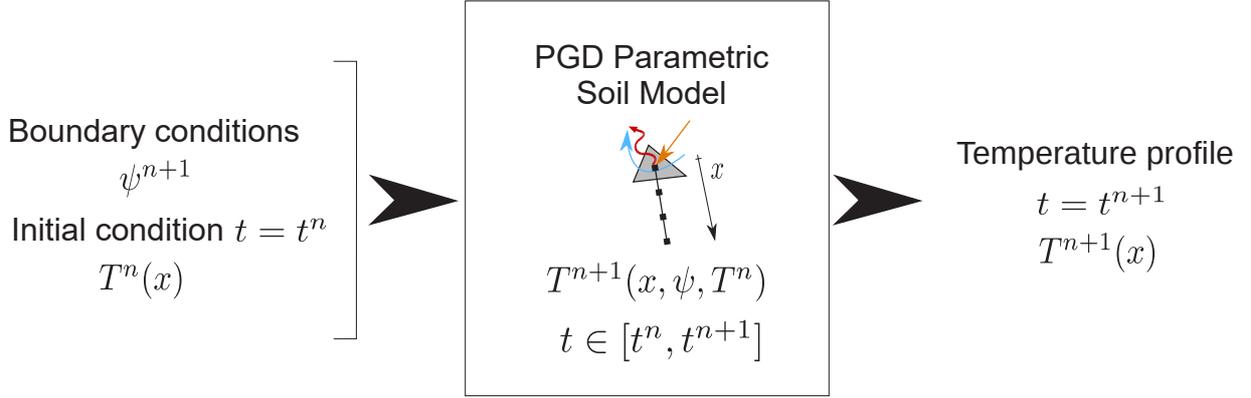


Figure 2: PGD Parametric soil model with the initial condition as a parameter

112 To achieve a universal solution to this problem, the model boundary conditions need to
 113 be defined as parameters. This means that the PGD parametric reduced-order model must
 114 be calculated for any boundary conditions over a predefined interval. Given the temperature
 115 profile at time step t^n and the boundary conditions at time step t^{n+1} as parameters, the
 116 model outputs the temperature profile at time step t^{n+1} (Figure 2). This previous temper-
 117 ature profile can be seen as an *initial condition* of the problem solved between t^n and t^{n+1} .
 118 The model can be developed as a space (x) - boundary conditions (BC) - initial condition
 119 (IC) compartmentalization of the solution.

120 The implicit semi-discretization in time of Eq. (1) yields:

$$T^{n+1} = T^n + \frac{\Delta t}{c} \frac{\partial}{\partial x} \left(k \frac{\partial T^{n+1}}{\partial x} \right), \quad (2)$$

with the following boundary conditions at $x = 0$:

$$-k \frac{\partial T^{n+1}}{\partial x} = \psi^{n+1}.$$

121 The challenge therefore is to compute a parametric model for the field T^{n+1} by solv-
 122 ing problem (2) in searching for a separate solution that depends on space (x) - boundary
 123 conditions (BC) - initial condition (IC) as follows:

$$T^{n+1} = \sum_{i=1}^{\mathcal{M}} X_i(x) G_i(\psi^{n+1}) F_i(T^n)$$

124 It may be speculated why a space-time-BC separation of the solution would not suffice. In
 125 our specific case, the BC are not constant over time. The surface energy balance is not the

126 same between time steps t^n and t^{n+1} . With a parametric space-time-BC, the model would
 127 compute the solution at t^{n+1} with the BC at time t^{n+1} , from the temperature profile at
 128 t^n . However, the previous temperature profile (at t^n) is calculated with different boundary
 129 conditions (at t^{n+1} and not at t^n). The parametric formulation of the problem thus needs
 130 to take into account the previous temperature profile as a parameter.

131 2.2.1. Use of the POD to parameterize the initial condition

132 As presented above, the parametric formulation of the soil problem requires taking the
 133 previous temperature profile into account as a parameter. Once discretized in space however,
 134 the temperature profile of the *initial condition* is no longer of infinite dimension: the descrip-
 135 tion of the previous temperature profile provides one piece of information per node, which
 136 implies inputting as many parameters in the PGD parametric model as the number of nodes
 137 in the grid, plus the boundary conditions and spatial coordinates. To avoid this tremen-
 138 dous number of parameters, the *initial condition* needs to be parameterized. Gonzalez et
 139 al. (2014, [20]) and Cueto et al. (2016, [18]) proposed implementing the Proper Orthogonal
 140 Decomposition (POD) method to parametrize the initial condition with a minimum number
 141 of parameters. The POD method extracts the relevant information from a set of *snapshots*
 142 by means of its projection into a smaller subspace. As a result, from a set of random data,
 143 the POD builds a deterministic representation of a dataset. This representation is built from
 144 the basis Φ . The ultimate goal is to retain a detailed representation of the dataset with a
 145 minimum or optimal number of modes in Φ . For these properties, the POD method param-
 146 eterizes the initial temperature profile. More details on the procedure for building the basis
 147 Φ can be found in Appendix B.

148 This method consists of seeking a set of basis functions Φ that approximate the tem-
 149 perature profile $T(x, t)$ from the eigenvalues and eigenmodes. The basis Φ is then used
 150 to parameterize the initial condition. For this purpose, the initial system of equations is
 151 projected into the reduced-order basis by simply performing the following change of variable:

$$T^n(x) \simeq \tilde{T}^n(x) = \sum_{j=1}^N \Phi_j(x) \zeta_j \quad (3)$$

152

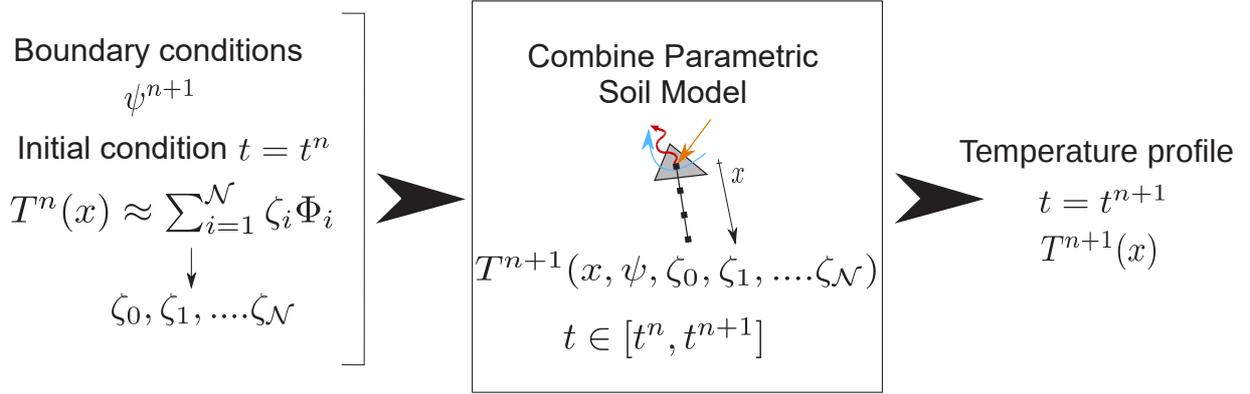


Figure 3: PGD Parametric soil model with the use of POD to configure the initial condition

153 In this manner, the temperature profile can be optimally parameterized with a minimum
 154 number of modes. The previous parametric problem is thus modified as follows:

$$T^{n+1} = \sum_{i=1}^{\mathcal{M}} X_i(x) G_i(\psi^{n+1}) F_i^1(\zeta_1) F_i^2(\zeta_2) \dots F_i^{\mathcal{N}}(\zeta_{\mathcal{N}})$$

155 in which ζ_j are the new parameters of the PGD model, as presented in Figure 3.

156 2.2.2. Building the PGD parametric solution

157 The PGD method approximates the solution to a problem as a finite sum of separable
 158 functions. Such functions are determined by means of an iterative procedure. The PGD can
 159 be described according to three main steps, i.e.:

- 160 1. initialization of the function basis,
- 161 2. enrichment of the basis functions through an iterative process,
- 162 3. solution convergence test.

163 The PGD parametric method will be described hereafter. For further details on the
 164 method and its developments, the interested reader may refer to [18, 19]. The model is
 165 developed as a space - BC - IC separation of the solution. The boundary condition is defined
 166 as a universal function ψ and could be:

- 167 • the DIRICHLET boundary condition in the ground,

168 • the ROBIN boundary condition at the surface.

We are now searching for the temperature profile:

$$T^{n+1}(x, T^n, \psi) = \sum_{i=1}^{\mathcal{M}} X_i(x) \otimes F_i(T^n) \otimes G_i(\psi)$$

169 where $T \in \Omega_x \times]0, \Delta t] \times \Omega_{T^n} \times \Omega_\psi$, with $\Omega_{T^n} = [T^{n-}, T^{n+}]$ and $\Omega_\psi = [\psi^-, \psi^+]$.
 170 The PGD approximation is the sum of \mathcal{M} functional products \otimes involving each function,
 171 hereafter denoted by a point in the equations. First, the initial condition T^n needs to be
 172 introduced explicitly into the weak form of the problem (Eq. (4)), yielding Eq. (5), for
 173 an implicit scheme with the test function u^* (Eq. (6)). This test function formulation (i.e.
 174 Galerkin formulation) is most frequently used in the literature:

$$\int_{\Omega_x \times \Omega_{T^n} \times \Omega_\psi} u^* \cdot \left(c \frac{\partial T}{\partial t} - \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) \right) dx \cdot dT^n \cdot d\psi = 0 \quad (4)$$

$$\int_{\Omega_x \times \Omega_{T^n} \times \Omega_\psi} u^* \cdot \left(c \frac{T^{n+1} - T^n}{\Delta t} - \frac{\partial}{\partial x} \left(k \frac{\partial T^{n+1}}{\partial x} \right) \right) dx \cdot dT^n \cdot d\psi = 0 \quad (5)$$

$$u^*(x, T^n, \psi) = X^*(x) \cdot F(T^n) \cdot G(\psi) + X(x) \cdot F^*(T^n) \cdot G(\psi) + X(x) \cdot F(T^n) \cdot G^*(\psi) \quad (6)$$

177 The separated representation is built with an iterative procedure that features two nested
 178 loops: the alternating direction strategy, and the enrichment process. At enrichment step m ,
 179 the first $m - 1$ terms have been computed. The new functions X_m , F_m and G_m must now be
 180 calculated according to:

$$T_m^{n+1}(x, T^n, \psi) = \sum_{i=1}^{m-1} X_i(x) \cdot F_i(T^n) \cdot G_i(\psi) + X_m(x) \cdot F_m(T^n) \cdot G_m(\psi) \quad (7)$$

181 A nonlinear problem must now be solved where the unknowns are the functions X_m , F_m
 182 and G_m . In this aim, an alternating direction, also called a fixed-point algorithm, will be
 183 used. Each function is randomly initialized and then solved by iteration. At enrichment step
 184 m and at iteration p of the fixed-point algorithm, we obtain the following temperature profile
 185 approximation:

$$T_m^{n+1,p}(x, T^n, \psi) = T_{m-1}^{n+1}(x, T^n, \psi) + X_m^p(x) \cdot F_m^p(T^n) \cdot G_m^p(\psi) \quad (8)$$

186 As mentioned above, the PGD algorithm is composed of two enrichment loops. Each step is
 187 summarized in Algorithm 1 and detailed thereafter.

Algorithm 1 Compute the PGD basis $\tilde{T}^{n+1}(x, T^n, \psi)$

while $m < m_{max}$ and $\frac{\|X_m(x).F_m(T^n).G_m(\psi)\|}{\|X_1(x).F_1(T^n).G_1(\psi)\|} < \epsilon$ **do**

First guess: $X_m^0(x)$, $F_m^0(T^n)$ and $G_m^0(\psi)$

while $p < p_{max}$ and $\frac{\|X_m^p(x).F_m^p(T^n).G_m^p(\psi) - X_m^{p-1}(x).F_m^{p-1}(T^n).G_m^{p-1}(\psi)\|}{\|X_m^{p-1}(x).F_m^{p-1}(T^n).G_m^{p-1}(\psi)\|} < \tilde{\epsilon}$ **do**

Compute X Using Eq. (11)

Compute F Using Eq. (12)

Compute G Using Eq. (13)

end while

Add functions to the basis: $X_m(x) = X_m^p(x)$, $F_m(T^n) = F_m^p(T^n)$, $G_m(\psi) = G_m^p(\psi)$

end while

$\tilde{T}^{n+1}(x, T^n, \psi) = \sum_{i=1}^{\mathcal{M}} X_i(x).F_i(T^n).G_i(\psi)$

188 First, the fixed-point algorithm is randomly initialized, then each term is calculated one
 189 after the other:

- 190 • $X_m^p(x)$ with $F_m^{p-1}(T^n)$ and $G_m^{p-1}(\psi)$ assumed to be known;
- 191 • $F_m^p(T^n)$ with $X_m^p(x)$ and $G_m^{p-1}(\psi)$ assumed to be known;
- 192 • $G_m^p(\psi)$ with $X_m^p(x)$ and $F_m^p(T^n)$ assumed to be known.

193 The alternating direction process stops once a fixed point has been reached. The criterion
 194 used to make this determination is defined in Eq. (9) with $\tilde{\epsilon}$ being a criterion defined by the
 195 user [19].

$$\frac{\|X_m^p(x).F_m^p(T^n).G_m^p(\psi) - X_m^{p-1}(x).F_m^{p-1}(T^n).G_m^{p-1}(\psi)\|}{\|X_m^{p-1}(x).F_m^{p-1}(T^n).G_m^{p-1}(\psi)\|} < \tilde{\epsilon} \quad (9)$$

196 Upon completion of the fixed-point algorithm, the functions are added to the basis:
 197 $X_m(x) = X_m^p(x)$, $F_m(T^n) = F_m^p(T^n)$, $G_m(\psi) = G_m^p(\psi)$. The enrichment process of
 198 the PGD basis stops when the following criterion ϵ , defined by the user, has been reached
 199 (Eq. (10)) [19].

$$\frac{\|X_m(x).F_m(T^n).G_m(\psi)\|}{\|X_1(x).F_1(T^n).G_1(\psi)\|} < \epsilon \quad (10)$$

200 **Alternation direction strategy**

201 The first step of the fixed-point algorithm has now been described; it consists of computing
 202 $X_m^p(x)$. Since the methodology is the same for computing $F_m^p(T^n)$ and $G_m^p(\psi)$, the two
 203 following steps will be developed in the Appendix.

204 *Computation of $X_m^p(x)$ from $F_m^{p-1}(T^n)$ and $G_m^{p-1}(\psi)$*

205

Eq. (7) is introduced into Eq. (5), along with the following test function:

$$u^*(x, T^n, \psi) = X_m^*(x).F_m^{p-1}(T^n).G_m^{p-1}(\psi) = X^*.F.G$$

206 We then obtain the following equation:

$$\begin{aligned} & \int_{\Omega_x \times \Omega_{T^n} \times \Omega_\psi} X^*.F.G \left(c \frac{X.F.G}{\Delta t} - \frac{\partial}{\partial x} \left(k \frac{\partial X}{\partial x} \right) . F . G \right) dx. dT^n. d\psi \\ & \quad - \int_{\Omega_x \times \Omega_{T^n} \times \Omega_\psi} X^*.F.G . c \frac{T^n}{\Delta t} dx. dT^n. d\psi \\ = & - \int_{\Omega_x \times \Omega_{T^n} \times \Omega_\psi} X^*.F.G \sum_{i=1}^{m-1} \left(c \frac{X_i.F_i.G_i}{\Delta t} - \frac{\partial}{\partial x} \left(k \frac{\partial X_i}{\partial x} \right) . F_i . G_i \right) dx. dT^n. d\psi \end{aligned}$$

207 Since all the functions depending on the parametric coordinate ψ and T^n are known, they
 208 can be integrated over their domain: Ω_{T^n} and Ω_ψ .

$$\left\{ \begin{array}{l} f_1 = \int_{\Omega_{T^n}} (F) dT^n \\ f_2 = \int_{\Omega_{T^n}} (F)^2 dT^n \\ g_1 = \int_{\Omega_\psi} (G) d\psi \\ g_2 = \int_{\Omega_\psi} (G)^2 d\psi \\ f_i = \int_{\Omega_{T^n}} (F . F_i) dT^n \\ g_i = \int_{\Omega_\psi} (G . G_i) d\psi \end{array} \right.$$

209 We derive the following simplified equation (11), which can be solved by any discretization
 210 technique. The finite difference method has been used herein, with a center second-order
 211 derivative scheme.

$$c \frac{X}{\Delta t} \cdot f_2 \cdot g_2 - \frac{\partial}{\partial x} \left(k \frac{\partial X}{\partial x} \right) \cdot f_2 \cdot g_2 = - \sum_{i=1}^{m-1} \left(c \frac{X_i}{\Delta t} \cdot f_i \cdot g_i - \frac{\partial}{\partial x} \left(k \frac{\partial X_i}{\partial x} \right) \cdot f_i \cdot g_i \right) + f_1 \cdot g_1 \cdot c \frac{T^n}{\Delta t} \quad (11)$$

212 With this same method, $F_m^p(T^n)$ can be computed from $X_m^p(x)$ and $G_m^{p-1}(\psi)$, producing the
 213 following algebraic equation (12), whose direct solution yields the function F . Details on this
 214 development can be found in Appendix C.1.

$$F \cdot \left(\frac{c}{\Delta t} x_2 g_2 - x_3 g_2 \right) = - \sum_{i=1}^{m-1} \left(c \frac{F_i}{\Delta t} x_{i,1} g_i - F_i x_{i,2} g_i \right) + x_1 g_1 c \frac{T^n}{\Delta t} \quad (12)$$

215 Moreover, $X_m^p(x)$ and $F_m^p(T^n)$ enable computing $G_m^p(\psi)$. Equation (13) presents the final
 216 algebraic equation; its direct solution yields the function G . Details on this development can
 217 be found in Appendix C.2.

$$G \left(\frac{c}{\Delta t} x_2 f_2 - x_3 f_2 \right) = - \sum_{i=1}^{m-1} \left(c \frac{G_i}{\Delta t} x_{i,1} f_i - G_i x_{i,2} f_i \right) + x_1 f_1 c \frac{T^n}{\Delta t} \quad (13)$$

218 2.3. Implementation methodology

219 The combined parametric soil model developed herein makes use of both POD and PGD
 220 methods and features an *offline/online* strategy. All steps are detailed in Figure 4. The POD
 221 method serves to configure the previous soil temperature profile, as presented in Figure 3. In
 222 order to perform this configuration, we first need a POD basis, which is built from a set of
 223 snapshots calculated by the LOM (via the finite difference method). This basis is calculated
 224 once *offline*. For each of the equations (11), (12), and (13), the previous temperature profile
 225 is projected into a POD basis by executing the following change of variable:

$$T^n \approx \sum_{i=1}^{\mathcal{N}} \zeta_i \Phi_i = \Phi \zeta$$

226 A minimum number of modes in the POD basis, \mathcal{N} , is defined to achieve the desired
 227 accuracy. Note that the number of modes has a direct influence on the number of parameters
 228 used in the PGD model. The parametric model can then be solved with the PGD Algorithm
 229 1 for:

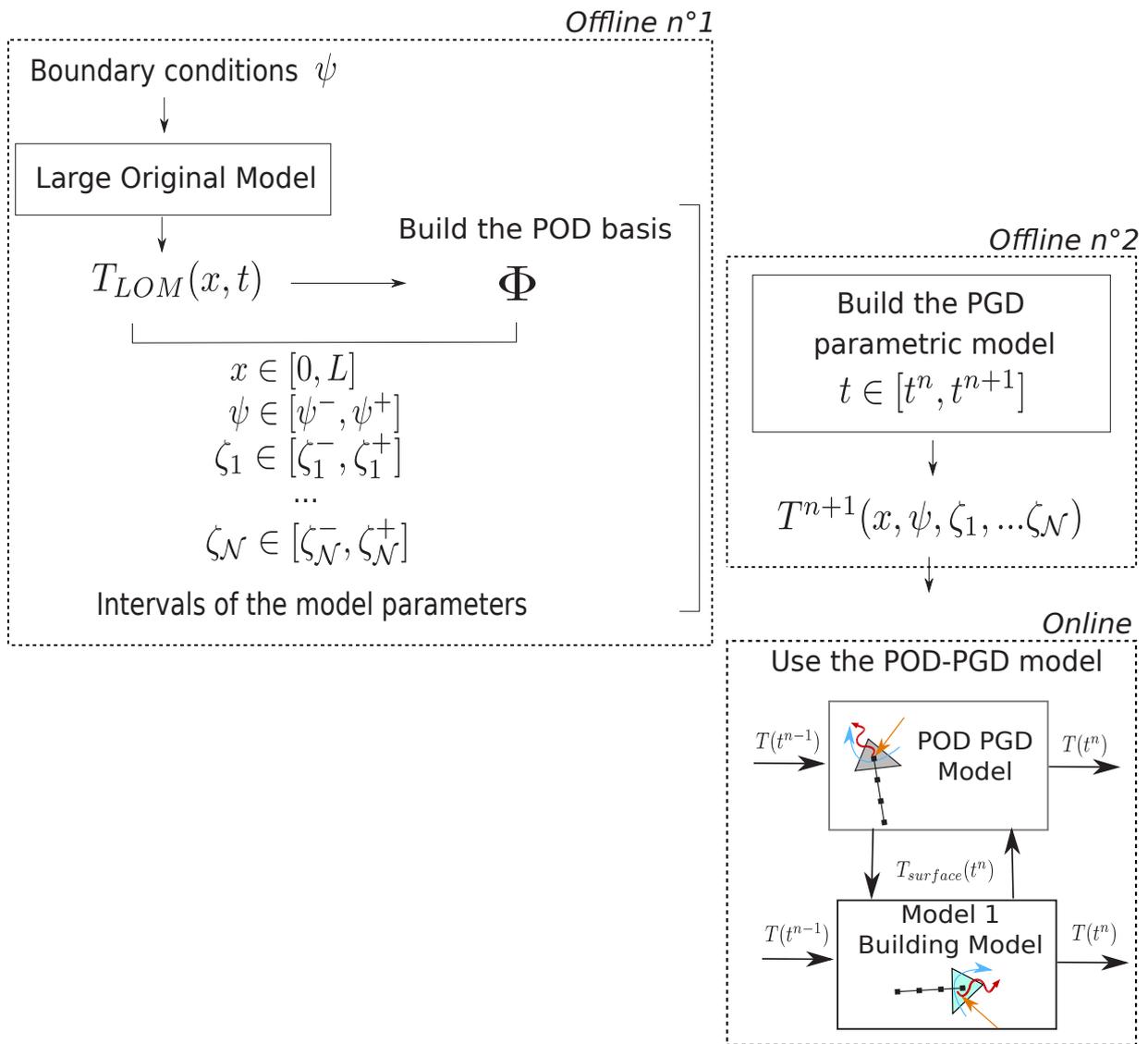


Figure 4: Implementation methodology of the model

- 230 • a spatial mesh,
- 231 • a time interval of (T^n, t^{n+1}) ,
- 232 • any values of the boundary condition ψ defined within a discretized interval $[\psi^-, \psi^+]$,
- 233 • each mode of the POD basis: i.e. $\zeta_1 \in [\zeta_1^-, \zeta_1^+], \dots, \zeta_{\mathcal{N}} \in [\zeta_{\mathcal{N}}^-, \zeta_{\mathcal{N}}^+]$.

234 Once the PGD parametric model has been built, it can be applied for any value within the
 235 previously defined intervals. Figure 3 summarizes the utility of the POD+PGD parametric
 236 model. The initial condition is projected onto the reduced basis Φ so as to identify the
 237 parameters ζ_i at time step T^n . Afterwards, the PGD modes are computed for the defined
 238 parameters x , ζ_i and ψ .

239 2.4. Assessment methodology

240 The global methodology used to assess this model is presented herein. The combined
 241 parametric model is applied to several study cases in order to evaluate the numerical method
 242 and the ability of the model to properly reproduce physical phenomena with or without
 243 influence of the urban environment. The results of the developed POD+PGD parametric
 244 soil model $T_{num}(x, t)$ are then compared to the results of a reference solution $T_{ref}(x, t)$ that
 245 could be analytical, measurements or the LOM results. For each case study, the indicator
 246 chosen is the ℓ_2 norm (or Root Mean Square Error) noted ε_2 ; it is computed as a spatial or
 247 time function by the following discrete ℓ_2 formulation, where N_x and N_t are the number of
 248 elements over each axis.

$$\varepsilon_2(t) = \sqrt{\frac{1}{N_x} \sum_0^{N_x} [T_{num}(x, t) - T_{ref}(x, t)]^2}$$

$$\varepsilon_2(x) = \sqrt{\frac{1}{N_t} \sum_0^{N_t} [T_{num}(x, t) - T_{ref}(x, t)]^2}$$

The global error is given by the maximum of the previous function $\varepsilon_2(t)$ and $\varepsilon_2(x)$ as described hereafter:

$$\varepsilon_\infty = \max_t(\varepsilon_2(x))$$

249 **3. Results**

250 *3.1. Evaluation of the numerical method with an analytical solution*

251 In this part, for validation purposes, the POD+PGD ROM will be applied to a case
 252 containing an analytical solution. The respective performances of this new model, the fi-
 253 nite difference model and the POD model on its own will be compared. The previously
 254 defined problem will be solved for a uniform slab with a thermal diffusivity of $6.10^{-7} m^2.s^{-1}$
 255 and conductivity of $1.5 W.m^{-1}.K^{-1}$. The slab body is initially at a constant temperature,
 256 $T(x, 0) = 0$. The temperature at $x = L$ is set at $T(L, t) = 0$. At the surface ($x = 0$), only
 257 a convective heat flux is considered, with: a cosine-periodic air temperature whose amplitude
 258 is $10^\circ C$, a period of 24 hours, and a convective heat transfer coefficient of $10 W.m^{-2}.K^{-1}$. No
 259 source term has been taken into account. The spatial domain $\Omega_x = [0, 1]$ is discretized with
 260 a uniform mesh $dx = 0.01 m$. The total simulation time lasts two days, with a time step
 261 of 15 minutes. Only the last day is studied, while the first day is considered as the period
 262 necessary to initialize the model. The problem is first solved by running the finite difference
 263 method. Then, the POD model is built on the previous results with four nodes ($\mathcal{N} = 4$).
 264 The PGD parametric model is built with both the initial condition and air temperature as
 265 parameters ($\psi = T_{air}$):

$$T^{n+1}(x, \zeta_1, \zeta_2, \zeta_3, \zeta_4, T_{air}) = \sum_{i=1}^{\mathcal{M}} X_i(x).C_i(\zeta_1).D_i(\zeta_2).E_i(\zeta_3).F_i(\zeta_4).G_i(T_{air}).$$

266 Each computed solution is compared to an analytical solution from the EXACT toolbox [21]
 267 described in the Appendix A. Figure 5 shows the evolution of temperature calculated by the
 268 analytical solution as well as by the three developed models. The temperature signal of the
 269 Finite Difference model and the reduced-order model (POD+PGD parametric) overlap. As
 270 revealed in these figures, the ROM solutions with 4 modes and the Finite Difference model
 271 provide similar results with $\epsilon_2 = 0.03^\circ C$ at the surface and $0.01^\circ C$ at a depth of $0.5 m$. Figure
 272 6 indicates the ℓ_2 error evolution for the various models compared to the analytical solution.
 273 The quantity ϵ_∞ equals $0.045^\circ C$ for the combined parametric model. Since the POD basis is
 274 used under similar conditions to those for the learning process, the results of the combined
 275 method provide a close fit. The third set of errors given in Figure 6 estimates the error due

276 to the POD parameterization of the initial condition, which remains below 0.016°C . For
 277 each model, the ℓ_2 error decreases with depth. This first case study illustrates the numerical
 278 behavior of the POD+PGD parametric model, which can be considered accurate enough for
 279 application to other situations.

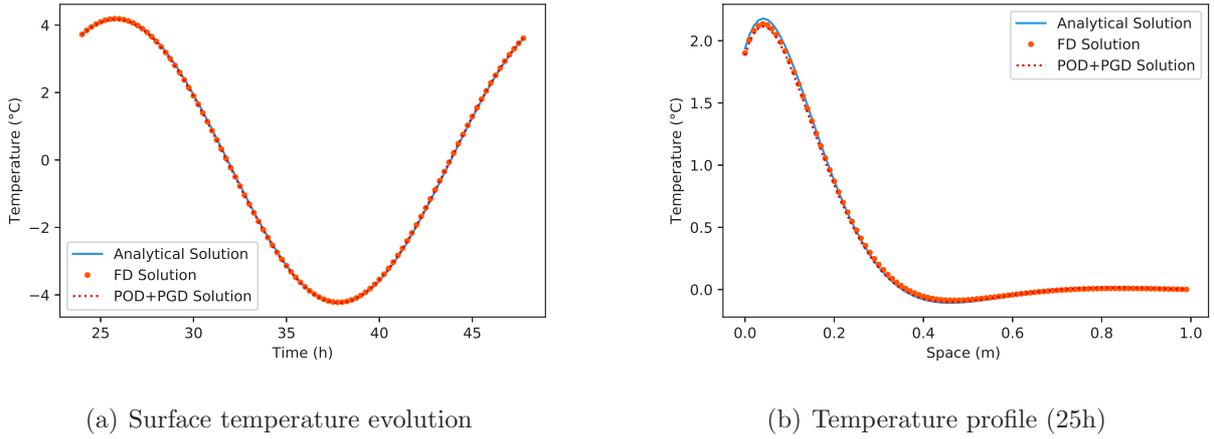


Figure 5: Temperature profile calculated by the analytical solution as well as by the FD and combined parametric models

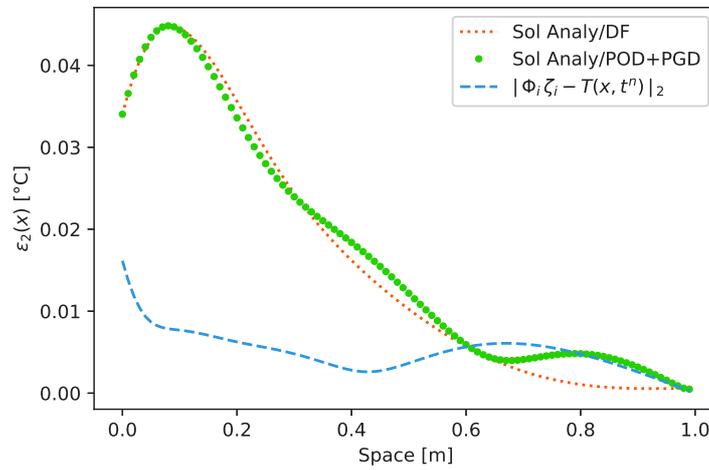


Figure 6: Evolution of the ℓ_2 error. First set: calculated between the Analytical Solution and the Finite Difference model. Second set: calculated between the POD+PGD parametric model and the Analytical Solution. Third set: estimate drawn from the error due to the POD parameterization of the initial condition

280 *3.2. Application to a case study independent of the influence of the urban environment*

281 The combined parametric model will be applied in this section to an asphalt parking
282 lot. This layout was chosen to avoid interactions with other surfaces due to solar and long-
283 wave radiative exchanges with surrounding vertical surfaces (building facades). Under these
284 conditions, the POD+PGD parametric model can be evaluated before its coupling with
285 the micro-climatic model. Results of the combined parametric model will be compared to
286 measurements conducted on this lot. The measurement campaign will be presented first,
287 then the model will be applied and assessed.

288 *3.2.1. Presentation of the measurement campaign*

289 Data from the ROSURE/HydroVille project [22] are used herein. The experimental site is
290 located near Nantes (France) and consists of an asphalt parking lot measuring 2500 m^2 . This
291 campaign has entailed surface and air temperatures as well as on heat flux measurements
292 during a warm summer period (June 2004). During the experiment, the parking lot was
293 watered to simulate artificial rain events. For the observations available during this campaign,
294 this paper focuses on all variables measured at the middle of the lot:

- 295 • surface and ground temperatures: vertical profiles at depths of 0, 1, 2, 3, 4, 5, 6, 10,
296 15, 24, 34, 50 and 75 cm;
- 297 • wind speed;
- 298 • radiation components.

299 The humidity and air temperatures were measured outside the lot. The data were collected
300 with a 1-min time step. The final data were averaged over 15-min intervals. More information
301 on this campaign can be found in [22].

302 *3.2.2. Combined parametric model set-up*

The simulation is run for the entire period from June 5th at midnight to June 14th mid-
night, with a 15 – *minutes* time step. A centimetric grid is used. The temperature profile
is initialized from ground temperatures measured on June 4th at 11:45 pm. The boundary
depth condition, which corresponds to the ground temperature at a depth of 75 *cm* has

Heat fluxes	Input data used to calculate each flux	Calculation Method
q_c	Measured wind speed	Correlation method
	Measured surface temperature	
	Measured air temperature	
q_{net}	Measured net radiative flux	Measurements
q_ℓ	Water height	Mass-transfer method
	Water-holding capacity of the surface	
	Measured air relative humidity	
	Measured air temperature	
q_w	Measured water temperature	Water layer energy balance
	Total measured total sprinkled water	

Table 1: Detail of each input data required to calculate the upper boundary condition

been set in accordance with the experimental data. At the surface, a NEUMANN boundary condition is considered from the surface energy balance detailed below:

$$-k \frac{\partial T}{\partial x} = q_{net} - q_c - q_\ell - q_w,$$

303 The convective heat flux q_c is calculated from the measured air and surface temperature
304 plus the wind speed using a correlation method with a characteristic length of 1 m . More
305 details on this method can be found in [10]. For the radiative heat fluxes q_{net} , the net heat
306 flux measurement is used. During a watering event, two heat fluxes are to be added to the
307 previous ones: a runoff convective heat flux q_w , and a latent heat flux q_ℓ . The computational
308 details of these two fluxes is described in [23]. The runoff convective heat flux is calculated
309 from the amount of water sprinkled during each watering event. Data acquired on June
310 7th, and 8th were used to calibrate the surface water holding capacity. The latent heat flux
311 is calculated from measured air characteristics (temperature, relative humidity). All of the
312 input data categories are listed in the Table 1.

313 The soil is composed of three different materials: 5 cm of asphalt, 45 cm of ballast, and
314 an altered natural mica-schist soil underneath. The soil composition and thermal properties
315 were not measured during the campaign; instead, they were calibrated according to the
316 observed soil thermal profile, thus reducing the difference between measured and simulated

317 surface temperature on the centimetric grid. The data measured on June 6th were used
318 for calibration purposes. From the measured temperature gradient, changes in soil thermal
319 properties in the first layer were identified (0–1 cm, 1–5 cm). The calibrated soil properties
320 are summarized in Table 2.

Layer depth m	Material Characteristics	Thermal conductivity $W.m^{-1}.K^{-1}$	Volumetric heat capacity $10^6 J.m^{-3}.K^{-1}$
0.00 - 0.01	Asphalt Concrete	2.5	2.3
0.01 - 0.05	Asphalt Concrete	2.5	2.1
0.05 - 0.50	Old Filled Ballast	1.8	2.3
0.50 - 0.75	Natural Soil	1.3	2.1

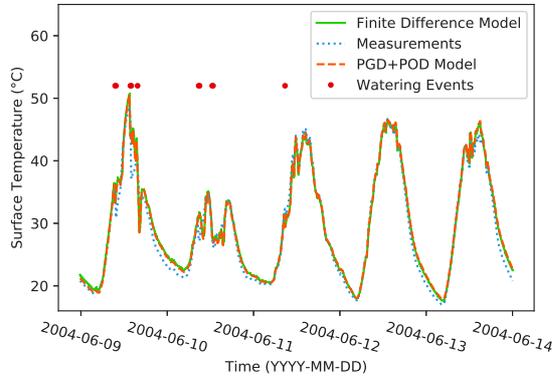
Table 2: Calibrated soil characteristics

321 As presented in Section 2.3, the problem is first solved using the finite difference method.
322 Next, the POD model is built on the previous results with four nodes ($\mathcal{N} = 4$). The PGD
323 parametric model is built with the *initial condition*, the surface heat flux q_a and the ground
324 temperature T_∞ all as parameters, whereas the PGD parametric model comprises 16 modes
325 ($\mathcal{M} = 16$). The combined parametric model is thus as follows:

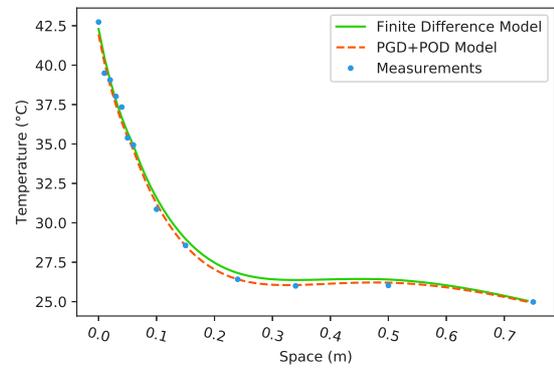
$$\tilde{T}^{n+1}(x, \zeta_1, \zeta_2, \zeta_3, \zeta_4, q_a, T_\infty) = \sum_{i=1}^{\mathcal{M}} X_i(x) \cdot C_i(\zeta_1) \cdot D_i(\zeta_2) \cdot E_i(\zeta_3) \cdot F_i(\zeta_4) \cdot G_i(q_a) \cdot H_i(T_\infty).$$

326 3.2.3. Combined parametric model assessment

327 The ROM model is then evaluated by comparing the time series of the measured and
328 calculated temperatures at the surface and at several depths within the soil. Since the
329 proposed model has combined an *a priori* method (PGD) with an *a posteriori* one (POD),
330 a learning process is required. This process consists of building the POD basis just once
331 and offline. The combined parametric model can then be used online, under different heat
332 boundary conditions. To obtain an efficient combined parametric model, this learning period
333 should be as short as possible. At first, in order to evaluate model performance under actual
334 conditions, the learning and simulation periods will be the same. This initial study is aimed
335 at assessing model behavior regardless of the influence from the learning period.



(a) Comparison of the surface temperatures from June 9th to 13th



(b) Comparison of the temperature profiles from June 9th at 11:45 am (watering event at 09:30 am)

Figure 7: Comparison of simulated and measured temperatures

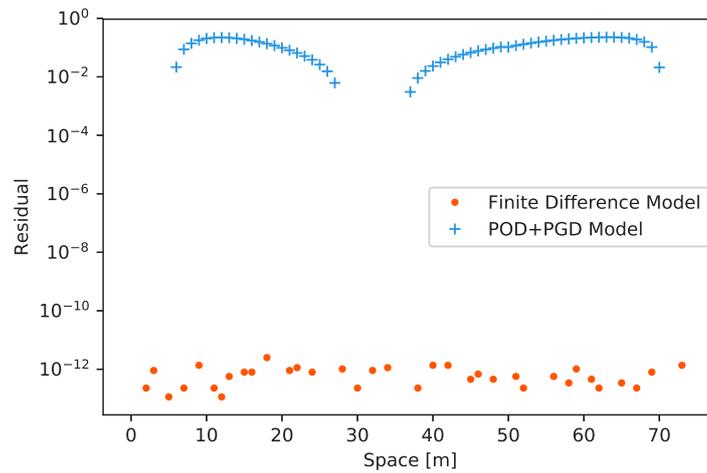


Figure 8: Comparison of the residual of the heat equation calculated for both methods at the last time step

336 In this instance, the ROM model is evaluated over an entire learning period. For both
337 models studied, the temporal variations in the surface temperature are well reproduced com-
338 pared to the measurement results (Figure 7). The combined parametric model signal is closer
339 to the measurement signal than that produced by the finite difference model. In considering
340 this observation, the combined parametric model might seem more accurate than the finite
341 difference model. If we were to compare the numerical residuals of both methods however,
342 the final residual given by the finite difference method would be smaller than that output
343 by the ROM (Fig. 8). According to these results, the PGD solution cannot then be more
344 accurate than the finite difference solution. Due to the calibration of material properties,
345 the finite difference model itself overestimates the surface temperature, whereas the reduced
346 order model underestimates it.

347 During watering events, the same shape is obtained yet with a time lag. This error is
348 independent of the reduced order model behavior itself but depends on the surface energy
349 balance. As described in [23], the surface energy balance during a watering event takes into
350 account additional heat fluxes: a latent heat flux and a runoff convective heat flux. The
351 dynamics of both these heat fluxes are complicated to reproduce during a watering event,
352 which explains the observed time lag. This discrepancy is due to an approximation of the
353 physical phenomena in the model and not in the numerical solution.

354 To determine overall ROM performance, four sets of errors are calculated. The results are
355 presented in Figure 9. The first set evaluates the overall accuracy of the combined parametric
356 model. The quantity is calculated between the POD+PGD and measured temperatures. For
357 the sake of comparison, a second set has been calculated between the finite difference model
358 and the measurements. These first two sets provide the total error due to the chosen numerical
359 scheme, the numerical method and the physical model. To evaluate the error due solely to
360 the numerical scheme and method, a third error set evaluates the loss in accuracy due to the
361 reduction in model order; $\epsilon_2(x)$ is calculated between the POD+PGD parametric model and
362 the finite difference model. The last set estimates the error due to POD parameterization of
363 the initial condition. These ℓ_2 errors have all been calculated from June 8th at 11:45 pm to
364 June 13th at the same time, outside the calibration period.

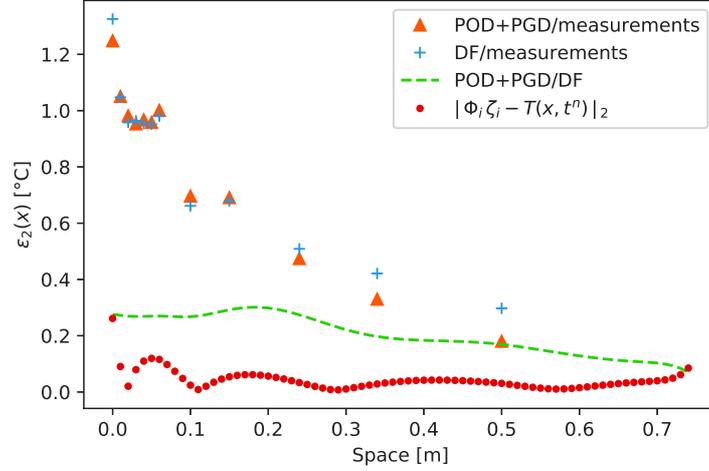


Figure 9: Evolution of the ℓ_2 error. First set: calculated between the POD+PGD parametric model and the measurements. Second set: calculated between the finite difference model and the measurements. Third set: calculated between the parametric model and the finite difference model. Fourth set: estimated error due to the POD parameterization of the initial condition.

365 This loss of accuracy due to the model order reduction remains less than 0.30°C . The
 366 error is greater within the first few centimeters, close to the surface boundary condition, and
 367 then decreases with depth. The combined parametric model accuracy depends directly on
 368 the POD basis accuracy (used to describe the previous temperature profile). The final set in
 369 Figure 9 shows that a portion of the error is due to the initial condition parameterization;
 370 this error prevails close to the boundary condition, where variations are steeper.

371 The $\epsilon_2(x)$ error between the POD+PGD parametric model and the measurements equals
 372 1.24°C at the surface, while it amounts to 1.32°C between the finite difference model and
 373 the measurements. The combined parametric model might appear to be more accurate, but
 374 as explained before the reduced-order model cannot be more accurate since the numerical
 375 residual of the POD+PGD method exceeds the finite difference residual. The model order
 376 reduction has consequences on the reproduction of daily peaks, specifically it underestimates
 377 the temperature signal amplitude. This same phenomenon could be observed with the finite
 378 difference model and a coarser grid.

379 After evaluating the numerical method with the analytical solution, this case study has
 380 evaluated both the numerical method and the physical model. Moreover, under actual con-

381 ditions, both models (POD+PGD and FD) are capable of reproducing the dynamics with
382 relatively good accuracy. Hence, the combined parametric model can be applied to more
383 complex situations, such as an urban scene.

384 *3.2.4. Influence the learning period*

385 As mentioned above, the accuracy of the combined parametric model depends on the
386 accuracy of the POD basis, which is directly determined by the learning period. In order to
387 study the influence of this parameter on model behavior, various learning periods are com-
388 pared. Figure 10 provides the evolution of the global error as a function of the number of
389 days used for the learning period. The error $\varepsilon_2(x)$ is calculated between the combined para-
390 metric model and the measurements (crosses) as well as between the combined parametric
391 model and the finite difference model (points). The first set indicates the evolution of the
392 global model error, which pertains to the numerical scheme, plus the methods errors and
393 physical model errors. The second set indicates the evolution of the error due specifically to
394 the learning period. Both sets are nearly constant over the number of days used to compose
395 the learning period. The number of days selected for the learning process has no influence
396 on the maximum observable error. The assumption could thus be made that using a longer
397 learning period will not improve the maximum model error.

398 The previous indicator however does not yield information on the ability of the model
399 to reproduce a specific dynamic. As such, we are proposing herein to study the model
400 response to various stresses. The dataset examined contains various boundary conditions with
401 sudden drops in the temperature signal due to watering events. Three learning conditions
402 are assessed: one day with no watering events (June 13th), one day with two watering events
403 (June 10th), and the full learning period. June 10th and 13th have been drawn from the
404 calibration period.

405 Figure 11 displays the $\varepsilon_2(t)$ of the different learning periods calculated between the
406 parametric model and the finite difference model over time. For these three periods, several
407 observation can be drawn. Over the entire period, the combined parametric model is more
408 accurate when dry days are considered (i.e. June 5 – 6th and 12 – 13th). Watering events are
409 more difficult to represent accurately. When the learning period is June 13th (see Fig. 11),

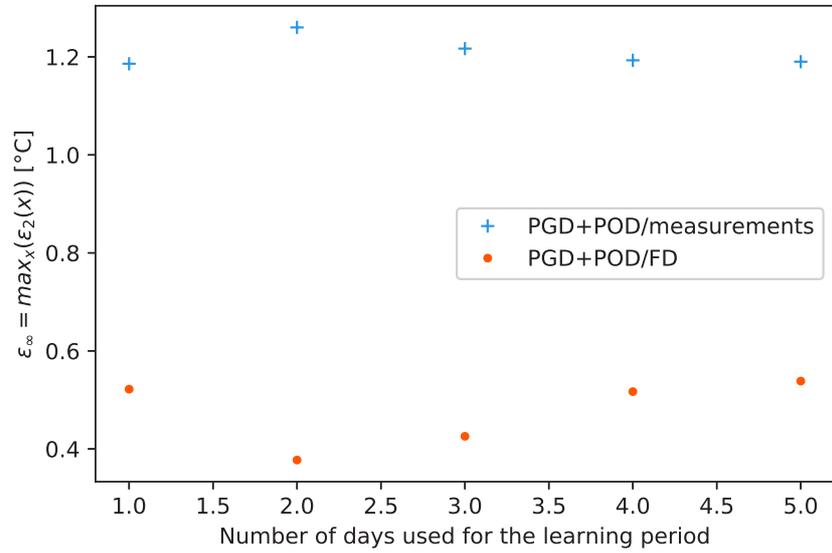


Figure 10: ε_∞ vs. number of days used for the learning period

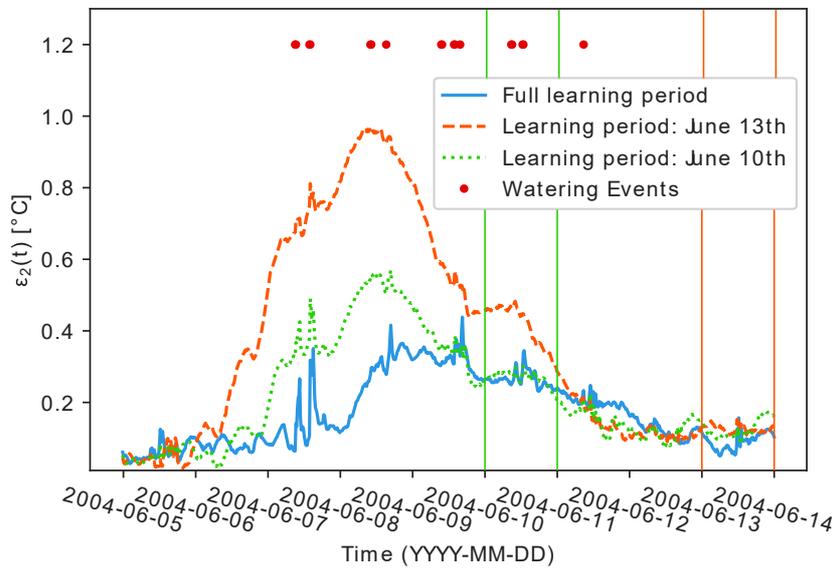


Figure 11: The RMSE calculated between the finite difference model and the POD+PGD parametric model for each temperature profile over time and for different learning periods

410 the POD+PGD parametric model is once again accurate under dry conditions (June 5 – 6th
411 and 12 – 13th). The learning period however has been reduced to just a single day, compared
412 to the previous 9 days with the full learning period. Nonetheless, days with watering events
413 still have a higher RMSE value. The combined parametric model is incapable of accurately
414 reproducing this type of boundary condition. Moreover, June 10th is used as the learning
415 period (green rectangle in Fig. 11) and includes watering events. The model therefore is
416 accurate when learning conditions are considered (June 10th). Yet aside from this period,
417 the model loses accuracy due either to dry days or a watering event.

418 Since the POD basis is sensitive to the learning period, the learning process and, in our
419 specific case, the period need to be carefully selected. The previous comparison illustrates
420 the difficulty involved in selecting a representative period for all weather conditions extending
421 over a longer time scale (season or year). Since it has been validated under actual boundary
422 conditions, the POD+PGD parametric model can now be applied to an urban scene in order
423 to study the thermal behavior of the model under new constraints.

424 *3.3. Practical application to an urban environment*

425 The ROM developed will now be applied to the case of a theoretical urban environment
426 consisting of two canyon streets. For this purpose, the ROM has been coupled with the
427 SOLENE-microclimat simulation tool [6, 7], through use of the ping-pong method [24], which
428 will be discussed in Section 3.3.3. The microclimate tool is used to take into account the
429 surrounding surface in the surface energy budget (i.e. short- and long-wave radiative budget).
430 The objective here is to study the behavior of the ROM model implemented within a complete
431 urban heat balance. In this pursuit, the results of the combined parametric model will be
432 compared to those of the finite difference model under various conditions. The model covers
433 all soil within the urban scene. The study focuses on three zones featuring various radiative
434 stresses. The influence of the area chosen to conduct the learning process will be analyzed,
435 and the computational cost of the various models will be compared.

436 *3.3.1. Description of the urban scene*

437 The urban scene is composed of a square and two canyon streets, with an aspect ratio
438 of $H/W = 1$ (H: building height; W: street width). The streets are oriented north-south

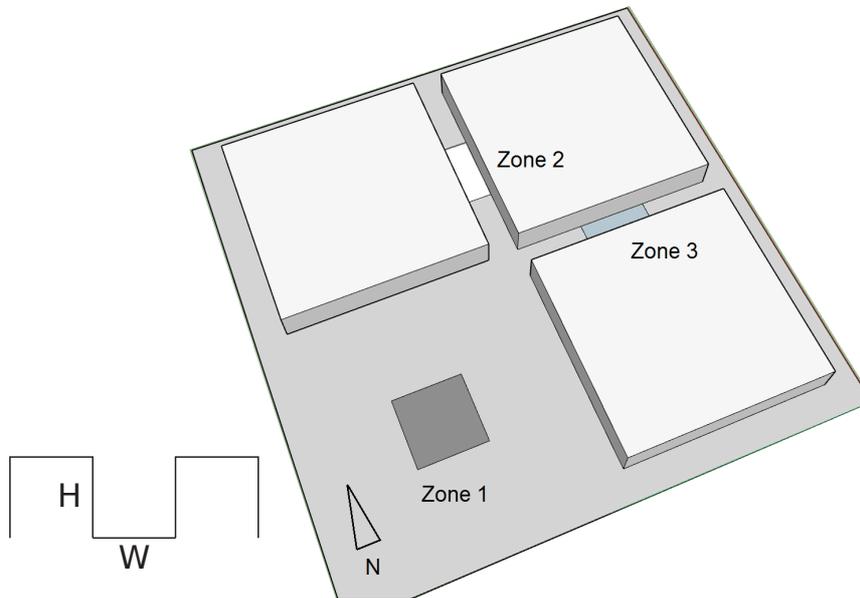


Figure 12: 3D overview of the urban scene

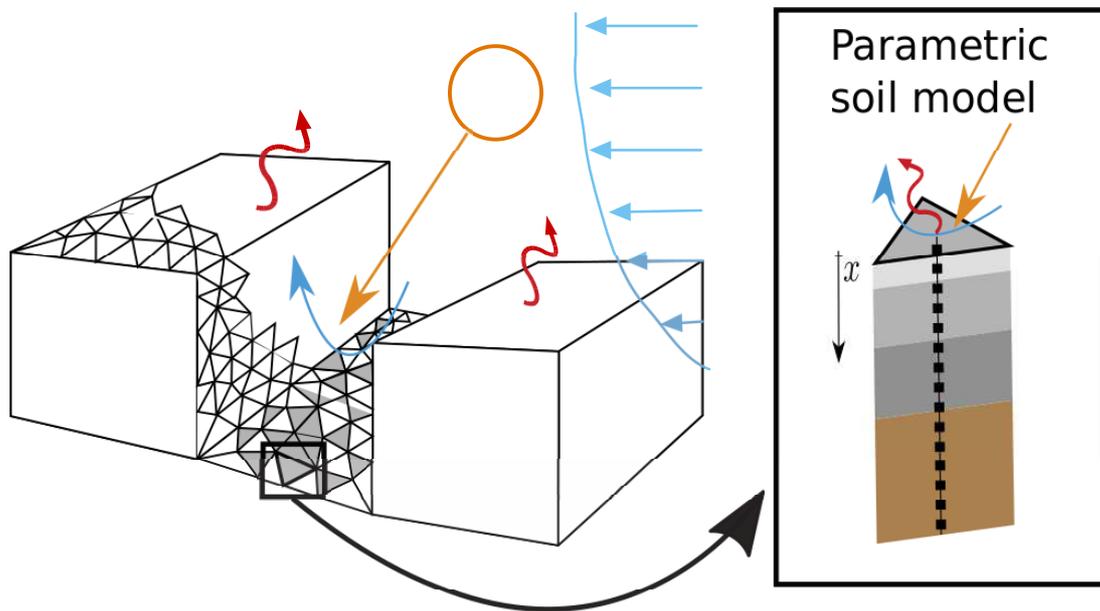


Figure 13: Mesh of the urban geometry. Each surface of the geometry is meshed with triangles, behind which a model has been implemented. In this specific case, the 1D combined parametric soil model is run for each triangle of the floor. For the other triangles (buildings), the 1D SOLENE-microclimat thermal model is employed (see [7] for further details)

Layer Number	Material Characteristics	Depth m	Thermal conductivity $W.m^{-1}.K^{-1}$	Volumetric heat capacity $10^6 J.m^{-3}.K^{-1}$
0	Asphalt	0.07	2.0	2.04
1	Grave	0.25	0.52	1.42
2	Soil	1.00	0.7	1.44

Table 3: Thermal characteristics of the soil for the canyon streets case study

439 and east-west. Figure 12 shows the scene configuration and Figure 13 the mesh, which is
440 built from triangles. The soil represents 32% of the total mesh content in the urban scene,
441 which comprises 7,372 triangles. The soil is considered to be an impervious urban surface;
442 its thermal properties are listed in Table 3. The soil albedo is 0.10 and its thermal emissivity
443 is 0.95. The buildings are represented by concrete boxes with 20 *cm* of concrete and 10 *cm* of
444 external insulation (rock wool). No vegetation has been considered in this case study.

445 The simulation is run for 6 days in 2010 from April 29th at 7 pm through May 6th at
446 6 am, with a one-hour time step. Only the last two days are used to evaluate the model,
447 while considering the first four days as the necessary model initialization period. Since the
448 SOLENE-microclimat model has been introduced here on a perfectly clear day, a two-day
449 period is sufficient to analyze model behavior under various stresses. Weather data acquired
450 in 2010 for the city of Nantes are used as input model data. These data were recorded by
451 the ONEVU (theIRSTV’s Nantes Metropolitan Environmental Observatory, Mestayer et
452 al., 2011 [25]) at the Pin Sec station. Among the observations available, the following data
453 are used as model inputs: air temperature, global and IR radiative fluxes. The combined
454 parametric soil model has been used for each soil triangle in the scene. Three specific zones
455 of this scene, with different net radiative heat flux balances, have been studied:

- 456 • The centre of the square (Zone 1 see Figure 12).
- 457 • The middle of the north-south canyon street (Zone 2 see Figure 12).
- 458 • The middle of the east-west canyon street (Zone 3 see Figure 12).

459 *3.3.2. Combined parametric model setup*

In order to model an entire urban scene, the surface heat balance of the combined parametric model needs to be adjusted to take the surrounding surfaces into consideration. For this purpose, the POD+PGD parametric model will be coupled with a microclimate tool. A Robin boundary condition has been considered; it depends on two heat fluxes: a convective heat flux between the surface and the atmosphere, and a net radiative heat flux. The convective flux is calculated with a constant convective heat transfer coefficient: $h = 10 \text{ W.m}^{-2}.\text{K}^{-1}$. The radiative balance between the soil and surrounding surfaces is calculated by the `SOLENE-microclimat` tool. As noted previously, the problem is first solved by the finite difference method, whereby the finite difference model was coupled with `SOLENE-microclimat` to simulate a day with a perfectly clear sky. Data from a single day are then used to calculate the POD basis Φ with five modes ($\mathcal{N} = 5$). The PGD parametric model is built from the *initial condition*, the net radiative heat flux q_{net} and the air temperature T_a as parameters. This model can be described by the following equation:

$$\tilde{T}^{n+1}(x, \zeta_1, \zeta_2, \zeta_3, \zeta_4, \zeta_5, q_{net}, T_a) = \sum_{i=1}^{\mathcal{M}} X_i(x) \cdot C_i(\zeta_1) \cdot D_i(\zeta_2) \cdot E_i(\zeta_3) \cdot F_i(\zeta_4) \cdot G_i(\zeta_5) \cdot H_i(q_{net}) \cdot I_i(T_a).$$

460 In the deep soil (below 1 meter), the temperature is assumed constant over the simulation
 461 period. Deep in the ground (several meters), the temperature remains constant from one
 462 season to the next and can be substituted by the average air temperature [1]. According to
 463 this assumption, the depth boundary condition has been set here at the mean air temperature
 464 signal over the entire period.

465 *3.3.3. Coupling method*

466 As described in the introduction, the `SOLENE-microclimat` tool is composed of several
 467 1D models assembled with a strong coupling method (i.e. the onion method). It was de-
 468 cided to couple the soil model extracted from the `SOLENE-microclimat` co-simulation loop.
 469 Along these lines, the numerical coupling technique employed was the ping-pong scheme,
 470 as described by Hensen (1995 [24]), which represents a weakly coupled strategy. The main
 471 idea here is to oscillate between the microclimate model and the parametric soil model. Each
 472 model uses the results from the other, as illustrated in Figure 14. First, `SOLENE-microclimat`

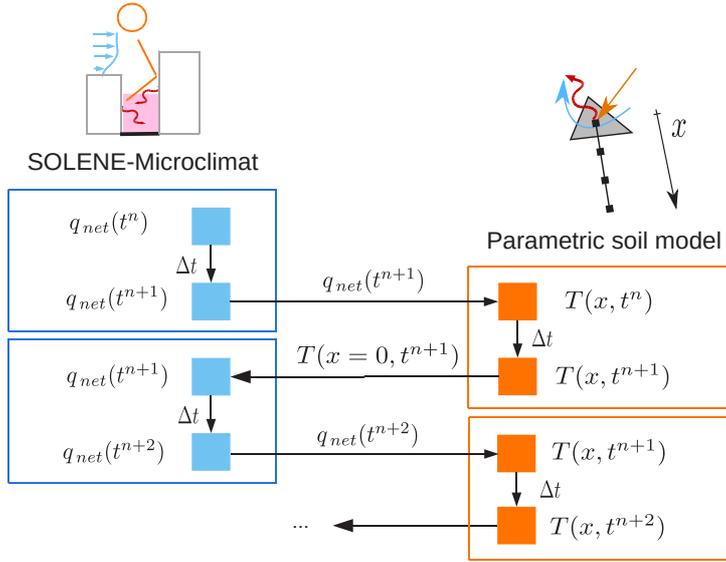


Figure 14: Co-simulation of the parametric soil model with SOLENE-microclimat, the net radiative heat flux $q_{net}(t)$ and $T(x = 0, t)$ as the soil surface temperature

473 calculates, by means of successive iterations (onion method), the street energy balance (short-
474 and long-wave and convection heat fluxes) until convergence at time step $n + 1$. The heat flux
475 balance $\phi_{K,L}(x, n + 1)$ is then used by the parametric soil model to calculate the temperature
476 of each surface $T_{soil}(x, n + 1)$. These data are then transferred to SOLENE-microclimat in order
477 to initialize the next time step ($n + 2$) of heat flux balance computation for the time step.
478 The performance of the combined parametric soil model is compared to the finite difference
479 soil model coupled to SOLENE-microclimat in the same manner. The results of the finite
480 difference soil model are adopted as the reference solution. Since the soil models have been
481 extracted from the SOLENE-microclimat co-simulation loop with the ping-pong coupling
482 method, the soil temperature cannot be updated upon each internal SOLENE-microclimat
483 iteration; moreover, the long-wave radiative heat flux cannot be updated with the calculated
484 soil temperature. Such is the main limitation of this coupling method. However, since the
485 coupling limitations are identical for the two outside soils, the error caused by this coupling
486 method on the long-wave radiative heat flux has no impact on the comparative results.

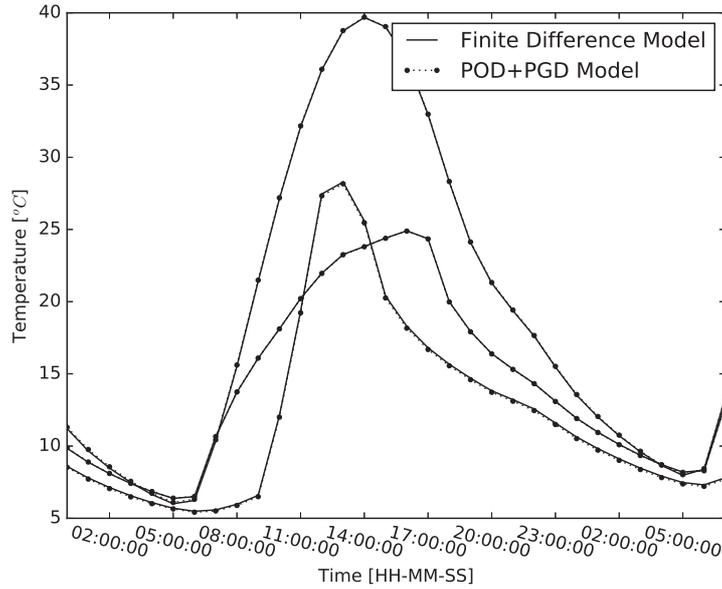


Figure 15: Comparison of the calculated mean surface temperatures over the three zones by both the finite difference and combined parametric soil models. Data from the corresponding zone have been used for the learning step.

3.3.4. Influence of the learning area

As discussed above, in order to obtain an efficient reduced-order model, the learning period needs to be as short as possible and representative of soil behavior. In an urban scene, each triangle has a unique surface heat balance that depends not only on meteorological conditions but also on its location (Fig. 13). The location of the triangles used to perform the learning process affects the dynamic model behavior. To evaluate the influence of these conditions on the model, several zones are compared with several learning areas. The aim here is to study the impact of the area assigned to perform the learning step on the results. As indicated in the introduction, surface temperature then becomes the key urban micro-climate variable; it drives the various heat fluxes at the urban surface. In order to evaluate the performance of this reduced model coupled with a microclimate tool, the surface temperature calculated by the combined parametric model is to be compared to that calculated by the finite difference model.

Three learning zones are compared. For each of them, the POD basis is built on the data calculated by the finite difference model for the considered zone. Since the scene is

502 composed of thousands of triangles, the mean temperature results over the zone are used to
 503 perform the learning step. Figure 15 displays the mean surface temperature calculated over
 504 each zone, with a learning stage performed on each corresponding zone. Results are given
 505 for a PGD basis with $\mathcal{M} = 38$. The mean temperature signals of each zone, calculated with
 506 both models, overlap. The ROM (POD+PGD) correctly reproduces the dynamics, with a
 507 POD basis of 5 modes.

Evaluation zone	Zone 1: the square	Zone 2: North-South street	Zone 3: East-West street
Learning zone used	$\varepsilon_2(x = 0)$	$\varepsilon_2(x = 0)$	$\varepsilon_2(x = 0)$
Zone 1	0.15	0.17	0.18
Zone 2	0.14	0.06	0.03
Zone 3	0.12	0.03	0.03

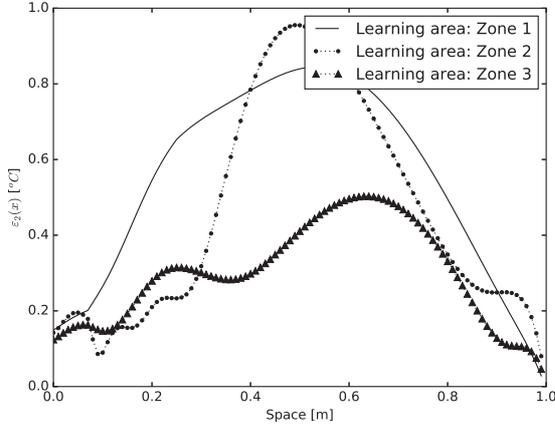
Table 4: $\varepsilon_2(x = 0)$ ($^{\circ}C$) calculated between the mean temperature output by the finite difference and combined parametric soil models for April 4th and 5th in using different learning zones

Evaluation zone	Zone 1: the square	Zone 2: North-South street	Zone 3: East-West street
Learning zone used	ε_{∞}	ε_{∞}	ε_{∞}
Zone 1	0.84	1.92	1.64
Zone 2	0.96	0.27	0.31
Zone 3	0.50	0.44	0.38

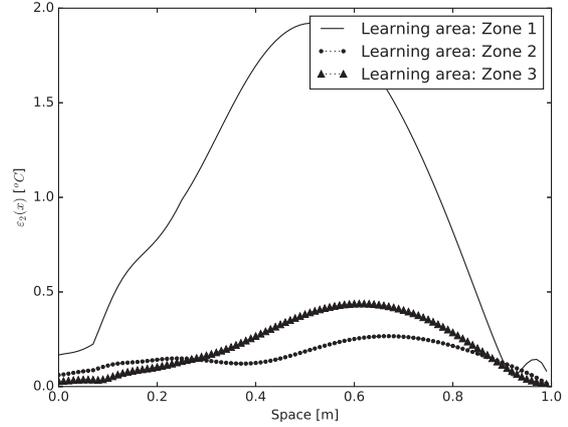
Table 5: ε_{∞} ($^{\circ}C$) calculated between the mean temperature output by the finite difference and combined parametric soil models for April 4th and 5th in using different learning zones

508 Lastly, in order to evaluate the behavior of the combined parametric soil model, the ε_{∞}
 509 error indicator is calculated from the output of the mean temperature of each zone, depending
 510 on the learning zone. The ℓ_2 error is also derived at the surface since the surface temperature
 511 is the key urban micro-climate variable. Tables 4 and 5 list all these results. To obtain
 512 a model as accurate as that presented above at the surface (see Fig. 9), the POD basis
 513 comprises 5 modes. For this POD basis and the PGD basis containing 38 modes, the RMSE
 514 of the mean surface temperature remains less than $0.20^{\circ}C$. Depending on the learning zone

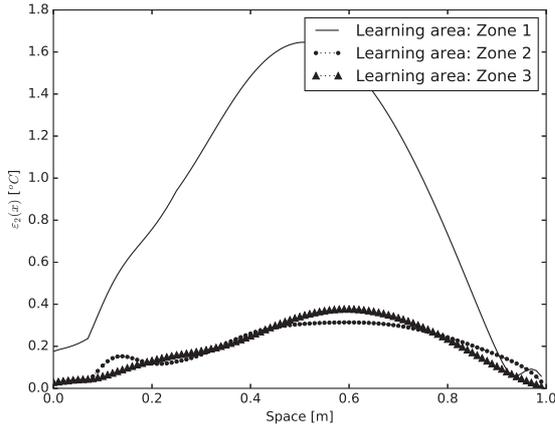
515 targeted, the loss of accuracy varies from $0.03^{\circ}C$ to $0.18^{\circ}C$ at the surface. The maximum
 516 RMSE varies from $0.27^{\circ}C$ to $1.92^{\circ}C$.



(a) Zone 1



(b) Zone 2



(c) Zone 3

Figure 16: Evolution of the $\varepsilon_2(x)$ error for the three zones relative to the learning area used (i.e. the figure in (a) provides the evolution of this $\varepsilon_2(x)$ error for zone 1. Each plot corresponds to one of the learning areas used)

517 Figure 16 shows the evolution of the $\varepsilon_2(x)$ error for all three zones relative to the learning
 518 area used. Results vary by learning zone. Zone 1 (Fig. 16(a)) is not affected by the shading.
 519 The temperature range then is most significant in this zone compared to the other zones
 520 ($37.0^{\circ}C$ amplitude). At the surface, the loss of accuracy increases when the learning process
 521 is performed on the streets (zones 2 and 3). The model does not easily reproduce a signal with

522 an amplitude larger than that learned. The two streets (zones 2 and 3) exhibit very different
 523 surface heat balances. The temperature signal of zone 2 (Fig. 16(b)) has an amplitude of
 524 $27.4^{\circ}C$. The temperature increases suddenly and then decreases gradually. However, the
 525 temperature signal of zone 3 (Fig. 16(c)) gradually increases and then decreases but with
 526 a smaller amplitude: $22.2^{\circ}C$. The dynamic of both temperature profiles is very distinct,
 527 which has consequences on the accuracy of the POD+PGD parametric soil model. For each
 528 zone, the error is smaller when the learning conditions are met: the calculated area is the
 529 same as that used during the learning stage. For the other zones, the loss of accuracy is
 530 greater. For example, the maximum Zone 1 error $\varepsilon_{\infty} = 0.84^{\circ}C$ occurs when the learning
 531 period is performed on this zone, and $0.1.92^{\circ}C$ when the learning process is performed on
 532 zone 2 (see Table 5). This finding illustrates that both the learning period and location must
 533 be representative of the soil thermal behavior in order to generate accurate results.

534 3.3.5. Calculation cost and time

535 The model is also evaluated with respect to its computational cost. Two methods are used
 536 herein to compare the combined parametric model with the finite difference model, namely
 537 the number of problem degrees of freedom, and CPU time. For both the finite difference
 538 and combined parametric models, the most expensive operation is to reverse the problem
 539 matrix. The number of problem degrees of freedom often yields direct information on the
 540 computational complexity of the problem and its cost [16]. With the finite difference model,
 541 a system of N_t time steps and N_x nodes needs to be solved. With the PGD parametric model,
 542 the size of the system depends on N_t and the number of POD modes N . The computational
 543 cost reduction can then be estimated by: $1 - \frac{N}{N_t}$. In our case study, the model was run for 5
 544 modes and 157 time steps (6 days), for a 96.5% cost reduction.

545 The previous method however does not take into account all the additional operations,
 546 like additions and multiplications, due to the PGD parametric model, the coupling, etc.
 547 Consequently, CPU time is used to produce an estimation of the actual drop in computational
 548 cost. The CPU times of all simulations (performed on the same computer) were calculated
 549 for both soil models (POD+PGD and FD); it includes all online operations due to the model
 550 and its coupling with SOLENE-microclimat. The simulation time was averaged over 11 runs

Step	CPU time for $\mathcal{M} = 38$
Offline no.1: Building the POD basis	503.5
Offline no.2: Building the PGD parametric model	375.0
Online : Use of the Finite Difference Model	503.4
Online : Use of the Combine Parametric model	99.4
Online Calculation cost reduction	80%

Table 6: Mean CPU computation time for each step (see Fig. 4 for a description of the offline/online steps)

551 for each soil model. The simulations performed with the finite difference model required on
552 average 503.4 sec, while the combined parametric model time amounted to 99.4 sec with 38
553 modes. The actual cost reduction for this urban scene is 80% with 38 modes. The number of
554 modes used in the PGD basis has little influence on the online computation time, yet it does
555 affect the offline steps and the accuracy of the PGD model. This computation time depends
556 on: the urban scene, the duration of the simulated period, the proportion of soil triangles in
557 the scene (32% in this case), and the specifications of the computer used. However, it does
558 not take into account the time spent offline to build the POD basis.

559 4. Conclusion

560 The main objective of this paper has been to propose a reduced urban soil model that
561 precisely reproduces the surface temperature and thermal heat flux exchanged between the
562 soil and the urban scene with a reduced computation time. In this aim, a combination of two
563 order-reduction methods has been proposed. The PGD method is used to build a parametric
564 solution, while the POD method configures the initial condition with a minimum number of
565 parameters. The compound method is then composed of an offline phase (learning step plus
566 creation of the parametric model) and an online phase (use of the parametric model).

567 The POD+PGD parametric soil model was first validated using a simple case with a
568 known analytical solution. The parametric model solutions with 4 modes and the finite dif-
569 ference model produced similar results with an RMSE of $0.03^{\circ}C$ at the surface and $0.01^{\circ}C$
570 at a depth of 0.5 m. This first case study illustrates the numerical behavior of the com-
571 bined parametric model, which has been considered accurate enough to be applied to other

572 situations.

573 The parametric model was then applied to an actual case study in an open space, more
574 specifically a parking lot. For both models (POD+PGD ROM, FD), the temporal temper-
575 ature variation at the surface and at several depths was well reproduced compared to the
576 measurements. The loss of accuracy due to model reduction for an entire learning period
577 remains below $0.30^{\circ}C$ (POD+PGD ROM/FD) at the surface with an RMSE of $1.24^{\circ}C$ with
578 the measurements (POD+PGD ROM/measurements).

579 Beyond the loss due to the PGD algorithm, this accuracy drop is due to the POD ba-
580 sis accuracy, which depends directly on the learning period. Various learning periods were
581 compared herein. As expected, the model is accurate when run under the same tempo-
582 ral boundary conditions. Complex signals, such as watering events, are more difficult to
583 model accurately, which is why selecting a short and representative learning period is key to
584 obtaining an efficient parametric model.

585 The combined parametric soil model was then used in the case of a theoretical urban
586 scene with two canyon streets; it was coupled with the `SOLENE-microclimat` simulation tool
587 using the ping-pong co-simulation scheme. For a POD basis containing 5 modes and a PGD
588 basis composed of 38 modes, the RMSE of the mean surface temperature remained less than
589 $0.20^{\circ}C$. Since each triangle of an urban scene has different loadings depending on its location,
590 the accuracy of the model varies with the location of the area used for the learning process.

591 These two case studies illustrate the difficulties encountered when selecting a represen-
592 tative learning period, which must be short and representative of boundary conditions (me-
593 teorological conditions, materials, location of the triangle, etc.) inherent in the simulated
594 soil. The selection of an efficient learning period - sample pair is a new challenge to ensuring
595 model efficiency.

596 In conclusions, the proposed ROM is able to accurately represent the thermal heat flux
597 exchanged between the soil and the urban scene. For our specific urban case study (30%
598 of soil triangles in the scene and a one-week simulation period), the computational cost

599 reduction amounts to 80%. And this time only takes into account the online part. Further
600 work must now be carried out to improve the offline part, i.e. the learning process.

601 A. Description of the analytical solution

602 The analytical solution $T_{AS}(x, t)$ of problem 1 from the EXACT toolbox [21] is described
603 hereafter.

The problem is normalised with the following dimensionless variables:

$$\tilde{x} = \frac{x}{L}; \tilde{t} = \frac{\alpha t}{L^2}; \tilde{\omega} = \frac{\omega \cdot L^2}{\alpha}; B = \frac{h_{conv} \cdot L}{k}$$

The solution is given by the following equation:

$$\begin{aligned} \frac{T_{AS}(x, t)}{T_\infty} = \text{Real} \left[B \cdot e^{i\tilde{\omega}\tilde{t}} \cdot \frac{(1 + R) \cdot [e^{-\tilde{\sigma}\tilde{x}} - e^{-\tilde{\sigma}(2-\tilde{x})}]}{2 \cdot \tilde{\sigma} \cdot (1 + R \cdot e^{-2\tilde{\sigma}})} \right] \\ - 2B \sum_{m=1}^{\infty} \frac{\beta_m^2 \cdot e^{-\beta_m^2 \tilde{t}}}{\beta_m^4 + \tilde{\omega}^2} \cdot \frac{[\beta_m \cos(\beta_m \tilde{x}) + B \sin(\beta_m \tilde{x})] \beta_m}{\beta_m^2 + B^2 + B} \end{aligned} \quad (\text{A.1})$$

604 with : $\sigma^2 = \frac{i\omega}{\alpha}; \tilde{\sigma} = \sigma L; R = \frac{\tilde{\sigma} - B}{\tilde{\sigma} + B}; -B = \beta_m \cot \beta_m$

605 The notation *Real* refers to the real part of the complex expression. β_m values are the positive
606 roots of $\beta_m \cot \beta_m = -B$.

607 B. Building the POD basis Φ

608 The POD method consists of searching for a set of basis functions Φ that approximates
609 the temperature profile $T(x, t)$ from the eigenvalues and eigenmodes (Eq. (3)). This method
610 follows three steps:

- 611 • First, the LOM is solved. Here, the finite difference model is used to calculate the
612 temperature profile in the soil for a given set of boundary conditions (q_a, T_a) .
- 613 • Then, from this dataset, the reduced-order model is built.
- 614 • Finally, this model can be used under a new set of conditions.

615 The temperature profile obtained from the LOM constitutes a collection of snapshots stored
616 in a matrix $\mathbf{Q}(N_x, N_t)$, in which each column represents a snapshot of the temperature profile
617 at a given time step. This matrix defines the terms of the learning process and has an impact

618 on the reduced model performance. For this reason, the snapshots must be representative of
619 the problem (boundary values, initial conditions, materials used). The basis Φ capturing the
620 greatest amount of energy from a system with a minimum number of degrees of freedom is
621 formed by the eigenvectors of the problem:

$$\gamma \cdot \Phi = \lambda \cdot \Phi \quad (\text{B.1})$$

622

$$\gamma = \mathbf{Q}\mathbf{Q}^T \quad (\text{B.2})$$

623

624 From the *snapshots* matrix \mathbf{Q} , the auto-correlation matrix γ can then be calculated.
625 Next, the eigenmodes (eigenvalues and eigenvectors) are calculated in solving the eigenvalue
626 problem (Equation B.1). The temperature profile is now approximated by \mathcal{N} eigenvectors
627 Φ_i associated with their \mathcal{N} eigenvalues λ_i with $N_x \geq \mathcal{N} \geq 1$.

$$\begin{aligned} \lambda_1 \geq \dots \geq \lambda_i \geq \dots \geq \lambda_{\mathcal{N}} \geq 0 \\ N_x \geq \mathcal{N} \geq 1 \end{aligned} \quad (\text{B.3})$$

628 Each eigenvalue represents the portion of energy captured by a mode. Several criteria may
629 be used to select the optimal number of modes. In most examples, an arbitrary threshold is
630 set depending either on the ratio between the first and current eigenvalues ($\lambda_{\mathcal{N}} > \eta \lambda_1$) or
631 on the portion of energy captured ($\frac{\sum_{k=1}^{\mathcal{N}} \lambda_k}{\sum_{i=1}^{N_x} \lambda_i} > 0.999$). In the field of fluid dynamics, Sirovich
632 et al. (1991 [26]) used a combination of both criteria. Moreover, according to Sempey
633 (2007, [27]), these criteria must be adjusted to each specific problem. The reduced basis Φ
634 of order \mathcal{N} can now be written in the following form:

$$\Phi = \begin{pmatrix} \Phi_1 & \Phi_2 & \dots & \Phi_{\mathcal{N}} \end{pmatrix} = \begin{pmatrix} \Phi_1(x_1) & \Phi_2(x_1) & \dots & \Phi_{\mathcal{N}}(x_1) \\ \Phi_1(x_2) & \Phi_2(x_2) & \dots & \Phi_{\mathcal{N}}(x_2) \\ \dots & \dots & \dots & \dots \\ \Phi_1(x_{N_x}) & \Phi_2(x_{N_x}) & \dots & \Phi_{\mathcal{N}}(x_{N_x}) \end{pmatrix} \quad (\text{B.4})$$

635

636 **C. Development of the alternating direction strategy**

637 *C.1. Computation of $F_m^p(T^n)$ from $X_m^p(x)$ and $G_m^{p-1}(\psi)$*

638 Eq. (7) is inserted into Eq. (5), with the following test function (Eq. (C.1)).

$$u^*(x, T^n, \psi) = X_m^p(x) \cdot F_m^*(T^n) \cdot G_m^{p-1}(\psi) = X \cdot F^* \cdot G \quad (\text{C.1})$$

639 We obtain the following equation:

$$\begin{aligned} & \int_{\Omega_x \times \Omega_{T^n} \times \Omega_\psi} X \cdot F^* \cdot G \left(c \frac{X \cdot F \cdot G}{\Delta t} - \frac{\partial}{\partial x} \left(k \frac{\partial X}{\partial x} \right) \cdot F \cdot G \right) dx \cdot dT^n \cdot d\psi \\ & \quad - \int_{\Omega_x \times \Omega_{T^n} \times \Omega_\psi} X \cdot F^* \cdot G \cdot c \frac{T^n}{\Delta t} dx \cdot dT^n \cdot d\psi \\ = & - \int_{\Omega_x \times \Omega_{T^n} \times \Omega_\psi} X \cdot F^* \cdot G \sum_{i=1}^{m-1} \left(c \frac{X_i \cdot F_i \cdot G_i}{\Delta t} - \frac{\partial}{\partial x} \left(k \frac{\partial X_i}{\partial x} \right) \cdot F_i \cdot G_i \right) dx \cdot dT^n \cdot d\psi \end{aligned}$$

640 Since all the functions that depend on parametric coordinate ψ and x are known, they
641 can be integrated over their domain: Ω_x and Ω_ψ .

$$\left\{ \begin{array}{l} x_1 = \int_{\Omega_x} (X) dx \\ x_2 = \int_{\Omega_x} (X)^2 dx \\ x_3 = \int_{\Omega_x} \left(X \cdot \frac{\partial}{\partial x} \left(k \frac{\partial X}{\partial x} \right) \right) dx \\ g_1 = \int_{\Omega_\psi} (G) d\psi \\ g_2 = \int_{\Omega_\psi} (G)^2 d\psi \\ x_{i,1} = \int_{\Omega_x} (X \cdot X_i) dx \\ x_{i,2} = \int_{\Omega_x} \left(X \cdot \frac{\partial}{\partial x} \left(k \frac{\partial X_i}{\partial x} \right) \right) dx \\ g_i = \int_{\Omega_\psi} (G \cdot G_i) d\psi \end{array} \right. \quad (\text{C.2})$$

642 We derive the following algebraic equation (12), whose direct solution yields the function F.

$$F \cdot \left(\frac{c}{\Delta t} x_2 g_2 - x_3 g_2 \right) = - \sum_{i=1}^{m-1} \left(c \frac{F_i}{\Delta t} x_{i,1} g_i - F_i x_{i,2} g_i \right) + x_1 g_1 c \frac{T^n}{\Delta t} \quad (\text{C.3})$$

643 *C.2. Computation of $G_m^p(\psi)$ from $X_m^p(x)$ and $F_m^p(T^n)$*

644 Eq. (7) is inserted into Eq. (5), with the following test function (Eq. (C.4))

$$u^*(x, T^n, \psi) = X_m^p(x) \cdot F_m(T^n) \cdot G_m^*(\psi) = X \cdot F \cdot G^* \quad (\text{C.4})$$

645 We obtain the following equation:

$$\begin{aligned}
& \int_{\Omega_x \times \Omega_{T^n} \times \Omega_\psi} X \cdot F \cdot G^* \left(c \frac{X \cdot F \cdot G}{\Delta t} - \frac{\partial}{\partial x} \left(k \frac{\partial X}{\partial x} \right) \cdot F \cdot G \right) dx \cdot dT^n \cdot d\psi \\
& \quad - \int_{\Omega_x \times \Omega_{T^n} \times \Omega_\psi} X \cdot F \cdot G^* \cdot c \frac{T^n}{\Delta t} dx \cdot dT^n \cdot d\psi \\
= & - \int_{\Omega_x \times \Omega_{T^n} \times \Omega_\psi} X \cdot F \cdot G^* \sum_{i=1}^{m-1} \left(c \frac{X_i \cdot F_i \cdot G_i}{\Delta t} - \frac{\partial}{\partial x} \left(k \frac{\partial X_i}{\partial x} \right) \cdot F_i \cdot G_i \right) dx \cdot dT^n \cdot d\psi
\end{aligned}$$

646 Since all the functions that depend on parametric coordinate x and T^n are known, they can
647 be integrated over their domain: Ω_x and Ω_{T^n} .

$$\left\{ \begin{array}{l}
x_1 = \int_{\Omega_x} (X) dx \\
x_2 = \int_{\Omega_x} (X)^2 dx \\
x_3 = \int_{\Omega_x} \left(X \cdot \frac{\partial}{\partial x} \left(k \frac{\partial X}{\partial x} \right) \right) dx \\
f_1 = \int_{\Omega_{T^n}} (F) dT^n \\
f_2 = \int_{\Omega_{T^n}} (F)^2 dT^n \\
x_{i,1} = \int_{\Omega_x} (X \cdot X_i) dx \\
x_{i,2} = \int_{\Omega_x} \left(X \cdot \frac{\partial}{\partial x} \left(k \frac{\partial X_i}{\partial x} \right) \right) dx \\
f_i = \int_{\Omega_{T^n}} (F \cdot F_i) dT^n
\end{array} \right. \quad (C.5)$$

648 We derive the following algebraic equation (13), whose direct solution yields the function G .

$$G \left(\frac{c}{\Delta t} x_2 f_2 - x_3 f_2 \right) = - \sum_{i=1}^{m-1} \left(c \frac{G_i}{\Delta t} \cdot x_{i,1} \cdot f_i - G_i x_{i,2} f_i \right) + x_1 f_1 c \frac{T^n}{\Delta t} \quad (C.6)$$

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