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# Wireless Broadcast with short labels

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**Abstract**—In this paper we study the broadcast problem in wireless networks when the broadcast is helped by a labelling scheme. We focus on two variants of broadcast: broadcast without acknowledgement (i.e. the initiator of the broadcast is not notified at the end of broadcast) and broadcast with acknowledgement. Our contribution is twofold. *First*, we propose label optimal broadcast algorithms in a class of networks issued from recent studies in Wireless Body Area Networks then we extend our solutions to arbitrary networks. We propose an acknowledgement-free broadcast strategy using 1-bit labels and broadcast with acknowledgement using 2-bits labels. In the class of level-separable networks our algorithms finish within  $2D$  rounds for both broadcast with and without acknowledgement, where  $D$  is the eccentricity of the broadcast initiator. *Second*, we improve a recent [11] labelling-based broadcast scheme with acknowledgement designed for arbitrary networks in terms of memory complexity.

**Index Terms**—Labelling Scheme, Broadcast, Wireless Networks

## I. INTRODUCTION

*Broadcast* is the most studied communication primitive in networks and distributed systems. Broadcast ensures that once a *source node* (a.k.a. the broadcast initiator) sends a message then all other nodes in the network should receive this message in a finite time. Limited by the transmission range, messages may not be able to be sent directly from one node to some other arbitrary node in the network. Therefore relay nodes need to assist the source node during the message propagation by re-propagating it. Deterministic centralized broadcast, where nodes have complete network knowledge has been studied by Kowalski *et al.* in [19]. The authors propose an optimal solution that completes within  $O(D \log^2 n)$  rounds, where  $n$  is the number of nodes in network and  $D$  is the largest distance from the source to any node of the network. The time lower bound for broadcast,  $\Omega(\log^2 n)$ , has been proved in [2] for a family of radius-2 networks. For deterministic distributed broadcast, assuming that nodes only know their IDs (i.e. they do not know the IDs of their neighbors nor the network topology), in [8] is proposed the fastest broadcast within  $O(n \log D \log D)$  rounds, where  $D$  is the diameter of network. The lower bound in this case, proposed in [9], is  $\Omega(n \log D)$ , where  $D$  is the largest distance from the source to any node of the network.

In wireless networks, when a message is sent from a node it goes into the wireless channel in the form of a wireless signal which may be received by all the nodes within the transmission range of the sender. However, when a node is

located in the range of more than one node that send messages simultaneously the multiple wireless signals may generate *collisions* at the receiver. The receiver cannot decode any useful information from the superimposed interference signals. At the MAC layer several solutions have been proposed in the last two decades in order to reduce collisions. All of them offer probabilistic guarantees. Our study follows the recent work that addresses this problem at the application layer. More specifically, we are interested in deterministic solutions for broadcasting messages based on the use of extra information or advice (also referred as *labelling*) precomputed before the broadcast invocation.

Labelling schemes have been designed to compute network size, the father-son relationship and the geographic distance between arbitrary nodes in the network (e.g. [1], [14] and [16]). Labelling schemes have been also used in [13] and [15] in order to improve the efficiency of Minimum Spanning Tree or Leader Election algorithms. Furthermore, [10] and [12] exploit labelling in order to improve the existing solutions for network exploration by a robot/agent moving in the network.

Very few works ( e.g. [18] and [11]) exploit labelling schemes to design efficient *broadcast* primitives. When using labelling schemes nodes record less information than in the case of centralized broadcast, where nodes need to know complete network information. Compared with the existing solutions for deterministic distributed broadcast the time complexity is improved. In [18] the authors prove that for an arbitrary network to achieve broadcast within constant number of rounds a  $O(n)$  bits of advice is sufficient but not  $o(n)$ . Very recently, a labelling scheme with 2-bits advice (3 bits for broadcast with acknowledgement) is proposed in [11]. The authors prove that their algorithms need  $2n - 3$  rounds for the broadcast without acknowledgement and  $3n - 4$  rounds for broadcast with acknowledgement in arbitrary network.

**Contribution:** Our work is in the line of research described in [11] and [18]. We first introduce a new family of networks, called level-separable networks issued from in Wireless Body Area Networks (e.g. [3], [5], [6], [4] and [7]). We then propose an acknowledgement-free broadcast strategy using 1-bit labels and a broadcast scheme with acknowledgement using 2-bits labels. In the class of level-separable networks our algorithms are memory optimal and terminate within  $2D$  rounds for both types of broadcast primitives, where  $D$  is the eccentricity of the broadcast source. Second, we address the arbitrary networks and improve the broadcast scheme with

acknowledgement proposed in [11] in terms of memory and time complexity by efficiently exploiting the 3-bits labelling encoding. Differently from the solution proposed in [11], our solution does not use extra local persistent memory except the 3-bits labels.

## II. MODEL AND PROBLEM DEFINITION

### A. Communication Model

We model the network as a graph  $G = (V, E)$  where  $V$ , the set of *vertices*, represents the set of nodes in the network and  $E$ , the set of *edges*, is a set of unordered pairs  $e = (u, v)$ ,  $u, v \in V$ , that represents the communications links between nodes  $u$  and  $v$ . In the following  $d(u)$  denotes the set of neighbors of node  $u$ .

We target wireless networks where due to the limitation of the transmission power, a node may not have connections with the other nodes in the network (i.e.,  $|d(u)| \leq |V| - 1$ ). However, we assume that the network is connected, i.e., there is a path between any two nodes in the network.

We assume that nodes execute the same algorithm and are *time synchronized*. The system execution is decomposed in *rounds*. When a node  $u$  sends a message at round  $x$ , all nodes in  $d(u)$  receive the message at the end of round  $x$ . Collisions occur at node  $u$  in round  $x$  if a set of nodes,  $M \subseteq d(u)$  and  $|M| > 1$ , send a message in round  $x$ . In that case it is considered that  $u$  has not received any message.

In the following we are interested in solving the *Broadcast problem*: when a source node sends a message, this message should be received by all the nodes in the network in finite bounded time.

### B. Level-Separable Network

In this section, we define a family of networks, *Level-Separable Network*, issued from WBAN area (e.g. [3], [5], [6], [4] and [7]). We say an arbitrary network is a Level-Separable Network if the underlay communication graph  $G = (V, E)$  of the network verifies the *Level-Separable* propriety defined below.

To define the *Level-Separable* propriety, we introduce some preliminary notations.

Let  $G(V, E)$  be a network and let  $s \in V$ , a predefined vertex, be the source node of the broadcast. Each vertex  $u \in V$  has a geometric distance with respect to  $s$  denoted  $d(s, u)$ . The eccentricity of vertex  $s$ ,  $\varepsilon_G(s)$ , is the farthest distance from  $s$  to any other vertex. In the rest of the paper we denote  $\varepsilon_G(s)$  by  $D$ .

**Definition 1 (Level).** Let  $G(V, E)$  be a network and  $s$  the source node. For any vertex  $u$  in  $G(V, E)$ , the level of  $u$  is

$$l(u) = d(s, u)$$

i.e., the level of  $u$  is its geometric distance to  $s$ . Let

$$S_i = \{u \mid u \in V, l(u) = i\}$$

denote the set containing all the vertices at level  $i$ .

**Definition 2 (Parents and Sons).** Let  $G(V, E)$  be a network. A vertex  $u$  is parent of vertex  $v$  (a vertex  $v$  is son of vertex  $u$ ) in graph  $G$  with the root source node  $s$ : if

$$l(v) - l(u) = 1 \wedge \{u, v\} \in E$$

Let  $S(u)$  ( $P(v)$ ) be the set of sons (parents) of  $u$  ( $v$ ). If  $v \in S(u)$  ( $u \in P(v)$ ), we say that  $u$  ( $v$ ) has  $v$  ( $u$ ) as son (parent).

*Level-Separable* propriety below defines how to filter nodes in the same level  $i$  into two disjoint subsets.

**Definition 3 (Level-Separable Subsets).** Given  $G(V, E)$  a network and the set  $S_i$  (the set of all vertices in the same level  $i$  of  $G$ ), the level-separable subsets of  $S_i$  are  $S_{i,1}$  and  $S_{i,2}$ , such that

$$S_{i,1} \cap S_{i,2} = \emptyset, S_{i,1} \cup S_{i,2} = S_i$$

There may be many possible pairs of  $S_{i,1}$  and  $S_{i,2}$  for a level  $i$ . Let  $T_i$  be the set of all possible pairs of *Level-Separable Subsets*:

$$T_i = \{(S_{i,1}^{(1)}, S_{i,2}^{(1)}), (S_{i,1}^{(2)}, S_{i,2}^{(2)}), \dots, (S_{i,1}^{(2^x-1)}, S_{i,2}^{(2^x-1)})\}$$

where  $(m)$  on right-top of each pairs represent the index of pairs (the  $m$ th pairs) in  $T_i$ .

**Definition 4 (Multi Parents Set).** Let  $G(V, E)$  be a network and let  $S_i$  contain all vertices at level  $i$ . The *Multi Parents Set*,  $F_i$  for any  $i > 1$ , contains vertices at level  $i$  that have more than one parent at level  $i - 1$ . We define  $F_i$  as:

$$F_i = \{u \mid u \in S_i, l(u) = i \wedge |P(u)| > 1\}$$

For level  $i = 1$ , as all vertices has only one parent, the root,  $F_1 = \emptyset$ .

**Definition 5 (Level-Separable Propriety).** Given an arbitrary graph  $G(V, E)$ , for all level  $i \in [1, D - 1]$ , where  $D$  is the eccentricity of source node,  $G$  verifies the *Level-Separable* propriety, if there exists pairs for every  $T_i$  (the set of all possible pairs of *Level-Separable Subsets* at level  $i$ ),  $(S_{i,1}^{(k)}, S_{i,2}^{(k)})$ , such that:

$$|P(u) \cap S_{i,1}^{(k)}| = 1, \forall u \in F_{i+1}$$

i.e., for every vertex  $u$  at level  $i + 1$  having multi-parents at level  $i$ ,  $u$  has only one parent in  $S_{i,1}$ .

Note that if  $F_{i+1} = \emptyset$ , then  $S_{i,1} = \emptyset$ . When  $S_{i,1}$  is fixed,  $S_{i,2}$  is  $S_i \setminus S_{i,1}$ .

**Definition 6 (Level-Separable Network).** A network  $G(V, E)$  is a *Level-Separable Network*, if its underlay graph verifies the *Level-Separable* propriety.

Note that *Level-Separable Graph* has similar flavor with *Bipartite Graph* [17]. A graph  $G = (V, E)$  is said to be *Bipartite* if and only if there exists a partition  $V = A \cup B$  and  $A \cap B = \emptyset$ . So that all edges share a vertex from both sets  $A$  and  $B$ , and there is no edge containing two vertices in the same set. A bipartite graph separates nodes into two independent sets. In a level-separable network we aim at separating nodes