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Wireless Broadcast with optimal labelling

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***Abstract*—In this paper we study the broadcast problem in wireless networks when the broadcast is helped by a labelling scheme. Our contribution is twofold. *First*, we propose label optimal broadcast algorithms in a class of networks issued from our recent studies in wireless body area networks. We refer to these networks (level separable networks). We focus on two variants of broadcast: broadcast without acknowledgement (i.e. the initiator of the broadcast is not notified at the end of broadcast) and broadcast with acknowledgement. We propose an acknowledgement-free broadcast strategy using 1-bit labelling and broadcast with acknowledgement using a 2-bit labelling. In the class of level-separable networks our algorithms are memory optimal and finish within $2D$ rounds for both broadcast with and without acknowledgement, where D is the eccentricity of the broadcast initiator. In the broadcast with acknowledgement, we trigger the acknowledgement message during the broadcast propagation such that no extra rounds are needed for sending the acknowledgement back to the source once the broadcast is finished. *Second*, we improve a recent [1] labelling-based broadcast scheme with acknowledgement designed for arbitrary networks in terms of memory and time complexity. That is, we better exploit the encoding of labels in order to not use extra memory to carry back the acknowledgement to the source. Nodes store only a 3-bits labelling.**

***Index Terms*—Labelling Scheme, Broadcast, Wireless Networks**

I. Introduction

Broadcast is the most studied communication primitive in networks and distributed systems. Broadcast primitive ensures that once a *source node* (a.k.a. the broadcast initiator) sends message to all other nodes in the network should receive this message in a finite time. Limited by the transmission range, messages may not be able to be sent directly from one node to some other arbitrary node in the network. Therefore relay nodes need to assist the source node during the message propagation by re-propagating it. Deterministic centralized broadcast, where nodes have complete network knowledge has been studied by Kowalski *et al.* in [2]. The authors propose an optimal solution broadcast that completes within $O(Dlog^2n)$ rounds, where n is the number of nodes in network and D is the largest distance from the source to any node of the network. The time lower bound for broadcast, $\Omega(log^2n)$, has been proved in [3] for a family of radius-2 networks. For deterministic distributed broadcast, assuming that nodes only know their IDs (i.e. they do not know the IDs of their neighbors nor the network topology), [4] propose the fastest broadcast within $O(nlogDloglogD)$ rounds, where D is the diameter of network. The lower bound in this case proposed in [5] is $\Omega(nlogD)$, where D is the largest distance from the source to any node of the network.

In wireless networks, when a message is sent from a node it goes into the wireless channel in the form of a wireless signal which can be received by all the nodes within the transmission range of the sender. However, when a node is located in the range of more than one node that send messages simultaneously the multiple wireless signals may generate *collisions* at the receiver. The receiver cannot decode any useful information from the superimposed interference signals. At the MAC layer several solutions have been proposed in the last two decades in order to reduce the collisions. All of them offer probabilistic guarantees. Our study follows the recent work that addresses this problem at the application layer. More specifically, we are interested in deterministic solutions for broadcasting messages based on the use of extra information or advise (also referred as *labelling*) precomputed before the broadcast is invoked.

Labelling schemes have been designed to compute network size, the father-son relationship and the geographic distance between arbitrary nodes in the network (e.g. [6], [7] and [8]). Labelling schemes have been also used in [9] and [10] in order to improve the efficiency of Minimum Spanning Tree or Leader Election algorithms. Furthermore, [11] and [12] exploit labelling in order to improve the existing solutions for network exploration by a robot/agent moving in the network.

Very few works (e.g. [13] and [1]) exploit labelling schemes to efficiently design *broadcast* primitives. When using labelling schemes nodes record less information than in the case of centralized broadcast, where nodes need to know complete network information. Compared with the existing solutions for deterministic distributed broadcast the time complexity is improved. In [13] the authors prove that for an arbitrary network to archive broadcast within constant number of rounds a $O(n)$ bits of advice is sufficient but not $o(n)$. Very recently, a labelling scheme with 2-bits advice (3 bits for broadcast with acknowledgement) is proposed in [1]. The authors prove that their algorithms need $2n-3$ rounds for the broadcast without acknowledgement and $3n-4$ rounds for broadcast with acknowledgement in arbitrary network.

Contribution: Our work is in the line of research described in [13] and [1]. We first introduce a new family of networks, called level-separable networks issued from our recent studies in wireless body area networks [14], [15], [16], [17] and [18]. We then propose an acknowledgement-free broadcast

strategy using 1-bit labelling and a broadcast scheme with acknowledgement using a 2-bit labelling. In the class of level-separable networks our algorithms are memory optimal and finish within $2D$ rounds for both types of broadcast primitives, where D is the eccentricity of the broadcast source. Second, we address the arbitrary networks and improve the broadcast scheme with acknowledgement proposed in [1] in terms of memory and time complexity by efficiently exploiting the 3-bits labelling encoding. Our solution, use no local persistent variables except the 3-bits labelling.

II. Model and Problem Definition

A. Communication Model

We model the network as a *graph* $G = (V, E)$ where V, the set of *vertices*, represents the set of nodes in the network and E, the set of $edges$, is a set of unordered pairs $e = (u, v)$, $u, v \in V$, that represents the communications links between nodes u and v. In the following $d(u)$ denotes the set of neighbors of node u.

We target wireless networks where due to the limitation of the transmission power, a node may not have connections with the other nodes in the network (i.e., $|d(u)| \leq |V| - 1$). However, we assume that the network is connected, i.e., there is a path between any two nodes in the network.

We assume that nodes execute the same algorithm and are *time synchronized*. The system execution is decomposed in $rounds$. When a node u sends a message at round x, all nodes in $d(u)$ receive the message at the end of round x. Collisions occur at node u in round x if a set of nodes, $M \subseteq d(u)$ and $|M| > 1$, send a message in round $(x - 1)$. In that case is considered that u has not received any message successfully.

In the following we are interested in solving the *Broadcast problem*: when a source node sends a message it should be received by all the nodes in the network in finite bounded time.

B. Level-Separable Network

In this section, we present a family of networks we called *Level-Separable Network*. We say an arbitrary network is a Level-Separable Network if the underlay communication graph $G = (V, E)$ of the network verifies the *Level-Separable* propriety defined below.

To define the *Level-Separable* propriety, we introduce some preliminary notations.

Let $G(V, E)$ be a network and let s, a predefined vertex $s \in V$, be the source node of the broadcast. Each vertex $u \in V$ has a geometric distance with respect to s denoted $d(s, u)$. $\varepsilon_G(s)$ is the eccentricity of vertex s, that is the farthest distance from it to any other vertex. In the rest of the paper we denote $\varepsilon_G(s)$ by D.

Definition 1 (Level)**.** *Let $G(V, E)$ be a network. For any vertex u in $G(V, E)$, where s is the source node, the $level$ of the node u is*

$$l(u) = d(s, u)$$

i.e., the level of u is its geometric distance to s. Let

$$S_i = \{u \mid u \in V,\ l(u) = i\}$$

denote the set containing all the vertices at level i.

Definition 2 (Parents and Sons)**.** *Let $G(V, E)$ be a network. A vertex u is parent of vertex v (a vertex v is son of vertex u) in graph G with the root source node s: if*

$$l(v) - l(u) = 1 \ \wedge \{u, v\} \in E$$

Let $S(u)$ ($P(v)$) be the set of sons (parents) of u (v). If $v \in S(u)$ ($u \in P(v)$), we say that u (v) has v (u) as son (parent).

Level-Separable propriety focus on how to separate nodes in the same level i into two disjoint subsets of S_i. We therefore define *Level-Separable Subsets* below.

Definition 3 (Level-Separable Subsets)**.** *Given $G(V, E)$ a network and the set S_i (the set of all vertices in the same level i of G), the level-separable subsets of S_i are $S_{i,1}$ and $S_{i,2}$, such that*

$$S_{i,1} \cap S_{i,2} = \varnothing,\ S_{i,1} \cup S_{i,2} = S_i$$

There may be many possible pairs of $S_{i,1}$ and $S_{i,2}$ for a level i. Let T_i be the set of all possible pairs of *Level-Separable Subsets*:

$$T_i = \{(S_{i,1}^{(1)}, S_{i,2}^{(1)}), (S_{i,1}^{(2)}, S_{i,2}^{(2)}), ..., (S_{i,1}^{(2^x-1)}, S_{i,2}^{(2^x-1)})\}$$

where (m) represent the index of pairs in T_i.

Definition 4 (Multi Parents Set)**.** *Let $G(V, E)$ be network and let $l_1 > 0$ and $l_2 > 0$ two successive levels in $G(V, E)$ ($l_2 - l_1 = 1$) and let the sets S_{l_1}, S_{l_2} be such that S_{l_1} contains all parents of all vertices in S_{l_2}, and S_{l_2} contains all sons of all vertices in S_{l_1}. The Multi Parents Set F_{l_2} at level l_2 is a subset of S_{l_2}. It contains all the vertices at level l_2, who have more than one parents in level l_1. For any level $i > 0$, we define F_i as:*

$$F_i = \{u \mid u \in S_i,\ l(u) = i \wedge |P(u)| > 1\}$$

Definition 5 (Level-Separable Propriety)**.** *Given an arbitrary graph $G(V, E)$, for all level $i \in [1, D - 1]$, where D is the eccentricity of source node, G verifies the Level-Separable property, if there exists pairs for every T_i (the set of all possible pairs of* Level-Separable Subsets *at level i), $(S_{i,1}^{(k)}, S_{i,2}^{(k)})$, such that:*

$$|P(u) \cap S_{i,1}^{(k)}| = 1,\ \forall u \in F_{i+1}$$

i.e., for every vertex u at level $i + 1$ having multi-parents at level i, u has only one parent in $S_{i,1}$.

Note that if $F_{i+1} = \varnothing$, then $S_{i,1} = \varnothing$. When $S_{i,1}$ is fixed, $S_{i,2}$ is $S_i \setminus S_{i,1}$.

Definition 6 (Level-Separable Network)**.** *A network $G(V, E)$ is a Level-Separable Network, if its underlay graph verifies the Level-Separable property.*

Note that *Level-Separable Graph* has similar flavor with *Bipartite Graph* [19]. A graph $G = (V, E)$ is said to be