

# BIAXIAL COMPRESSION TESTS ON HOPKINSON BARS

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Pierre QUILLERY**

*LMT-Cachan, Mechanics & Material team, Dynamics unit*

# Layout:

- **Industrial context**
- **Hopkinson set-up working**
- **Experimental results**
- **Conclusions and prospects**

Multiaxial dynamic loadings occur in many industrial cases.

For instance, automotive crashes:



crash-box crushing (shock absorber)

foam submitted to a complex stress state

[Durrenberger 2007]

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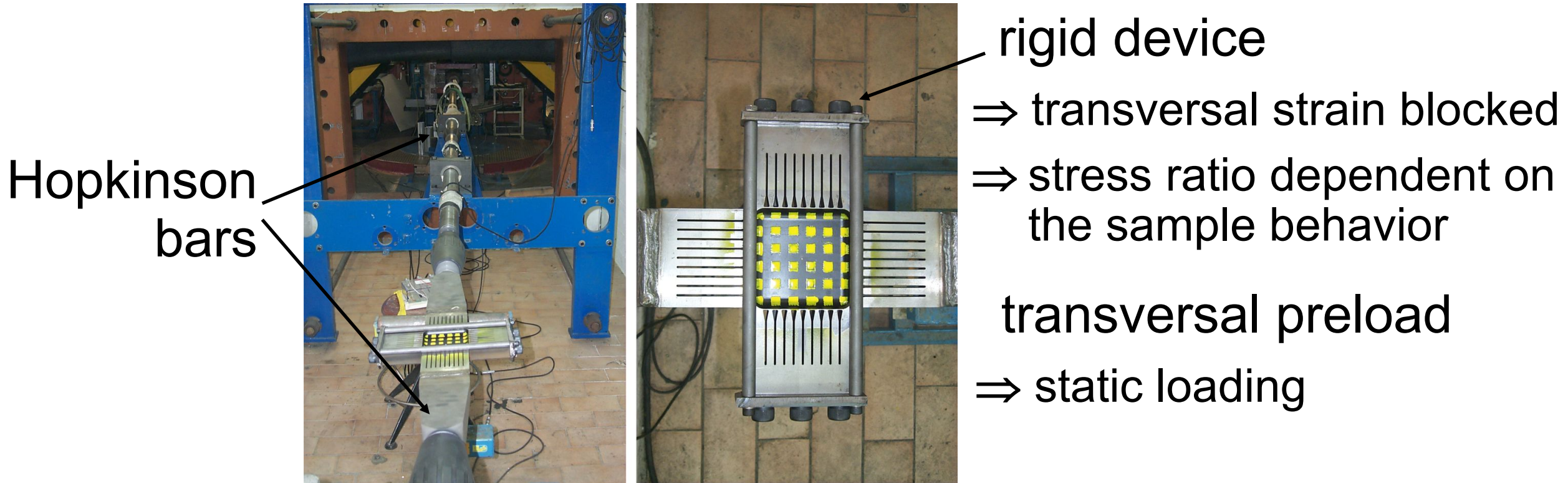
foam submitted to a complex stress state

[Durrenberger 2007]

Hopkinson bars: most convenient dynamic set-up (impact of a striker)

Problem: How to obtain a multiaxial loading from an initial uniaxial loading?

# Overview focused on dynamic biaxial compression set-ups:



[Albertini 2014]

Overview focused on dynamic biaxial compression set-ups:  
Perpendicular Hopkinson bar devices:



[Hummeltenberg 2012]

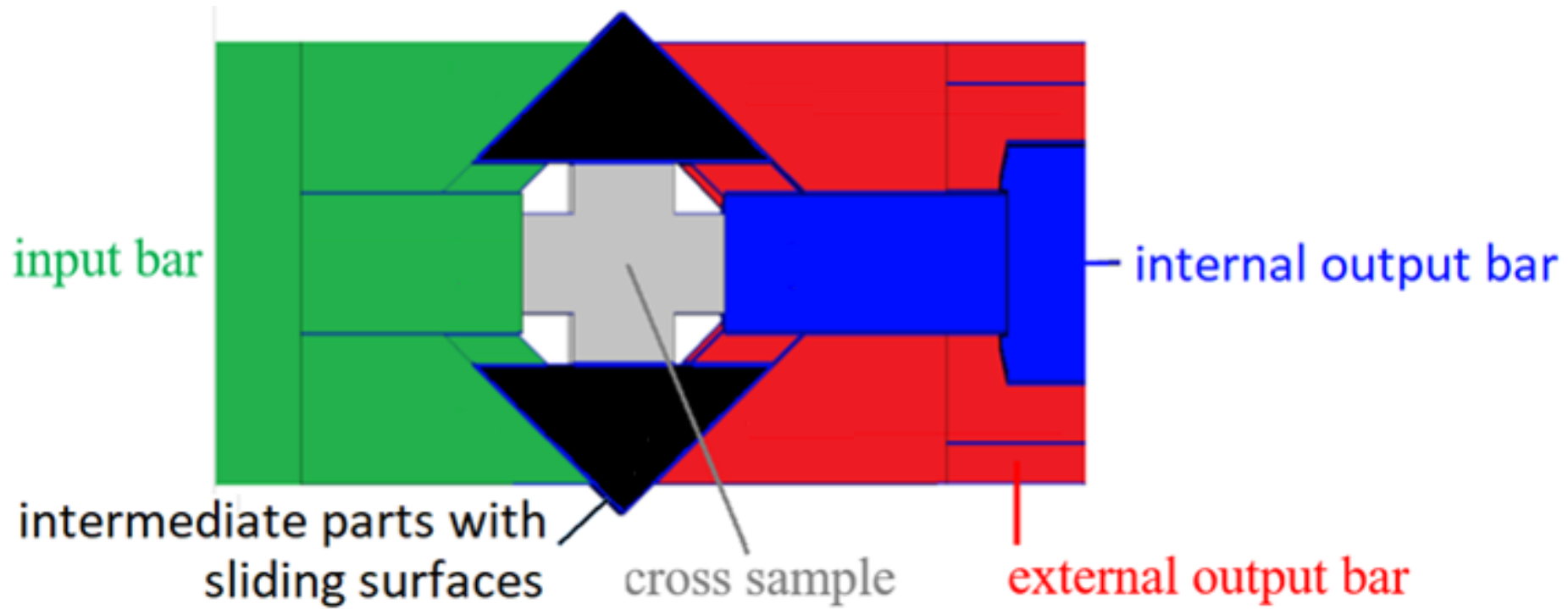
accuracy of a few tenth of ms on the impacts instants needed  
⇔ accuracy of a few mm on the strikers positions needed

# Layout:

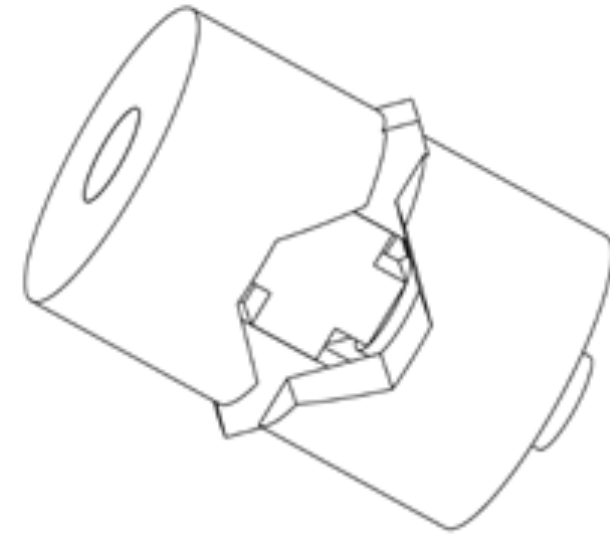
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Cut view:

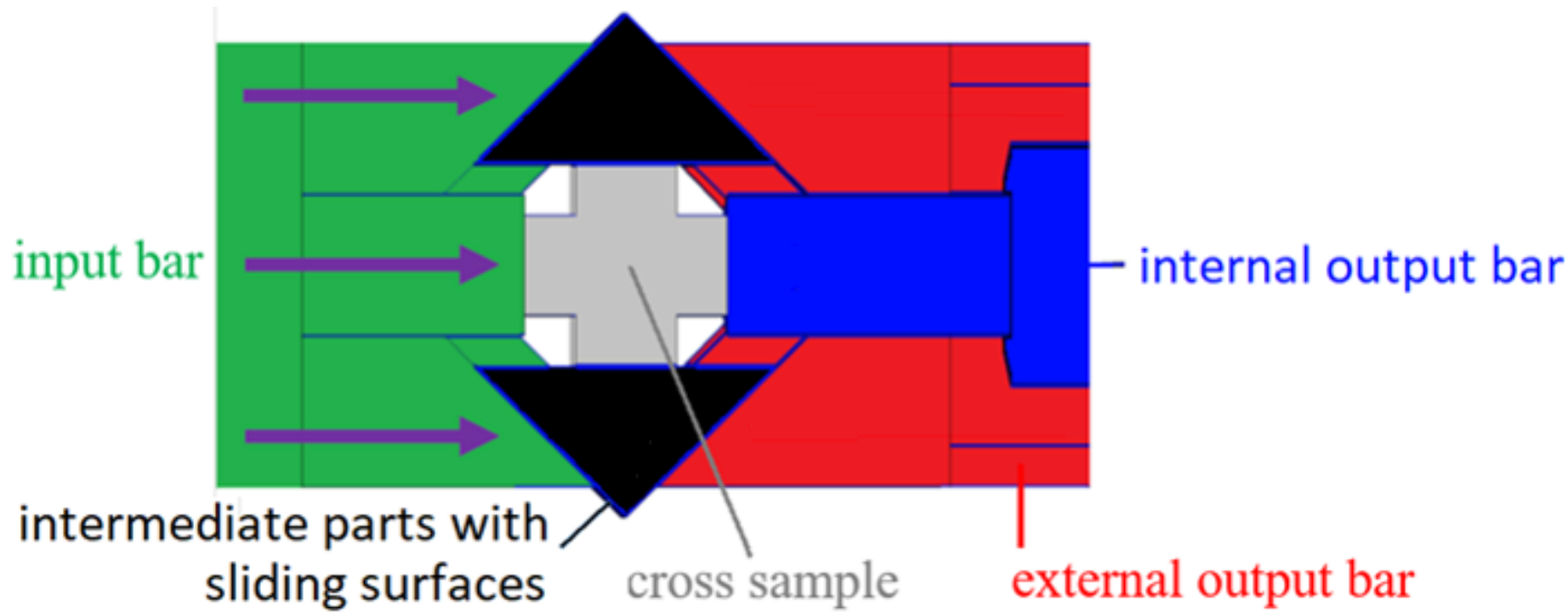


3D view:

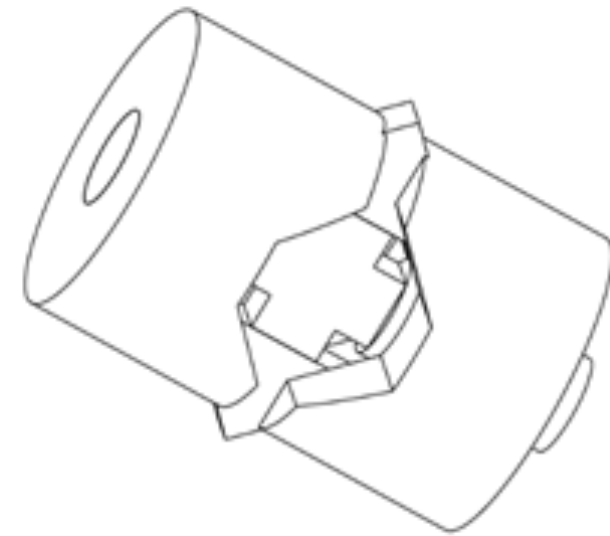




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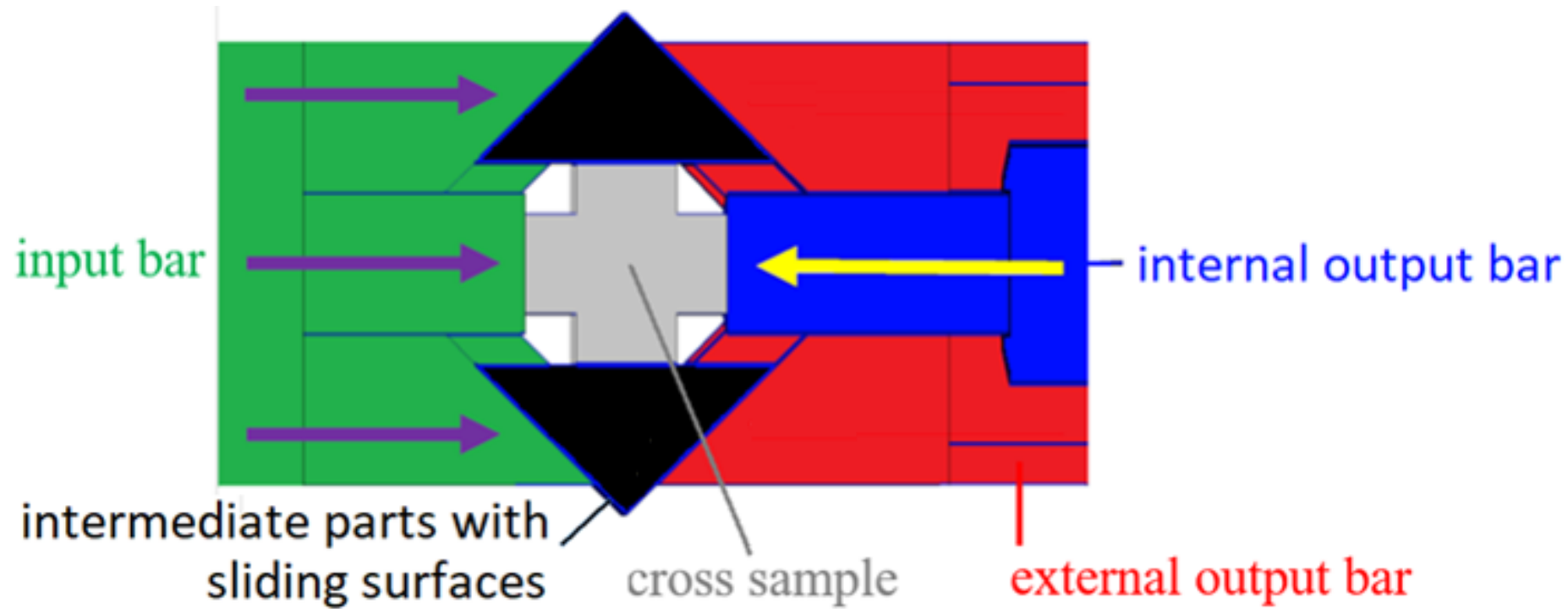


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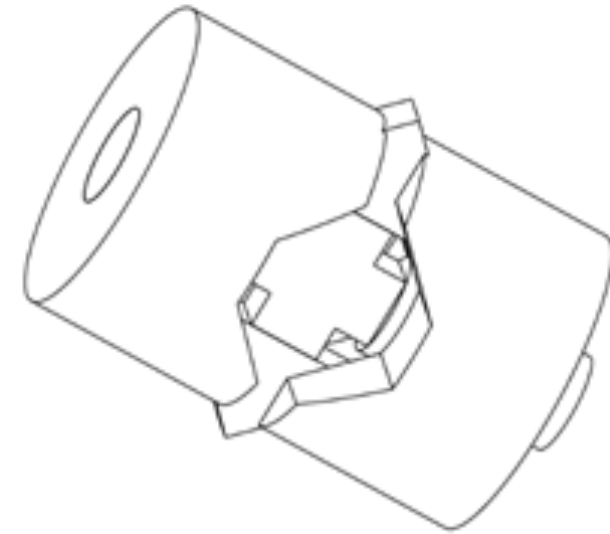


single striker + single input bar  $\Rightarrow$  no problem due to shifting

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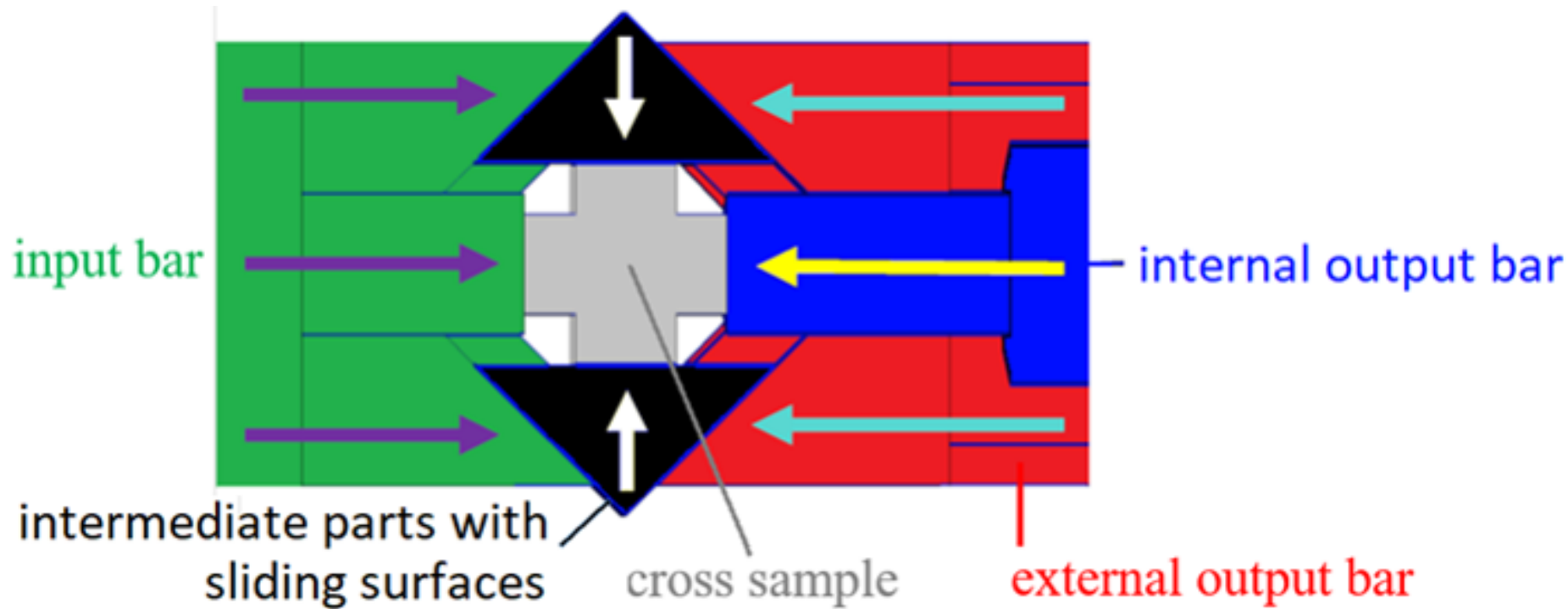


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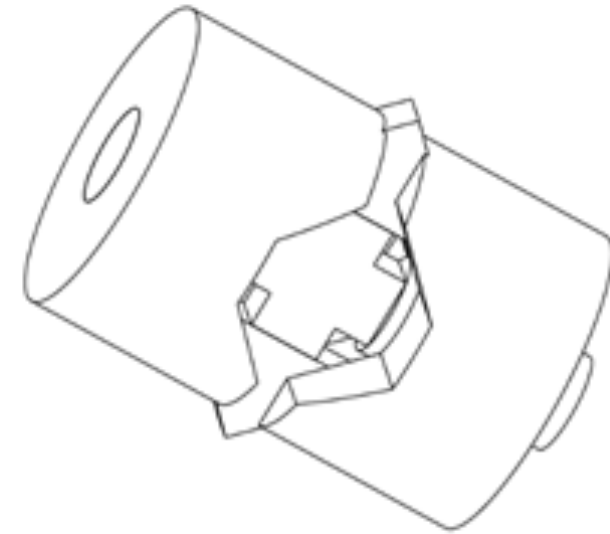


single striker + single input bar  $\Rightarrow$  no problem due to shifting  
internal output bar inertia (impedance)  $\Rightarrow$  sample axial loading

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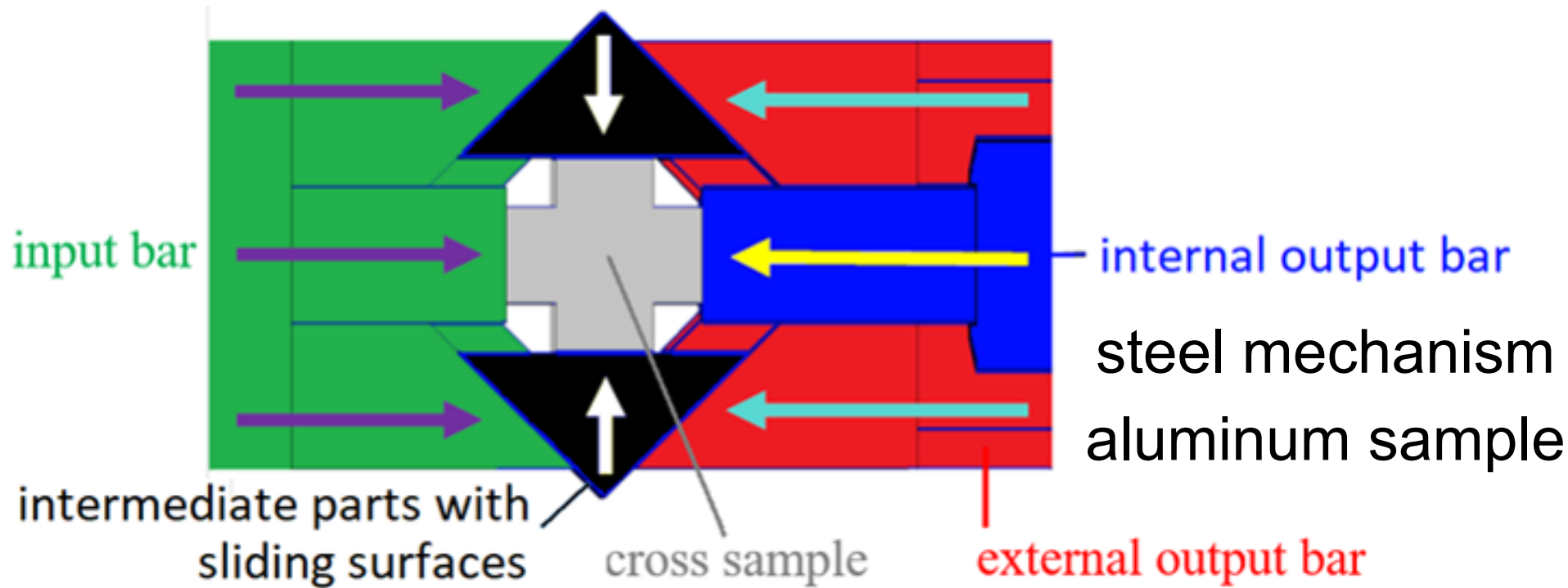


3D view:

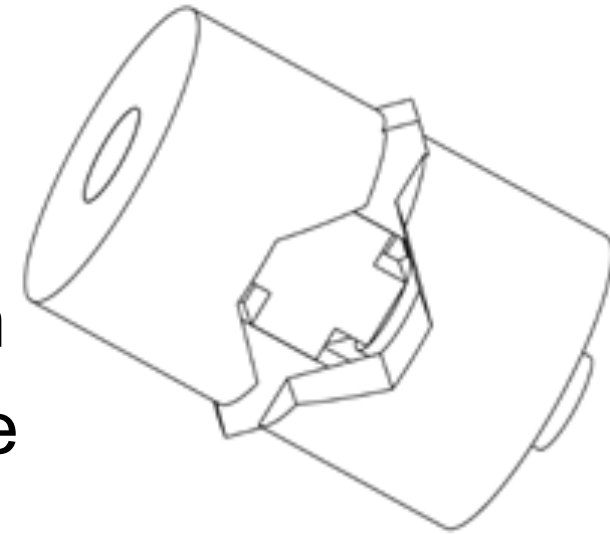


single striker + single input bar  $\Rightarrow$  no problem due to shifting  
internal output bar inertia (impedance)  $\Rightarrow$  sample axial loading  
external output bar inertia (impedance)  $\Rightarrow$  sample transversal loading

Cut view:



3D view:



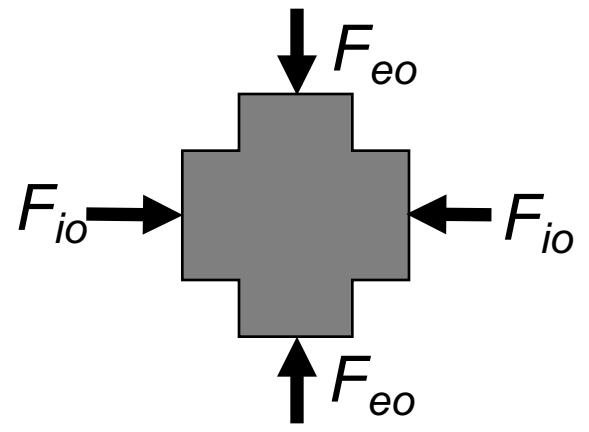
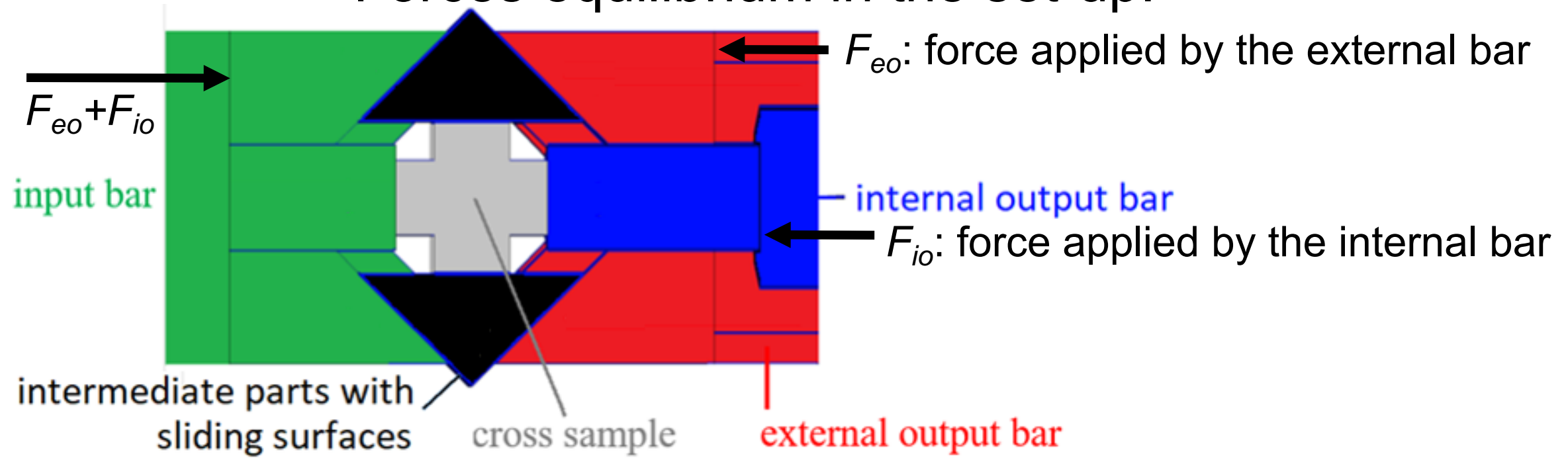
external diameters: 22 mm

internal diameter of the external output bar: 18 mm

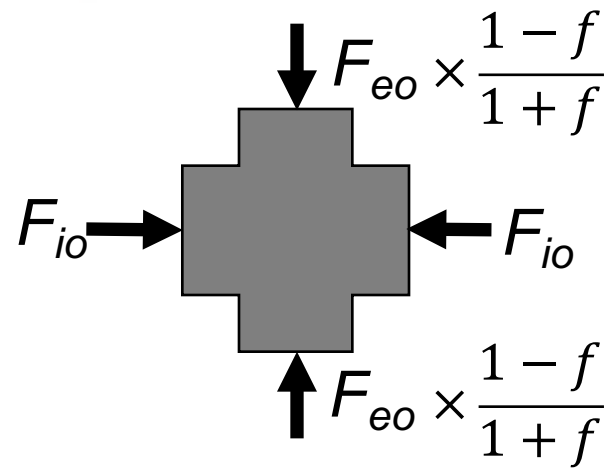
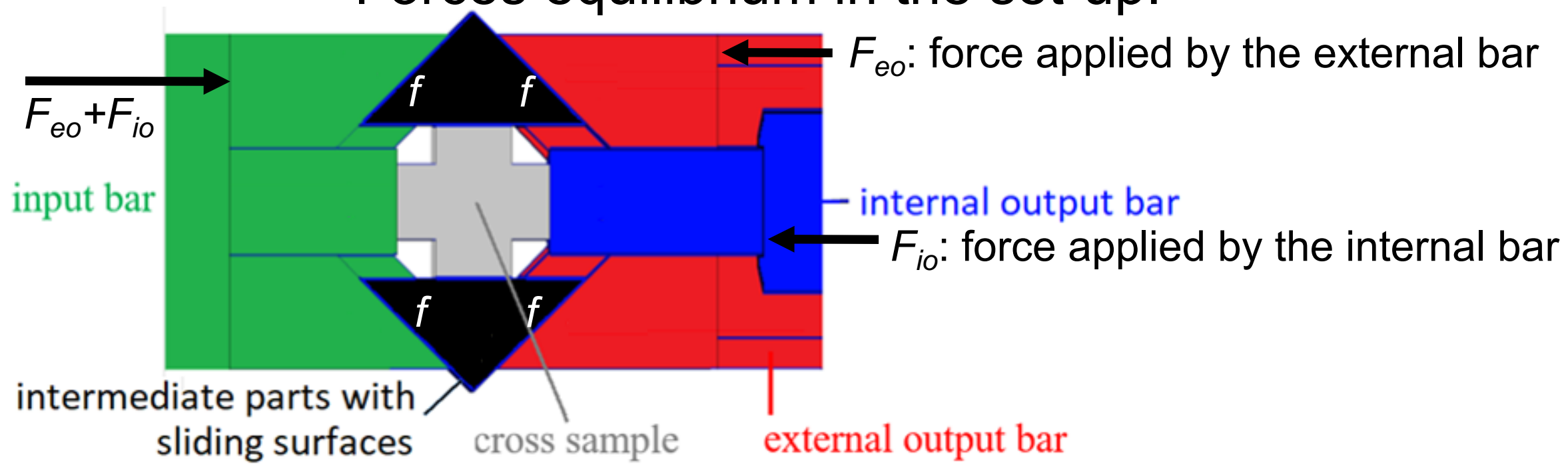
diameter of the internal output bar: 12 mm

sample size (boundary to boundary): 10 mm × 10 mm, thickness: 5 mm

### Forces equilibrium in the set-up:



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by taking account of the friction  
 $f$ : friction coefficient

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## Measurements:



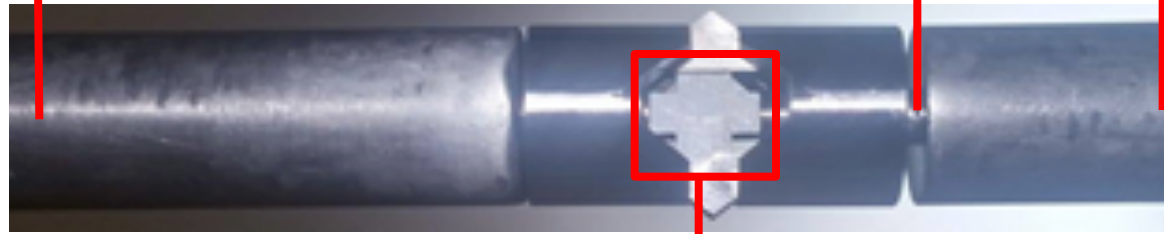
strain gauges glued on the 3 bars  
frequency: 500 kHz



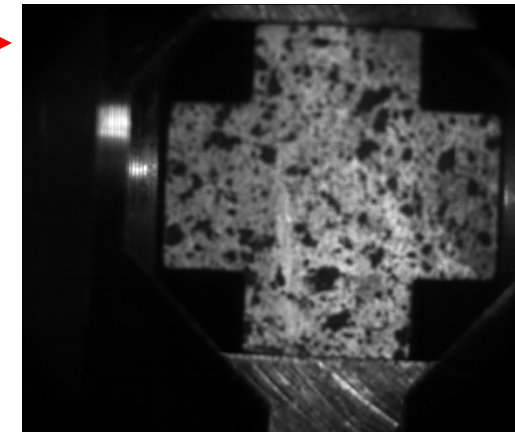
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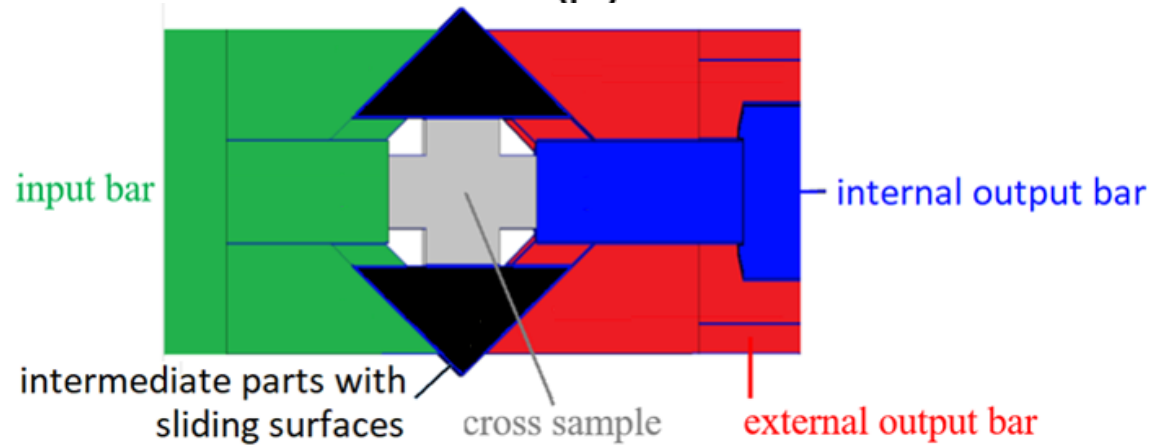
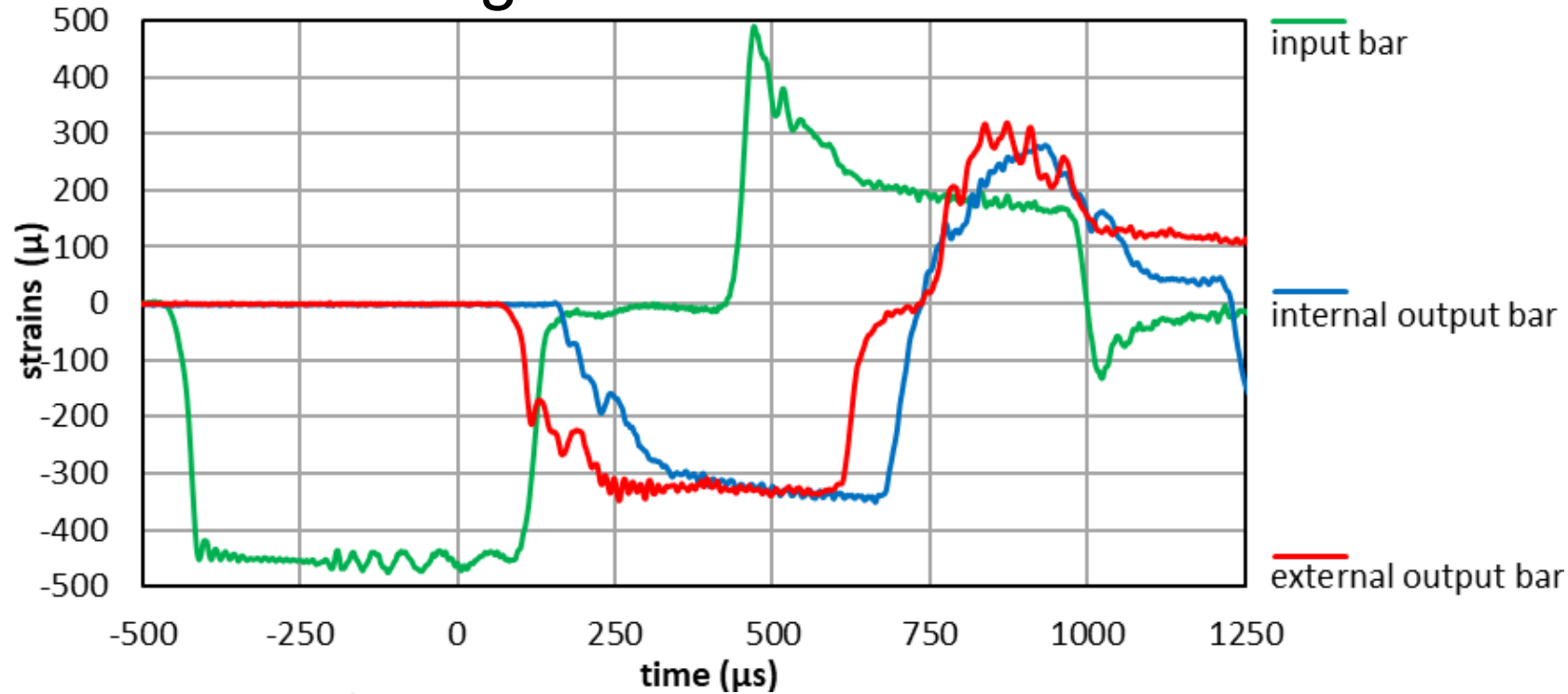
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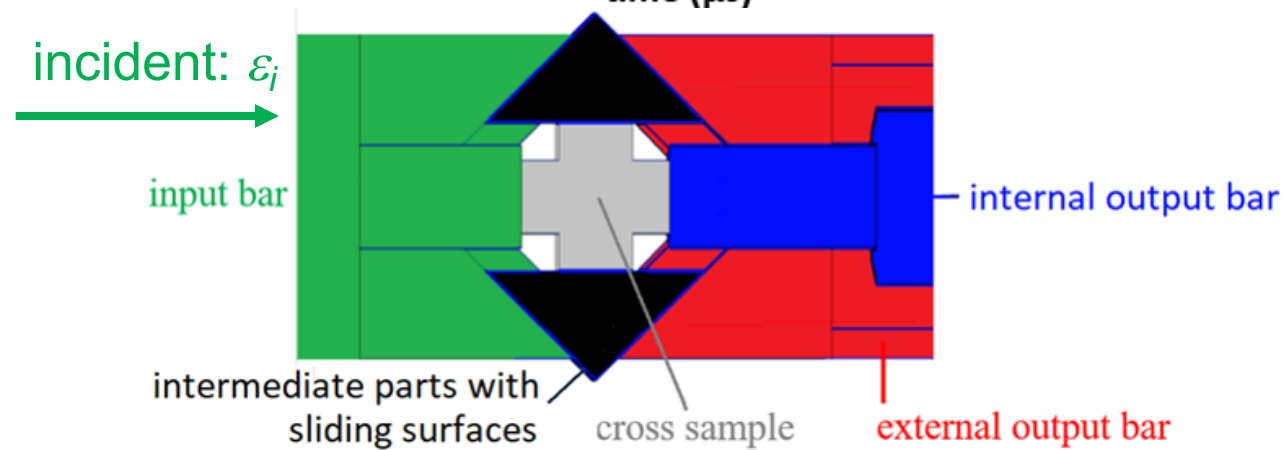
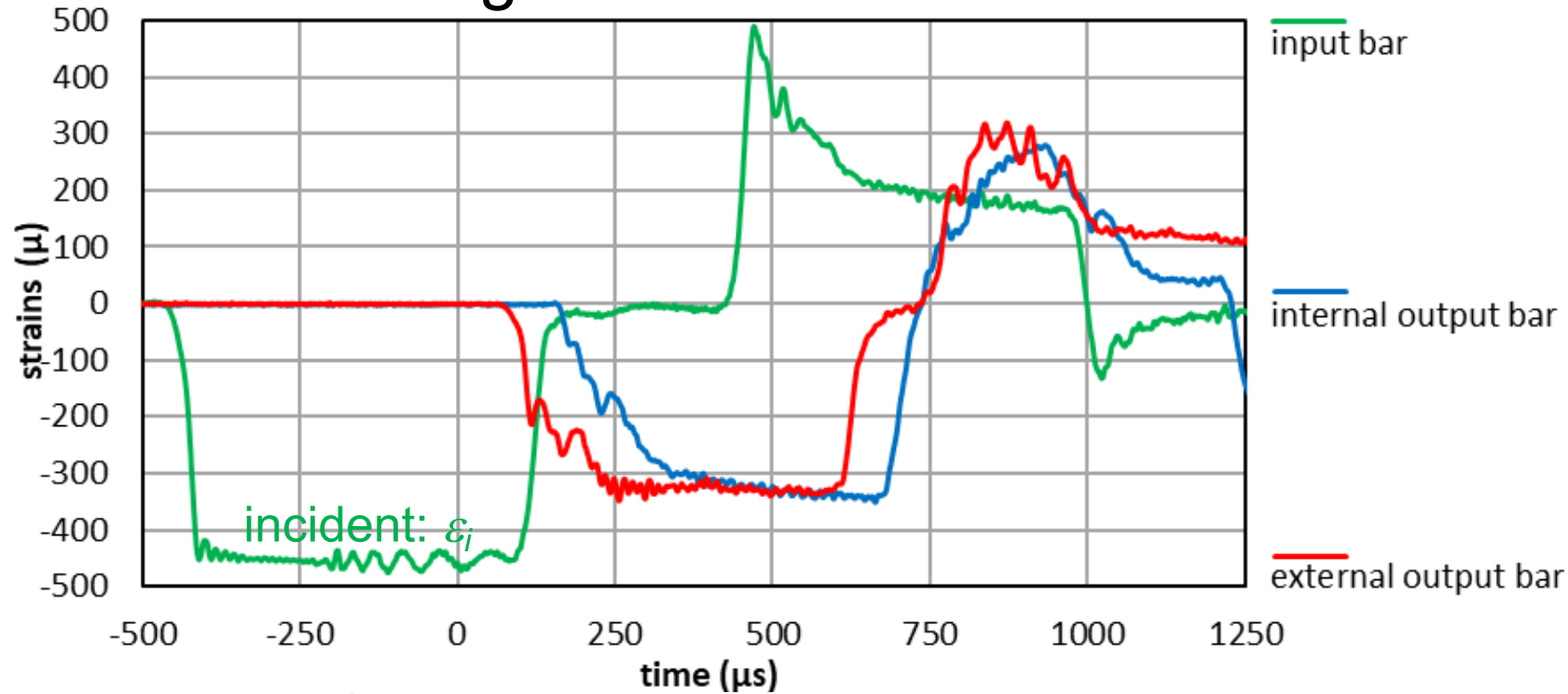
speckled sample observed by a camera  
frequency: 15 kHz  
resolution: 768 pixels  $\times$  648 pixels



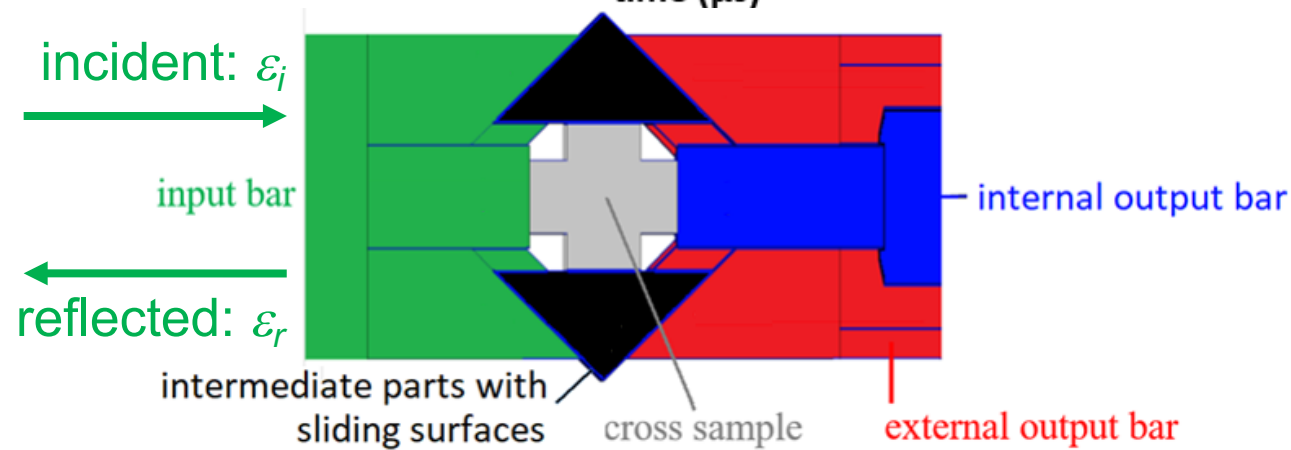
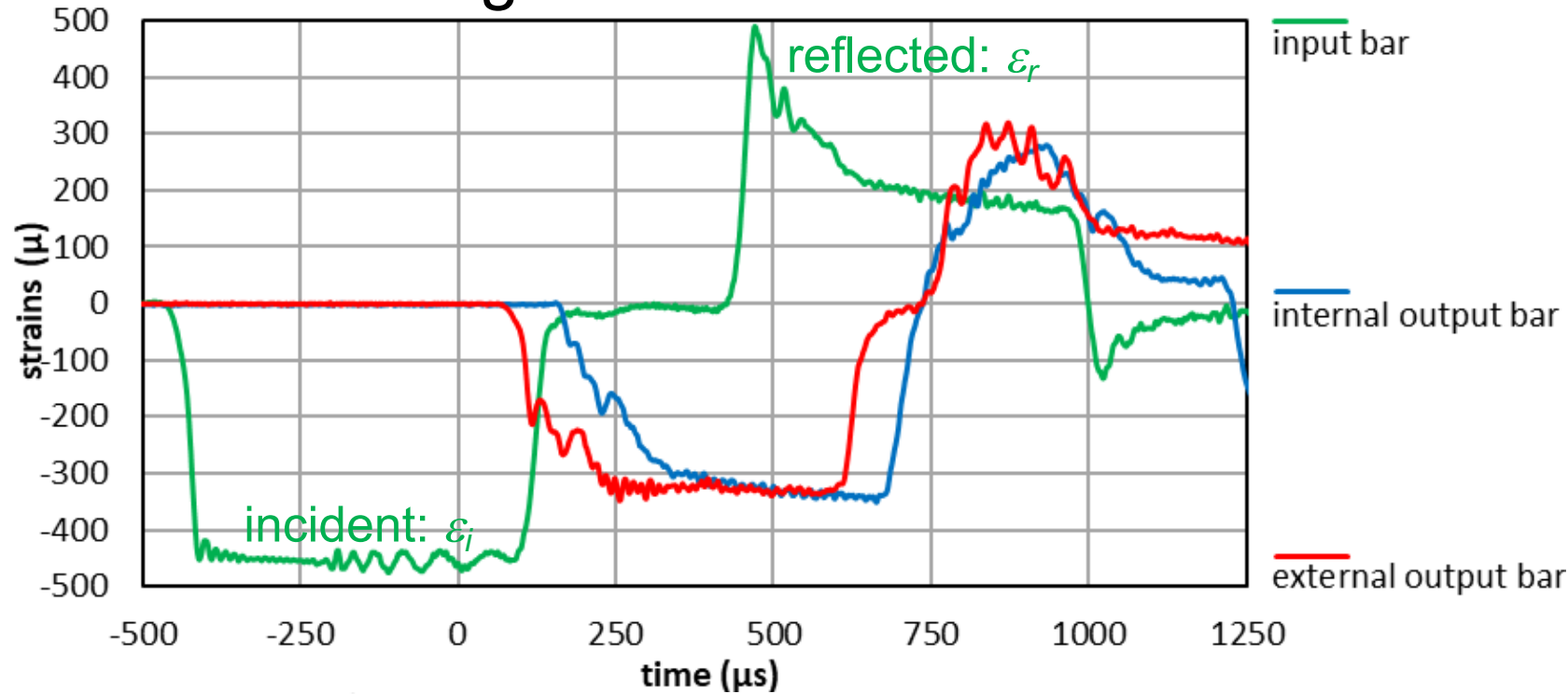
## Gauges raw measurements:



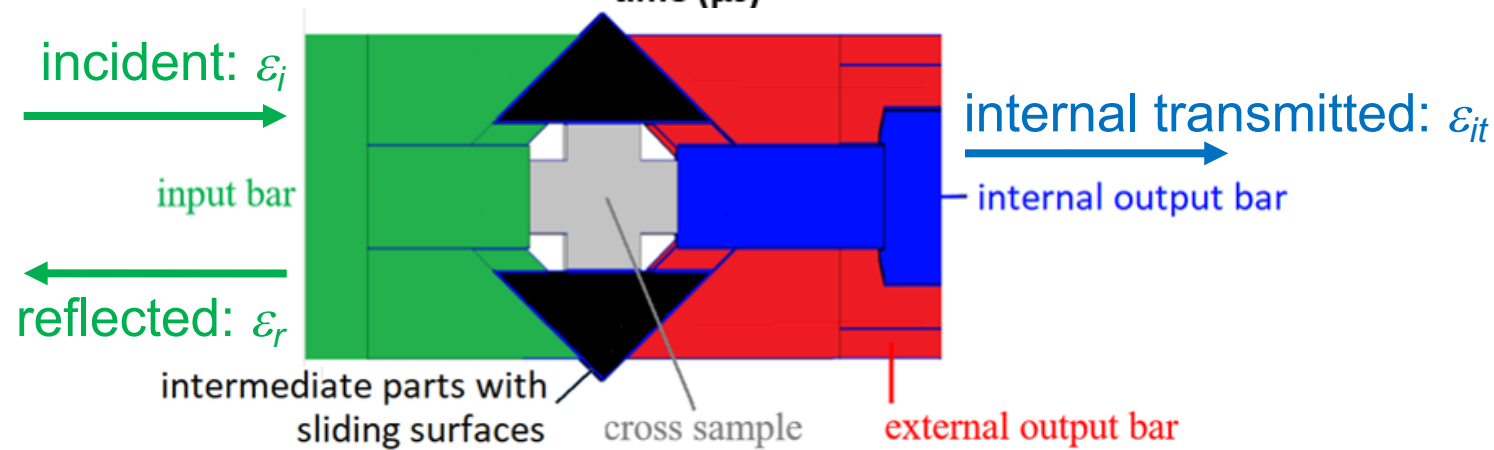
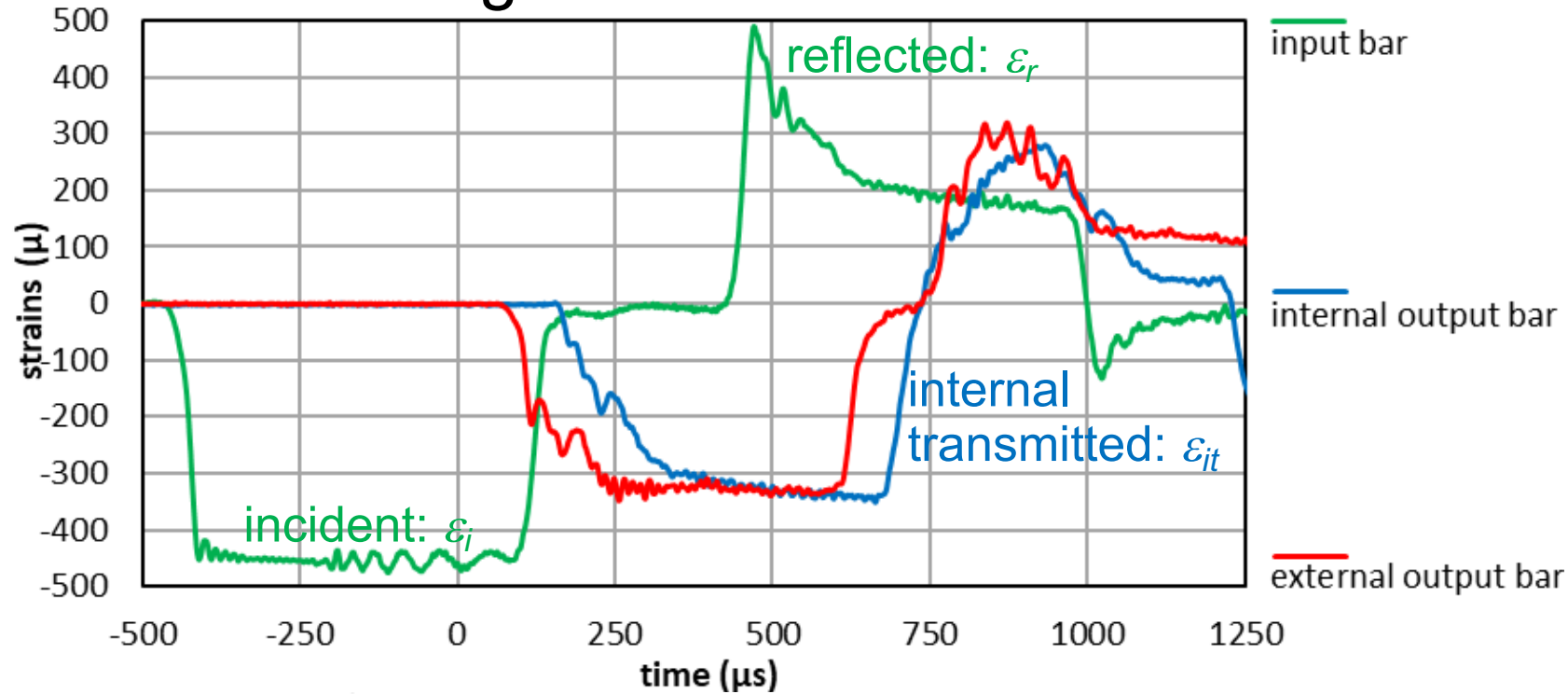
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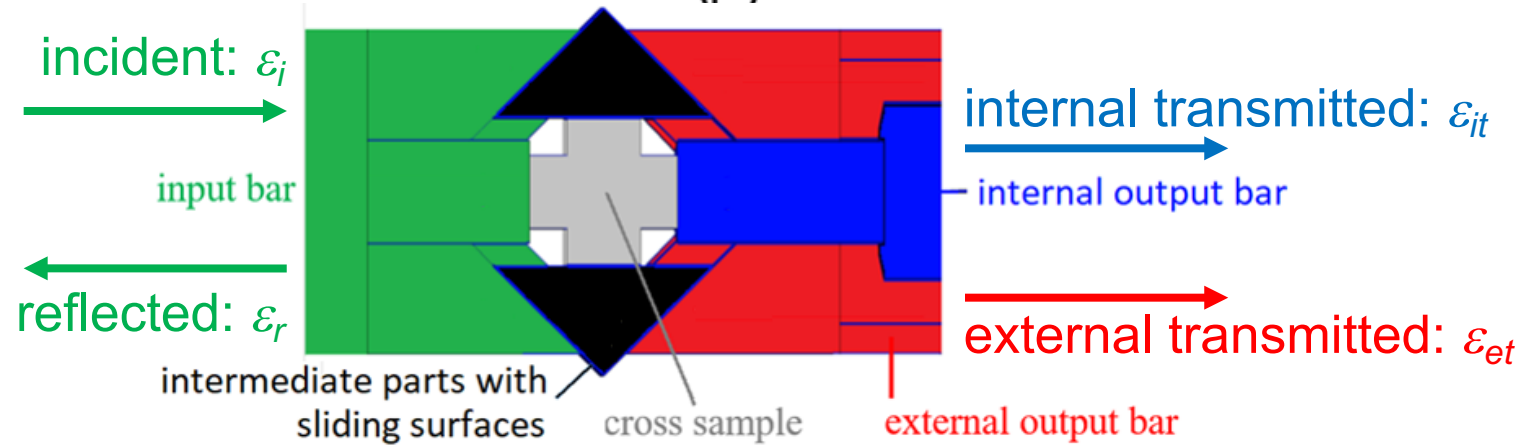
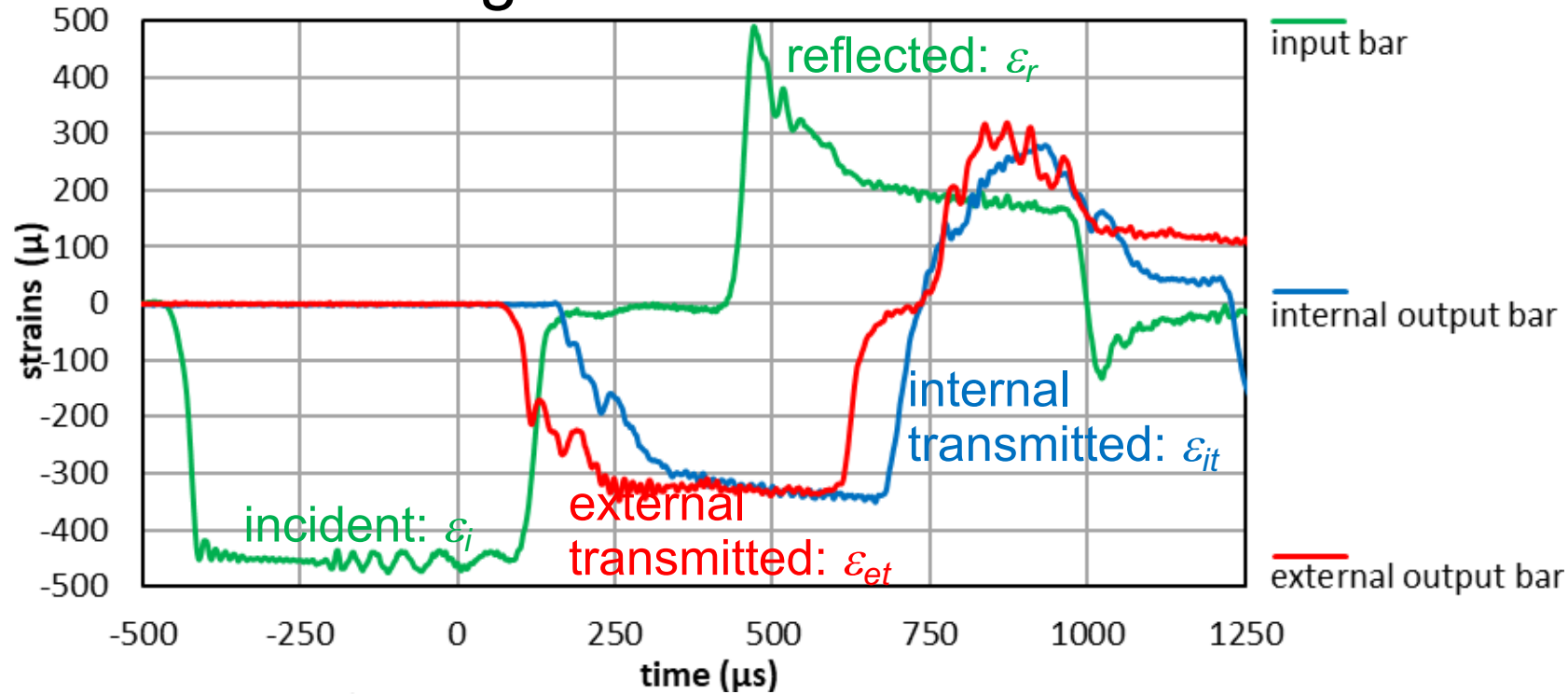
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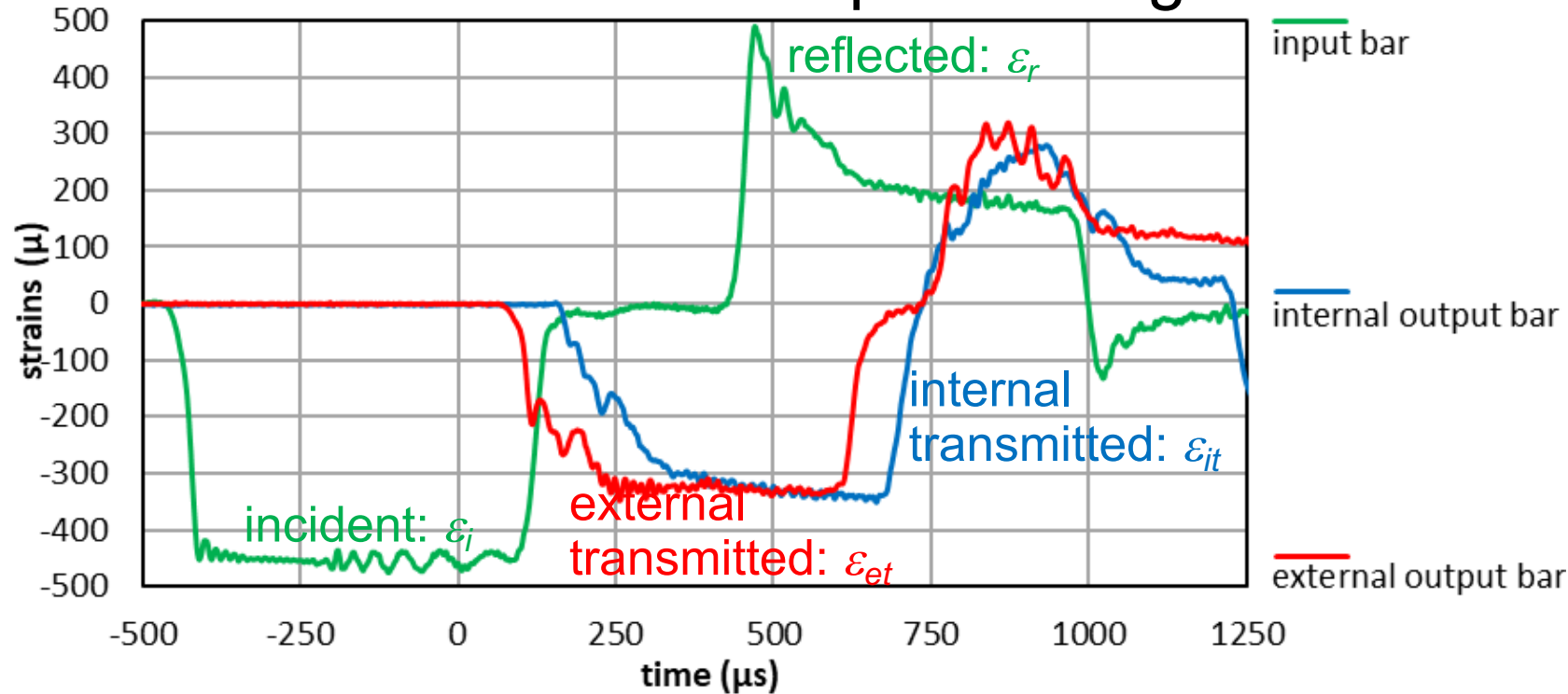


## Gauges raw measurements:





## Measurements processing:



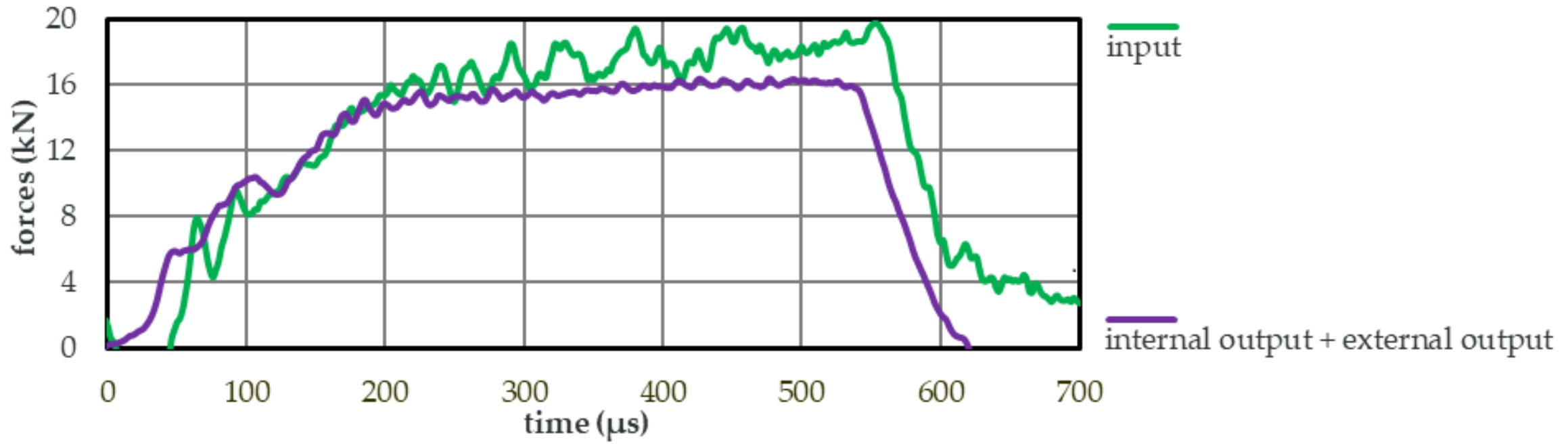
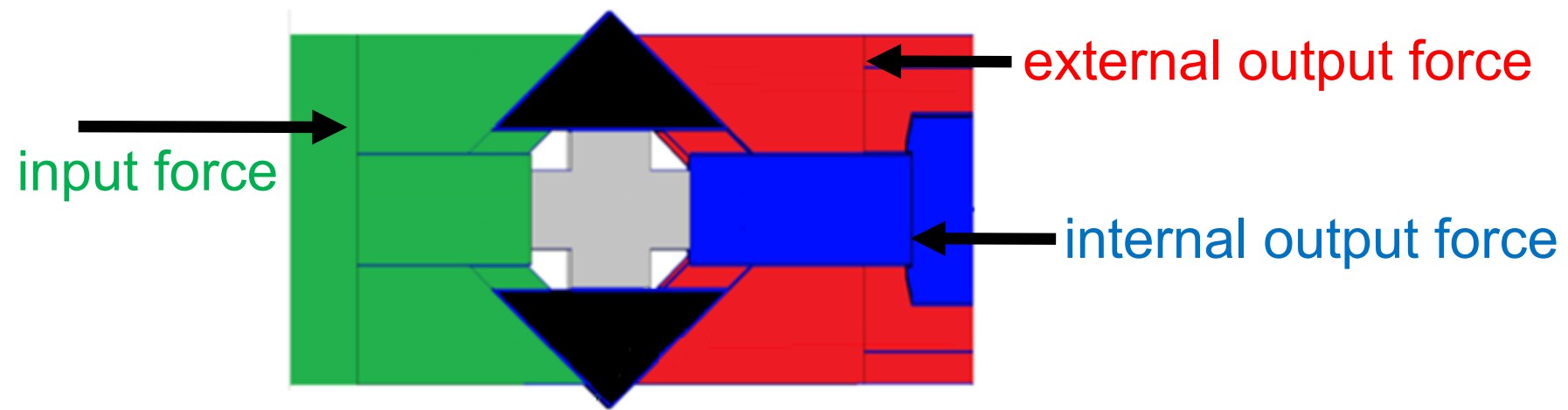
wave transportation (time shifting of  $\varepsilon_i$ ,  $\varepsilon_r$ ,  $\varepsilon_{it}$ ,  $\varepsilon_{et}$ ) + Hopkinson formulae

⇒ forces and velocities at the bars interfaces

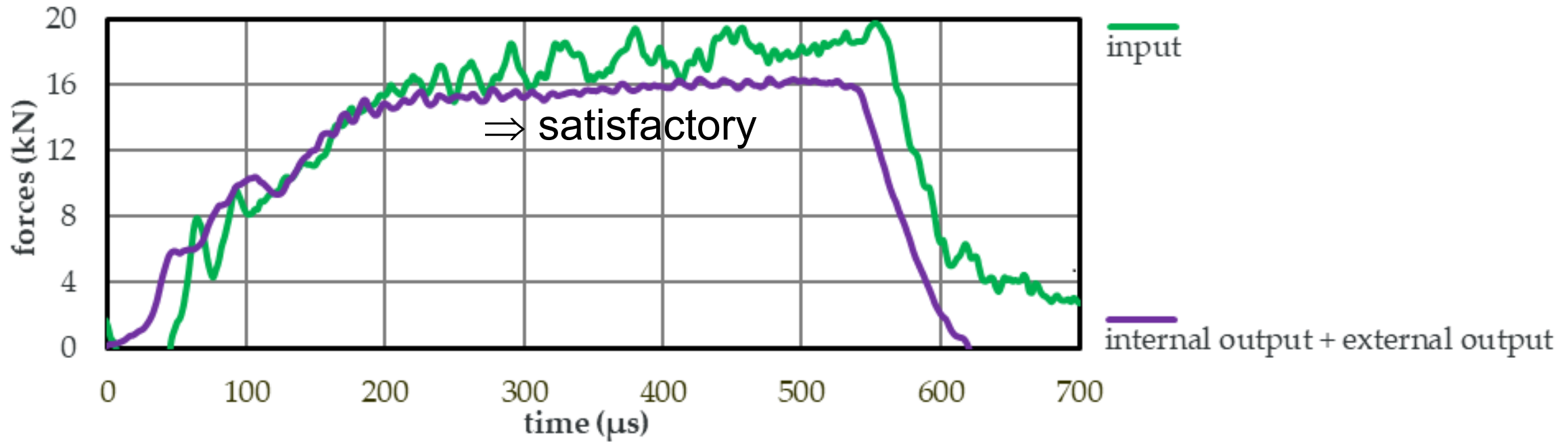
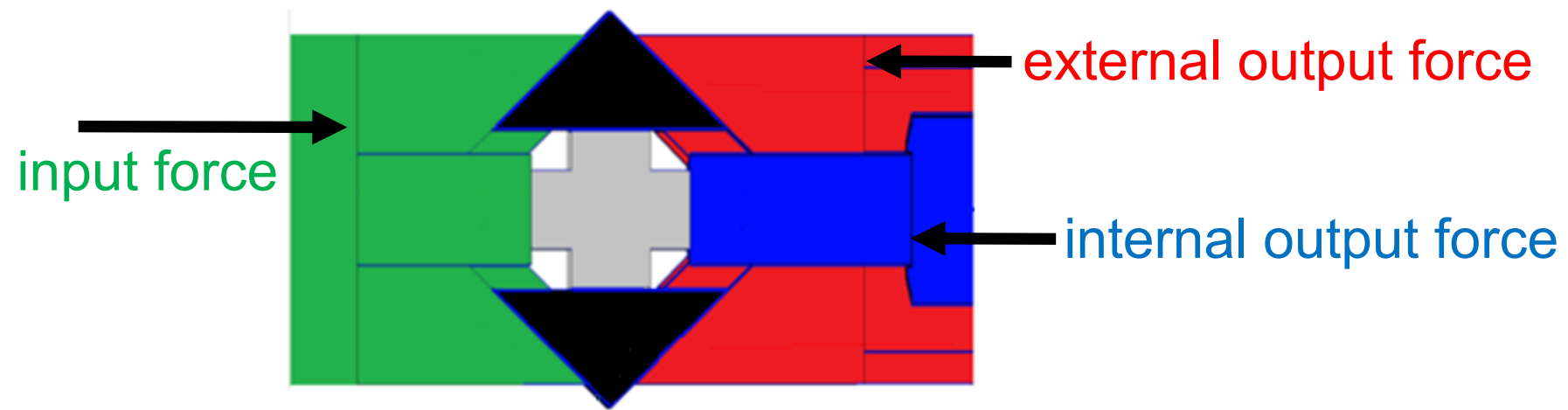
velocities at the bars interfaces + numerical integration

⇒ displacements at the bars interfaces

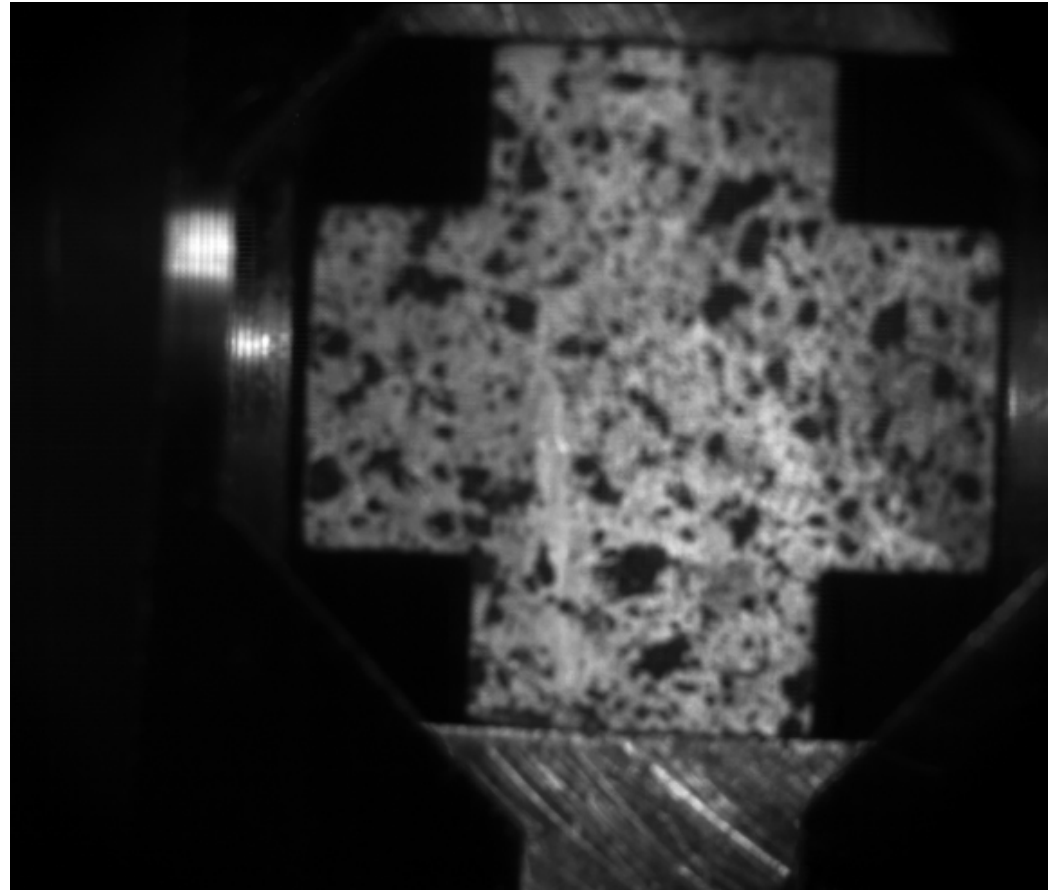
### Forces equilibrium:



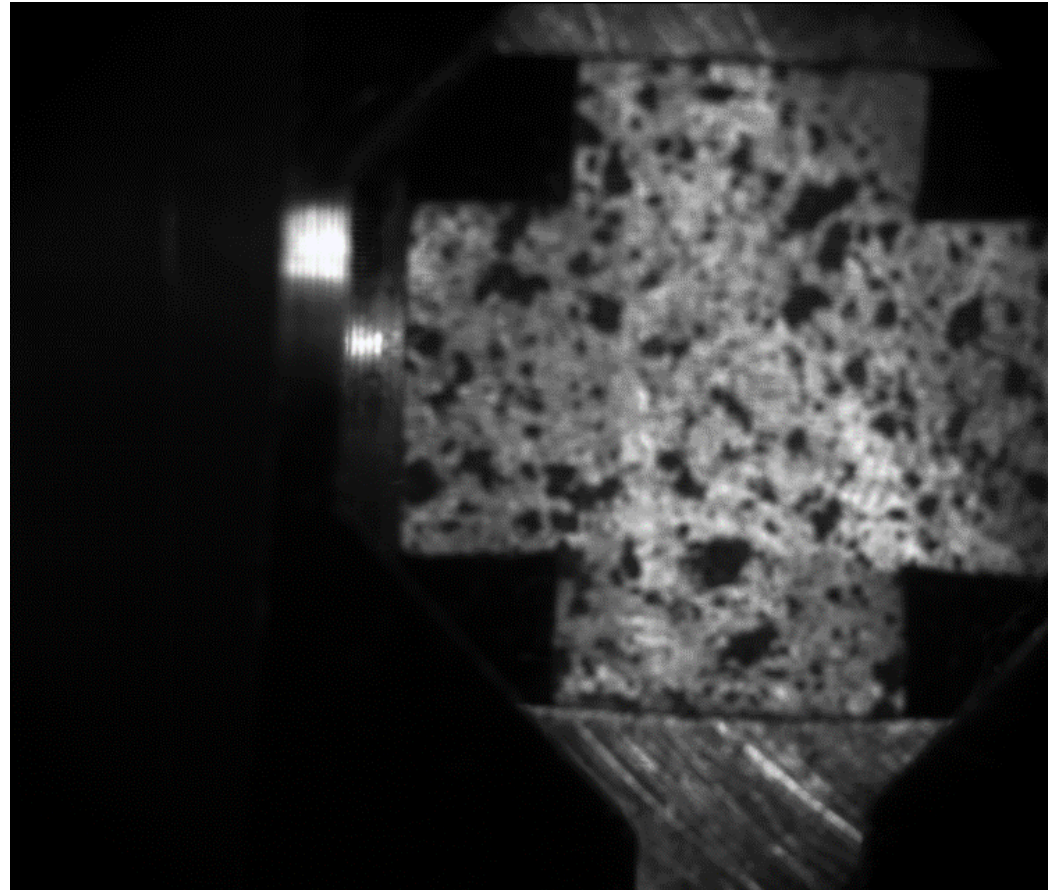
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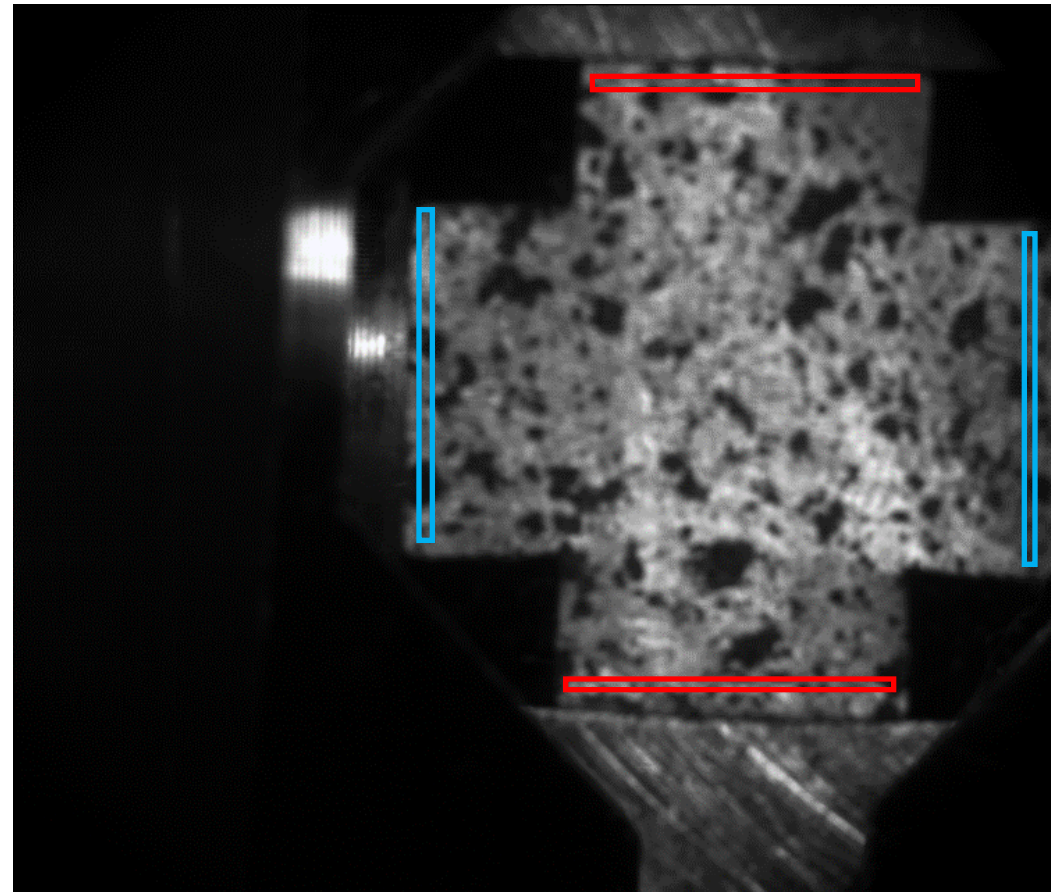
## Digital Image Correlation (Correli RT3):



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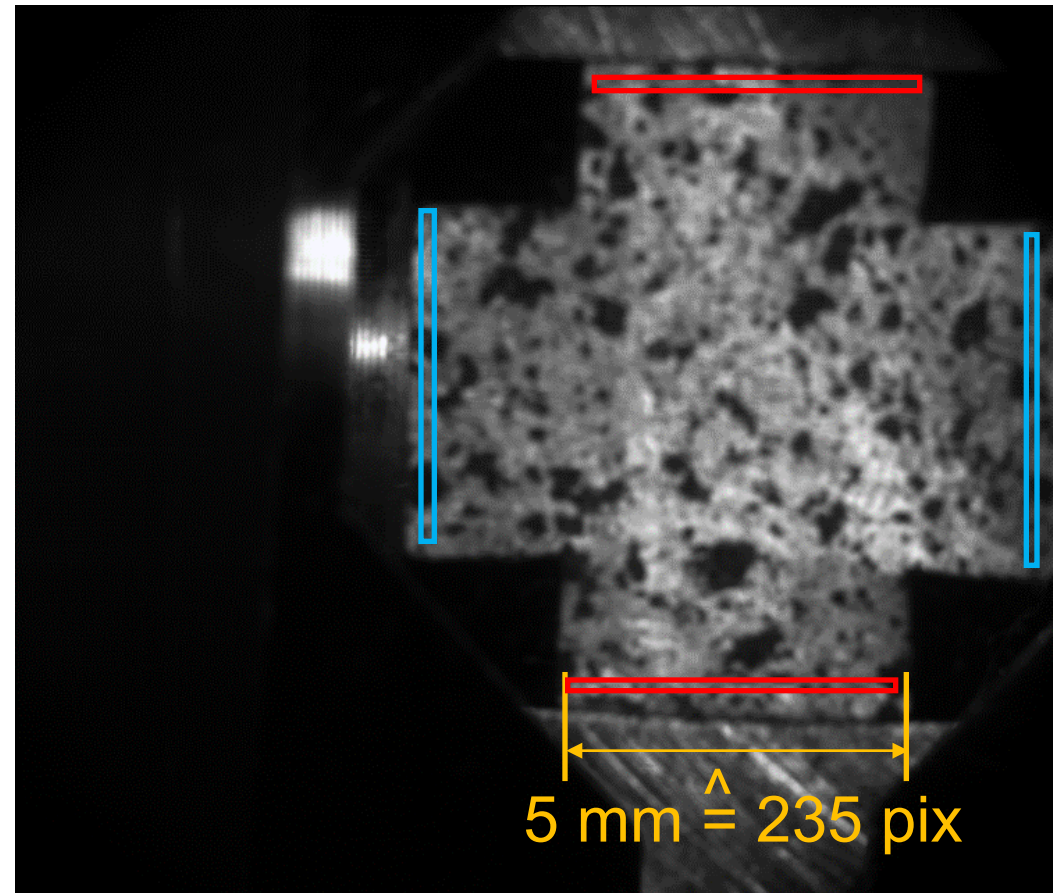
Axial displacement

Transversal displacement

⇒ displacement field calculation in **pixels**



## Digital Image Correlation (Correli RT3):



Axial displacement

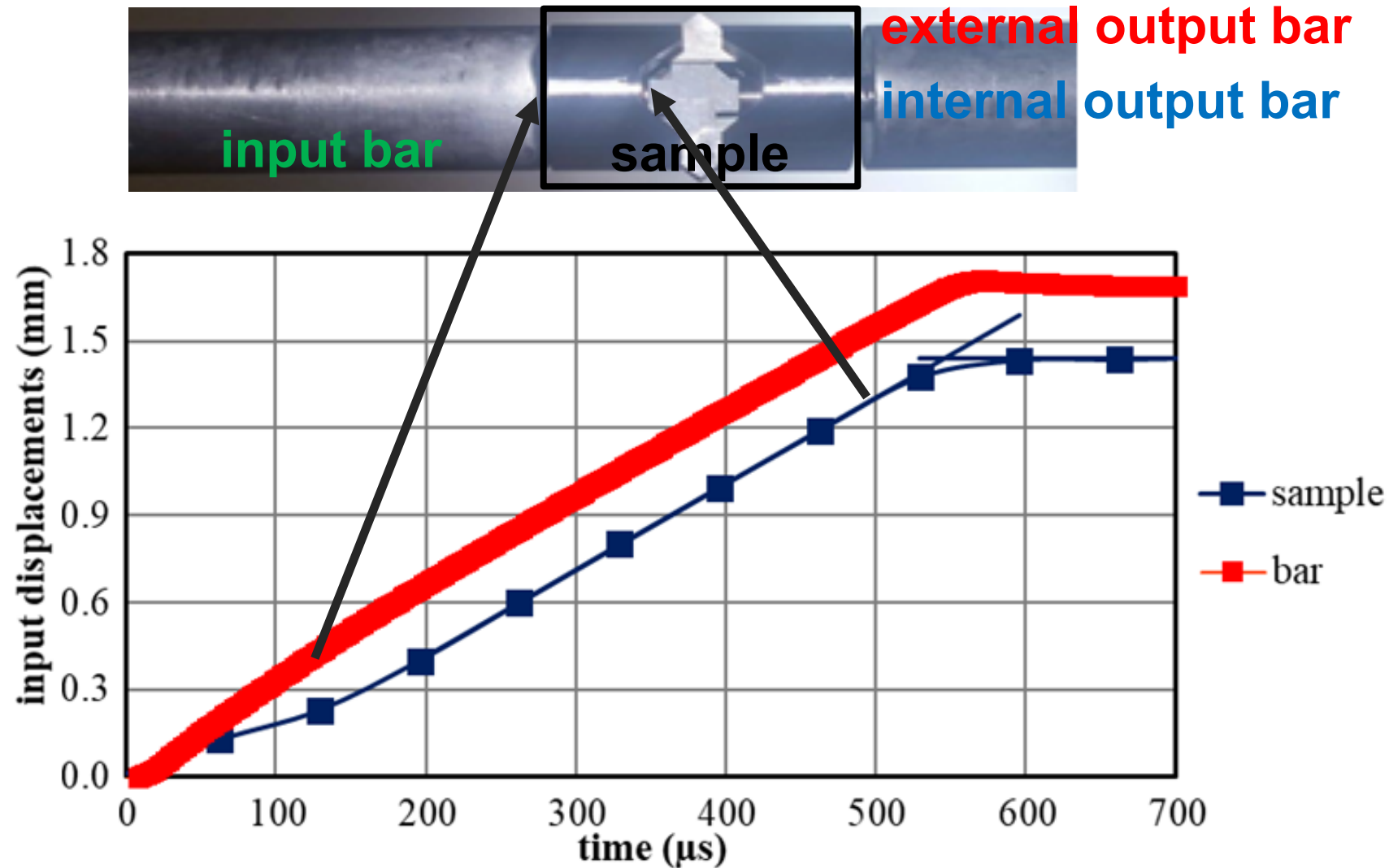
Transversal displacement

⇒ displacement field calculation in **pixels**

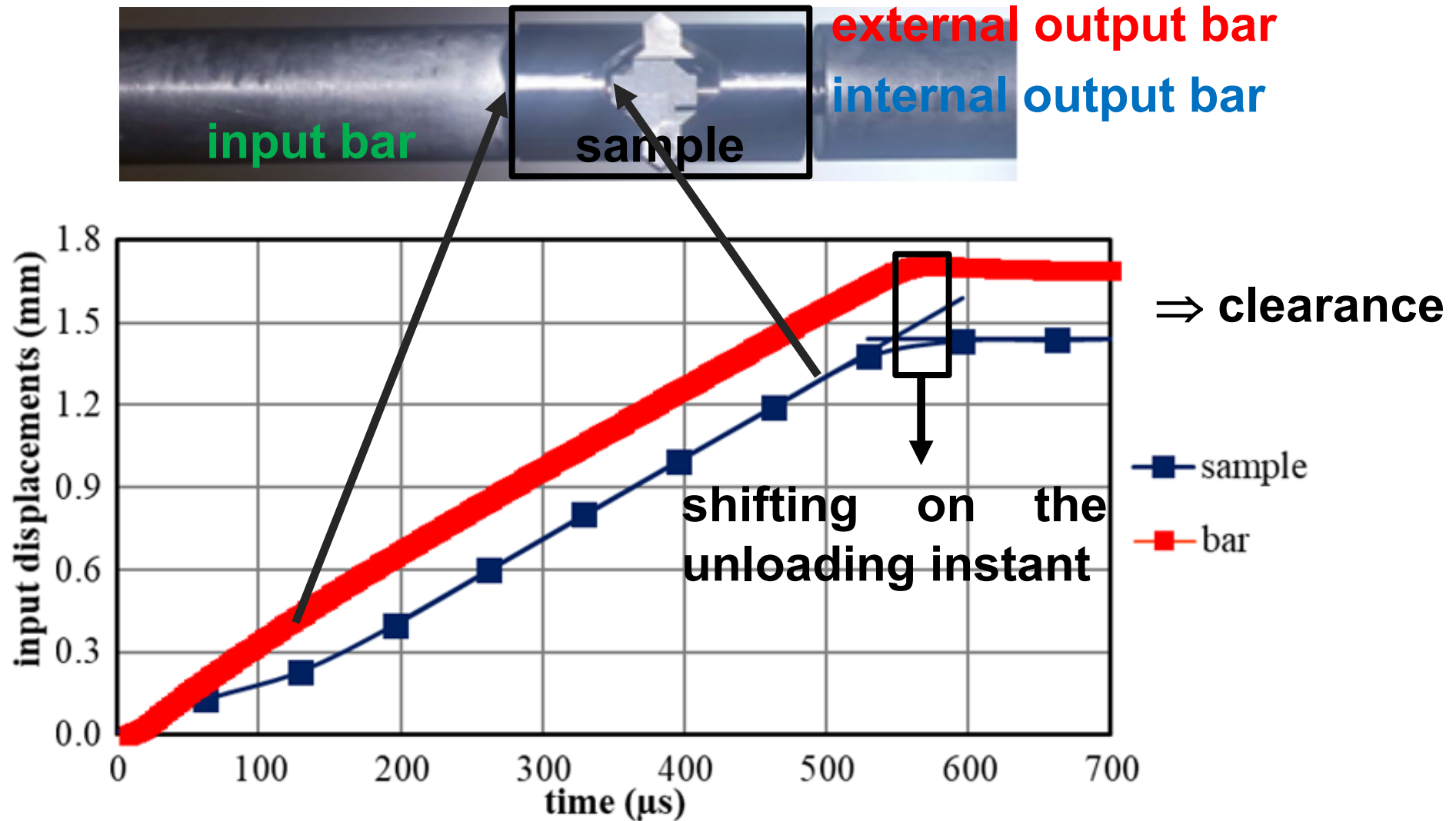
⇒ displacement field calculation in **mm**



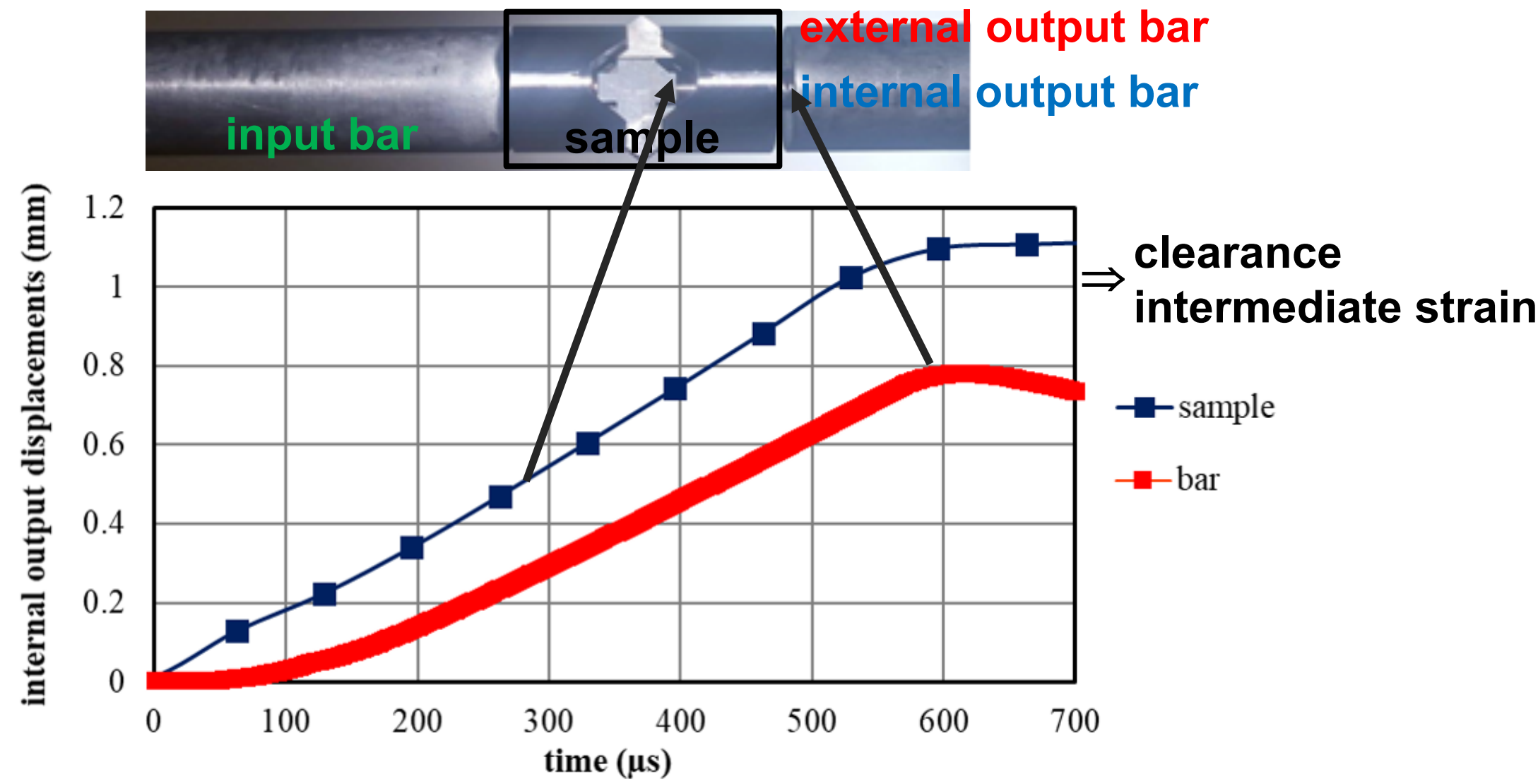
## Comparison between bars and sample boundaries displacements:



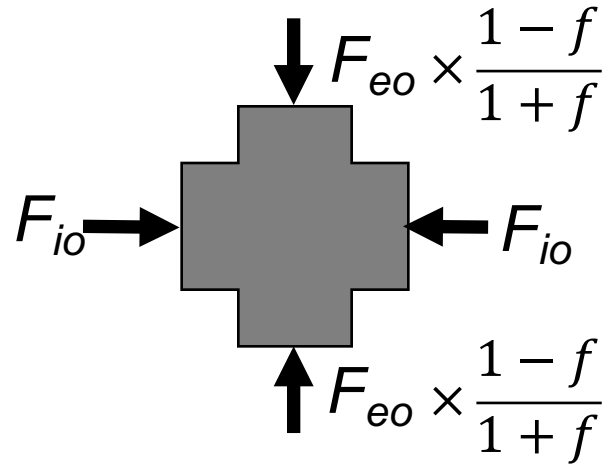
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## Determination of the stress at the 4 boundaries:



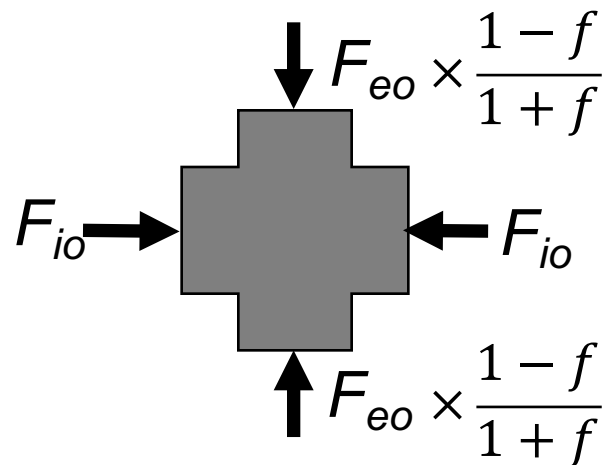
$F_{io}$ : force applied by the internal bar

$F_{eo}$ : force applied by the external bar

5 mm × 5 mm: arm section

$f$ : friction coefficient

### Determination of the stress at the 4 boundaries:



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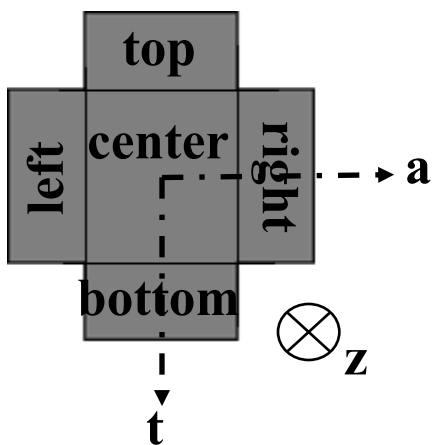
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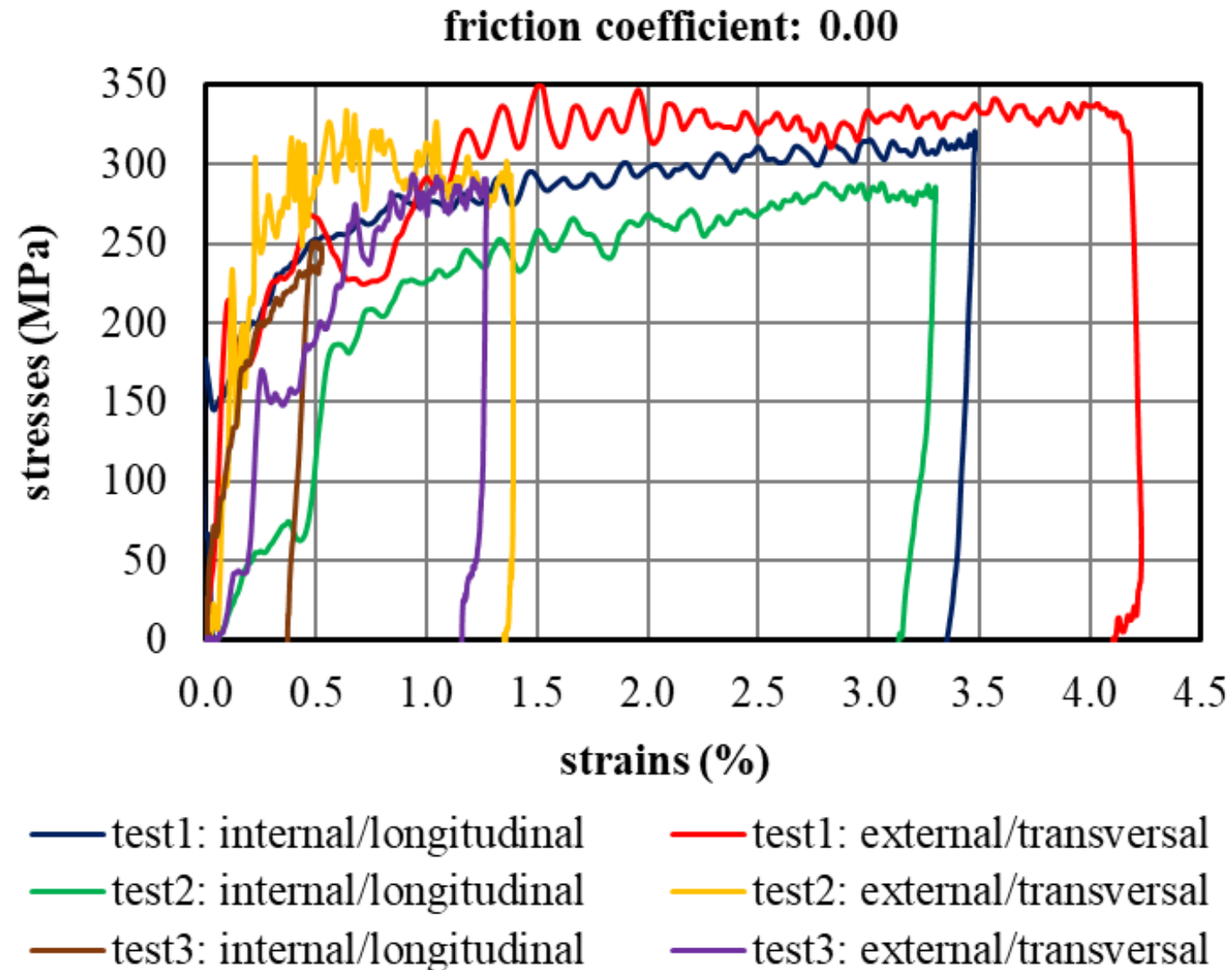
In the case of a Von-Mises plastic threshold  $\sigma_{thr}$ :

⇒ At the end of the test:

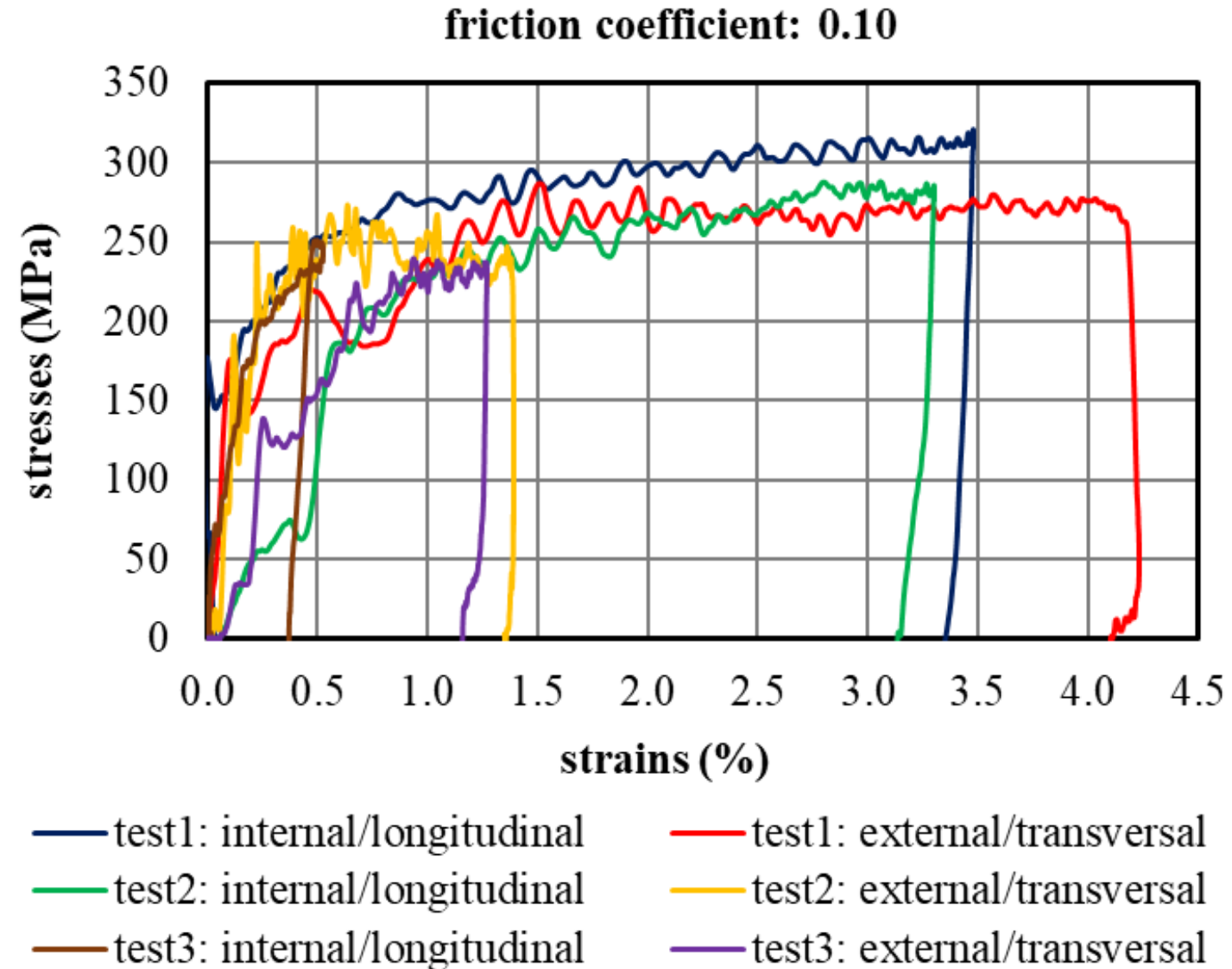
stress ratio = 1, whatever the strain ratio



From the stresses given by the gauges and the stains given by the camera:

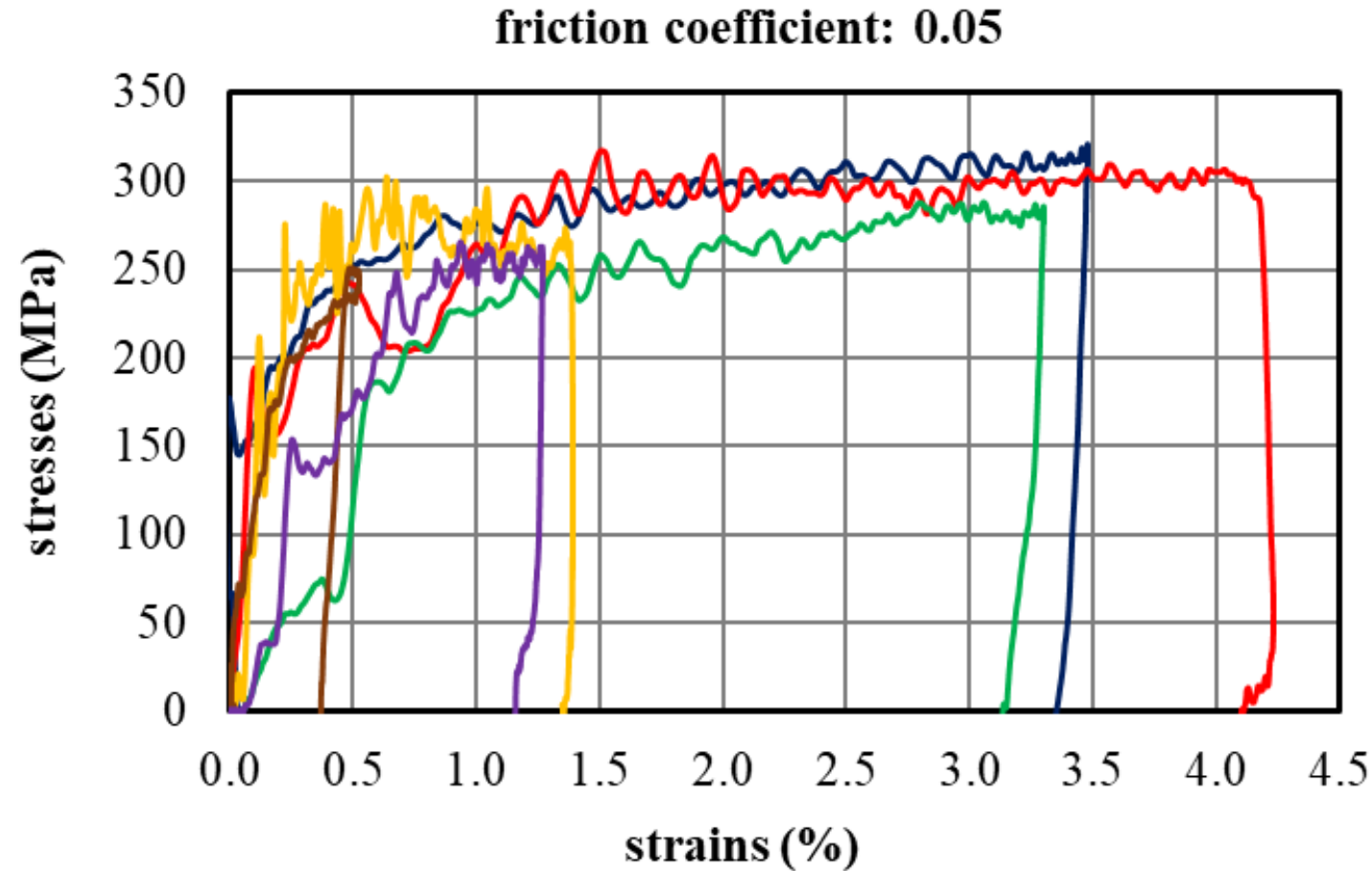


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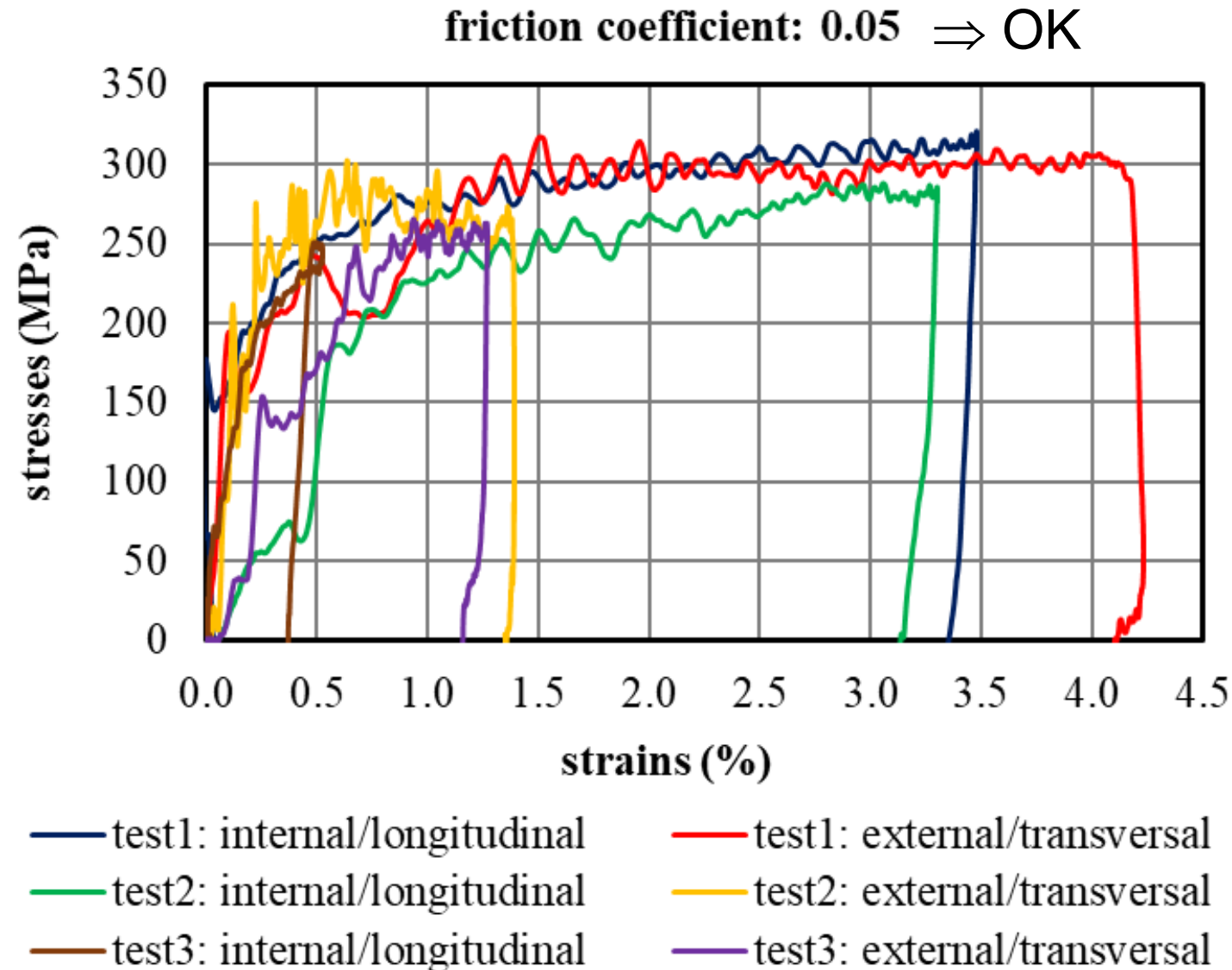


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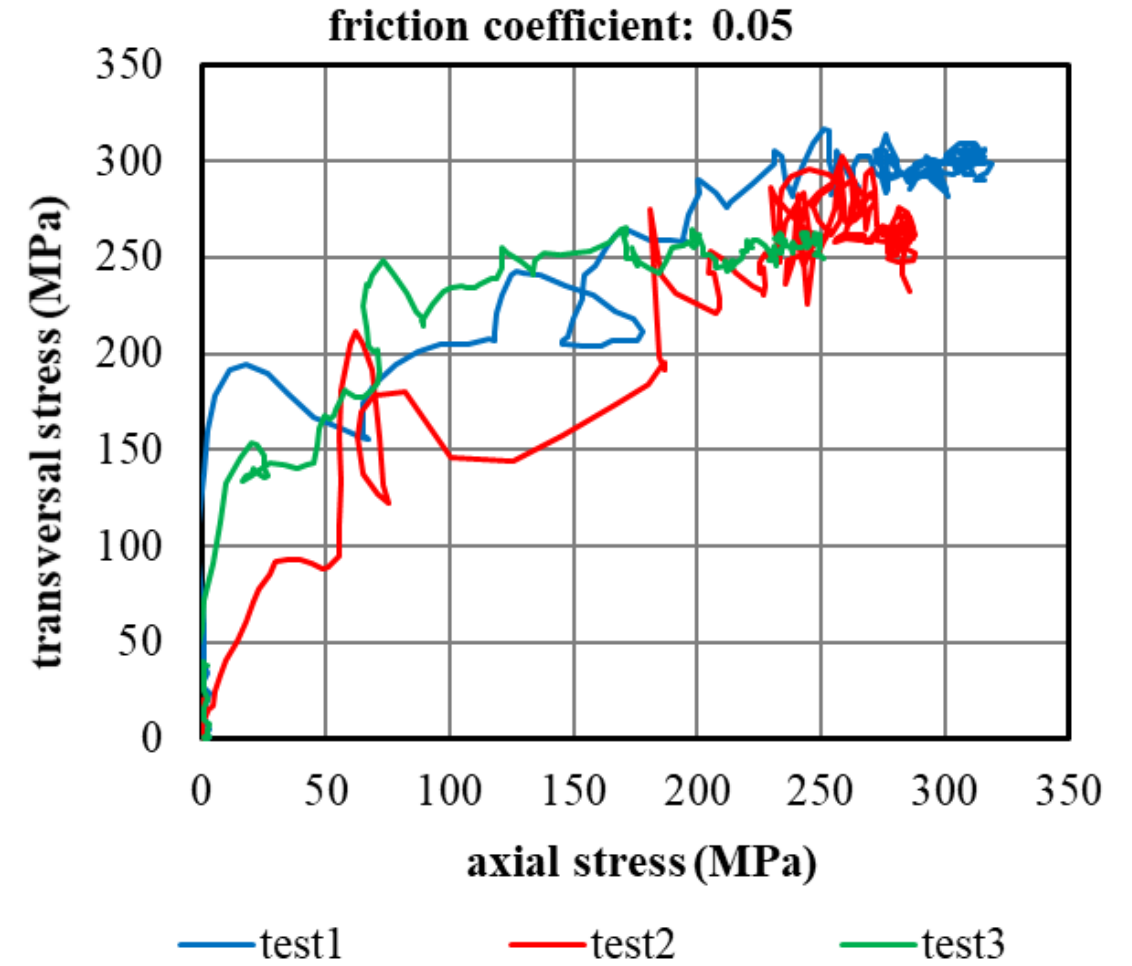
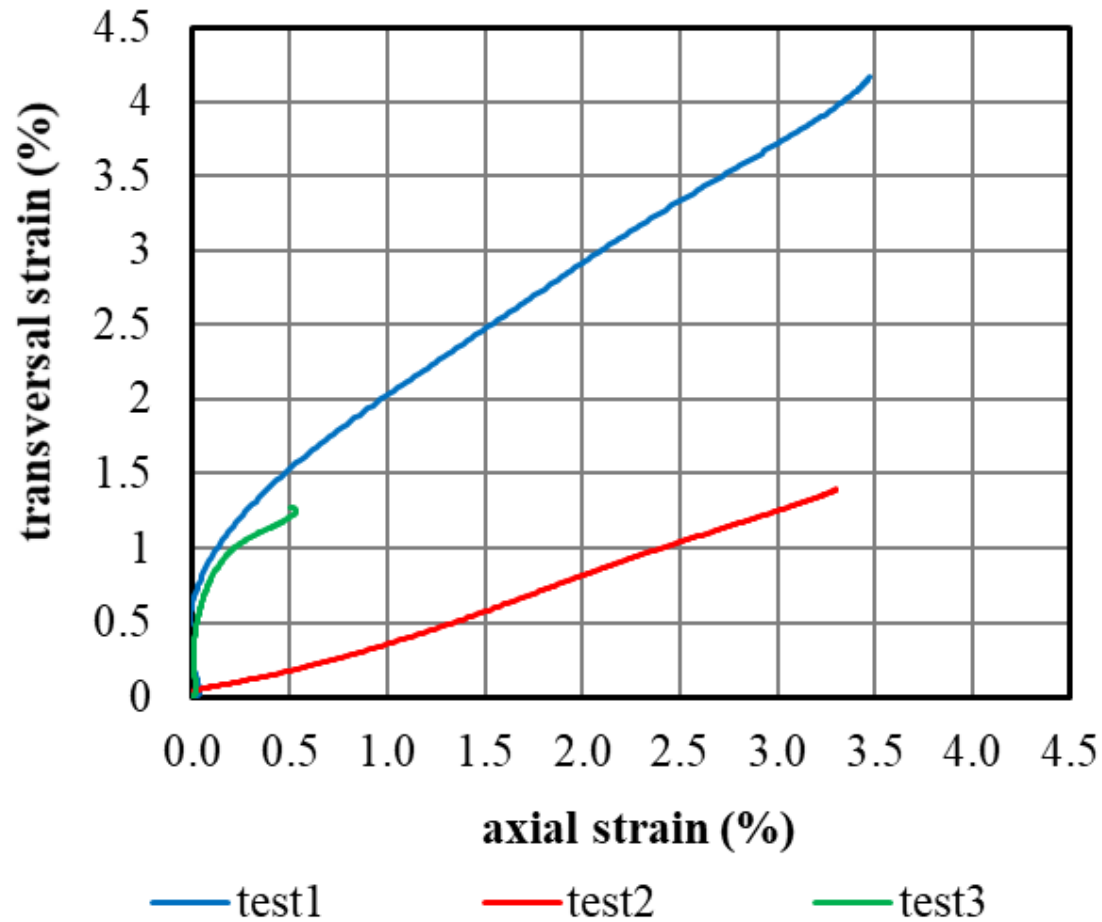


- |                                |                               |
|--------------------------------|-------------------------------|
| — test1: internal/longitudinal | — test1: external/transversal |
| — test2: internal/longitudinal | — test2: external/transversal |
| — test3: internal/longitudinal | — test3: external/transversal |

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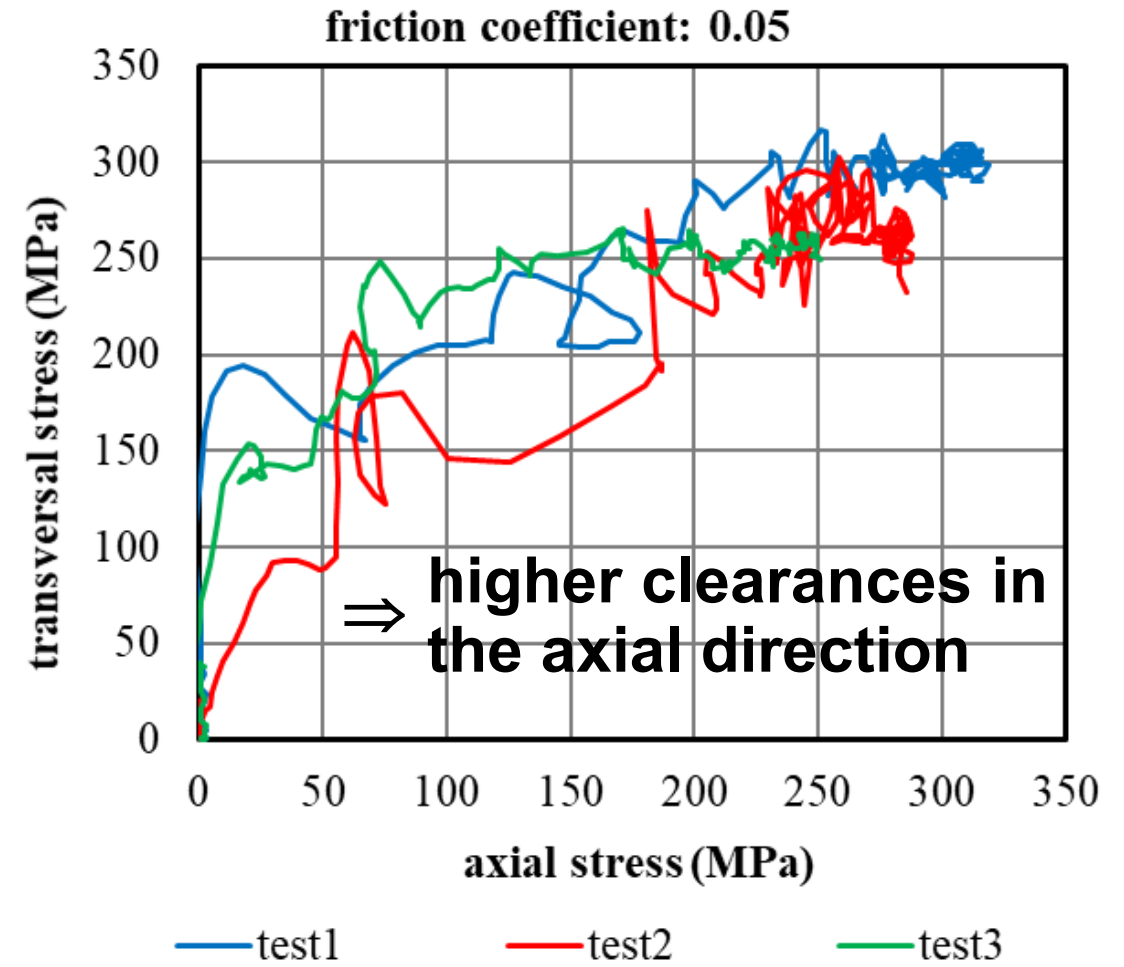
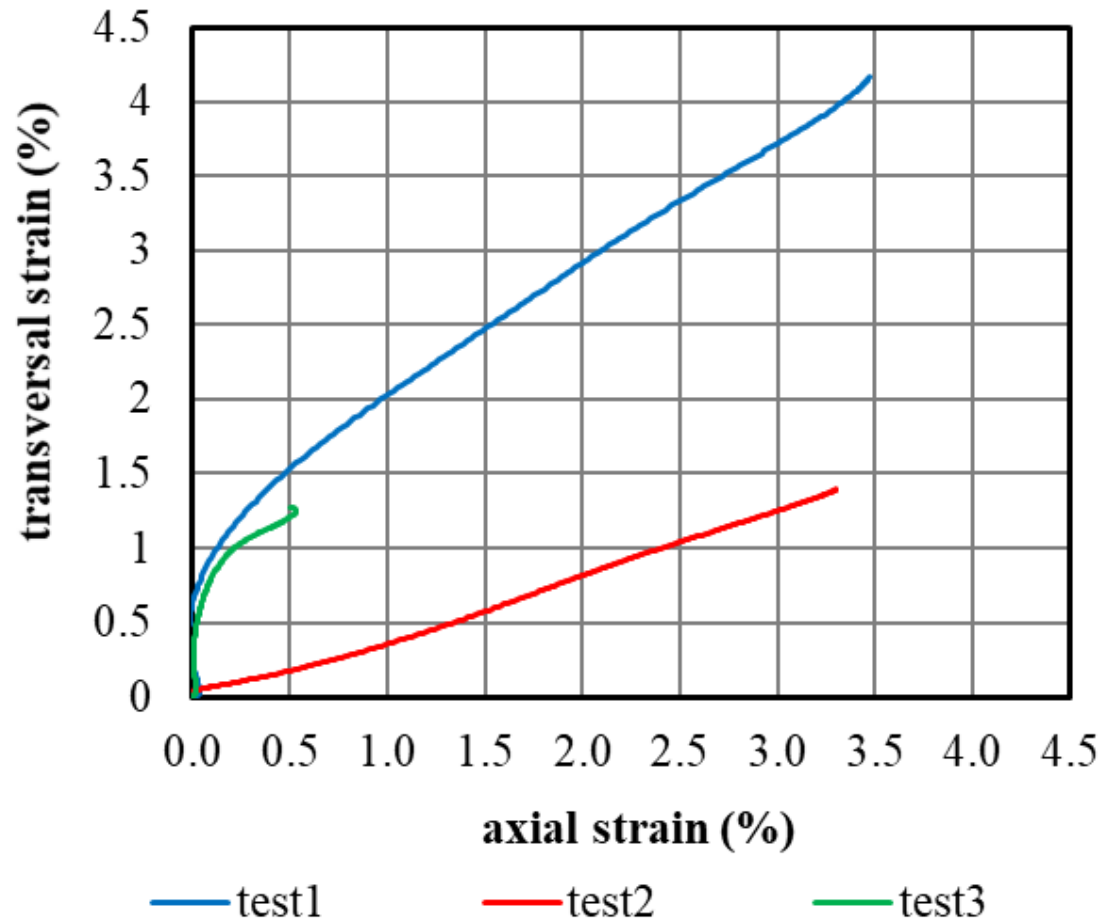


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Loading paths (without unloading):



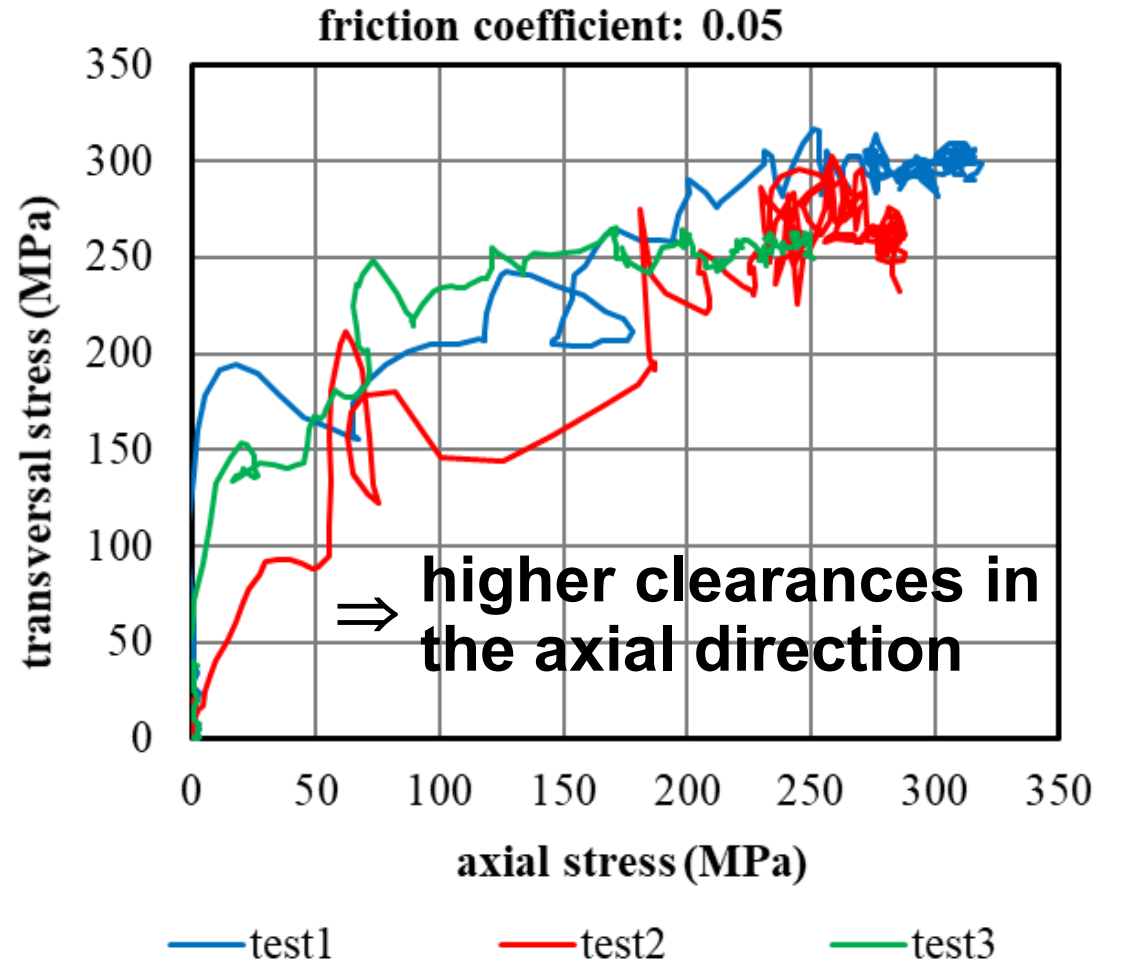
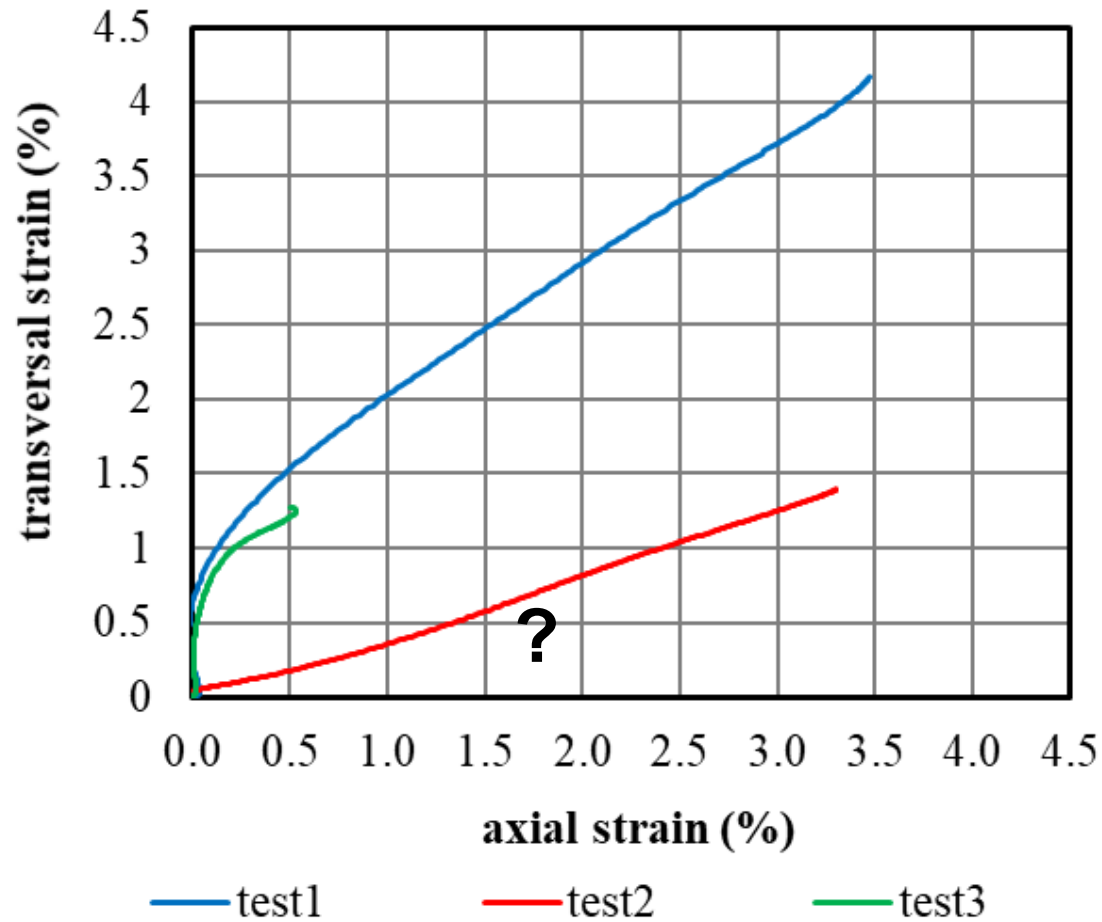
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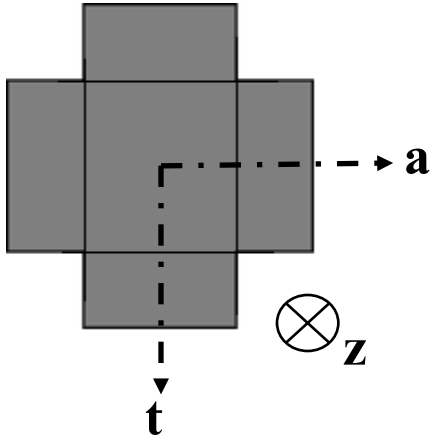
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Loading paths (without unloading):



## Sample behavior in its elastic phase:

Strains as functions of the stresses:

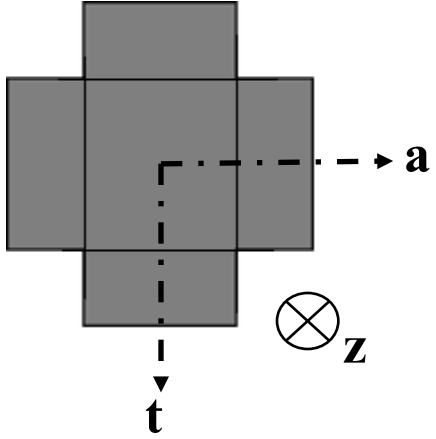


$$\begin{cases} \varepsilon_a = \frac{\sigma_a - \nu\sigma_t}{E} \\ \varepsilon_t = \frac{\sigma_t - \nu\sigma_a}{E} \end{cases} \quad \begin{array}{l} E: \text{homogenized Young modulus} \\ \nu: \text{homogenized Poisson's ratio} \end{array}$$

$\Rightarrow$  axial stress-strain law  $\neq$  transversal stress-strain law

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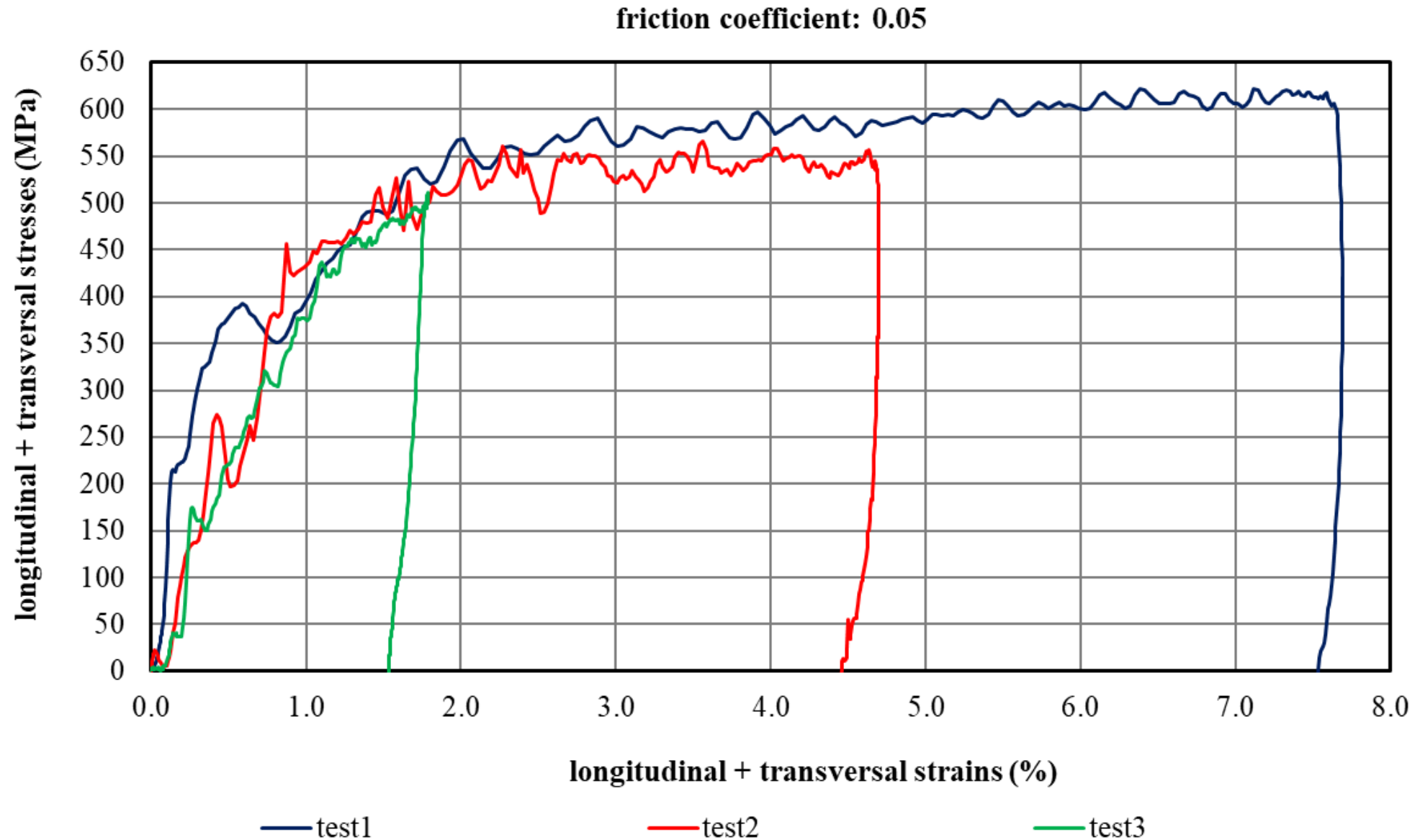
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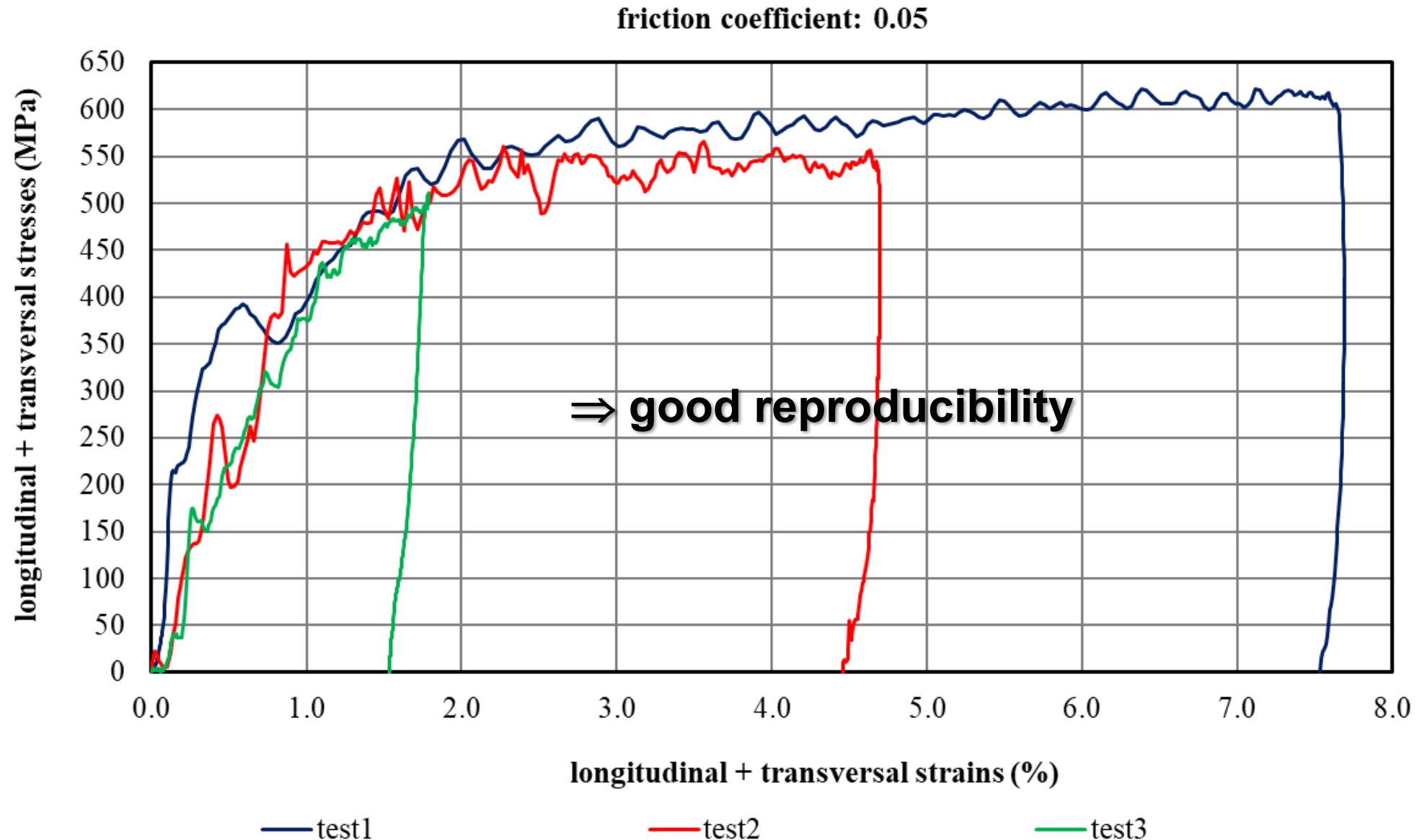
$$\Rightarrow \sigma_a + \sigma_t = \frac{E}{1 - \nu} (\varepsilon_a + \varepsilon_t)$$



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Design of a dynamic biaxial compression set-up.

3 tests conducted with a good reproducibility.

Axial measurements reliability similar to those of uniaxial tests.

Transversal measurements dependent on the estimated friction coefficient.

Loading path dependent on the clearances.

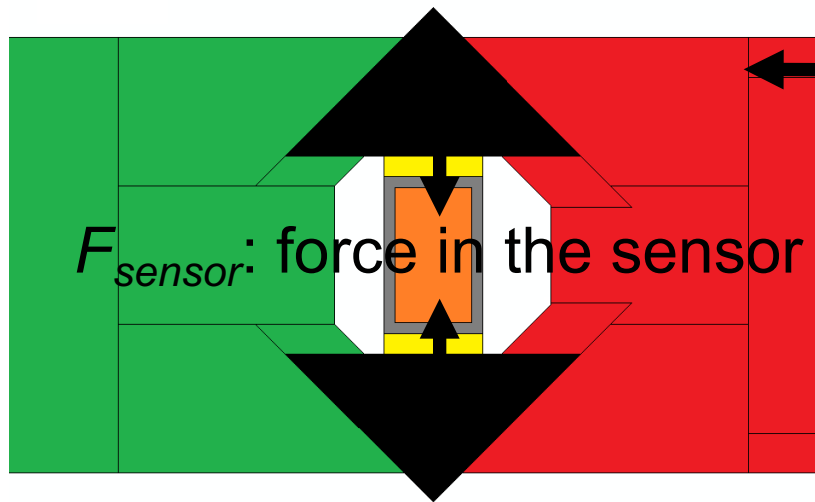
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$F_{eo}$ : force applied by the external bar

$$\frac{F_{sensor}}{F_{eo}} = \frac{1 - f}{1 + f}$$

The friction could be measured from this dedicated test.

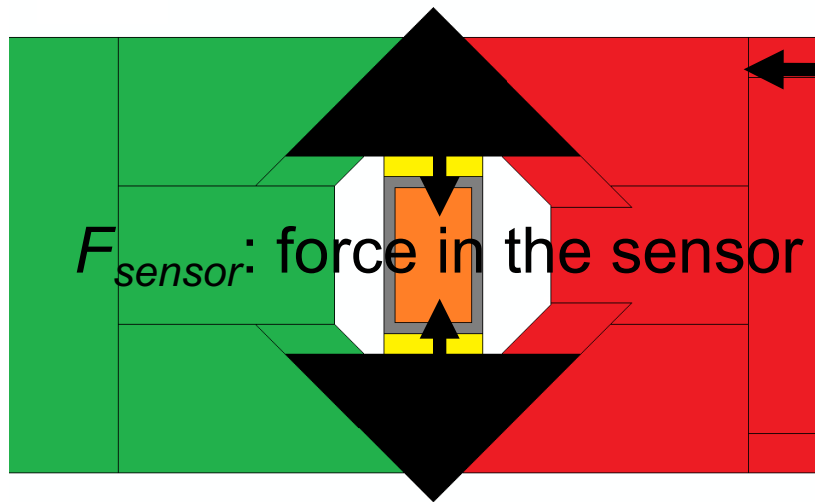
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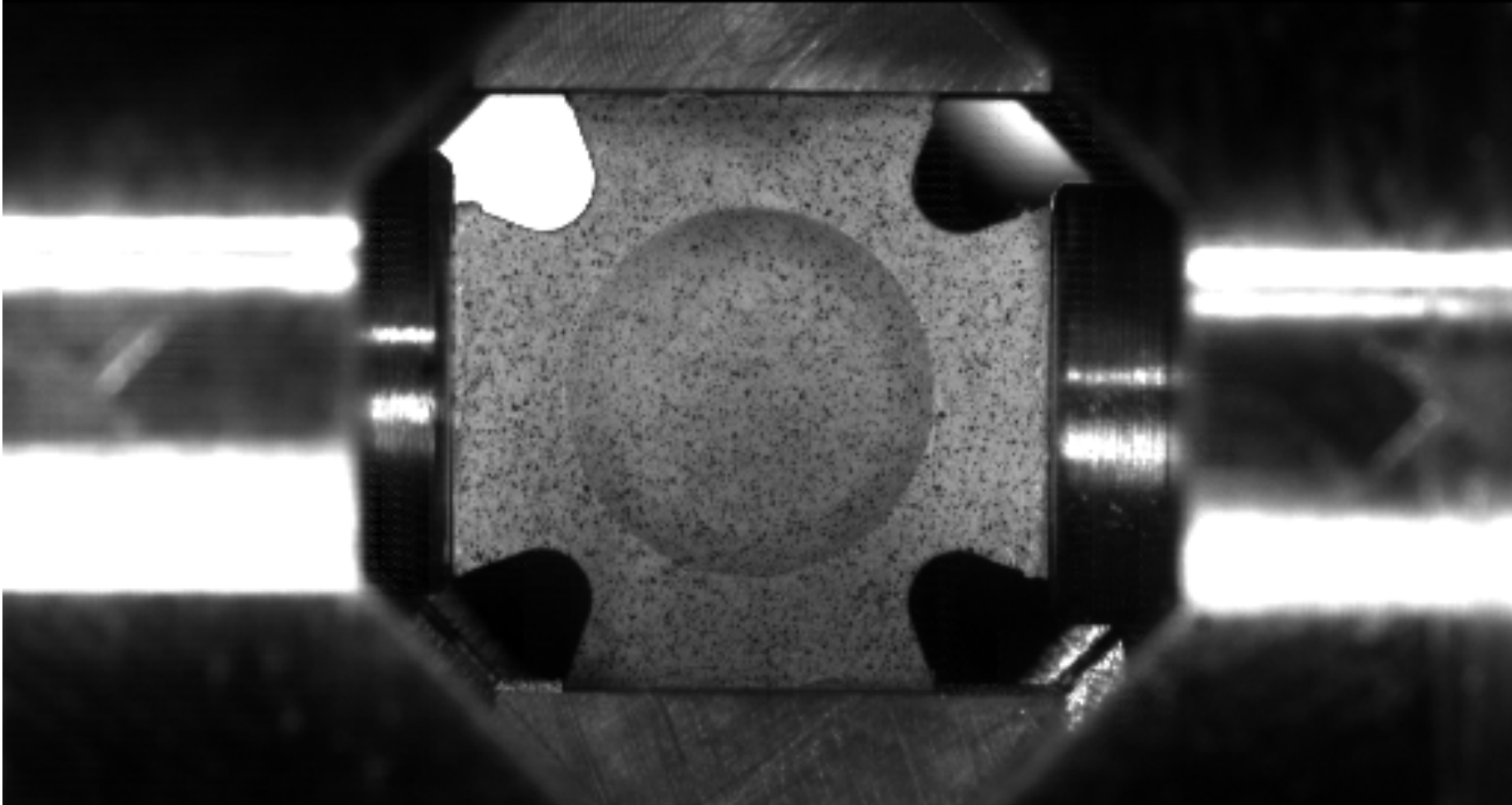
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The set-up has been used for material characterization.

FASTCAM SA5 mode... 50000 i/s  
512 x 272  
+00:00:00.00030  
Milieu  
Date : 2017/6/6  
1/50000 sec  
image : 15  
Time : 18:50



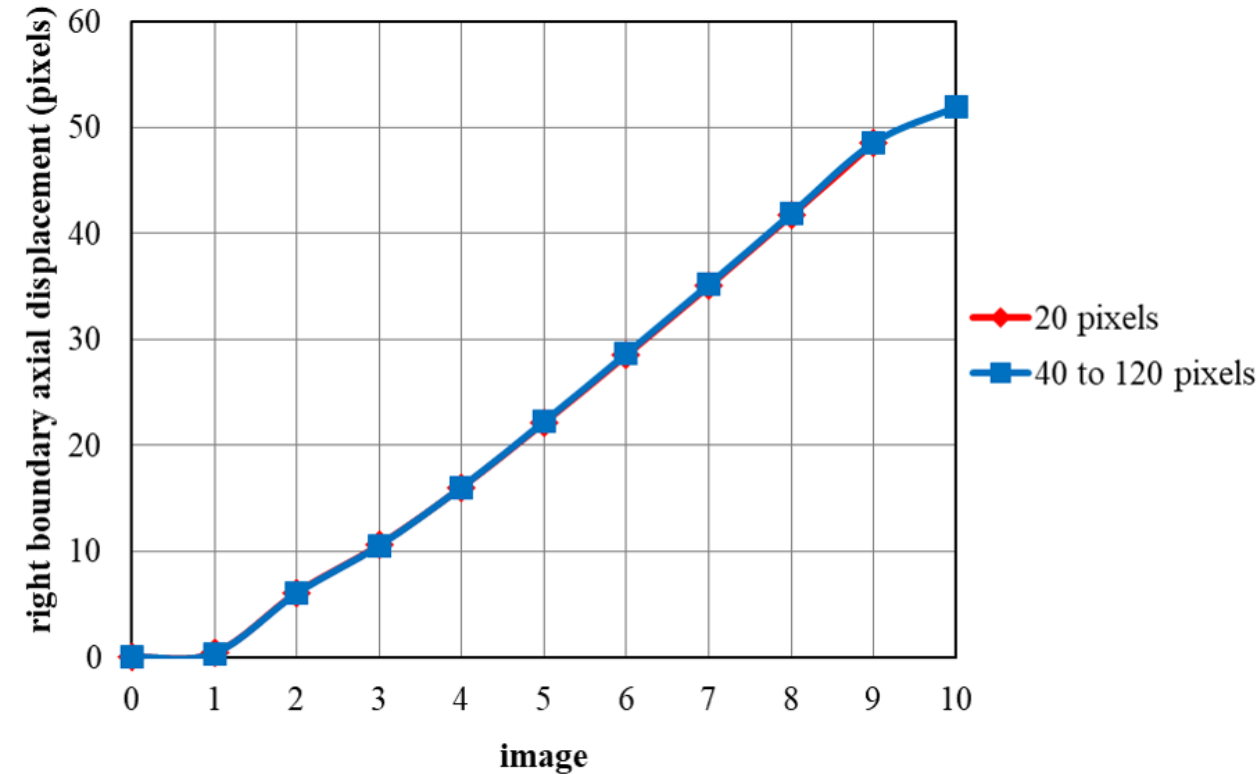
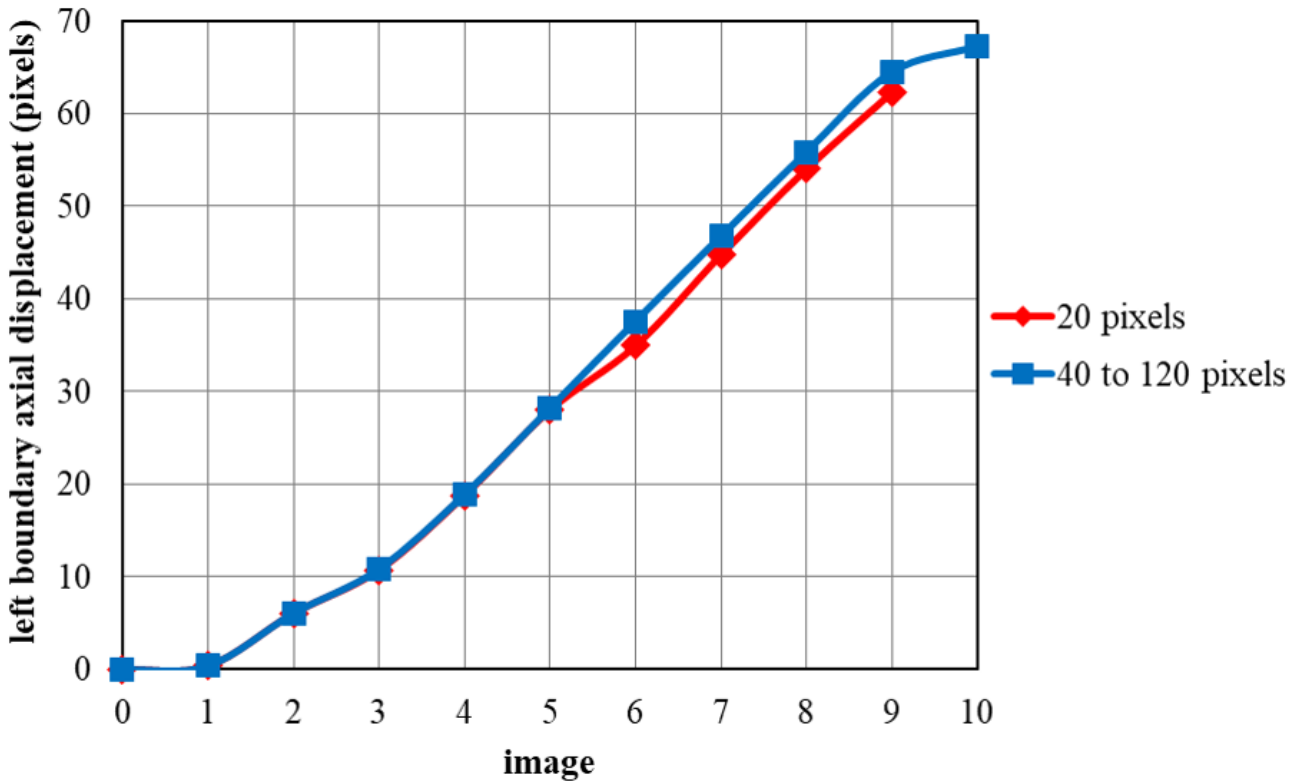


**Thank you for your attention.**

**Questions?**

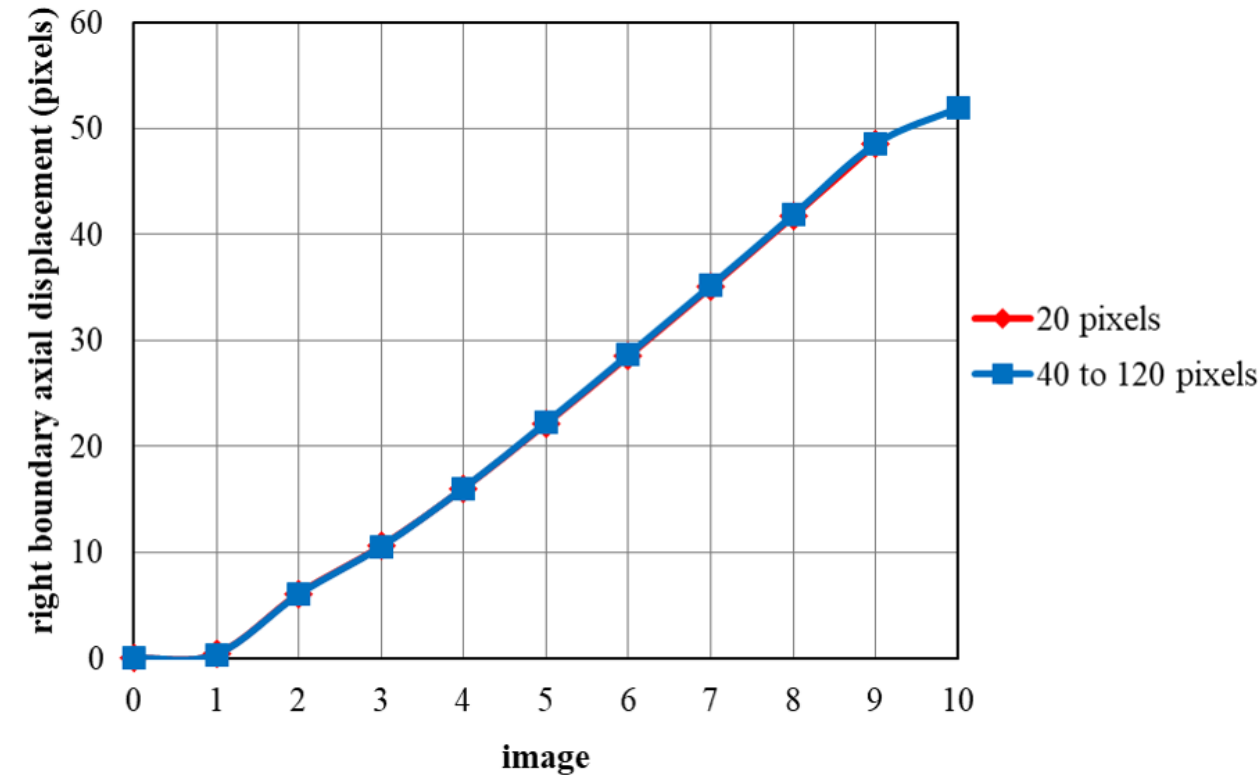
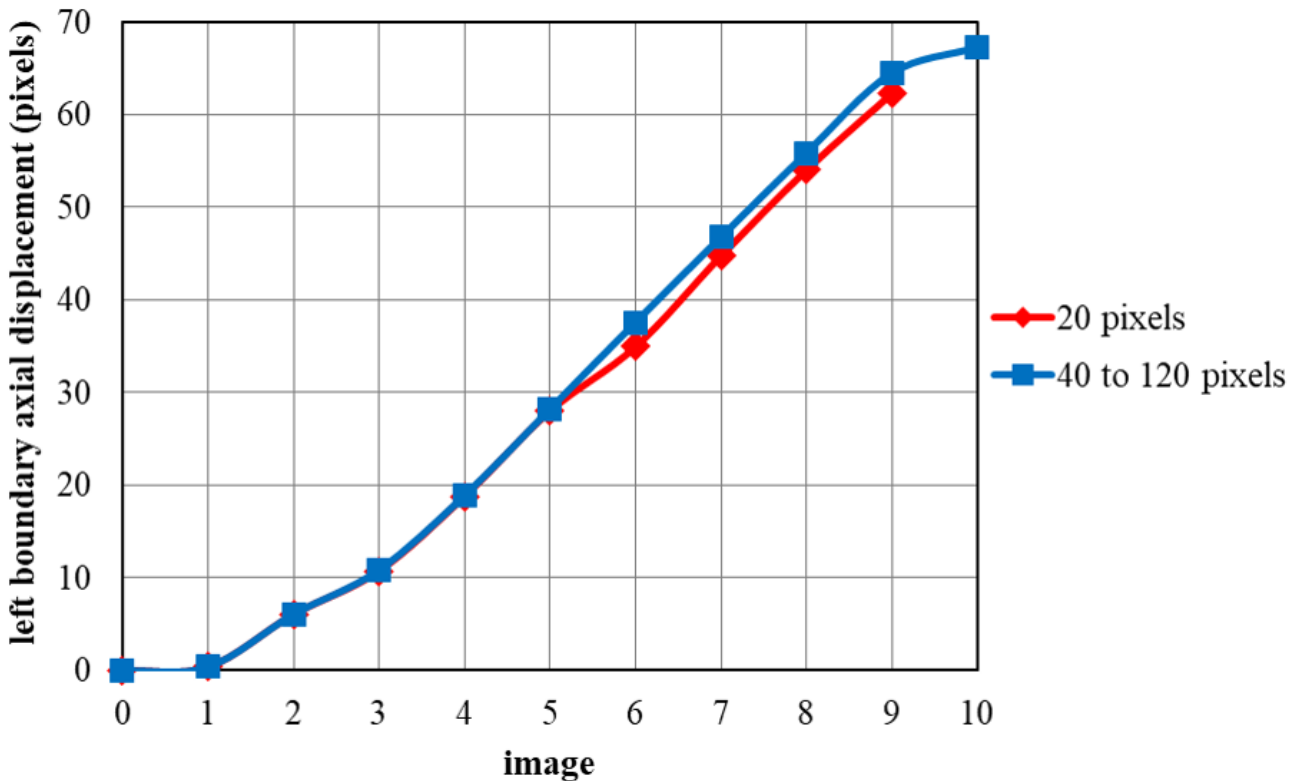
## Digital Image Correlation (Correli RT3):

displacements evolutions depending on the regularization length:



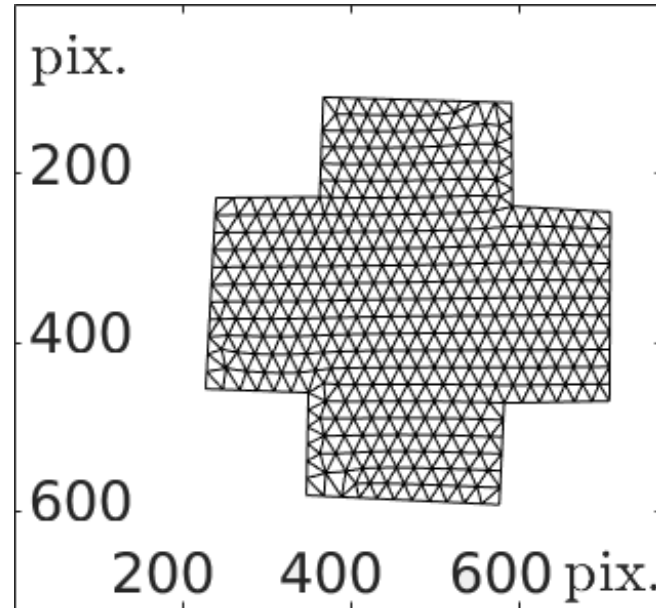
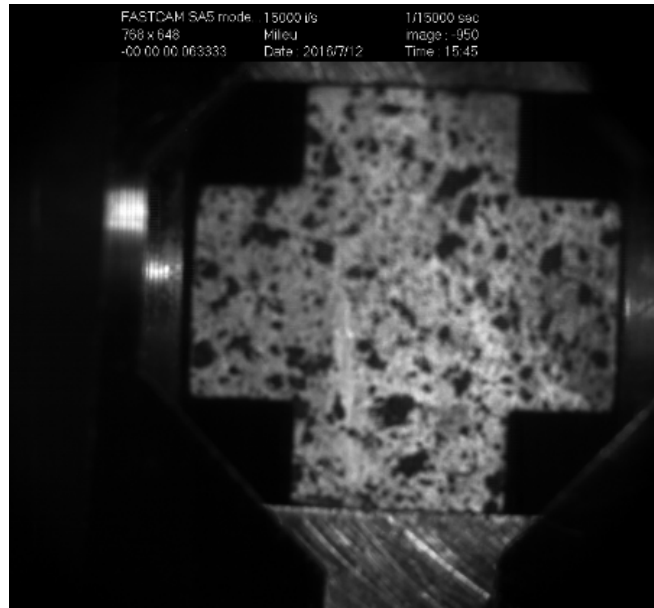
## Digital Image Correlation (Correli RT3):

displacements evolutions depending on the regularization length:



⇒ convergence beyond 40 pixels (2 times the element size)

## Digital Image Correlation (Correli RT3):



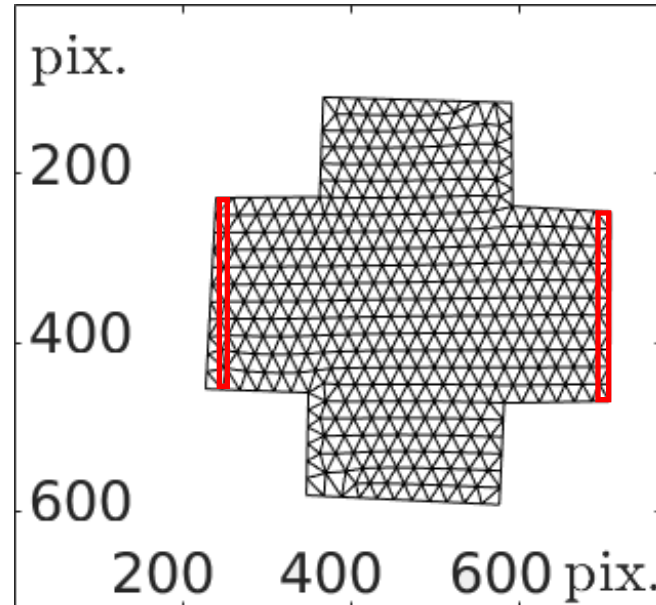
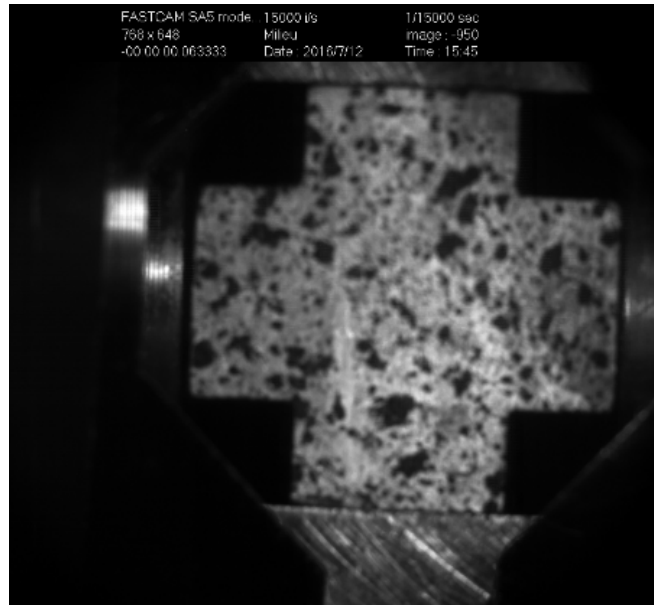
### Mesh:

- triangular 3 noded elements (T3)
- element size: 20 pixels

### Elastic regularization:

- 0.49 Poisson's ratio (incompressible displacement field)
- regularization length: 20 pix - 120 pix

## Digital Image Correlation (Correli RT3):



### Mesh:

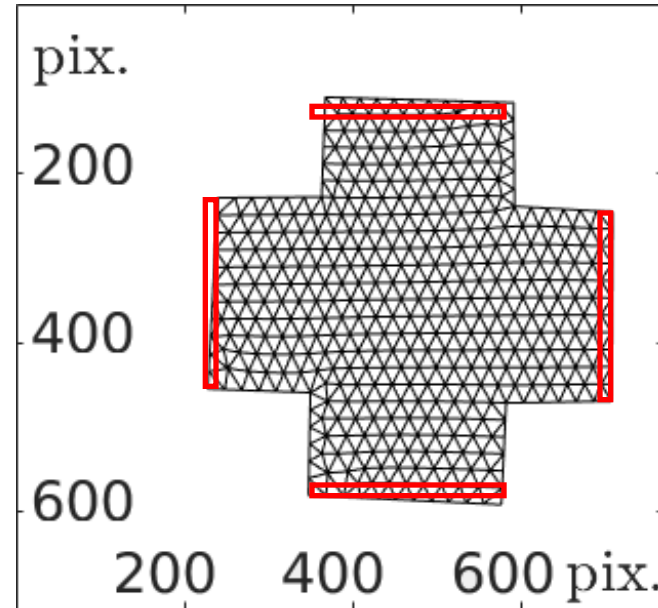
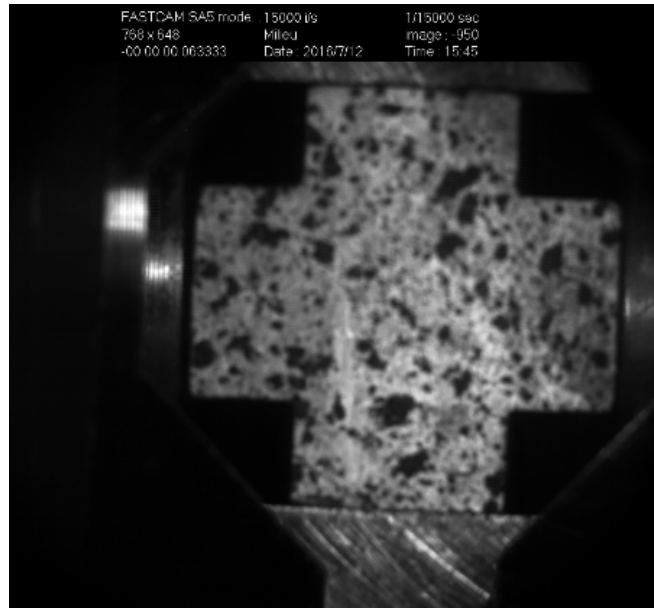
- triangular 3 noded elements (T3)
- element size: 20 pixels

### Elastic regularization:

- 0.49 Poisson's ratio (incompressible displacement field)
- regularization length: 20 pix - 120 pix

axial mean displacement on the left & right boundaries estimated over a 5-pix thickness

## Digital Image Correlation (Correli RT3):



### Mesh:

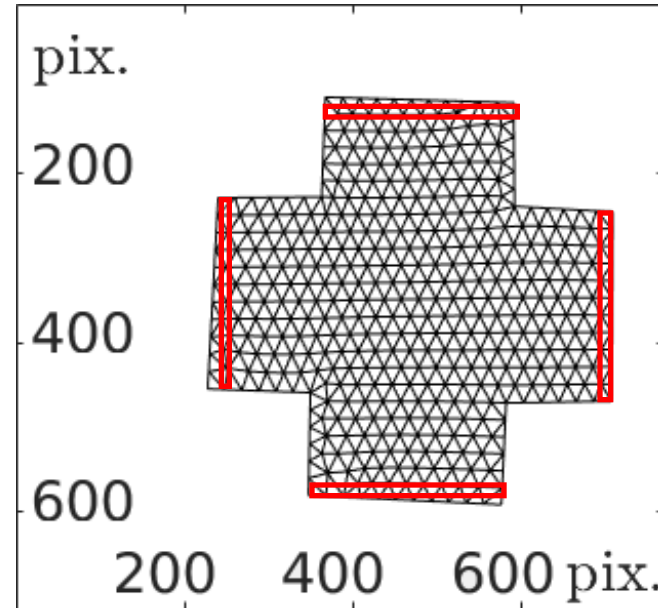
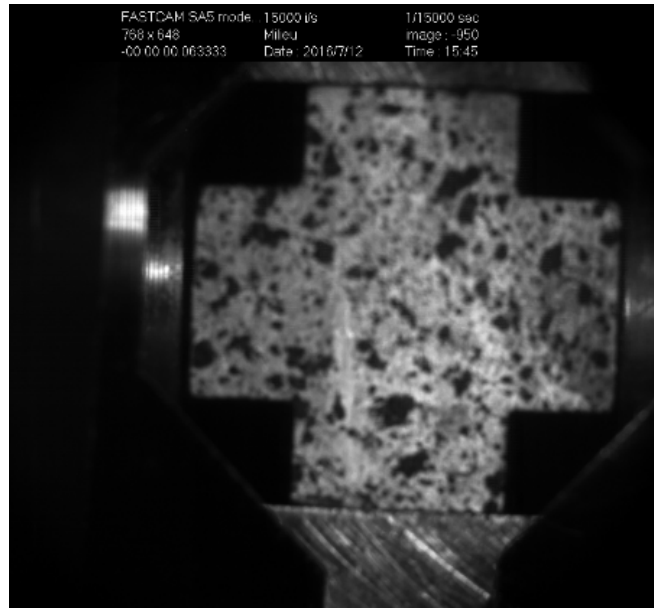
- triangular 3 noded elements (T3)
- element size: 20 pixels

### Elastic regularization:

- 0.49 Poisson's ratio (incompressible displacement field)
- regularization length: 20 pix - 120 pix

axial mean displacement on the left & right boundaries estimated over a 5-pix thickness  
transversal mean displacement on the top & bottom boundaries estimated over a 5-pix thickness

## Digital Image Correlation (Correli RT3):



Mesh:

- triangular 3 noded elements (T3)
- element size: 20 pixels

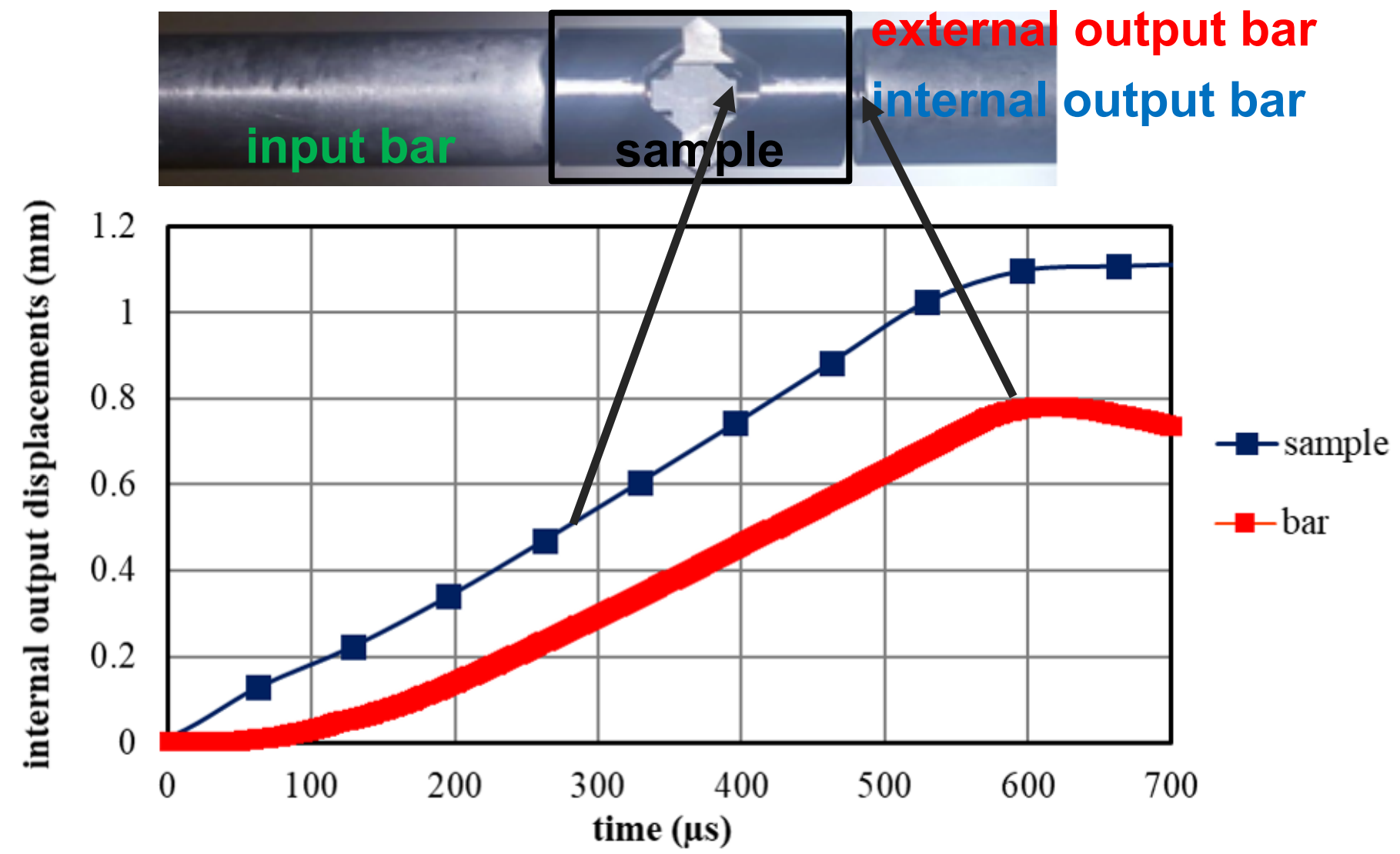
Elastic regularization:

- 0.49 Poisson's ratio (incompressible displacement field)
- regularization length: 20 pix - 120 pix

axial mean displacement on the left & right boundaries estimated over a 5-pix thickness  
transversal mean displacement on the top & bottom boundaries estimated over a 5-pix thickness

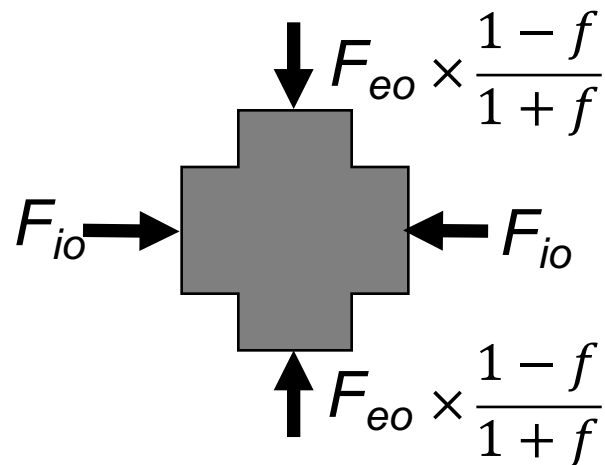
⇒ estimation of the mean axial and transversal strains

# Comparison between bars and sample boundaries displacements:





### Determination of the stress at the 4 boundaries:



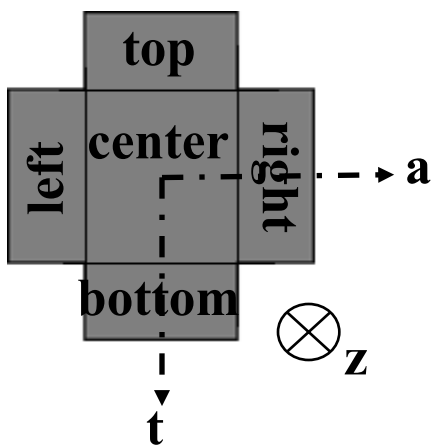
$F_{io}$ : force applied by the internal bar

$F_{eo}$ : force applied by the external bar

5 mm × 5 mm: arm section

$f$ : friction coefficient

In the case of a Von-Mises plastic threshold  $\sigma_{thr}$ :



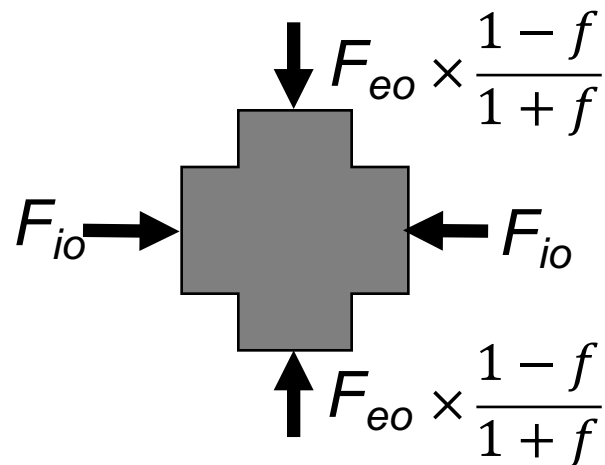
In the center:

stresses: 
$$\begin{pmatrix} \sigma_{thr} & 0 & 0 \\ 0 & \sigma_{thr} & 0 \\ 0 & 0 & 0 \end{pmatrix}_{a,t,z}$$

strain rates: 
$$\begin{pmatrix} \dot{\epsilon}_{cen} & 0 & 0 \\ 0 & \dot{\epsilon}_{cen} & 0 \\ 0 & 0 & -2\dot{\epsilon}_{cen} \end{pmatrix}_{a,t,z}$$

$\forall \dot{\epsilon}_{cen}$

### Determination of the stress at the 4 boundaries:



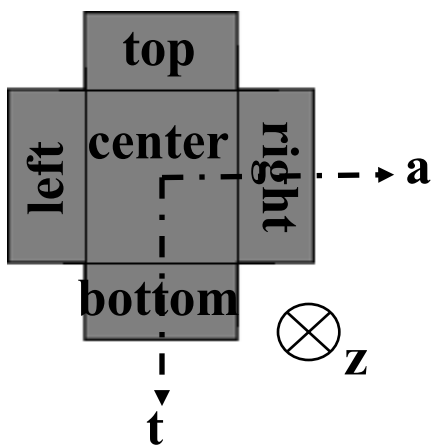
$F_{io}$ : force applied by the internal bar

$F_{eo}$ : force applied by the external bar

5 mm × 5 mm: arm section

$f$ : friction coefficient

In the case of a Von-Mises plastic threshold  $\sigma_{thr}$ :



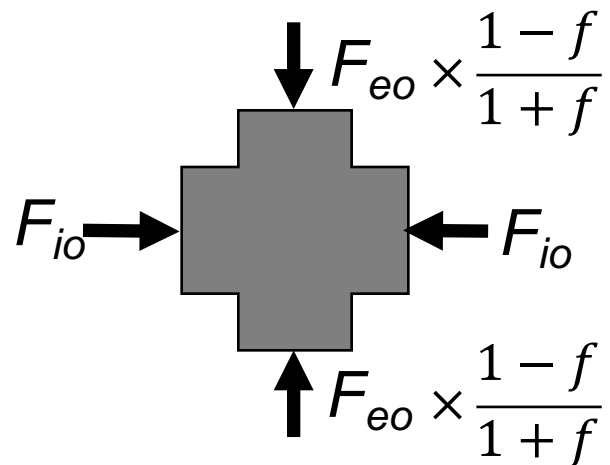
In the top arm:

stresses: 
$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & \sigma_{thr} & 0 \\ 0 & 0 & 0 \end{pmatrix}_{a,t,z}$$

strain rates: 
$$\begin{pmatrix} -0.5\dot{\epsilon}_{top} & 0 & 0 \\ 0 & \dot{\epsilon}_{top} & 0 \\ 0 & 0 & -0.5\dot{\epsilon}_{top} \end{pmatrix}_{a,t,z}$$

$\forall \dot{\epsilon}_{top}$

### Determination of the stress at the 4 boundaries:



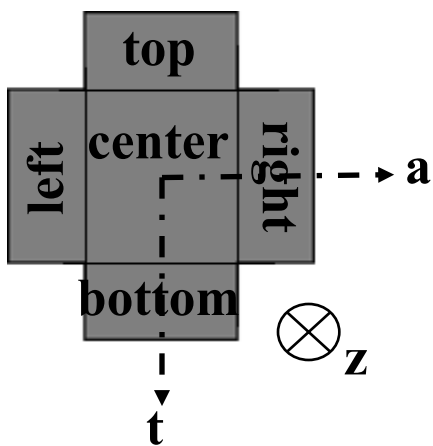
$F_{io}$ : force applied by the internal bar

$F_{eo}$ : force applied by the external bar

5 mm × 5 mm: arm section

$f$ : friction coefficient

### In the case of a Von-Mises plastic threshold $\sigma_{thr}$ :

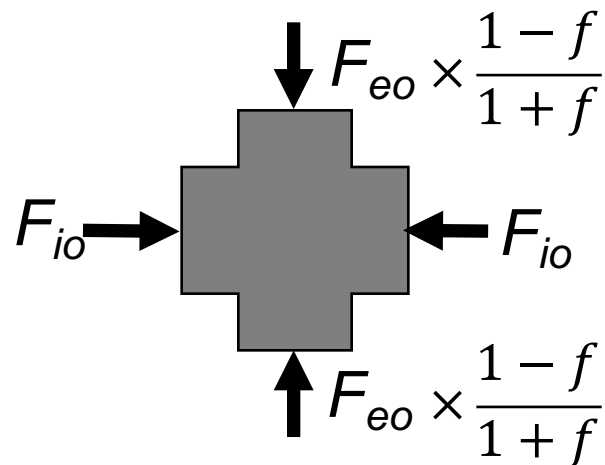


In the bottom arm:

stresses: 
$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & \sigma_{thr} & 0 \\ 0 & 0 & 0 \end{pmatrix}_{a,t,z}$$

strain rates: 
$$\begin{pmatrix} -0.5\dot{\epsilon}_{bot} & 0 & 0 \\ 0 & \dot{\epsilon}_{bot} & 0 \\ 0 & 0 & -0.5\dot{\epsilon}_{bot} \end{pmatrix}_{a,t,z}$$
  
 $\forall \dot{\epsilon}_{bot}$

### Determination of the stress at the 4 boundaries:



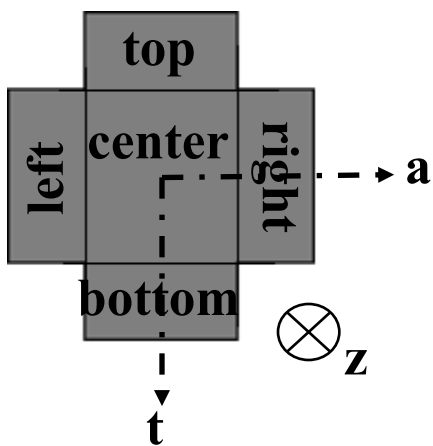
$F_{io}$ : force applied by the internal bar

$F_{eo}$ : force applied by the external bar

5 mm × 5 mm: arm section

$f$ : friction coefficient

### In the case of a Von-Mises plastic threshold $\sigma_{thr}$ :

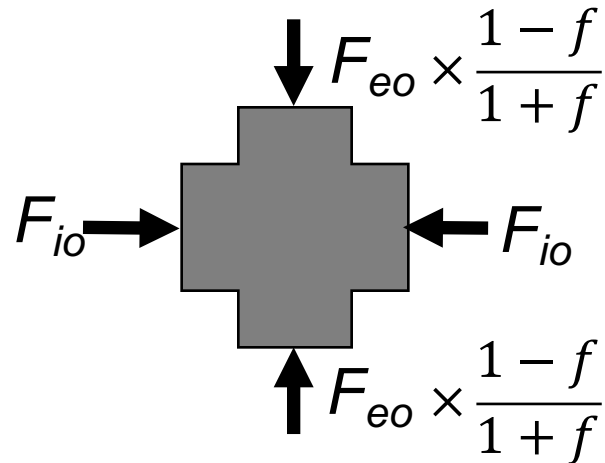


In the left arm:

stresses: 
$$\begin{pmatrix} \sigma_{thr} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{a,t,z}$$

strain rates: 
$$\begin{pmatrix} \dot{\epsilon}_{left} & 0 & 0 \\ 0 & -0.5\dot{\epsilon}_{left} & 0 \\ 0 & 0 & -0.5\dot{\epsilon}_{left} \end{pmatrix}_{a,t,z}$$
  
 $\forall \dot{\epsilon}_{left}$

### Determination of the stress at the 4 boundaries:



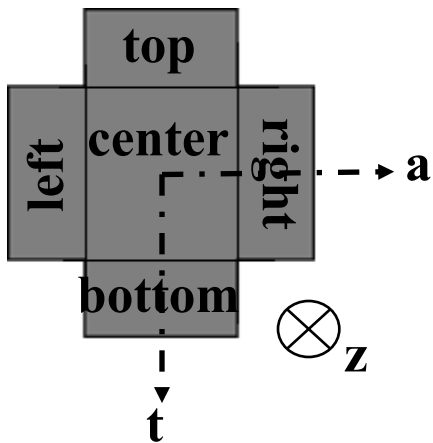
$F_{io}$ : force applied by the internal bar

$F_{eo}$ : force applied by the external bar

5 mm × 5 mm: arm section

$f$ : friction coefficient

### In the case of a Von-Mises plastic threshold $\sigma_{thr}$ :



In the right arm:

stresses: 
$$\begin{pmatrix} \sigma_{thr} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{a,t,z}$$

strain rates: 
$$\begin{pmatrix} \dot{\epsilon}_{rig} & 0 & 0 \\ 0 & -0.5\dot{\epsilon}_{rig} & 0 \\ 0 & 0 & -0.5\dot{\epsilon}_{rig} \end{pmatrix}_{a,t,z}$$

$\forall \dot{\epsilon}_{rig}$