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A Mathematical Formulation and Tabu Search Approach for the Road-Rail Assignment Problem

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Abstract: Inspired from the digital internet, the Physical Internet (PI) is defined as a global standardized and interconnected logistics system. This paper studies the Road-Rail assignment problem which consists on assigning inbound trucks to the PI-docks and assigning the unloaded PI-containers to the train's wagons. A mixed integer linear programming mathematical model (MILP) is proposed with the objective to minimize both the number of used wagons and the internal travel distance by the PI-containers from the PI-docks to the wagons. Moreover, a heuristic based on the first fit algorithm is proposed for generating an initial grouping of containers and a tabu search meta-heuristic is used after to find better solutions. Finally, the proposed methods are tested on several instances and the obtained results are presented.

Keywords: Physical Internet, Road-Rail PI-hub, MILP Mathematical Model, first fit algorithm, Tabu Search.

1 Introduction

1.1 Physical Internet

Recently, a novel Supply Chain Management paradigm named Physical Internet (PI) has attracted the attention of both the academic and the industrial community. Physical Internet is defined as a worldwide interconnected open logistics system aiming to change the way goods are handled, moved, stored and shipped based on the metaphor of the Digital Internet. The Physical Internet is based on the physical and digital inter-connectivity of the supply chain by encapsulating freight in modular PI-containers as data packets in the TCP-IP protocol. The objective of the Physical Internet is the sustainability of the logistics systems in three aspects: economical, environmental and social (Montreuil et al. 2010; Meller et al. 2012).

Sarraj et al. (2014); Montreuil et al. (2010) presented the three key elements to exploit the Physical Internet: PI-containers, PI-movers and PI-nodes. PI-containers are standardized containers and they can be handled and stored by different companies. PI-movers are used to move and handle PI-containers such as PI-vehicles (PI-trucks, PI-left ...) and PI-carriers (PI-wagons, PI-trailers ...). The PI-nodes are locations for receiving, storing and transferring PI-containers between PI-movers. The PI-nodes can be PI-transits, PI-switches, PI-bridges or PI-hubs

As the literature of Physical Internet is steadily growing, several papers reviewed the previous contributions on the Physical Internet; for instance, Treiblmaier et al. (2016) reviewed papers on the Physical Internet, and categorized them depending on the research methodologies (Quantitative modeling, Case studies, Simulation ...).

1.2 Road-Rail PI-hubs

This paper focuses on the Road-Rail PI-hub which is designed to transfer PI-containers from trains to other trains, from trucks to trains and from trains to trucks. Road-Rail PI-hubs are composed from three main PI-sorters: Rail-Road PI-sorters to transfer PI-containers from wagons to trucks, Road-Rail PI-sorters to transfer PI-containers from trucks to wagons and Rail-Rail PI-sorter to transfer PI-containers between trains.

The Rail-Road PI-hub was addressed by many researchers; for instance, Pach et al. (2014) proposed different grouping strategies for loading PI-containers in the trucks. Their study showed the effect of the grouping of PI-containers on minimizing the evacuation time in the Road-Rail PI-hub. In addition, Walha et al. (2014) studied the Rail-Road PI-hub allocation problem, which considers the train unloading section of the PI-hub that is used to transfer PI-containers from wagons to trucks. Their paper proposed also a mathematical formulation and a heuristic to find the best assignment of trucks to docks while minimizing the internal distance covered by the PI-containers from wagons to trucks.

In another work, Walha et al. (2016) suggested a simulated annealing meta-heuristic for solving the Rail-Road allocation problem to minimize the distance travelled by the PI-containers. They also proposed a multi-agent based approach which generates solutions while considering perturbations. The dynamic approach takes into consideration the filling rate and the travelled distance.

In this paper, a mathematical model of the Road-Rail PI-hub assignment problem is proposed. The objective is to minimize the number of used wagons and the internal travel distance of the PI-containers from trucks to wagons. The main constraint is that the PI-containers with the same destination must be loaded and grouped in consecutive wagons. Also, a truck has to free the dock once all the PI-containers are unloaded. A first fit based heuristic and a tabu search meta-heuristic are proposed to solve the Road-Rail PI-hub assignment problem. The objective of the first fit algorithm is to find an initial grouping of containers while keeping PI-containers with the same destination in consecutive wagons. The obtained grouping and the truck assignment are then improved using a tabu search meta-heuristic based on several local search moves.

The remaining of this paper is organized as follows. The description of the problem and the proposed MILP mathematical formulation of the Road-Rail PI-hub assignment problem are detailed in sections 2 and 3 respectively. In section 4, the first fit based heuristic and the tabu search meta-heuristic are presented. Finally, the obtained results on several instances are summarized in section 5.

2 Problem description

In this paper, we consider a Road-Rail PI-hub assignment problem. In their paper, Walha et al. (2014) formulated mathematically the allocation of the PI-containers to the Rail-Road section (Section 1 in Figure 1), where the PI-containers are unloaded from the wagons and transferred through the PI-Sorter and then loaded into the outgoing trucks.

Our research focuses on the Road-Rail section (Figure 1) which deals with the transfer of the PI-containers from trucks to wagons. The train is composed of thirty empty wagons. Each five wagons are considered as a block. The wagons will be loaded with the PI-containers that are unloaded from the trucks.

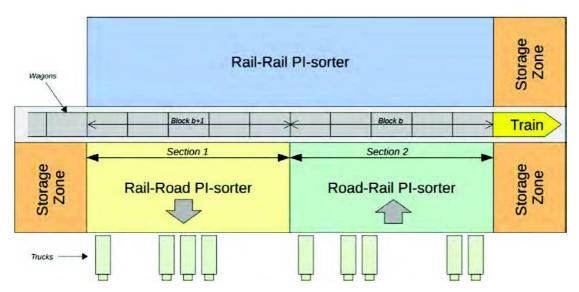


Figure 1: Overview of the Road-Rail PI-hub Layout

Once the train enters the Road-Rail PI-hub, the first block of five wagons is assigned to the Road-Rail PI-sorter section which is dedicated to receive PI-containers from trucks and load them into the train's wagons. PI-containers with the same destinations must be loaded in the same consecutive wagons so that in the next visited Road-Rail PI-hub the assignment of the outgoing trucks to the docks become easier since PI-containers with the same destination are grouped in consecutive wagons. Once the first block of wagons is filled with PI-containers, the next block will be assigned to the unloading Road-Rail section, until all the trucks are unloaded.

The main assumptions considered in this paper for the Road-Rail assignment problem are the following:

- Inbound trucks can unload containers with different lengths, and each one of those containers has a specific destination;
- PI-containers with the same destination must be loaded in consecutive wagons;
- Each one of the train's wagons must load only PI-containers that have the same destination:
- For simplification, one block of 5 wagons is considered for loading the PI-containers;

The lengths of PI-containers considered in this paper are: 1.2m, 2.4m, 3.6m, 4.8m, 6m and 12m. The useful length of the truck is 13.5m. The number of docks in the Road-Rail section is 28. For the wagons, the useful length is 18m (the full length of the wagon is 20m).

3 Mathematical formulation

In this section, the Road-Rail assignment problem is formulated as a mixed integer linear programming MILP model.

The mathematical model has two main decisions for the Road-Rail assignment problem:

- the assignment of the trucks to the docks;
- the assignment and the grouping of the PI-containers in the wagons.

Input parameters 3.1

N: total number of containers;

K : number of docks;

D: number of destinations;

W: number of wagons to load with PI-containers;

H : number of trucks;

Q: wagon's capacity (useful length);

i: indices of the containers;

k: indices of the docks:

d: indices of the destinations;

w: indices of the wagons;

h: indices of the trucks;

 P_k : position of the center of the dock k starting from the right axis of the Road-Rail PI-sorter

 R_w : position of the center of the wagon w starting from the right axis of the Road-Rail PIsorter zone;

 L_i : length of container i;

$$A_{hi} = \left\{ \begin{matrix} 1, & \text{ if the container i is in the truck h} \\ 0, & \text{ Otherwise} \end{matrix} \right.$$

$$S_{di} = \begin{cases} 1, & \text{if d is the destination of the container i} \\ 0, & \text{Otherwise} \end{cases}$$

 α : weighting factor for the number of used wagons;

 β : weighting factor for the total distance traveled by containers;

 $M: A \text{ big positive number}, M \ge \max(W, 5*20m).$

3.2 Decision variables

Binary variables:

$$x_{iw} = \begin{cases} 1, & \text{if the container i is assigned to the wagon w} \\ 0, & \text{Otherwise} \end{cases}$$

$$_{w} = \begin{cases} 0, & \text{Otherwise} \end{cases}$$

$$y_{hk} = \begin{cases} 1, & \text{if the truck h is assigned to the dock k} \\ 0, & \text{Otherwise} \end{cases}$$

$$y_{hk} = \{0, Otherwise\}$$

$$u_w = \begin{cases} 1, & if the wagon w \text{ is used} \\ 0, & Otherwise \end{cases}$$

$$z_{iwk} = \begin{cases} 1, & \text{if the container i is in a truck that is assigned to the dock k,} \\ & \text{and the container i is assigned to the wagon w} \\ 0, & \text{Otherwise} \end{cases}$$

$$e_{wd} = \begin{cases} 1, & \text{if d is the destination of the wagon w} \\ 0, & \text{Otherwise} \end{cases}$$

Continuous variable:

 d_{iw} : distance traveled by the container i to the wagon w;

3.3 Objective function

The objective of the MILP model is to minimize the weighted sum of both the number of used wagons and the total internal traveled distance of PI-containers:

Minimize:

$$\alpha \sum_{w=1}^{W} u_w + \beta \sum_{i=1}^{N} \sum_{w=1}^{W} d_{iw}$$
 (1)

Where α and β are the weighting factors for the number of used wagons and the total distance traveled by containers respectively.

3.4 Constraints

$$\sum_{w=1}^{W} x_{iw} = 1 \quad (\forall i = 1 \dots N)$$
 (2)

$$\sum_{i=1}^{N} x_{iw} L_i \le Q \quad (\forall w = 1 \dots W)$$
(3)

$$x_{iw} + x_{jw} \le \sum_{d=1}^{D} S_{di} S_{dj} + 1 \quad (\forall i, j = 1 ... N, \forall w = 1 ... W, i \ne j)$$
 (4)

$$\sum_{h=1}^{H} y_{hk} \le 1 \quad (\forall \ k = 1 \dots K)$$
 (5)

$$\sum_{k=1}^{K} y_{hk} = 1 \quad (\forall h = 1 \dots H)$$
 (6)

$$x_{iw} \le u_w \quad (\forall i = 1 \dots N, \forall w = 1 \dots W)$$

$$(7)$$

$$e_{wd} \le S_{di} + 1 - x_{iw} \quad (\forall i = 1 ... N, \forall w = 1 ... W, \forall d = 1 ... D)$$
 (8)

$$u_w = \sum_{d=1}^{D} e_{wd} \quad (\forall w = 1 ... W)$$
 (9)

$$|w_1 - w_2| + 1 \le \sum_{w=1}^W e_{wd} + M \left(2 - \left(e_{w_1 d} + e_{w_2 d} \right) \right)$$

$$(\forall d = 1 \dots D, \forall w_1, w_2 = 1 \dots W, w_1 \ne w_2)$$
(10)

$$|w_1 - w_2| + 1 \le \sum_{w=1}^{W} u_w + M \left(2 - \left(u_{w_1} + u_{w_2}\right)\right)$$

$$(\forall w_1, w_2 = 1 \dots W, w_1 \neq w_2) \tag{11}$$

$$u_1 = 1 \tag{12}$$

$$d_{iw} \ge |P_k - R_w| - M \left(1 - z_{iwk}\right)$$

$$(\forall i = 1 ... N, \forall w = 1 ... W, \forall k = 1 ... K, \forall h = 1 ... H)$$
(13)

$$z_{iwk} A_{hi} \le y_{hk}$$
 $(\forall i = 1 ... N, \forall k = 1 ... K, \forall h = 1 ... H, \forall w = 1 ... W)$ (14)

$$\sum_{k=1}^{K} z_{iwk} = x_{iw} \quad (\forall i = 1 ... N, \forall w = 1 ... W)$$
(15)

 $x_{iw}, y_{hk}, u_w, z_{iwk}, e_{wd} \in \{0, 1\}$

$$(\forall i = 1 ... N, \forall w = 1 ... W, \forall k = 1 ... K, \forall h = 1 ... H, \forall d = 1 ... D)$$
(16)

$$d_{iw} \ge 0 \quad (\forall i = 1 \dots N, \forall w = 1 \dots W) \tag{17}$$

Assignment and capacity constraints:

In constraint (2), each container i unloaded from the truck must be assigned to only one wagon w. Constraint (3) ensures that the wagons capacity Q is not exceeded. Constraint (4) ensures that two containers i and j with different destinations cannot be loaded in the same wagon. In constraint (5), two trucks cannot be assigned to the same dock at the same time. Constraint (6) ensures that each truck is assigned to only one dock. In constraint (7), if a container is assigned to a wagon, this wagon is used. Constraint (8) sets the destination for a wagon if there is any container assigned to it. Constraint (9) ensures that if a wagon is used it must have a destination. In constraints (10), the wagons which have the same destination must be consecutive.

Constraint (11) ensures that all the used wagons must be consecutive in order to avoid empty wagons between the used ones (Figure 2). Constraint (12) ensures that the wagons are used starting from the first wagon.

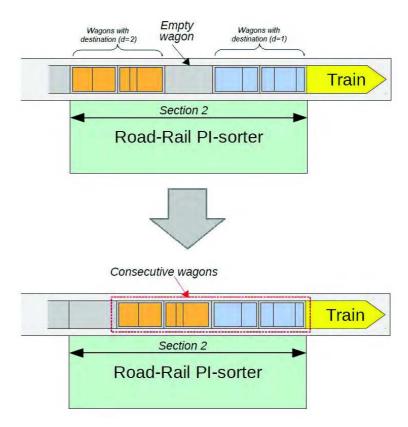


Figure 2: All the used wagons must be consecutive

Distance constraints:

Constraint (13) calculates the distance traveled by a container from the dock to the wagon. Constraints (14) and (15) calculate the value of the variable z_{iwk} which is used for calculating the distance in constraint (13). Constraint (16) ensures that the variables x_{iw} , y_{hk} , u_w , z_{iwk} and e_{wd} are binary. Constraint (17) ensures that the distance d_{iw} is positive.

The minimum value of the parameter *M* is:

$$M \ge \max(W, 5 * 20m)$$

Where W is the total number of wagons in the train, and 5*20m is the length of the road-rail section which can handle 5 wagons at a time (20m is the total length of one wagon and the useful length of the wagon is 18m). Indeed, in constraints (10) and (11) the minimum value of M is the total number of the wagons W. However, in constraint (13) the minimum value of M is the total length of the five wagons (length of the road-rail section).

4 Proposed approaches for solving the Road-Rail assignment problem

In this section, a tabu search meta-heuristic is proposed to solve the model. The solving process starts by assigning the containers to the wagons using the first fit bin packing algorithm of Johnson (1973) to generate an initial grouping of the containers. Then, the tabu search improves the grouping and finds all the possible combinations (Figure 3). The next step of the tabu search is to find the best assignment of the trucks to the docks while minimizing the number of used wagons and the distance travelled by the containers.

4.1 Initial grouping

In a first place, an initial grouping of the containers is generated based on the first fit bin packing algorithm (Johnson, 1973). After selecting a destination, each container with the selected destination is loaded in the first available wagon. Once the capacity of the wagon is exceeded, the next wagon is selected. The heuristic ends when the number of available wagons is reached.

4.2 Tabu search

The proposed tabu search is composed of three steps (Figure 4):

- Improving the initial grouping of the first fit algorithm;
- Generating all the possible combinations of wagons of the initial grouping;
- Improving the trucks' assignment.

4.2.1 Improving the initial grouping of the first fit algorithm

The first step of the tabu search is to improve the initial grouping of the containers. At each iteration of this step, two local search moves are performed with different probability for each move $(P_1 + P_2 = 1)$:

- Insert container i in a different wagon with a probability P_1 .
- Swap container i with another container from a different wagon with a probability P_2 .

Those two local search moves are also considered in the third step while re-assigning the trucks to the docks.

4.2.2 Generating all the possible combinations of wagons

In the second step, the algorithm generates all the possible combinations of the best grouping found in the first step. For example as shown in figure 3, the blue containers with destination (d = 1) can be placed in different wagons on the train. It is important to mention that while changing containers positions, the wagons on which the containers with the same destination are placed must be kept consecutive (Figure 3).

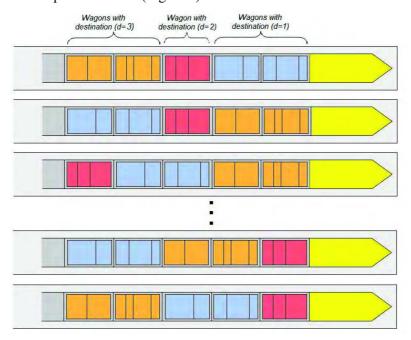


Figure 3: Example of grouping solutions generated in the second step of the tabu search

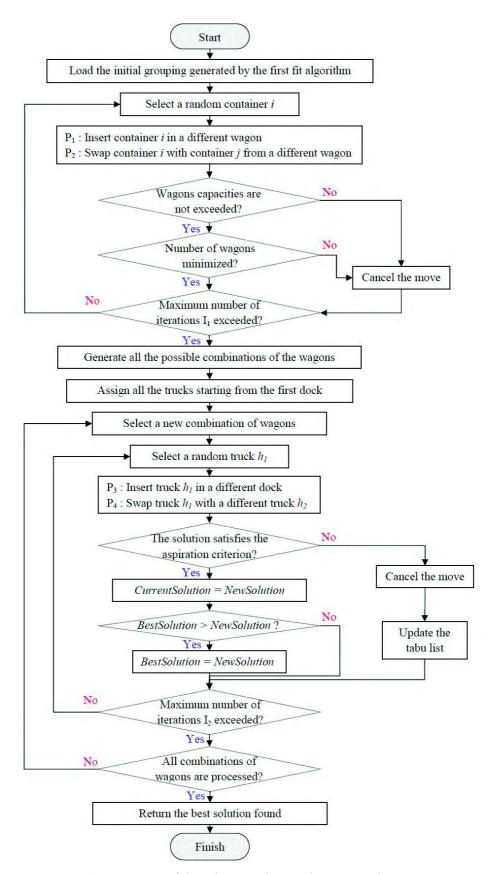


Figure 4: An overview of the tabu search meta-heuristic solving process

4.2.3 Improving the trucks' assignment

The last step is to improve the assignment of the trucks to the docks. For each combination of groupings, a random truck is selected. Then, the same local search moves of the first step are also used in the truck assignment with different probabilities $(P_3 + P_4 = 1)$:

- Insert truck h in a different dock with a probability P₃.
- Swap truck h with another truck from a different dock with a probability P₄.

The next step is to check if the aspiration criterion is satisfied. In this study, the aspiration criterion, which is used to prevent the algorithm from getting stuck in local minima, is the deviation between the current best solution S1 and the new solution found S2.

$$A = \frac{Objective(S2) - Objective(S1)}{Objective(S1)}$$

If the deviation is lower than A, the move will be kept and the solution is accepted as the current solution. Otherwise, the move will be canceled and added to the tabu list. Afterwards, if the current solution is better than the best solution found, the current solution is considered as the current best solution found. An overview of the tabu search solving process is presented in figure 4.

4.2.4 Illustrative example

An illustrative example is detailed in table 1 which shows the containers that are initially loaded in the trucks with their different indices and destinations.

Total Trucks Containers lengths and (container's number, destination) length 1.2 m 6 m 2.4 m 3.6 m 1 13.2 m (1, 1)(2, 2)(3, 1)(4, 2)4.8 m 2.4 m 6 m 2 13.2 m (5, 1)(6, 2)(7, 1)4.8 m 1.2 m 2.4 m 2.4 m 1.2 m 1.2 m 3 13.2 m (8, 1)(9, 2)(10, 3)(11, 1)(12, 2)(13, 3)1.2 m 1.2 m 3.6 m 1.2 m 6 m 4 13.2 m (14, 1)(15, 1)(16, 1)(17, 2)(18, 3)4.8 m 1.2 m 1.2 m 3.6 m 2.4 m 5 13.2 m (19, 2)(20, 1)(21, 2)(22, 2)(23, 1)4.8 m 2.4 m 6 m 6 13.2 m (24, 2)(25, 2)(26, 3)2.4 m 4.8 m 2.4 m 1.2 m 7 10.8 m (27, 2)(28, 3)(30, 2)(29, 3)90 m Total

Table 1: Length and destination of the containers per each truck

The containers indices are presented for each wagon. As can be seen in figure 5, unlike the first fit based heuristic, the tabu search meta-heuristic is able to assign all the containers to the five wagons while respecting the capacity and the destination constraints.

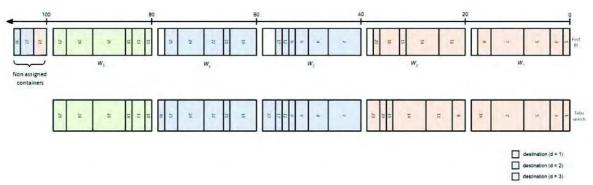


Figure 5: An illustrative example of the grouping of containers

5 Experimental results

The proposed mathematical model is validated on small instances using CPLEX Concert Technology which was able to find optimal solutions only for small instances (table 2). The first fit based heuristic and the tabu search meta-heuristic are implemented in C++. All the tests are performed on a PC Intel^(R) Core^(TM) i3 CPU 2.40 GHz with 4 GB of RAM. A maximum number of iterations without improvement is set for the first and the third step of the tabu search ($I_I = 50000$ and $I_2 = 100000$). Indeed, if no improvement is found after this number of iterations, the algorithm stops, and the best solution found is returned. Those values are fixed after a tuning of parameters.

The tabu search is tested on several randomly generated instances. The total useful length of the 5 wagons is 90 m (5 * 18 m = 90 m). As described previously, wagons with the same destination must be consecutive. Moreover, a wagon cannot load containers with different destinations. The lengths of the containers are uniformly generated among the following values: $\{1.2m, 2.4m, 3.6m, 4.8m, 6m, 12m\}$. The obtained results are summarized in tables 2 and 3 (used wagons and the traveled distance of the containers).

Table 2: Comparing CPLEX and tabu search on small instances

			CPLEX (Optimal)			Tabu Search			
N	D	Н	Used	Distance	CPU	Used	Distance	CPU	Distance
			Wagons		Time (s)	Wagons		Time (s)	Gap (%)
6	2	2	2	23.571	160.416	2	23.571	1.387	0.000%
		3	2	22.857	177.054	2	22.857	1.476	0.000%
	3	2	3	43.572	1220.394	3	43.572	2.707	0.000%
		3	3	42.143	819.489	3	42.143	3.650	0.000%
8	2	2	2	45.714	1094.823	2	46.427	2.177	1.560%
		3	2	64.285	3347.694	2	64.285	3.515	0.000%

Table 3: Results obtained with the tabu search on large instances

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N	D	Н	Used Wagons	Distance	CPU Time (s)					
	2	5	4	28.572	4.566					
10		8	4	51.429	6.899					
10	3	5	4	62.858	4.822					
		8	4	47.142	7.107					
	2	7	5	36.782	7.733					
15		10	5	23.214	5.356					
13	3	7	5	51.784	8.514					
		10	5	70.356	9.451					

In table 2, CPLEX was able to find the optimal solution only for small instances. Tabu search found optimal solution for five small instances (table 2). Since CPLEX and tabu search used the same number of wagons, the gap between the distances is presented in the last column in table 2. For large instances in table 3, since no solution found by CPLEX within a time limit of 1 hour, only tabu search results are presented.

As it can be seen in tables 2 and 3, regarding the number of containers, while increasing the number of destinations the total distance travelled by the containers is also increased.

6 Conclusion

In this paper, a mixed integer linear programming MILP formulation of the Road-Rail assignment problem was proposed. The objective of the model was presented as a weighted sum of two objectives: the number of used wagons and the total travel distance by the PI-containers from the docks to the wagons. A first fit based heuristic and a tabu search metaheuristic were suggested to solve the proposed MILP model. Finally, the proposed tabu search was tested on several instances and gave good quality results.

Future works will be conducted on optimizing the formulation of the proposed mathematical model to solve large instances. More tests will be conducted to test the robustness of the proposed methods on multiple instances.

7 Acknowledgements

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