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Principles towards justification: A focus on teacher actions

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The aim of this paper is to understand how a proposed set of design principles regarding tasks and teacher actions provide students with opportunities to justify. We see justification as a reasoning process that relies on mathematical concepts, properties, procedures, ideas, and, in some situations, particular cases. The teaching intervention, part of a design-based research, is carried out in a grade 7 class of an experienced teacher in nine lessons about linear equations. Data is gathered by classroom observations (video and audio recorded) and a researcher logbook. Data analysis takes into account a set of design principles and a framework regarding students' justifications. The results show that paths of teacher's actions that rely on the design principles enable students to present rather complete justifications based on logical coherence and on mathematical aspects of the situation.

Keywords: Reasoning skills, teaching practices, teacher-student interaction.

Introduction

Developing students' mathematical reasoning is an important aim of teaching and learning mathematics. Students' engagement in reasoning processes allows them to move from using procedures with little or no understanding towards envisioning mathematics as a logical, interrelated, and coherent subject. We consider reasoning as making justified inferences (Brousseau & Gibel, 2005), using processes as formulating questions and solving strategies, formulating and testing generalizations and other conjectures, and justifying them. In this paper, we focus on justification as a central reasoning process. Enhancing students' mathematical reasoning in the classroom requires the set-up of challenging learning environments that go much beyond proposing students to solve exercises using well-known procedures. In order to better understand how teachers may foster students' mathematical reasoning, we conduct a design-based research (Cobb, Jackson, & Dunlap, 2016) that relies on whole class mathematical discussions triggered by exploratory tasks as privileged moments to promote students' mathematical reasoning. In this paper, we aim to understand how a set of design principles regarding tasks and teacher actions that focus on justification might promote students' justifications during whole class discussions.

Students' justifications

In the classroom, justifying, particularly justifying conjectures and generalizations, is a reasoning process that rarely emerges spontaneously. Often, students accept conclusions such as conjectures and generalizations without feeling the need to test or justify those (Henriques, 2010). In many situations, students focus mostly on what is familiar or on ideas that they superficially recall, paying little or no attention to the mathematical properties or concepts implicated (Lithner, 2000, 2008). However, justifying is a reasoning process central to mathematics learning, as it allows students to connect mathematical ideas, concepts, and objects, to present arguments to support statements and conjectures, to solve problems and to develop new mathematical ideas (Brodie, 2010). We consider

justifying as a reasoning process and as the way to prove statements by relying on concepts, properties, procedures, and mathematical ideas and, in some situations, on particular examples.

Justifications in the classroom can occur at different levels regarding formality and complexity. Brousseau and Gibel (2005) propose three different levels regarding the formality of a justification: Level A – Justification that is not formally presented, but that might be associated with the student's actions as a model of his/her action; Level B – A formal but incomplete justification with inferences based only implicitly in elements of the situation or on what is considered to be shared knowledge; Level C – A formal justification based on a sequence of related inferences, with explicit reference to the situation or to what is considered to be shared knowledge. The concept of formal justification referred in these three levels is not necessarily the same as a formal justification in mathematics related to a mathematical proof, but rather to what is considered to be formal in a specific situation, namely accordingly to the grade level and the knowledge of students. However, as students advance through their schooling, formal justifications should be increasingly more formal from a mathematical standpoint, being sometimes equivalent to proofs or to significant parts of proofs. Drawing upon the classifications of Lannin (2005) and Carraher, Martinez and Schliemann (2008), it is possible to consider six levels of complexity: Level 0 – no justification, when the answers do not include a justification; Level 1 – Appeal to external authority, when the justification refers to other individual or reference material; Level 2 – Empirical evidence, when the justification is based in particular examples; Level 3 – Logical coherence, when justification is based on logic; Level 4 – Generic example, when the justification is deductive, but stated in relation to a particular situation; Level 5 – Deductive justification, when the validity of the justification is based on a deductive argument that is independent from the particular cases or examples. At all these levels, a justification presented by a student may be correct, partially correct, or incorrect. Thus, it is important that the students understand what validates a justification and reject justifications based on authority, perception and common sense (Lannin, Ellis, & Elliot, 2011).

Tasks and teacher actions to enhance justification

Students learn to reason by “reasoning and by analyzing the achieved reasoning processes” (Ponte & Sousa, 2010, p. 32). Therefore, to enhance justification processes it is necessary to provide situations in which students have the possibility to justify their answers. As such, in mathematics teaching and learning, and particularly to enhance students' mathematical reasoning, a central aspect is the design of suitable tasks. It is important to understand the nature of those tasks, the ways students engage in them, and the interactions that may emerge (Brodie, 2010).

Several studies refer that problem solving and exploratory tasks have potential to develop students' mathematical reasoning (e.g., Francisco & Maher, 2011; Henriques, 2010). However, it is not necessary or even appropriate that all tasks involve questions at a high challenging level (Brodie, 2010). Such challenge may be infeasible due to time constraints and may lead to students' demotivation and loss of interest. Moreover, while designing a task, its structure and level of challenge should be considered according to the students to whom it is going to be proposed. In addition, we note that, just by themselves, exploratory tasks and problems, are not sufficient to foster students' mathematical reasoning. Teacher actions emerge as equally central to provide situations that promote students' mathematical reasoning. Regarding teacher actions to enhance

justifications, Bell (2011) highlights that the teacher should help students to make sense of justifications, ask for alternative justifications, emphasize what validates a justification, and focus on the explanation of “why”. Also, it is important that the teacher encourages the students to share their ideas and various versions of their reasoning, seeking to consider students’ incorrect or partial contributions and to broaden their valid contributions (Brodie, 2010).

Methodology: Design, participants, data analysis

This paper originates from a broader research study that aims to develop a local theory about enhancing students’ mathematical reasoning in the classroom, following a design-based research (Cobb et al., 2016). In order to do so, we established several design principles (Cobb et al., 2016), i.e., heuristics that structure the intervention, based on the research literature and on a previous cycle of experimentation focusing on tasks and teacher actions to enhance students’ mathematical reasoning. Four of these principles specifically focus on justification. One principle refers to task design and states that tasks must include questions that ask for a justification of answers or of solving processes. The other three principles concern teacher actions and indicate that the teacher must promote situations that prompt students to (a) justify and present alternative justifications; (b) identify valid and invalid justifications, indicating why; and (c) share ideas, namely by accepting and valuing incorrect or partial contributions, deconstructing, supplementing, or clarifying them.

This is the third cycle of design, after a first cycle that took place in lessons about sequences and a second cycle in lessons about linear equations. This third cycle took place in a public school in a grade 7 class with 27 students (12-13 years old), throughout nine lessons about linear equations. A detailed plan of each lesson was prepared considering tasks specifically designed to promote students’ mathematical reasoning and considering possible teacher actions. Each lesson plan was proposed by the first author and discussed in detail with the teacher, who made all the amendments and adjustments that she felt necessary considering the class characteristics and the available resources. The participating teacher was selected because of her experience and her availability to consider changes on her practice. All participants in this study are volunteers, provided an informed consent, and their names are fictitious.

Data analysis is centered on the design principles regarding tasks and teacher’s actions and also on students’ justifications. The episode that we present is from lesson eight that aimed to lead students to be able to relate equations and functions. This lesson was directly observed and video and audio recorded and notes were made in a researcher’s logbook.

An episode about equations

Task and context

The episode presented in this paper focuses on the first part of the task proposed in lesson eight of the nine lessons that constitute the linear equations unit. This segment of the task (Figure 1) aims to lead students to establish a procedure to figure out the intersection point of two functions. Earlier in the school year, the students learned about algebraic and geometric representations of linear functions, with no participation from the researchers.

Frances received a plant as a gift and she registered its growth. Simon thought it was a really nice idea and, on the same day, bought a plant and also registered its growth. The functions that follow represent the height of both plants on their first days with the students:

$$\text{Frances' plant: } f(x)=0.4x$$

$$\text{Simon's plant: } s(x)=0.2x+2.2$$

1. Represent graphically the functions f and s .
2. Based on the previous representations, indicate on which day the plants have the same height.
3. Consider the comment: "Graphs are not necessary to know on which day the plants have the same height. Knowing the functions that represent the height of each plant is enough to find out when they are equal". What would be the other approach to figure out the day when the plants have the same height? Justify your answer.

Figure 1: Proposed task about functions and equations

In the first two questions of this task, the students can support their answers by using GeoGebra app, as this particular school has iPads available by request. This was not the first time that the students used either the tablets or GeoGebra. Taking into account the design principle regarding task design, question 3 asks for a justification.

Justifying based on knowledge about functions

At the beginning of the lesson, the teacher asks a student to read the questions to the class and clarifies the aims of the task and the tools to use. Then, students work autonomously, in pairs, for a couple of minutes. After inserting the algebraic expressions of the functions in GeoGebra, some of them state that the plants have the same height on the 11th day. The teacher begins the whole class discussion by asking for a justification to that answer:

Teacher: How do you know that it is on the 11th day? (Several students raise their hands in order to answer.) Isa.

Isa: Because, if we check, both straight lines intersect in 11.

The teacher's invitation to justify (principle (a)) led Isa to justify her answer to question 2 based on her previous knowledge about functions. This justification is incomplete regarding referencing "in 11", however, it refers to elements of the situation, namely, the graph representations of both functions and the intersection point. Thus, Isa presents a generic justification regarding the available data (level 4 justification).

Aiming to complete Isa's answer (principle (c)), the teacher revoices this student's answer leading to a more accurate justification:

Teacher: In 11...

Isa: In point 11.

Teacher: In point 11?

Gabriel: Abscissa.

Teacher: In the point with abscissa 11.

By referring to parts of students' answers, the teacher implicitly identifies what is invalidating the justification (principle (b)), and based on the students' answers, the teacher highlights what completes the justification (principles (b) and (c)).

After validating Isa's answer, the teacher decides to go further on justifying, asking for another justification (principle (a)):

Teacher: Why am I looking at the intersection in x -axis . . . For the value in x -axis?

Isa: Because x [-axis] is the axis of objects...

Teacher: Yes... And how do I know if I am looking for an object or looking for an image?

Isa's justification relies on mathematical concepts (level 3 justification), however, her statement is not sufficient to provide a justification in this specific situation as it does not relate to the context of the problem. Once again, the teacher validates a partial contribution from Isa and encourages the students to complete that contribution (principles (b) and (c)). Another student tries to justify, but he does not add any information to what Isa has already said. Then, Gabriel participates in the discussion:

Gabriel: I think it is because the height is in... In... I just forgot the name.

Teacher: The axis...

Gabriel: The ordinate, the ordinate axis, and the days are in the abscissa [axis].

At this point of the discussion, Gabriel adds some relevant information to the justification, by relating objects and images of these functions to the context of the situation (level 4 justification). Despite this relevant relation, the required justification is still incomplete, and so the teacher continues on encouraging students to justify (principle (c)):

Teacher: And how do I know that days, in this particular case, are objects and heights are images?

Gabriel: Because there is... I forgot it...

Leonardo: Why it is that way, isn't it? Let me reason the other way around... If the height would be there [in x -axis]...

As the students struggle to address the teacher's question without being able to justify (level 0 justification), the teacher gives some more information in order to complete the justification (principle (c)):

Teacher: What do the functions s and f represent?

Several students: Height.

Teacher: Plant's height, right? Depending on what?

Several students: Time.

Teacher: The time that elapses, in days. OK, very well.

This information provided by the teacher leads the students to easily identify dependent and independent variables, thus completing the required justification (level 4 justification).

Both this and the previous justifications in this segment rely on students' prior knowledge about functions and emerge during the whole class discussion supported by the teacher's actions based on the defined design principles.

Justifying based on knowledge about equations

Right after discussing question 2, the teacher introduces question 3. At this point, a student immediately proposes a strategy to solve this question. This leads the students to engage in a new segment of whole class discussion, without having time to work autonomously on this question:

Teacher: Now, pay attention to question 3, because... (Santiago raises his hand). Tell me.

Santiago: So teacher, we have that thing that was G.C.D. (M.D.C. in Portuguese), I believe it was... Multiple (regarding the M in M.D.C)...

As the teacher allows Santiago to intervene, he brings to the discussion a strategy based on a mathematical concept that was not expected in this situation. Despite seeming a senseless idea, the teacher lets him go on with his explanation (principle (c)):

Teacher: Greatest common divisor?

Santiago: Yes, something like that. Can't we use it to answer to when do they intersect? . . . I can't recall it, but wasn't there something in common? Doing each number and then...

By allowing Santiago to justify it is possible to understand that, despite incorrect, the student's justification relies on an idea with some logical coherence (level 3 justification). Thus, both in G.C.D. and in intersecting functions one is trying to find "something in common", as he refers. At this point, the teacher poses more questions in order to deconstruct the misconceptions about G.C.D. which leads the other students to identify Santiago's strategy as not fitting to this situation.

After clarifying that, Clara presents her strategy:

Clara: We can use an equation (referring to $0.4x=0.2x+2.2$), and the number that we get is the day they have [the same height].

...

Teacher: What are you expecting as a solution of this equation?

Several students: 11.

Teacher: 11. So, confirm that.

Evoking the information obtained in the previous questions, the teacher supports Clara's strategy to solve this equation and, by asking to confirm the result, she prompts the students to justify (principle (a)) that 11 is the solution of the mentioned equation. Students do this in autonomous work, and then Daniel intervenes:

Daniel: Teacher, it isn't.

Teacher: It isn't? So, solve the equation over there (on the board).

As Daniel solves the equation on the board, the teacher realizes that he has just missed an x in one of the steps and, by following his solving process (principles (b) and (c)), the justification based on procedures is properly achieved (level 4 justification).

In this segment of the discussion, justifications, either valid or invalid, are based on knowledge about mathematical concepts. These justifications emerge when teacher's actions rely mostly on

encouraging students to share ideas and completing those ideas.

Conclusion

All the situations in the episode that we analyze were prompted by the proposed task. This task, focused on making sense of the relationships between equations and functions, provides an opportunity for students to develop a procedure to find where two functions intersect. This underscores the idea that collective activity in whole class discussions enable students to share, debate and clarify their reasoning and, in particular, their justifications (Galbraith, 1995).

This study shows that, if particular teacher action paths that rely on the design principles are followed, justifications are likely to emerge in whole class discussions. In this episode, when the principle regarding asking for a justification is enacted, the students present justifications. These justifications are based on previous knowledge about mathematical concepts or ideas or on known mathematical procedures. Thus, those are justifications based on logic or deductive justifications stated in relation to a particular situation. However, these justifications are often incomplete and sometimes incorrect, and, as it has been seen in previous research (Galbraith, 1995), the use of available information about a certain mathematical concept or idea is not always adequate given the definitions or assumptions of the task. When the justification is incomplete, the teacher tends to encourage the students to complete the justification, validating or invalidating their statements only implicitly. Depending on her appraisal of the support that the students need to mobilize their knowledge, the teacher provides them with more or less information. By relying on these principles, the complete justification emerges from the whole class discussion. When an invalid justification is at stake, and according to the defined principles, the teacher values students' contributions and keeps on encouraging them to present their ideas, leading them to present justifications based on logical coherence or on mathematical procedures. In these situations, where a student's justification is incorrect, teacher's actions strive either to abandon that justification and to focus on an alternative justification or, if possible, to adjust it to its correctness.

In this particular episode, students' justifications, despite sometimes incomplete or invalid, tend to be reasonably formal as they are based on mathematical aspects of the situation. Also, in the context of a whole class mathematical discussion based on the design principles, those justifications emerge often as justifications in a logical coherence level and, as students continue to add information, those justifications became generic example justifications. As this episode illustrates, in order to provide students with opportunities to move in-between levels of justification, it is not enough to ask students to justify and validate their justifications, but also to accept and value partial and incorrect justifications. Thus, the presented paths are likely to provide promising environments to develop students' justifying abilities, hence to be better prepared to deal with mathematical proof later in their schooling.

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