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Tool-based argumentation

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The study presented in this report is part of a research project concerning the mediation of artifacts in teaching and learning geometry. In this paper we analyze the first step of our research which concerns the student-pantograph interaction and the identification of the math laws incorporated in the machine. During this interaction we are specifically interested in arguments that students produce for supporting their claims. Tools, especially mathematical machines, may support argumentation processes focusing either on the structure of the machine, or to the embodied math concepts that emerge from the machine's movement. Our research has shown that these arguments hold mainly on the topological conception of geometric figures.

Keywords: Argumentation, instrumentation, figural concept, topology, pantograph.

Introduction

Mathematics is in close relation with material and non- material artifacts. Artigue (2002, p. 245) points out that *"*the development of mathematics has always been dependent upon the material and symbolic tools available for mathematics computations*"*. Teaching and learning geometry may be mediated by visual or design artifacts. Research on the use of artifacts in Geometry teaching as means to facilitate understanding and learning, has tended to focus mainly on technology integration into curriculum- such as computer software packages focusing on how learning takes place when students use such artifacts. In addition to the use of information technology in schools, the MMLab researchers (e.g., Bartolini Bussi, 2010; Mariotti et al., 1997) have recommended and investigated from an epistemological and pedagogical aspect the use of mechanical artifacts -mathematical machines- as a way to generate mathematical ideas or concepts in the classroom. The geometrical machine *"*is a tool that forces a point to follow a trajectory or to be transformed according to a given law*"* (Bartolini Bussi & Maschietto, 2008). These machines (for example pantographs) are linkages that allow the implementation of geometrical transformations, such as symmetry, reflection, translation, and homothety.

The study presented in this report is part of a research project concerning the teaching of geometry at an upper secondary school in Greece (early 2016). Our research project was conducted in the framework of an attempt to incorporate artifacts that bear geometrical machine characteristics, in the instruction of Euclidean geometry.

Theoretical framework

The theoretical framework of the instrumental approach was used for analyzing our observations (Verillon & Rabardel, 1995). According to this approach, the artifact is the material or symbolic object, while the instrument is defined as a mixed entity made up of both artifact and utilization schemes. In order for an artifact to lead to the development of an instrument, "the user has to develop mental schemes, which involve skills to use the artifact […] the birth of an instrument requires a process of appropriation, which allows the artifact to mediate the activity. This complex process is

called the instrumental genesis" (Drijvers & Trouche, 2008, p. 370). "The instrumental genesis, is a two sided process. On the one side, the construction of schemes is oriented toward the use of the artefact: the instrumentalisation. On the other side, the construction of schemes is oriented toward the task to be achieved: the instrumentation." (Goos et al., 2009, p.313). In our study we investigate the instrumentalisation process, i.e. the discovery of the elements and qualities of the artifact by the user. For our purpose, teaching homothety, the pantograph was the most convenient tool. Following Drijvers and Trouche (2008, p.369) we consider that the utilization schemes students construct during the instrumentalisation process, contain operational invariants that consist of – explicit or implicit – knowledge in the form of concepts-in-action or theorems-in-action.

Martignone and Antonini (2009) analyze more specifically the pantograph utilization schemes. They identified the "utilization schemes linked to the components of the articulated system (as the constraints, the measure of rods, the geometric figures representing a configuration of rods, ect.) and the utilization schemes linked to the machine movements" (p. 1253). In the second case, they distinguished two main sub-families: (1) the utilization schemes aimed at finding particular configurations obtained stopping the action in specific moments (limit zones; generic or particular or limit configurations) and (2) the utilization schemes aimed at analysing invariants or changes during continuous movements (wandering, bounded, guided; of particular configuration; between limit configuration; of dependence or in the action zones) (p. 1254). They have also conducted research on the argumentations produced in activities employing the pantograph (Antonini & Martignone, 2011). They distinguish between arguments about a) the drawings traced by the machine, b) the movement of the machine (as some dynamic properties of the articulated system), and c) the structure of the machine. Arguments about the structure are distinguished between referring to the figural aspect of the machine and the conceptual component of the geometric figure, which they discern in the structure.

In this paper we analyze the kind of argumentation produced by 16-17 year old students during the phase of investigating (a) the structure of a pantograph and (b) the configurations and (con) formations produced by the movements of its structural components.

Our research hypothesis was that the argumentation produced by the students, is in close relation with the machine's characteristics: students' explanations in the form of concepts or theorems in action are the exteriorization of precise utilization schemes developed by investigating the structure and the movement of the pantograph.

Methodology

The first step of our research, that is the subject of this paper, concerns the student-pantograph interaction. 26 students of an 11th grade class (16-17 years old), of different learning abilities and interests, took part voluntarily in the experiment. Participating students worked in 6 groups (4 groups of 4 people, and 2 groups of 5 people). All the participating students had no prior experience with any artifact, except for compasses and rulers. Two meetings were carried out with the groups, of four hours in totals, and members of two groups (A and B) were interviewed. The working sessions and interviews were audio recorded, and afterwards transcribed. The transcripts, visual material (photographs), and written reports of the groups constitute the data for the analysis.

The artifact, with which the students were asked to work, is a geometrical machine (linkage) with the characteristics of a pantograph, specifically a version of Scheiner's pantograph (Figures 1, 2). The building blocks of the pantograph model were two equally-sized wooden rods 30 cm long [OD=AE=AC=BD], held together by the links/pivots [A, L, D, and K] in the middle, thus forming a parallelogram [ALDK]. The rods had notches allowing reassembly of the linkage while maintaining its properties, provided that the links were placed in such a way that the ratios of the lengths of rod's parts were equal in each rod. The pantograph's linkage was mounted on a wooden platform (60cmx60cm).

Figure 1: The linkage

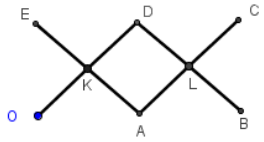

Figure 2: Schematic representation of the linkage

The tasks given to be treated by students concerned exploring the pantograph's structure and the investigation of special configurations and formations produced by the movements of the structural components of the linkage. The choice of the tasks was made following the distinction by Martignone and Antonini (2009) of the utilization schemes during pantograph exploration: the utilization schemes linked to the components of the articulated systems and the machine movements. The students invited to observe carefully the articulated system and to describe elements and characteristics of its structure such as length relationships and the mode of the rods' connection; to try to detect schematic shapes, properties and relationships that comprise its form and to create schematic representations of the articulated system (forms of system).

The analysis of the transcript was done following the classification of Martignone and Antonini (2009) about mathematical machine utilisation schemes and the kinds of arguments students use during the exploration of a geometrical machine, as a pantograph (Antonini & Martignone, 2011). In our research we examine utilization schemes linked to the structure of the machine in a static and in a dynamic status (: movement of the machine). For the first case (: static structure), hereafter, we use the symbols SF and SC, for the figural aspect of the machine and the conceptual component of the geometric figure which students discern in the structure, respectively. For the second case (: dynamic structure) we use the symbols MF and MC. In fact, in this second case we investigate the way students justify the embodied mathematics in the structure of the machine.

In this paper, we present and analyze the arguments produced by Group A -four girls hereby referred to as S1, S2, S3 and S4- as they investigate the configurations produced by the movements of the pantograph's structural components. Apart from space constraints, the omitted group (B) was already familiar with the abstract math concepts involved, as opposed to group A whose gradual discovery of the tool yielded high resolution into the thought process addressed by our research hypotheses.

Analysis of a transcript

The students of Group A took advantage of the capability of the linkage pivots to alter its form by opening and closing its parts, identified the property of the midpoint for the position of the pivots (as K, L in Figure 2) and inferred the equality of their lengths (equal rods).

S1: We identify a rhombus configuration (Figure 4), because the sides are equal as half of equal segments (SF, MC).

The student S1 perceives the components of the machine's structure as geometric objects and identifies in them geometrical relations. The equality of segments arose as an ascertainment while opening and closing the linkage. She uses the definition of rhombus (: because the sides are equal*)* and together with the figural aspect of the linkage's structure (: half of equal segments) to argue that the quadrilateral ALDK is a rhombus (Figure 2).

S2: Can we mention implied properties too?

Interviewer: Describe what you consider important.

This encouragement led the students to operate the artifact more dynamically, not only by opening and closing the rods, but also exploring characteristics and properties of specific conformations and support their claims, taking advantage of the capabilities provided by the pivots.

S2: Isosceles triangles are formed (*with her finger traces on the artifact the triangles OKA and ALB in* Figure 2) … if we assume that the articulated system can close its ends *(she points at the end of the rods and moves them until the linkage is closed,* Figure 3*)* and if we assume that it has a base because those (she *points at the pivots K and L,* Figure 2) are midpoints of equal sides (MF, MC).

The student executes the motion mentally ("if we assume … that it closes … and if it possesses a base"). They envision triangles in the linkage structure, though triangles do not exist. For these students the triangles they refer to are figural concepts (Fischbein, 1993). The students imagined the triangle and the reason that it is isosceles by moving the rods so that one rod meets the other (MF), while they deduce the equality of sides as halves of equal segments (MC).

S3: Maybe they're not triangles because they don't close?

The dimensions (thickness) of the rods don't allow them to coincide. The limitations of the artifact create a conflict between the figural aspects of the structure and the conceptual aspects of the geometrical figures of the articulated system. The students doubt whether they can actually consider it as a triangle.

S1: If we move the linkage, in a special position, we have a square (MC).

Figure 3: Closed linkage

Figure 4: The rhombus

Figure 5: The square

The student mentions the word "square" without justification. They have been taught that square is a special case of the rhombus. By moving the artifact, they predict that for a specific position of the rods, a square will be formed (Figure 5). Their square is of a conceptual nature. At the same time, it has an intrinsic figural nature: only while referring to the artifact one may consider operations like "if we move….we have a square". As a matter of fact, the square to which they refer cannot be considered as either a pure concept or a mere concrete representation.

Interviewer: How do you know it's a square?

S1: We have a right angle.

Interviewer: How do you know the angle is right?

S1: Since we have the capability of opening and closing an angle, then it can take all values between 0 and 180 … then definitely one of these values will be 90 … so one (angle) will be 90 degrees (MC).

The students imagine through the rod motion a continuous creation of angles between 0°-180° and that obviously 90° will exist as mean of the interval, and consequently will correspond to an angle.

S3: It is not just one particular (right) angle … but anyone… for a specific position of the rods (MC).

Here is shown clearly the conceptual aspect of the argument. To the students the angle is not a characteristic of a static position of the artifact, but is dynamically created independently of the nature and position of the artifact on the planar surface (generalization). The right angle has been disconnected from the particular tool and is being described dynamically through its measure as a specific value in the interval 0°-180°.

S4: (*She opens and closes the rods and observes where the ends move and where the joint*) … the ends of the rods (the points O, A and B in Figure 2) … remain always on the same line as O (MF).

Interviewer: How do you know that those are on the same line?

S2: From the triangles […]

Interviewer: You assume that the base of one isosceles is an extension of the other, how do you know?

The student assumes that the bases of two isosceles triangles are on the same line. The researcher's intervention is critical. She is suspecting that parallelism will allow her to transfer angles so as to justify the conjecture of the points' co-linearity.

S1: Those (*points at pairs of opposite rods*) … are parallel … they are always parallel (MF)… from construction … because their distance remains always equal.

Parallelism is suggested to the students by the artifact's structure, and is reinforced during the artifact's movement. Substantiation of equal distance bears is theoretically unfounded.

The fact that the distance between the rods is always the same does not arise from a mathematical justification, but from analysis of the tool's structure. The student considers the tool as embodiment of some geometrical properties. From the moment they regarded the square as a special case of rhombus, they pointed out the constant distance (: opposite sides of the square) between the lines containing opposite rods, a fact that leads them to parallelism. In fact the student overgeneralizes (Gärdenfors, 2004, p.151) the equal distances in the case of the square, to any other position of the rods. She implies that in any configuration (even in the case of rhombus) the distances are equal.

Interviewer: How do you know this distance is always equal?

S1: The distance from here (*the student opening two fingers represents the supposed distance between two opposite rods*) is always equal to this one (DH=DZ, in Figure

6) and are equal in all positions (Figure 7) … same with this one (Figure 8-a square) … they can also be unequal, of course they're not always the same but they're equal in every position (MF) and the maximum distance is when it (the tool) forms a square (MC).

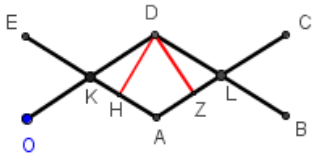

Figure 6

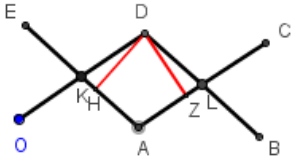

Figure 7

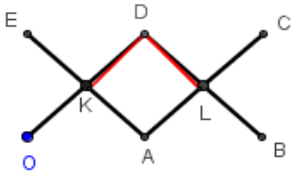

Figure 8

Figures 6,7,8: Schematically represent the conformations of the artifact that the student trace on the drawing paper.

The student's spatial conception is topological in nature. This conception appears first as an overgeneralization, and following the interviewer's persistence, it is expressed clearly through the movement of the artifact. For S1 rhombus and square are topologically equivalent (homeomorphic), leading to the conservation of equal distances (Figures 6, 7, 8). Piaget and Inhelder (1967) consider the structures of topology, to be the origin of the ontogenesis of spatial thinking.

Interviewer: Why is it the maximum distance?

S1: Because if I go over here (*she refers to her equivalent to Figure 6 drawing on the board*) from a right triangle (*shows on the drawing the triangle DZL to which she's referring*) I have a smaller distance, because this distance (DZ) is the smallest, as it's a leg … so these form this angle equal to this angle (*referring to the right angles of the right triangles DHK and DZL in her drawing*).

The argument the student formulates in her attempt to justify why the distance of opposite rods becomes maximum, in the case of a square, has two components. One concerns a geometrical property (: the perpendicular segment is shorter than any oblique) (MC), while the second component is based on figural characteristics related to the conformations of the tool (rhombus –square) (MF). In line with Radford (2003), we could consider the student's argument as a factual generalization: "A factual generalization is a generalization of actions in the form of an operational scheme that remains bound to the concrete level." (Radford, 2003, p.65).

Interviewer: But why are they parallel?

S3: Since the general shape is a rhombus (MC) and the distances between them (*meaning the opposite sides*) are always equal (MC).

The students identify the characteristics of a geometrical problem they have tackled in the past (distances of opposite sides in a rhombus are equal), but being unable to give a geometric proof, remain on a topological approach.

Discussion and conclusions

Our research hypothesis was that the argumentation produced by the students, is in close relation with the machine's characteristics. In fact, in agreement with the findings of Antonini and Martignone

(2011) students produce arguments both on a figural and a conceptual level even for the same investigation. The arguments used by the students are supported by the linkage's continuous movements in the action zones or between limit configurations.

Students produce arguments based on the figural characteristics of the tool mainly in two cases: when the structure of the tool renders their observation probable (3 collinear points), or when the tool produces the "proof" mainly through the linkage's continuous movements (: the triangle is isosceles, because the rods overlap). The figural characteristics make some facts "obvious" for the students to the point to give to the observer the impression of lack of substantial comprehension of geometrical definitions. For example, the students, in spite of recognizing the rhombus, do not readily deduce the parallelism of its side, and seek more complex proof, mainly via the artifact's attributes.

The instrument's structure may favor the theorems-in-action formulation (Fischbein, 1993), providing convincing argumentation: an angle is right, as it can be constructed by the artifact on a specific moment of its movement. Although the students were familiar with elementary proof in the framework of Euclidean geometry, their way of thinking was mainly topological. It seems that the mathematical tool, through its capacity for motion, favors such approaches. This was demonstrated not only in the justification that an angle is right, but also in the justification of two lines' parallelism. This fact implies the benefit of artifact use in Geometry teaching, before a formal introduction to the concept of proof.

Nevertheless, the tool's restrictions (e.g. rods not fully overlapping), create a conflict between the figural aspects of the structure and the conceptual aspects of the geometrical figures of the articulated system. For example, when trying to superimpose the wooden sides of the triangle in order to check if they are equal, students face the tool's restriction. Nevertheless, they can imagine it as an isosceles triangle. Those restrictions have two outcomes: a positive one being that students are forced to think in a more abstract manner and the negative one that the restrictions may lead them to false conclusions, giving the occasion of fruitful discussions.

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