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Looking for the roots of an argument: Textbook, teacher, and student influence on arguments in a traditional Czech classroom

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As part of a larger investigation aimed at getting a deeper insight into how particular teacher beliefs influence the role of the teacher, the students, and textbook materials in arguments that take place in one classroom, this study shows specific teacher beliefs that determine the role of each of the two other factors: the students' contributions and the textbook influence. This paper presents findings observed in a case study of a teacher who holds more traditional beliefs about teaching and learning of mathematics, in a 7th grade classroom. Namely, I present cases of conflict in preferences for particular warrant forms between: a) the teacher and the textbook authors b) the teacher's own beliefs, and c) the teacher and the students. I then interpret these in terms of the teacher's particular beliefs and show how they affect the theoretical model.

Keywords: *Mathematical argument, teacher mathematical beliefs, textbook curriculum.*

Introduction

Whether providing a mathematical proof of a theorem, explaining a formula, or a solution to a word problem, arguments are an inseparable part of mathematics teaching and learning. A lot has been said about argumentation practices and norms that guide those practices in various contexts. Literature has focused on the role of argumentation in textbooks, uncovering differences among textbooks in terms of arguments presented (Stacey & Vincent, 2009; Thompson, Sharon, & Johnson, 2012; Zalska, 2012), opportunities for arguments (Stylianides, 2009), and in providing teachers with support in argument-based tasks (Stylianides, 2008). The differences become even more complex when researchers describe mathematics teachers' beliefs concerning argumentation in their classrooms (e.g., Staples & Bartlo, 2012), the acceptability of certain types of arguments (Biza, Nardi, & Zachariades, 2009), and students' perceptions and preferences of arguments (e.g., Levenson, 2013; Levenson et al., 2006).

But what specific arguments actually take place when the teacher's beliefs meet the textbook authors' and the students' in a classroom? What is the role of the textbook and what role do students' contributions play in the class conducted by a teacher with a specific set of beliefs?

Theoretical framework

According to Remillard's (2005) review, the textbook curriculum's role is important but the levels of participation in the intended curriculum vary greatly. The general model holds that a teacher selects tasks from the text, designs their implementation, supplements it with other tasks, and, finally, improvises based on the student contributions (Remillard, 2005). Sherin and Drake (2006) further find that teachers approach these activities in different ways and link these to teachers' experiences as mathematical learners. Schoenfeld (2010) argues that the actions an individual takes can be explained by their enacted beliefs, goals, and resources (including knowledge).

The individual students' mathematical knowledge and their perceptions of the expectations put on arguments they produce, as well as their own preferences and beliefs, can differ from their teacher's

expectations (Planas & Gorgorió, 2004; Levenson et al., 2006). The students' contributions, requests or choices of arguments are the result of their own knowledge, beliefs, and goals; they have their weight in the negotiation of socio-mathematical norms regarding mathematical arguments and explanations.

Based on the above literature, I adapt Remillard's (2005) model to propose a framework for studying the potential influence of the three main participants on the arguments, or the "enacted" arguments. Namely, the model theorizes that: 1) the curriculum may: provide examples of, requests and opportunities for arguments (tasks) to be enacted; it may also provide guidance for the teacher, based on the textbook authors set of beliefs, resources and goals; 2) the teacher may, based on their own set of beliefs, resources and goals: evaluate (select), design and provide the examples, requests and opportunities for particular arguments and may need to make immediate decisions about arguments prompted by students, and 3) the students' actual requests for arguments, for clarifications of arguments, as well as their own arguments or claims, which in turn are given by their own beliefs, goals and resources.

In my research, I focus on the similarities and differences between the interactions of the three participants in this model, in classrooms with teachers with different general beliefs about mathematics and its teaching and learning. Further, I investigate what specific teacher beliefs underlie the particular interactions when it comes to specific types of arguments or warrants. In this paper, I present some findings in the case of a teacher who holds a set of beliefs that tend to be associated with traditional views.

Participants and data

Karen is an experienced mathematics teacher who was identified, within a broader investigation (see Zalska & Tumova, 2012), as a teacher with strong utilitarian beliefs about mathematics education. Two extensive interviews as well as short post-lesson interviews were conducted with Karen to infer her professional beliefs as well as her own intentions and interpretations of events in the lesson. She had been working with her class for almost two years prior to the data collection, to ensure that social norms in the classroom have been established. The number of students ranged from 15 to 18, with about an equal distribution of male and female students. The class use a main-stream textbook series, one of the most popular ones in the country. Karen was among the teachers who approved the choice of the textbook in her school, and her students each have a copy of it.

The data consists of interview and lesson audio-recordings, fully transcribed, and photographs from series of five lessons that Karen taught on the topic of percent. The analyzed textbook text included the unit on percent and the corresponding text in the teacher's book where authors provide teachers with commentaries for particular parts of the text and the tasks.

Data analysis

In order to be able to establish differences between mathematical arguments, I will adopt the following terms from the widely used Toulmin's model in the following way: a (mathematical) *argument* denotes a sequence of statements (including visual statements) that is provided with the intention to show that a mathematical *claim* (specific or general) is true (or not). In this study, arguments include explaining of an answer to a problem, as well as the working out of the answer to a problem. A *warrant* is one such statement that directly supports the claim. In the context of a

classroom, it is a statement that does not require further explanation, i.e. is accepted as true. I will consider two arguments to be different if they contain different forms of warrants (e.g. representations) or a different sequence of warrants.

The textbook data was analyzed in accordance with the theoretical framework: the arguments that were provided were analyzed in terms of warrant forms and sequences. The tasks were analyzed as requests and opportunities for arguments (towards a claim that contains a problem's solution). There were no specific requests for arguments.

The transcript of the lessons and the text was first analyzed for episodes of argumentation to establish specific context for argumentation and social norms in the classroom. Next, the identified episodes of argumentation were broken down to individual arguments and warrants and warrant forms were identified in order to investigate where differences between arguments were present. The kind of student and teacher participation on the argument was also taken into account, in order to separate the cases of arguments provided by the teacher (i.e. when Karen elicited an argument step by step and students only provided the final part of a requested warrant) from those suggested by students themselves.

The arguments observed were then compared to the examples of arguments in the textbook, comparing warrants and warrant forms. Further, the relevant part of teacher's manual was analyzed for commentaries and any additional rationale given a particular argument in order to get insight into the text author's beliefs. Karen's own comments about particular arguments and warrant forms, in class and during interviews, were also analyzed to gain insight into the beliefs behind her decisions.

In this paper, I present the instances when an argument chosen by Karen did not correspond with a) the textbook, b) her own belief about mathematics, and c) her students' contributions. I selected them to illustrate the choices made by Karen, to pinpoint her specific beliefs, linking them with the students' and textbook influence.

Efficiency and insight: Karen and the textbook

The arguments that Karen exemplifies in her classes when she teaches her students to solve problems involving percent differ from those in the textbook in two aspects. The textbook introduces the rectangular representation (see Figure 1) as part of problem-solving, a form of warrant(s); the authors sketch out the known and unknown quantities.

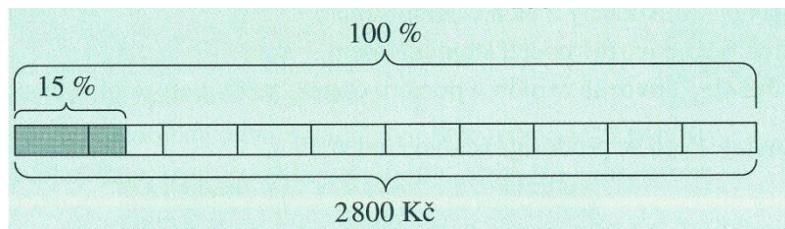


Figure 1: A rectangular representation of a 15% percent discount

Similarly, the textbook introduces one method for solving word problems with percent. The authors base the arguments on the concept of direct proportion, in particular, on the fact that the percent part changes in the same ratio as the percent. This idea is then used as a warrant in the method of the ratio-based rule of three (see Figure 2), which is explained and practiced in an earlier chapter in the book, the unit on ratios.

Zkus nejdříve podle obrázku odhadnout výsledek.
K přesnému výpočtu použijeme trojčlenku.

počet procent	4	100 %	...	125 Kč	2	základ
	3	108 %	...	x Kč	1	procentová část

$$\begin{array}{rcl}
 x : 125 & = & 108 : 100 \\
 \frac{x}{125} & = & \frac{108}{100} \\
 x & = & 125 \cdot \frac{108}{100} \\
 x & = & 125 \cdot 1,08 \\
 x & = & 135
 \end{array}$$

Obchodník prodává trička po 135 Kč.

Figure 2: The rule-of-three method

In contrast, Karen does not use the rectangular representation at any moment in her classes. The arguments that she does show students are given names ("one percent", "with a decimal", and "ratio") and referred to as "methods". The majority of warrants for methods are based on the multiplicative relationship of percent part and the base, and on the definition of one percent, as corresponding to one hundredth, either as a fraction or decimal.

In the authors view, in the teacher's book, the geometrical representation helps students to get a better insight into the problem. Similarly, the authors assign the use of the ratio warrant the prominent role of helping students to get an insight into the problem.

This belief about a need to understand the problem through the use of a particular method or warrant seems to collide with Karen's beliefs about what is important for her students. Rather, she values efficiency and straightforwardness in problem-solving. Hence, she introduces neither the rectangular representation nor the rule-of-three arguments when solving word problems in her teaching. In fact, she discourages her students from using it (albeit acknowledging its existence and its effectiveness):

Teacher: Someone mentioned a third method, in case you study from your textbook, [I don't recommend it, only if someone gets] really lost and needs a crutch [...] but in the time you write it all out (referring to the method), you might as well have finished other three problems [using the other methods].

Choosing not to justify – Karen's beliefs in conflict

The below example of a dialogue gives us a sense of how Karen's beliefs about the need to provide mathematical arguments for methods and general mathematical statements manifest themselves when the class discuss the percent – decimal relationship.

Teacher: So, if we have 18% (writing on the board), how do we get a decimal?

Students: Eighteen divided by 100.

Teacher: We divide by 100. Why? Because 18% is 18 hundredths (writing $18\% = 18/100 = 0.18$ on the board), to divide by a 100 means 18 hundredths.

Karen expressed her belief in having the responsibility to provide students with justification of mathematical statements. This responsibility is felt even in the one moment in the observed lessons when Karen acknowledges that she doesn't know how to provide a mathematical argument for the procedure, and states that students just "have to remember". The problem Karen posed to class is: "From a class of 22 students, six participated in a math competition. What percent of the class was that?" Karen goes on to exemplify two methods for solving the argument.

Teacher: The first one is the 1% method. Again, I think that this method is more convenient and easier... ok, what's the base in this problem?

Students: [suggest ideas]

Teacher: Yes, base or 100% is 22 pupils. There are 22 pupils [She writes a record of the solution on the board, writes "1% = ".]. Now, we'll calculate, Ada?

Ada: 1% will be 0.22. [Karen writes this on the board.]

Teacher: Now you just have to remember that the percent, [...] I don't know how to help you remember ... you need to remember. You can calculate the percent this way [...] we divide the percent part we want to express in percent by one percent.

The argument that she is reluctant to share with her students is in fact the ratio argument used in the rule-of-three method: firstly, that the *percent part* : *percent* ratio is a constant, and for all non-zero real numbers a , b , c , and d , if $a : b = c : d$ then $a = c \cdot b / d$. Clearly, this presents a conflict of beliefs for her, and she chooses not to present the argument, because this, in her mind, is too complicated and not possible to grasp with their current knowledge, especially for some students.

In the textbook, authors let the reader observe the first warrant through a series of examples, and then simply refer to the rule-of-three as practice established in the previous unit (on proportion). However, in the teacher's book they also admit that the equivalence of the two equations is, as yet in the curriculum, inaccessible to students and has not been established with students at this particular stage.

The stronger and the weaker: students' and Karen's preferences

The following passages will show examples when different arguments are provided by students. The exchanges take place at the beginning of the second lesson, students were converting a series of fractions into percent. They had just converted $4/5$ by expanding to tenths and then hundredths. Now Sam tries to convert $3/8$ in the same way:

Sam: I'll multiply the fraction by twelve and a half.

Teacher: Why twelve and a half?

Sam: Because if I multiplied 8 times 125 [unintelligible]

Teacher: So by 125, right?

Sam: But that will be a thousand, so ...

Teacher: Doesn't matter. But (writing on the board) 8 times 125 is 1000. What is 3 times 125?

Student: 375.

Sam is trying to expand the fraction to hundredths (realizing that expanding by 125 and simplifying to hundredths is the same as expanding by 12.5) but the teacher feels that this is not straightforward and accessible to all pupils, so she takes over and breaks the argument down. After a few more simple problems, where students don't need to calculate, they are asked to convert the fraction $9/40$. At first, a student (Will) suggests to reduce by two and expand by five. Then he adds:

Will: Or multiply (sic) by two and a half.

Teacher: Excellent, two and a half. Do you [all] agree?

Kim: And couldn't you expand to thousands?

Teacher: Also. And if you were to do that, by what number would you expand?

Kim: So, that would be times ... (thinking) ... two hundr ...

Teacher: Twenty five. Either, as Will said, we expand by two and a half, which is not very common, (she turns to the board and writes) if we want hundredths in the denominator we expand by two and a half (she writes this on the board), do you agree? Forty times two and a half is one hundred, right? And the numerator ... 18 and 4 and a half [...] 22 and a half. So what percent is $9/40$?

Students: Twenty two and a half.

Teacher: Or, as Kim said, expand by 25 (she writes on the board), the numerator (sic) is 1000, do you agree? And the denominator (sic) is ...

Students: 225.

Teacher: And we got the same thing, 22.5 %.

At this point, Karen allows a student (Will) to carry out an argument that is (like Sam's) based on expanding by decimals, but this time the student breaks it down into two warrants first, and Karen praises it. Will feels encouraged to suggest expanding by a decimal. Finally, another student supplies an argument based on the expansion to thousands (which had been shown by Karen before, see the transcript above). Both methods are now endorsed by the teacher, publicly, as valid arguments, and demonstrated on the board. When Karen summarizes these approaches, however, she qualifies Will's solution as "not very common".

Conclusions

The above examples illustrate how the enacted arguments were influenced by the three participants, the teacher, the textbook, and the students. Even though Karen was the most influential provider of mathematical arguments, arguments that were made in the classroom included students' own warrants, and became accepted as correct and valid by the teacher. At the same time, even as Karen acted as the decision-maker when it comes to choosing what representations are useful in warrants, i.e. efficient, for her class, what was her choice not to include the textbook's geometrical representation warrants based on? Clearly, the textbook does not give it a utilitarian value, i.e. it does not provide opportunities for its direct use, and makes the representation void of value, outside the possible provision of better insight, as the authors claim, but Karen did not find the claim convincing enough. In that sense, her decision was very much determined by two factors: a) by her pedagogical

content belief about the efficiency of a certain type of arguments and b) by the problems (opportunities for arguments rather than argument forms themselves) presented by the textbook authors in the unit. The second factor, in turn, is given weight by Karen's utilitarian view of the goals of mathematics education, i.e. being able to correctly solve problems provided by the curriculum.

The case of the rule-of-three method is perhaps even more interesting, especially as the ratio warrant that underlies it is also at the heart of a method Karen presents when she shows the procedure for finding the percent in a word problem, but decides that the justification is not straightforward enough for her class, and backs the procedure up with her own authority. What made her do that? When asked about the need to mathematically justify mathematical statements, Karen conceded that not all arguments are accessible to students (or not all of them). As I showed above, the textbook authors also use a warrant that they acknowledge is out of the students' immediate reach. Again, we observe similar tendencies, and at the same time it appears that in this case Karen's perception of her students' abilities accounted for her decision not to justify.

In her classes, Karen also allowed students to provide arguments that she had not intended to take place, and accepted them as long as they were mathematically correct. At the same time, she manipulated such publicly expressed arguments according to her perception of accessibility to all students and made frequent evaluative comments about the methods and arguments, labeling them as efficient, common practice, convenient, easier, or universal. This qualitative evaluation springs from her beliefs about her students' mathematical ability and what it means to be good in mathematics: in her view, some students are better at understanding the problem, and innately capable of finding and choosing the most efficient, original, or convenient method, an attribute she also gives mathematicians in general. For the others, she needs to show simply which method to use, and they need to learn it by solving many similar problems, i.e. for some students drilling is the only way to succeeding in mathematics. The episodes seemed to confirm that this belief corresponds with the students' contributions: the weaker students would rely on arguments promoted by Karen, while students who feel confident in their own warrants, could keep using their own.

In terms of the teacher's influence, it appears that the teacher is independently imposing her own beliefs that are very local, e.g. the choice of method, but the choice of representation is also clearly determined by the curriculum (and its tasks) and beliefs that are much more global. Further, the teacher's choice of not justifying mathematically can be caused by her own belief but also reinforced by similar examples in the textbook. Finally, the students' arguments are evaluated by the teacher in terms of their mathematical correctness, their efficiency, and their accessibility to all other students (as perceived by the teacher). They are then often re-formulated by the teacher, which potentially reinforces the dependency of the weaker students on the teacher's choice of argument.

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