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Identifying phases and activities in the proving process of first-year undergraduates

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It is well known that undergraduates commonly have to deal with great difficulties in constructing proofs, especially at the beginning of their mathematical studies (e.g. Weber, 2001). In the presented study, students' approaches to proving are analysed from a process-oriented perspective. The primary aim is to empirically confirm a process-oriented model for the proving competence of undergraduates. This model may be used to analyse proving processes at an individual level and, therefore, to gain more detailed information about proving process and its phases in general.

Theoretical framework

The study is based on a theoretical model of the proving process that describes different phases and activities in proof construction and which is mainly following the considerations of Boero (1999). Boero (1999) describes a proving process that starts with conjecturing and exploration activities and ends up in selecting arguments and linking them to a deductive chain. However, proving tasks at university level often consist of a statement estimated to be true, especially during the first year of studies. The construction of a proof is in this case rather aimed at justification instead of conjecturing and problem exploration. According to this, we suggest the following variation of Boero's model.

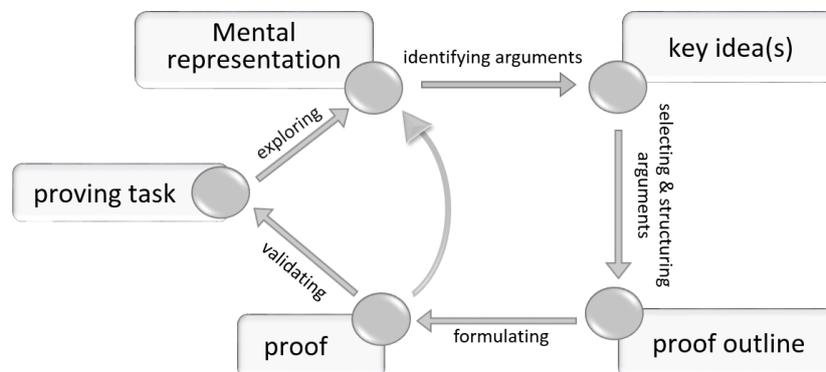


Figure 1: Proving cycle

This model differs from the existing model in three ways:

1. The model focuses on proving activities concerned with the justification of a given statement. In particular conjecturing activities are excluded. However, reducing the process described above does not mean to exclude all exploration activities, but locating them at the beginning of the proof construction. Leaning on Reusser's (1997) approach of a situation model, a mental representation of the given statement is estimated to be developed by exploring the proving task. This representation could affect the proving process in a meaningful way.
2. The model includes validating activities at the end of the process. In this phase, which is already implicitly considered by Boero (1999), the final proof is reviewed regarding content, structure and linguistics. Besides, further (shorter or more elegant) proofs can be considered.

3. The underlying structure of the model is a cycle. This kind of structure provides the assumption that proving processes are not supposed to be linear. In fact, the proving process is shaped by interruptions, revisions and turns.

Research question and method

The modifications lead to the following research question: Is the proving cycle an appropriate tool for analysing proving processes? That means, is it possible to reconstruct the different phases and activities stated in the proving cycle empirically? Is there in particular evidence for the existence of an exploration phase? In accordance with the research question, a qualitative study has been designed with the purpose to provide evidence of the proving cycle and to gain more detailed information about the different phases. Therefore, first year undergraduates and first year pre-service mathematics teachers (grammar school) are asked to work on proving tasks in the field of real analysis. To encourage the participants to talk about their ideas and approaches, the working processes are organized in pairs. The proving process of each pair is videotaped, transcribed and finally encoded according to Mayring's (2007) structuring content analysis. The coding is based on a system of categories, which consists of the five theoretical stated phases in the proving cycle.

Results

The analysis of data from six cases shows that the system of categories seems to be well suited to describe proving approaches of undergraduates. Each of the suggested phases could be empirically confirmed in nearly all cases. However, formulating a precise and clear proof is an activity, which is sometimes omitted. An exploration phase could be reconstructed in all cases, although it varies in quality and quantity. The structure of the analysed proving processes is linear insofar as many phases could be reconstructed in the suggested order and turns mainly concern consecutive phases. Only in those cases, where the identified key ideas turn out to be inadequate, the proving process starts with repeated exploration cyclically. As there is no need for further categories the proving cycle can be used as a tool for analysing proving processes. Additionally, this tool can serve as a basis for deeper inductive investigations of the proving competence from a process-oriented perspective.

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