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# Activity recognition from stride detection: a machine learning approach based on geometric patterns and trajectory reconstruction

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**Abstract**—In this paper, an algorithm for activity recognition is proposed using inertial sensors worn on the ankle. This innovative approach based on geometric patterns uses a stride detector that can detect both normal walking strides and atypical strides such as small steps, side steps and backward walking that existing methods struggle to detect. It is also robust in critical situations, when for example the wearer is sitting and moving the ankle, while most algorithms in the literature would wrongly detect strides. A technique inspired by Zero Velocity Update is used on the stride detection to compute the trajectory of the device. It allows to compute relevant features for the activity recognition learning task. Compared to most algorithms in the literature, this method does not use fixed-size sliding window that could be too short to provide enough information or too long and leads to overlapping issue when the window covers two different activities.

## I. INTRODUCTION

In the last decade, human activity recognition (HAR) has become an important field of research in a health-care context. While vision-based techniques work with intrusive equipments [1], inertial data analysis for activity recognition is one of the most important challenge with the emergence of accelerometers and gyrometers in daily life connected objects (wearable sensors, smartphones).

In this context, Sysnav has developed an ankle worn system based on magneto-inertial sensors [2], [3]. The device was designed to evaluate the physical condition of subjects suffering from pathologies associated with movement disorders such as neuromuscular diseases. The system is used as biomarker [4] for computing secondary outcome measures in clinical trials [5]. Compared to classical outcome measures, these relevant variables (stride length, stride speed) have the advantage of being calculated in a home environment over long periods. Indeed, classical tests for outcome measures are performed at the hospital and can be biased by the controlled environment aspect and the motivation of the patient. The variables provided by the four stairs test (time for climbing four stairs), ten meters run test (time for running ten meters) and the six minutes walk test (distance covered in six minutes

by walking) can be impacted by the form of the day without being correlated with patient’s condition.

In this work, we describe a method for activity recognition (AR) in order to compute pertinent stride variables related to the three previous tests (walking, stairs and running). It requires to detect precisely when a stride occurs in the recordings and its activity. Most of the papers in the literature use AR algorithms based on fixed-size sliding window combined with Hidden Markov Model [6] or machine learning [7], [8], [9]. In this case, errors appear at the beginning or at the end of the activities, when the window overlaps the end of one activity and the beginning of the next one. In other cases, the window length may be too short to provide the best information for the recognition process. Moreover this approach does not allow to detect individual strides. A few methods of stance detection (Figure 1) have been proposed in the literature by tuning thresholds to determine the start and the end of the walking phases [10], [11], [12], or using machine learning techniques on the frequency characteristics of the signals [13], [14]. These methods show good results when it is known that the pedestrian is walking but fail in a lot of real life situations. Indeed, several foot movements in sitting position for example are wrongly detected as strides.

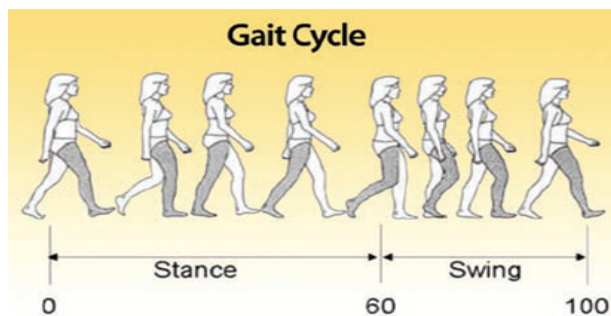


Fig. 1: Gait cycle (www.optogait.com).

In this paper we describe our step detector for AR which is built on a machine learning algorithm and the innovative

modeling of the swing phase. It is combined with a technique inspired by Zero Velocity Update (ZUPT) [15], [16], [17], [18] which is an effective method to limit the accumulation of integration errors for trajectory reconstruction. Indeed, The strategy which consists in the integration of the accelerations after removing the gravity and the speeds to compute a trajectory rapidly cumulates large errors. The ZUPT technique consists in correcting the speed drift by estimating the speed of the ankle when the foot is on the ground during the walk and then integrates the data only between two ZUPTs. As our step detector provides the ground phases, we can reconstruct the trajectory of the ankle during the stride which is a precious information for AR.

## II. STRIDE DETECTOR

The first step of the algorithm is to select intervals that may correspond to strides in the inertial data (acceleration and angular velocity in three dimensions). The system is worn around the ankle as shown in the Figure 2.



Fig. 2: Body frame definition.

In the default placement the  $Z$  axis is aligned with the leg and the  $X$  axis is aligned with the foot. However the device may be upside down and may turn around the ankle. Consequently, we do not use the three dimensions of the data but the norms for the interval segmentation. A combination of criteria on the accelerometer norm (close to one g, peak) and gyrometer norm (local minimum) is used to define the contact of the foot with the ground, the start and the end of the stride. The threshold values have to be sufficiently wide to detect all types of strides (small steps, running, stairs etc.). However many intervals are wrongly selected when the wearer is moving its ankle but not walking. The goal is now to select among these intervals which ones are true strides. We adopt a statistical learning approach to answer this problem.

A group of people of various ages and heights, were filmed practicing several activities while wearing the system. From the intervals selection above, a learning set is built using video control. A binary label is affected to each interval indicating if it is a stride (1) or not (0). Furthermore, if the interval is the stride, an other variable  $k$  marking the activity is added: "atypical step" that includes small step, side step, backward walking etc., "walking", "running", "climbing stairs" and "descending stairs".

Activity	Label $k$
Atypical strides	1
Walking	2
Running	3
Climbing stairs	10
Descending stairs	-10

TABLE I: Label definitions for activities.

Our database contains about 6000 positive intervals and also about 6000 negative intervals. In the following, we aim to compute relevant variables for a supervised machine learning algorithm in this binary classification problem.

### A. Sensors alignment

The previous interval segmentation uses norms only because of the body frame orientation issue. To access to the information held in the three dimensions, we have to compute the rotation that aligns the sensors on the orientation defined in Figure 2. Several strides were recorded for seven activities (backward walking, left and right side stepping, walking, running, climbing and descending stairs), ensuring that the system was placed as defined in Figure 2. From these records, the gyroscope data was used to define a reference pattern (in three dimensions) for each activity. The goal is to fit the gyroscope data of a new candidate stride, performed with any orientation, to each reference pattern by allowing a rotation.

Since the calculation method is the same for the seven 3D reference patterns, we will not indicate the activity in the following section. Let  $Y = (Y_1 \dots Y_n)$  with  $Y_i \in \mathbb{R}^3$  the 3D reference pattern data of size  $n$  on the three gyroscope axes and  $(X_1 \dots X_n)$  the gyroscope data of a new candidate stride. First, it is necessary to bring the observed data to the same number of samples as the reference pattern by a cubic spline interpolation [19] on each axis. Let  $X = (X_1 \dots X_n)$  be the vector of the observations. We want to compute the rotation  $R$  that minimizes  $\sum_{i=1}^n \omega_i \|RX_i - Y_i\|_2^2$ . The coefficients  $\omega_1, \dots, \omega_n$  are the weights given to the samples of the stride ( $\sum_{i=1}^n \omega_i = 1$ ). In this paper we set higher weight values to the samples in the middle of the stride to avoid side effects. Indeed, data in the end can be noisy by the contact of the foot with the ground. Moreover, the foot movement on the ground during the stance phase is less specific to the activity than during the swing phase.

**Property 1.** — Given  $X_i \in \mathbb{R}^3$  and  $Y_i \in \mathbb{R}^3$  for all  $i$  in  $\llbracket 1, n \rrbracket$ . The solution of the problem

$$R^* = \operatorname{argmin}_{RR^t=I, \det(R)=1} \sum_{i=1}^n \omega_i \|RX_i - Y_i\|_2^2,$$

is given by

$$R^* = VU^t, \quad (1)$$

where  $V$  and  $U$  are the unitary matrices of the decomposition into singular values of  $XWY^t$ , and  $W = \operatorname{diag}(\omega_1, \dots, \omega_n)$  with  $\sum_{i=1}^n \omega_i = 1$ .

We compute this rotation matrix for the seven 3D reference patterns to be sure that at least one good alignment has been computed. Moreover, residuals of these seven calculations are pertinent variables for AR. Indeed, the smaller are the residuals for one pattern, the likelier the stride is performed with the same activity. In the following section, we assume that the rotated data of the stride are in the reference frame defined in Figure 2.

### B. Swing modelling

We saw (Figure 1) that the cycle of a stride is divided into two phases: swing and stance. During the swing phase, moving the foot forward creates a distinctive pattern in the  $Y$  axis of the gyroscope (Figure 3). In this paper we call forward swing the sequence where the values remain negative. The aim of this

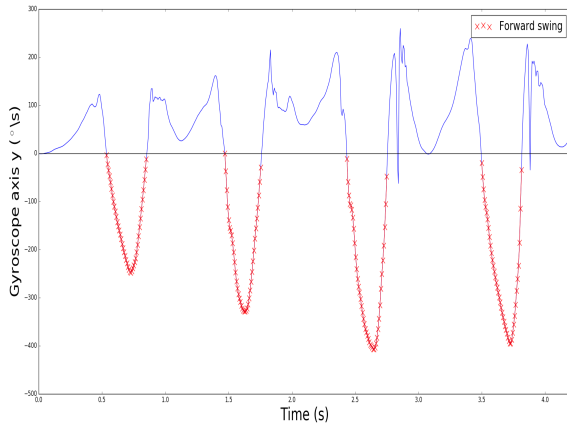


Fig. 3: Forward swing.

section is to model the forward swing. We want to compute a 1D reference pattern that defines the gyroscope data on the  $Y$  axis of the forward swing for five activities: atypical strides, walking, running, climbing and descending stairs (Table I).

Let  $N_k$  the number of strides of each activity  $k$ . We note  $f_{l,k}$  the observed function associated to the gyroscope data on the  $Y$  axis of the  $l^{th}$  on  $N_k$  forward swing of the activity  $k$  and  $f_k$  the unknown function associated to the reference pattern of the activity  $k$ , defined on the interval  $[0, 1]$  in  $\mathbb{R}$ . We assume that the 1D reference pattern we want to compute can be approached with an error  $\epsilon_l$  by all the  $f_{l,k}$  functions of the same activity by multiplying them with a real coefficient  $a_{l,k}$

$$f_k = a_{l,k} \times f_{l,k} + \epsilon_{l,k}. \quad (2)$$

We assume that the functions belong to a function space  $E$  with its norm  $\|\cdot\|$ . The observations are the functions  $f_{l,k}$  and we want to compute the estimators  $\hat{f}_k$  and  $\hat{a}_{l,k}$  which are

computed by least squares minimization under constraints  $(\mathcal{P})$  for all  $k$  in  $\{1, 2, 3, 4, 10, -10\}$  and for all  $l$  in  $\llbracket 1, N_k \rrbracket$ :

$$\hat{f}_k = \underset{f_k \in E, \|f_k\|=1}{\operatorname{argmin}} \sum_l \|\hat{a}_{l,k} f_{l,k} - f_k\|, \quad (3)$$

$$\hat{a}_{l,k} = \underset{a_{l,k} \in \mathbb{R}_+^*}{\operatorname{argmin}} \|a_{l,k} f_{l,k} - \hat{f}_k\|. \quad (4)$$

To solve the problem  $(\mathcal{P})$ , we consider an orthonormal basis  $(e_1, \dots, e_p)$  for the norm  $\|\cdot\|$ . In practice, we use Lagrange polynomials [20] but other basis can be selected such as Fourier basis. We note:

$$\hat{f}_k = \sum_{u=1}^p \hat{\gamma}_u e_u, \quad (5)$$

$$f_{l,k} = \sum_{u=1}^p \alpha_{l,u} e_u. \quad (6)$$

**Property 2.** — Given  $\Lambda_l = (\alpha_{l,1}, \dots, \alpha_{l,p})^t$  and the symmetric matrix  $A$  defined by  $A_{ij} = \sum_l \frac{\alpha_{l,i} \alpha_{l,j}}{\|\Lambda_l\|_2^2}$ , the solution of the problem  $(\mathcal{P})$  is given by:

$$\hat{f}_k = \pm \sum_{u=1}^p \omega_{p,u} e_u, \quad (7)$$

and

$$\hat{a}_{l,k} = \frac{\sum_{u=1}^p \alpha_{l,u} \hat{\gamma}_u}{\sum_{u=1}^p \alpha_{l,u}^2}, \quad (8)$$

where  $\omega_p$  is the eigenvector of  $A$  associated to its greater eigenvalue.

As the forward swing is defined by negative values, we choose in practice the solution of Equation 7 that takes negative values. The functions  $\hat{f}_k$  are computed once for all. We can now extract the forward swing of a new stride (negative values of the  $y$  axis gyroscope data), compute the multiplier coefficient (Equation 8) using  $\hat{f}_k$  coefficients and compare it to the 1D reference patterns. The residuals are relevant information for AR as for the alignment step. The smaller are the residuals for one pattern, the likelier the stride is performed with the same activity.

### C. Performances

The database contains 6213 intervals that do not correspond to strides (label 0) and 5964 stride intervals (label 1) divided into 5 different activities (Table I). For each element, 2695 features are computed:

- Frequency domain: from the norm of both accelerometer and gyroscope, features were computed in the time and frequency domains: maximum, mean, standard deviation, root mean square, interquartile range, Fast Fourier Transform...
- Alignment: using the alignment correction, the rotated gyroscope data of the interval is compared to the 3D reference pattern of each activity:  $\sum_{i=1}^n \omega_i \|RX_i - Y_i\|_2^2$ . Features are computed from the residuals during the stance and swing phases.

- Swing: if the interval is a stride, at least one rotation transforms the data so that the extracted forward swing phase is visible on the  $Y$  axis of the gyroscope (Figure 3). This forward swing is compared to the five 1D reference patterns  $\hat{f}_k$  (Equation 7) with the corresponding coefficient  $\hat{a}_{l,k}$  (Equation 8):  $\|\hat{a}_{l,k}f_{l,k} - \hat{f}_k\|$  for  $k$  taking values in  $\{1, 2, 3, 10, -10\}$  (Table I). The residuals are used as features.

We want to build a binary classifier that decides if one interval is a stride. Several supervised statistical learning algorithms have been tested, notably random forests which are known to perform well in large dimensions, Support Vector Machine (SVM), LASSO regression and boosting algorithms such as Adaboost and GBT (Gradient Boosting Tree [21]). We evaluated their performance using the cross-validation method (10-fold cross-validation [22]). The chosen algorithm with the best results is GBT.

The goal of boosting is to iteratively focus on observations that are difficult to predict. For GBT, the general idea is to compute a series of (very weak) decision trees [23] and to aggregate the results to minimize a cost function. Let  $g_n$  be the prediction function at the iteration  $n$ . We compute a new tree  $Tree_{n+1}$  which estimates the value of the cost function on  $g_n$ . The new prediction function  $g_{n+1}$  is then defined by  $g_{n+1} = g_n + \lambda_{rate} Tree_{n+1}$  where  $\lambda_{rate}$  is a real constant to be tuned to avoid overfitting.

The cross-validation results using GBT are presented in the following confusion matrix (Table II).

	Predicted 0	Predicted 1
Actual 0	6195	18
Actual 1	19	5945

TABLE II: Confusion matrix for stride detection.

The global error is about 0.3%. The distribution of the false negatives is presented in the Table III.

	Atypical step	Walking	Running	Climbing stairs	Descending stairs
FN	$\frac{19}{1152}$	$\frac{0}{1206}$	$\frac{0}{1334}$	$\frac{0}{1100}$	$\frac{0}{1155}$

TABLE III: False negatives distribution.

Our algorithm made no mistake on the "walking", "running" and "stairs" strides. On "atypical" strides, it achieved a detection error rate of 1.6% while most existing methods described in the literature do not detect them.

### III. ACTIVITY RECOGNITION

At this stage, we could build on the 5964 strides a function for the activity label prediction (Table I) with the same features above. It achieves good results but to improve them we take advantage of Sysnav technology for dead reckoning that enables trajectory reconstruction from a stride detection. The technique is inspired by Zero Velocity Update (ZUPT) and has shown good results on challenging situations [24]. On Figure 4, we can see that the sequence of three strides is correctly detected. The red points are the segmentation given by the

stride detector algorithm and in blue the trajectory computed. the trajectory is drawn in an arbitrary reference but we can

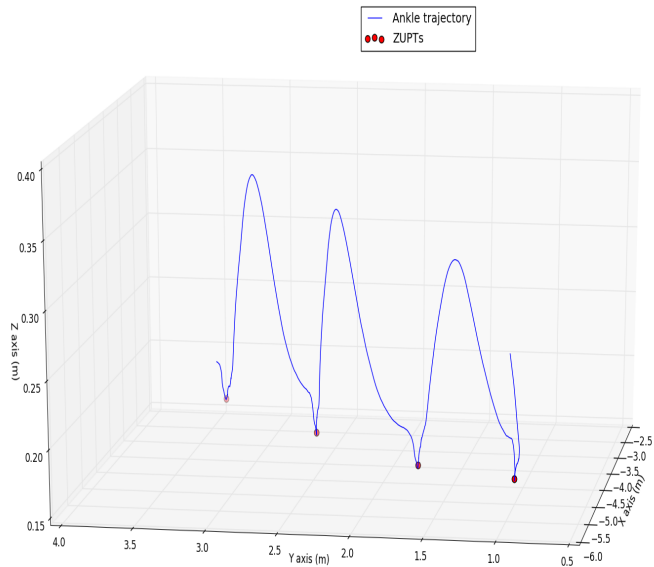


Fig. 4: Computed device trajectory.

extract information by considering the relative evolution. This technique provides also the speeds in the three dimensions and the angles evolution of the device that have characteristic patterns according to the activity performed. In the end, we compute 510 additional features used for the AR task.

The goal is then to build a classifier that recognizes the activity of each detected stride. We have tested several supervised learning algorithms for multi-class classification (five classes for five activities) on the database. Once again GBT provides the best results using for the 10-fold cross-validation. The confusion matrix is presented in the following Table IV.

	Predicted 1	Predicted 2	Predicted 3	Predicted 10	Predicted -10
Actual 1	1138	14	0	0	0
Actual 2	17	1185	0	2	2
Actual 3	0	0	1334	0	0
Actual 10	0	2	0	1098	0
Actual -10	0	0	0	0	1155

TABLE IV: Confusion matrix for AR.

The global score is about 99.4%. The difference between "atypical step" and "walking" is hard to define, especially for a small forward step. As even the labelling decision by the video viewer is difficult, it is not surprising to have most of the errors in the Table IV between these two classes.

We can now compute the entire algorithm for AR on the recordings of clinical studies. The overall algorithm (Algorithm 1) is described in pseudocode. The goal is to compute relevant variables on walking, running, and stairs daily episodes related to the tests performed by the patients at the hospital.



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**Algorithm 1: AR algorithm**

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**Input :** Recording of the system worn at the ankle  
**Output:** Activity of each detected stride

```
1 Calibration of the data
2 Detection of the candidate step intervals
3 foreach interval do
4   Computation of the frequency domain features
5   Computation of the rotations
6   foreach activity rotation do
7     Comparison with the 3D reference pattern
8     if negative values on y gyroscope axis then
9       foreach ID reference pattern do
10        Computation of the multiplying
11         coefficient (Equation 8)
12        Comparison with the reference pattern
13       end
14     end
15   end
16   GBT binary classification
17   if interval classified as a stride then
18     ZUPT on the start and the end of the interval
19     Data integration between the two ZUPTs
20     Computation of the trajectory features
21     GBT multi-class classification
22   end
```

---

#### IV. CONCLUSION AND FUTURE WORK

This paper describes an algorithm that allows to detect when a stride occurs (start and end points) with its activity from inertial sensors worn on the ankle. The stride detector is divided in for main stages:

- The selection of candidate intervals that may correspond to strides.
- The calculation of a rotation applied on the data in order to work in the same frame for all records. This stage is built on fitting the gyroscope data with 3D geometric patterns.
- The extraction of the forward swing on the gyroscope axis  $Y$ . These data are then fitted with 1D reference patterns.
- The binary classification of the intervals using the Gradient Boosting Tree algorithm with features computed along the previous points.

For normal walking, it has shown good results achievable with existing algorithms. But the stride detector described in this paper also has a good sensitivity for atypical strides such as small steps, side steps and backward walking contrary to most algorithms proposed in the literature. Moreover these approaches are likely to produce detection errors when the system wearer is moving his foot but not walking (e.g. sitting). This is as problem as non walking motion would be considered for the AR task.

The stride detector is used in combination with a ZUPT inspired method for the ankle trajectory reconstruction. It allows

the calculation of trajectory features such as the height and the length of the stride. This technique provides also the speeds and the angles evolution of the device in three dimensions that are used to compute additional features for AR. We use the Gradient Boosting Tree algorithm for this 5-class supervised classification problem ("atypical step", "walking", "running", "climbing stairs" and "descending stairs"), achieving a global score about 99.4% for the 10-fold cross-validation. This method has the advantage of not using fixed-sized sliding window which is a hard parameter tuning problem as a too big value leads to overlapping issue (2 different activities covered by the window) and a too small one does not provide enough information for AR.

In a future work, we could improve the trajectory reconstruction by adjusting the estimation of the ankle speed when the foot is on the ground. A better ankle trajectory will also improve the AR task. Moreover, we could build a classifier that uses the big amount of data provided by the clinical trials for its learning. In our paper, we assume that the GBT function behaves the same way on our database and on the home recordings. However we could imagine that in home situation, the wearer performs strides that are completely different to any instance of the database. In this case, we have no guarantee on the performance of the prediction function. There are algorithms that take into account all non-labeled observations in the learning process (transductive learning).

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