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NUMERICAL STUDY OF AN OSCILLATOR SUBMITTED  
TO UNILATERAL CONSTRAINTS AND DRY FRICTION

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This lecture introduces to the numerical use of a formulation recently proposed by the author (Moreau, 1985, 1986) for the dynamics of systems with a finite number of degrees of freedom, involving unilateral contacts and Coulomb friction.

The main feature of this approach is that Coulomb law is treated as a *multivalued conical relation*, without the traditional splitting of vectors into their normal and tangential components. This, together with the unilaterality of contact, results in the expression of the system dynamics through a *measure differential inclusion* which covers smooth motions as well as collisions or the other catastrophic events classically possible in the presence of Coulomb friction.

In the occurrence of a collision, the shock is treated as *inelastic*, i.e. immediately after impact, the normal component of the relative velocity of colliding bodies at the contact point equals zero.

The numerical technique rests on the time discretization, in some semi-implicit way, of the measure differential inclusion (see also Jean and Moreau, 1987).

The displayed example may be seen as a model of a piece of chalk driven at an angle on a blackboard. According to common experience, this may produce a creaking noise ; in a different frequency range, a chattering motion of the chalk may also be observed, imprinting a dotted line on the blackboard.

The piece of chalk is conceived as a rigid body  $B$ , assumed to only perform motions parallel to the  $(x,y)$  plane. All elements are visualized in this plane. This body is confined in the region  $y \geq 0$  by the rigid wall  $y = 0$  representing the blackboard, fixed in the  $(x,y)$  reference frame. The possible contact involves Coulomb friction with coefficient 0.5. The part of the boundary of  $B$  susceptible to contact is represented as an arc of circle with center  $A$  and radius  $b$ .

The action of the hand holding the chalk is schematized as follows : a point  $D$  of  $B$  (located, in the displayed cases, on the line drawn through  $A$  and through the mass center  $G$ ) experiences an elastic pull-back, centered at some external point  $C$  with prescribed uniform motion. Here  $C$  is supposed to move on the line  $y = 2.7$  with velocity 30 to the right. The

pull-back force has its  $(x, y)$ -components proportional to the corresponding components of vector  $\vec{DC}$ ; the respective stiffness coefficients equal 200 for the  $x$ -component and 2000 for the  $y$ -component. In addition the hand is assumed to exert on  $B$  a constant torque  $C_r$ ; in the figured case, negative values of  $C_r$  tend to favor the contact of  $B$  with the wall.

The plotter draws, at constant time intervals, a line with origin A and directed to G. The unilateral constraint makes A lie in half-plane  $y-b \geq 0$ ; this point belongs to the boundary line  $y-b=0$  as long as B remains in contact with the wall.

Two patterns of steady oscillation are presented, respectively corresponding to different negative values of  $C_y$ . The other parameters are fixed as follows:  $b = 1$ ,  $AG = 1.732$ ,  $GD = 1.6$ ; the moment of inertia of  $B$  about the axis perpendicular at  $G$  to the plane of figure equals 1; the mass of  $B$  equals 1.

The upper drawing corresponds to  $C_c = -700$ . This torque is enough for the contact to persist all throughout the motion. One observes angular oscillations; also oscillations with lower frequency affect the space-lag from the pull-back center  $C$ , as measured for instance by the variable  $s = x(C) - x(G)$ . The time-step length equals 0.002; position is plotted every 4th step.

Immediately below, there is drawn, for the same motion, the trajectory of the point with coordinates  $x(G) - x(C)$  and  $y(G)$ . The "doubly periodic" character of the motion is apparent; the trajectories, in the 6-dimensional space of the position and velocity variables, are coiled on some invariant torus. In fact, since this case involves persistent contact and sliding velocity with constant direction, the inherent non differentiability of the unilateral constraint and of the Coulomb law are masked. So the system behaves as a classical smooth nonlinear oscillator.

On the contrary, the bottom drawing, corresponding to  $C_r = -200$ , displays a periodic motion with intermittent contact. Recall that shocks are treated as inelastic, so the break of contact is not properly a bounce.

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