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► **To cite this version:**

Alexandre Goldsztejn, Stéphane Caro, Gilles Chabert. A parametric Kantorovich theorem with application to tolerance synthesis. The 18th International Symposium on Scientific Computing, Computer Arithmetic, and Verified Numerical Computations, Sep 2018, Tokyo, Japan. hal-01863731

**HAL Id: hal-01863731**

**<https://hal.science/hal-01863731v1>**

Submitted on 29 Aug 2018

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# A parametric Kantorovich theorem with application to tolerance synthesis

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**Keywords:** Kantorovich theorem, global optimization, tolerance synthesis

## 1 Problem and contribution

We consider a system  $f(x, q, p) = 0$  of  $n$  equations and  $n$  unknowns, denoted by  $x \in \mathbb{R}^n$ , with two kinds of parameters:  $q \in \mathbb{R}^m$ , interpreted as controlled parameters (called commands for short), and  $p \in \mathbb{R}^q$ , interpreted as design geometric parameters with some uncertainties. For a fixed command value  $q$ , we call the solutions  $x_0$  of  $f(x_0, q, 0) = 0$  the *nominal solutions* corresponding to the command  $q$ , while  $x$  satisfying  $f(x, q, p) = 0$  for  $p \neq 0$  is called a perturbed solution. The nominal solutions of interest are furthermore subject to given constraints  $g(x, q) \leq 0$ , in particular encoding domains for  $x$  and  $q$ . We aim at bounding rigorously the worst case distance from any nominal solution satisfying  $g(x_0, q) \leq 0$  to its corresponding perturbed solutions. This is done in two steps: First, we determine a maximal norm  $\bar{\epsilon}$  on the perturbations such that for all perturbations in  $\{p \in \mathbb{R}^q : \|p\| \leq \bar{\epsilon}\}$  the correlation between any nominal and its perturbed solutions is non-ambiguous (a uniqueness condition on the existence of the perturbed solution inside a neighborhood of the nominal solution will be proved). Second, we compute an upper-bound on the distance between the nominal solution and its perturbed solution. Both a crude constant upper-bound  $\bar{\epsilon}$  and a sharp upper-bound  $\epsilon(\|p\|)$  that depend on the norm of the perturbation. Notably, the provided sharp upper-bound will converge to zero as the norm of the perturbation converges to zero.

We propose a parametric Kantorovich theorem, which will achieve these two tasks. The idea is to compute worst case Kantorovich constants with respect to parameters  $q$  using a branch and bound algorithm dedicated to nonlinear nonsmooth global optimization. A rigorous first order model of the dependence with respect to parameters  $p$  is used to enforce the convergence of the error upper-bound. Details about these developments can be found in [1]. We provide here a different point of view, in particular emphasizing the reason why not using the interval Newton operator, in spite of its theoretical superiority on Kantorovich theorem in this context [2].

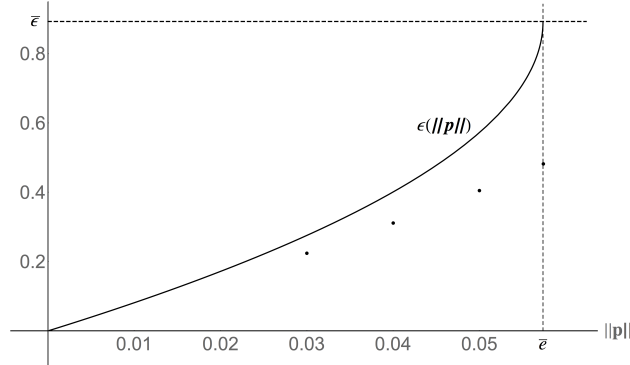


Figure 1: The upper bound  $\epsilon(\|p\|)$  (dots represent exact maximal error, showing the overestimation of  $\epsilon(\|p\|)$ ).

## 2 Case study

The PRRP manipulator is modeled by the following equation:

$$(x - a + p_1)^2 + (q - b + p_2)^2 = (l + p_3)^2, \quad (1)$$

where  $x$  is the pose,  $q$  is the command, parameters values are  $a = 1$ ,  $b = 1$  and  $l = 3$ , and  $p_i$  are uncertainties acting on them. The constraints  $g$  are  $2 \leq x \leq 3 \wedge 3 \leq q \leq 4$ . The proposed parametric Kantorovich theorem proves that every nominal pose has a unique corresponding perturbed pose for uncertainties satisfying  $\|p\| \leq 0.057$ , and that for these uncertainties the distance between the nominal pose and the perturbed pose is smaller than  $\epsilon(\|p\|)$ , with  $\epsilon(\|p\|)$  shown on Figure 1. We can see that  $\epsilon(\|p\|)$  actually converges to zero when  $\|p\|$  converge to zero. Experiments on parallel manipulators up to 3 degrees of freedom, i.e.,  $n = 3$  and  $m = 3$ , have been successfully conducted so far.

## References

- [1] Alexandre Goldsztejn, Stéphane Caro, and Gilles Chabert. A three-step methodology for dimensional tolerance synthesis of parallel manipulators. *Mechanism and Machine Theory*, 105:213 – 234, 2016.
- [2] Arnold Neumaier and Shen Zuhe. The Krawczyk operator and Kantorovich's theorem. *Journal of Mathematical Analysis and Applications*, 149(2):437 – 443, 1990.