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► **To cite this version:**

Alexandre Goldsztejn, Stéphane Caro, Gilles Chabert. A parametric Kantorovich theorem with application to tolerance synthesis. The 18th International Symposium on Scientific Computing, Computer Arithmetic, and Verified Numerical Computations, Sep 2018, Tokyo, Japan. hal-01863731

HAL Id: hal-01863731

<https://hal.science/hal-01863731v1>

Submitted on 29 Aug 2018

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A parametric Kantorovich theorem with application to tolerance synthesis

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Keywords: Kantorovich theorem, global optimization, tolerance synthesis

1 Problem and contribution

We consider a system $f(x, q, p) = 0$ of n equations and n unknowns, denoted by $x \in \mathbb{R}^n$, with two kinds of parameters: $q \in \mathbb{R}^m$, interpreted as controlled parameters (called commands for short), and $p \in \mathbb{R}^q$, interpreted as design geometric parameters with some uncertainties. For a fixed command value q , we call the solutions x_0 of $f(x_0, q, 0) = 0$ the *nominal solutions* corresponding to the command q , while x satisfying $f(x, q, p) = 0$ for $p \neq 0$ is called a perturbed solution. The nominal solutions of interest are furthermore subject to given constraints $g(x, q) \leq 0$, in particular encoding domains for x and q . We aim at bounding rigorously the worst case distance from any nominal solution satisfying $g(x_0, q) \leq 0$ to its corresponding perturbed solutions. This is done in two steps: First, we determine a maximal norm $\bar{\epsilon}$ on the perturbations such that for all perturbations in $\{p \in \mathbb{R}^q : \|p\| \leq \bar{\epsilon}\}$ the correlation between any nominal and its perturbed solutions is non-ambiguous (a uniqueness condition on the existence of the perturbed solution inside a neighborhood of the nominal solution will be proved). Second, we compute an upper-bound on the distance between the nominal solution and its perturbed solution. Both a crude constant upper-bound $\bar{\epsilon}$ and a sharp upper-bound $\epsilon(\|p\|)$ that depend on the norm of the perturbation. Notably, the provided sharp upper-bound will converge to zero as the norm of the perturbation converges to zero.

We propose a parametric Kantorovich theorem, which will achieve these two tasks. The idea is to compute worst case Kantorovich constants with respect to parameters q using a branch and bound algorithm dedicated to nonlinear nonsmooth global optimization. A rigorous first order model of the dependence with respect to parameters p is used to enforce the convergence of the error upper-bound. Details about these developments can be found in [1]. We provide here a different point of view, in particular emphasizing the reason why not using the interval Newton operator, in spite of its theoretical superiority on Kantorovich theorem in this context [2].

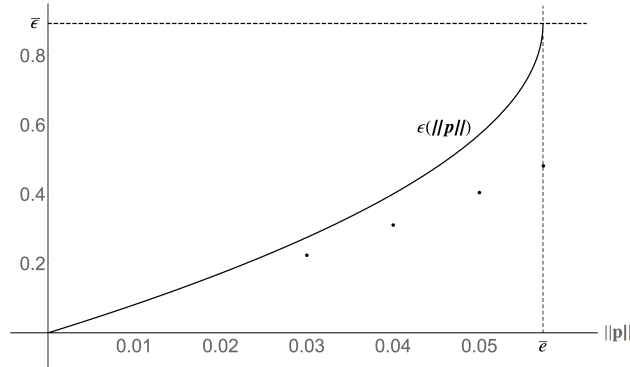


Figure 1: The upper bound $\epsilon(\|p\|)$ (dots represent exact maximal error, showing the overestimation of $\epsilon(\|p\|)$).

2 Case study

The PRRP manipulator is modeled by the following equation:

$$(x - a + p_1)^2 + (q - b + p_2)^2 = (l + p_3)^2, \quad (1)$$

where x is the pose, q is the command, parameters values are $a = 1$, $b = 1$ and $l = 3$, and p_i are uncertainties acting on them. The constraints g are $2 \leq x \leq 3 \wedge 3 \leq q \leq 4$. The proposed parametric Kantorovich theorem proves that every nominal pose has a unique corresponding perturbed pose for uncertainties satisfying $\|p\| \leq 0.057$, and that for these uncertainties the distance between the nominal pose and the perturbed pose is smaller than $\epsilon(\|p\|)$, with $\epsilon(\|p\|)$ shown on Figure 1. We can see that $\epsilon(\|p\|)$ actually converges to zero when $\|p\|$ converge to zero. Experiments on parallel manipulators up to 3 degrees of freedom, i.e., $n = 3$ and $m = 3$, have been successfully conducted so far.

References

- [1] Alexandre Goldsztejn, Stéphane Caro, and Gilles Chabert. A three-step methodology for dimensional tolerance synthesis of parallel manipulators. *Mechanism and Machine Theory*, 105:213 – 234, 2016.
- [2] Arnold Neumaier and Shen Zuhe. The Krawczyk operator and Kantorovich's theorem. *Journal of Mathematical Analysis and Applications*, 149(2):437 – 443, 1990.