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On the self-similarity of dissipation elements in homogeneous isotropic decaying turbulence

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Abstract

The self-preservation of a passive scalar in homogeneous isotropic decaying turbulence is investigated by the method of dissipation elements (L. Wang and N. Peters, *J. Fluid Mech.*, 554 (2006): 457-475). The method of dissipation elements is used to perform a space-filling decomposition of the scalar field into smaller sub-units. These sub-units are parameterized by their length ℓ and the difference $\Delta\phi$ of the scalar field between the ending points. It is shown that statistics of dissipation elements depend only on the mean length ℓ_m as relevant characteristic length scale. The analysis is based on a highly resolved direct numerical simulation.

1 Introduction

The concept of self-preservation has played an important role in shaping the understanding of turbulent flows. Several theoretical and experimental problems in turbulence research have been approached by self-preservation hypotheses (Antonia et al. 2003, George 1992, Hinze 1975). The assumption of complete self-preservation imposes certain constraints on the dynamics of the flow, allowing to express one-point or two-point statistics by choosing appropriate unique scales. The first self-preservation hypothesis was put forward by von Karman & Howarth (1938), stating that correlation functions of the velocity field $u(\mathbf{x}, t)$ can be expressed by choosing a single characteristic length scale $l(t)$ and a single characteristic velocity scale $u(t)$, i.e.

$$\langle u(\mathbf{x}, t)u(\mathbf{x} + \mathbf{r}, t) \rangle = u^2(t)f(r/l(t)), \quad (1)$$

where $f(r/l(t))$ is the two-point velocity correlation function and the angular brackets denote ensemble-averaging. However, many flows do not satisfy complete self-preservation. The term partial self-preservation refers to flows in which self-preservation is valid for a restricted range of scales only, requiring at least two different velocity and length scales to describe statistical quantities. Determining these scales and its scaling is of high relevance for understanding and modeling turbulence.

Another approach in turbulence research is to investigate the dynamics or shape of geometric objects, like vortex tubes or dissipation elements. The concept of dissipation elements has been developed by Wang

& Peters (2006, 2008, 2013) to study turbulent mixing. Dissipation elements are irregular in shape and appear as coherent structures in turbulent scalar fields. They are constructed by calculating gradient trajectories of the scalar field: starting from every point in the field, a gradient trajectory in ascending direction will reach a (local) maximum point, while in descending direction it will reach a (local) minimum point. The ensemble of all trajectories sharing the same two ending points defines a finite volume, which is called a dissipation element. A schematic illustrating the definition of dissipation elements is shown in fig. 1. By construction, dissipation elements are space-filling, which implies that all parts of the flow are covered when computing statistics over dissipation elements. Moreover, dissipation elements are self-contained, which means that each material point is included once and only once in the decomposed object. Dissipation elements may be characterized by simple parameters derived from the information at the ending points. Wang & Peters (2006, 2008) proposed to use the scalar difference

$$\Delta\phi = \phi_{\max} - \phi_{\min} \quad (2)$$

and the linear length ℓ between the ending points. It is important to emphasize the difference between the scalar difference $\Delta\phi$ of dissipation elements and the scalar increment $\delta\phi = \phi(\mathbf{x} + \mathbf{r}) - \phi(\mathbf{x})$ used in classical theories of turbulence, cf. Moin & Yaglom (1975). The scalar increment $\delta\phi$ is computed continuously along a straight line and the separation distance r between the two points is fixed in advance. While, on the other hand, for dissipation elements the scalar difference $\Delta\phi$ is conditioned on local extremal points which are connected through gradient trajectories. The length scale ℓ results from the turbulent field itself, rather than being prescribed externally. This way of averaging is therefore able to capture information about the local structure of a turbulent field, which is otherwise averaged-out, cf. Gauding et al. (2015).

The objective of this study is to use the concept of dissipation elements to analyze scalar mixing in homogeneous isotropic decaying turbulence from a self-preservation perspective. The remainder of the paper is as follows. A brief introduction of the direct numerical simulations (DNS) on which the analysis is based is presented in sec. 2, and the analysis of the DNS by the

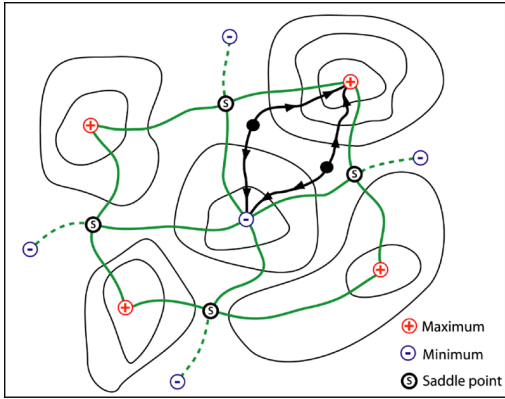


Figure 1: Illustration of the concept of dissipation elements in a two-dimensional field. Two gradient trajectories that connect local extremal points are depicted by black solid lines. The boundaries of individual dissipation elements are shown by green lines. Note that the ensemble of dissipation elements is space-filling.

method of dissipation elements is given in sec. 3.

2 Direct numerical simulations

Highly resolved direct numerical simulations (DNS) of decaying homogeneous isotropic turbulence has been carried out. The following paragraph briefly summarizes the main characteristics of the DNS. More details about the numerical procedure and the parallelization approach are given by Gauding et al. (2015, 2016). The DNS solves the incompressible Navier-Stokes equations in a triply periodic cubic box with size 2π by a pseudo-spectral method. Following the approach by Mansour & Wray (1994), the Navier-Stokes equations are formulated in spectral space as

$$\frac{\partial}{\partial t} (\hat{u}_i \exp(\nu \kappa^2 t)) = \exp(\nu \kappa^2 t) P_{ij} \hat{H}_j, \quad (3)$$

where \hat{H}_j is the Fourier transform of the non-linear terms and $P_{ij} = \delta_{ij} - \kappa_i \kappa_j / \kappa^2$ is the projection operator that imposes incompressibility. In eq. (3), the wave-number vector is denoted by $\boldsymbol{\kappa}$, \hat{u} denotes the Fourier transform of the velocity field, and ν is the kinematic viscosity. An integrating factor technique is used for an exact integration of the viscous terms. Temporal integration is performed by a low-storage, stability preserving, third-order Runge-Kutta scheme. The non-linear term is computed in physical space and a truncation technique with a smooth spectral filter is applied to remove aliasing errors. Characteristic properties of the DNS are summarized in table 1. A necessary constraint that has to be satisfied by the DNS is an adequate resolution of the smallest scales. Following Mansour & Wray (1994), we require that $\kappa_{\max} \eta \geq 1$, where $\eta = (\nu^3 / \langle \varepsilon \rangle)^{1/4}$ is the Kolmogorov length scale and κ_{\max} is the largest resolved wave-number. A grid resolution of 4096^3 points is used to appropriately account for both small and large scales.

Table 1: Characteristic properties of the DNS. The r.m.s. of the velocity field is defined as $u' = (\frac{2}{3} \langle k \rangle)^{1/2}$.

grid size N^3	4096 ³
peak wave-number κ_p	15
viscosity ν	0.00045
Reynolds number $u'_0 / (\nu \kappa_p)$	382.5
$u'_0 = u'(t=0)$	2.58
Schmidt number $Sc = \nu / D$	1

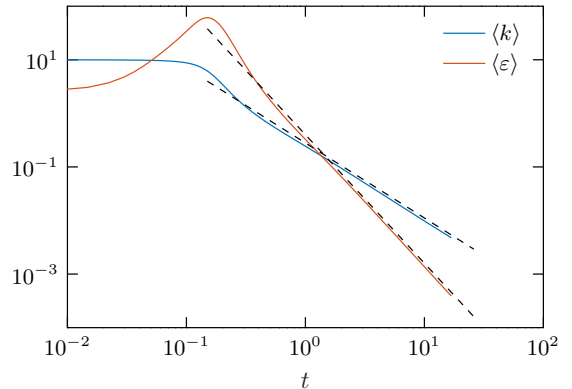


Figure 2: Temporal evolution of $\langle k \rangle$ and $\langle \varepsilon \rangle$ obtained from DNS. The black dashed lines refer to the analytical scaling exponents of $-10/7$ and $-17/7$, respectively.

The initial field is generated in spectral space to be random and isotropic and to satisfy incompressibility. It obeys a prescribed energy spectrum of the von Karman type, i.e.

$$E(\kappa, 0) \propto \kappa^4 \exp\left(-2\left(\frac{\kappa}{\kappa_p}\right)^2\right), \quad (4)$$

where $\kappa_p = 15$ is the wave-number at which the maximum of the initial spectrum $E(\kappa, 0)$ occurs. This value for κ_p is a compromise between limiting the confinement effect through the finite size of the computational domain and the goal of reaching a high Reynolds number. Prescribing a κ^4 energy spectrum at the large scales is equivalent to a conservation of Loitsyansky's integral and thus

$$\langle u^2 \rangle l^5 = \text{constant}. \quad (5)$$

Equation (5) requires a temporal decay of the mean turbulent energy and the mean dissipation as $\langle u^2 \rangle \propto t^{-10/7}$ and $\langle \varepsilon \rangle \propto t^{-17/7}$, respectively. A comparison of these scaling laws with the present DNS gives an excellent agreement, cf. fig. 2. It is important to note, that von Karman turbulence is not completely self-preserving. Complete self-preservation would require the mean turbulent energy to decay as $\langle u^2 \rangle \propto t^{-1}$.

In this work, a passive scalar in homogeneous isotropic decaying turbulence is analyzed by the method of dissipation elements. To this end, an additional advection-diffusion equation for scalar fluctuations is

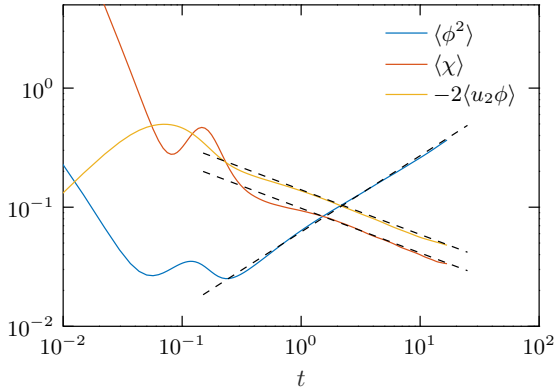


Figure 3: Temporal evolution of the scalar variance $\langle \phi^2 \rangle$ and its production $-2\Gamma\langle u_2\phi \rangle$ and dissipation $\langle \chi \rangle$.

solved, i.e.

$$\frac{\partial \phi}{\partial t} + u_i \frac{\partial \phi}{\partial x_i} = D \frac{\partial^2 \phi}{\partial x_i^2} - 2\Gamma\langle u_2\phi \rangle, \quad (6)$$

where D is the molecular diffusivity. A uniform mean scalar gradient Γ is imposed on the scalar field. In this configuration, the turbulence is decaying while the scalar experiences a continuous injection of energy at the large scales. The scalar variance $\langle \phi^2 \rangle$ is governed by the equation

$$\frac{\partial \langle \phi^2 \rangle}{\partial t} = -2\Gamma\langle u_2\phi \rangle - \langle \chi \rangle, \quad (7)$$

where χ is the mean scalar dissipation, defined as

$$\langle \chi \rangle = 2D \left\langle \left(\frac{\partial \phi}{\partial x_i} \right)^2 \right\rangle, \quad (8)$$

Equation (7) shows that the change of the scalar variance is determined by a balance between production $-2\Gamma\langle u_2\phi \rangle$ and dissipation $\langle \chi \rangle$. The temporal evolution of the terms of eq. (7) is shown in fig. 3.

3 Dissipation element analysis

In this section, we present the analysis of the DNS by dissipation elements with a special emphasis on the question whether statistics of dissipation elements are self-preserving.

3.1 The length distribution of dissipation elements

Two characteristic parameters, the length ℓ and the scalar difference $\Delta\phi$ have been defined to statistically represent dissipation elements. The linear length ℓ between extremal points provides a measure for the length scales present in turbulent fields. As ℓ is a random variable, its probability density function (pdf) is of interest. Firstly, fig. 4 (top) shows the length distribution $P(\ell)$ for different time steps, where it is observed that the curves do not collapse. For later times,

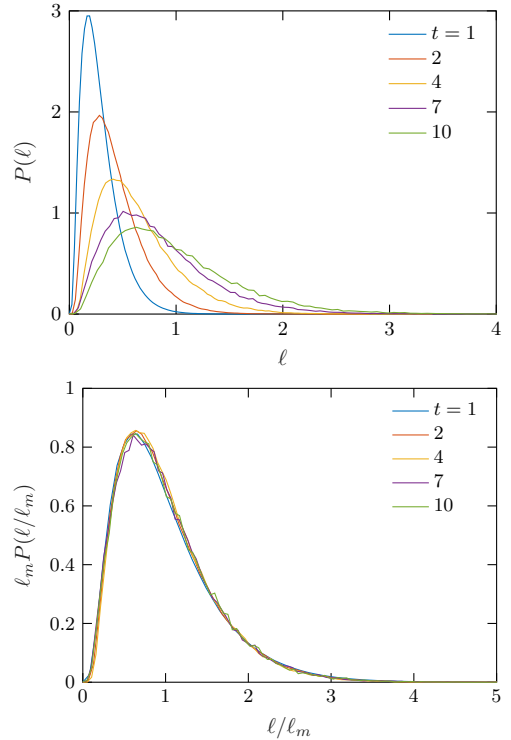


Figure 4: Length distribution of dissipation elements for different time steps.

the maximum of $P(\ell)$ is shifted towards larger scales. In other words: the mean length ℓ_m of dissipation elements, which is related to $P(\ell)$ by

$$\ell_m = \int_0^\infty \ell P(\ell) d\ell, \quad (9)$$

increases during decay, which is in agreement with the increase of all other length scales. The pdfs are strongly non-gaussian and decline steeply towards the origin as small dissipation elements are annihilated by molecular diffusion. Secondly, fig. 4 (bottom) shows the length pdf normalized with the respective mean length ℓ_m . By normalization with a single parameter, namely, the mean length ℓ_m , the marginal pdfs of the length collapse for all time steps to a single curve. This finding suggests that the normalized pdfs of the length in decaying turbulence are self-preserving.

In fig. 4, we have shown that the mean length ℓ_m is the only relevant length scale for the distribution of the length of dissipation elements. To obtain the dimensional pdf $P(\ell)$ we need to know how the normalization quantity ℓ_m scales with time. This question is addressed by fig. 5, where the temporal evolution of ℓ_m is compared with the scaling exponents of the Taylor length scale λ and the Kolmogorov length scale η , which are $\lambda \propto t^{1/2}$ and $\eta \propto t^{0.6}$, respectively, cf. Ishida et al. (2006) and Krogstad & Davidson (2010). Over approximately one decade of decay, ℓ_m is proportional to η , and turns into a Taylor-scaling, $\ell_m \propto \lambda$, at late times. The departure from the Kolmogorov scaling

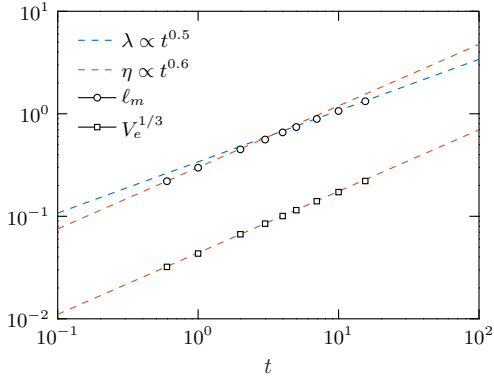


Figure 5: Temporal evolution of the mean length ℓ_m and the mean volume V_e of dissipation elements. As reference, the scaling exponents of the Taylor length scale, $\lambda \propto t^{1/2}$, and the Kolmogorov length scale, $\eta \propto t^{0.6}$ are shown.

at late times might be a result of confinement effects as ℓ_m attains a size which is close to the computational domain.

The temporal evolution of dissipation elements in decaying turbulence is governed by a complex process, involving slow and fast events, cf. Wang & Peters (2006). Slow events change the shape of dissipation elements through strain and diffusion by moving the extremal points relatively to each other. Fast events can be divided into cutting and reconnection events that result from a topology change of the scalar field and lead to a generation and annihilation of dissipation elements. A budget equation for the number of dissipation elements n_e can be derived and reads

$$-\frac{1}{n_e} \frac{\partial n_e}{\partial t} = \Lambda_1/n_e - \Lambda_2, \quad (10)$$

where Λ_1 and Λ_2 denote the frequencies of generation and annihilation, respectively. In decaying turbulence, the Reynolds number decreases and the annihilation process prevails over the generation process. Table 2 confirms that the number of dissipation elements n_e decreases significantly with time and consequently the mean volume of dissipation elements $V_e = (2\pi)^3/n_e$ increases. Figure 5 indicates that $V_e^{1/3} \propto \eta$. Further statistics of dissipation elements are summarized in table 2 for different time steps. For reference, table 2 provides also the Taylor-based Reynolds number Re_λ , the Taylor length scale $\lambda = (10\nu\langle k \rangle / \langle \varepsilon \rangle)^{1/2}$, and the integral length scale l_t , defined as

$$l_t = \frac{3\pi}{4} \frac{\int \kappa^{-1} E(\kappa) d\kappa}{\int E(\kappa) d\kappa}. \quad (11)$$

3.2 The joint distribution of $\Delta\phi$ and ℓ

Turbulent flows are characterized by a complex spatial structure with an interaction between various length scales. Useful information about the spatial structure

Table 2: Characteristic properties of the DNS at different time steps.

t	1	2	4	7	10
Re_λ	51.8	44.9	38.6	34.4	32.2
$\langle k \rangle$	0.2433	0.094	0.035	0.016	0.010
$\langle \varepsilon \rangle$	0.325	0.065	0.012	0.003	0.001
l_t	0.522	0.670	0.854	1.042	1.174
λ	0.058	0.080	0.112	0.149	0.179
ℓ_m	0.298	0.449	0.658	0.890	1.064
n_e	3050211	831991	243961	89841	48555
V_e	0.0001	0.0003	0.0010	0.0028	0.0051

of turbulent flows can be obtained by two-point statistics as they capture not only the local but also the non-local phenomena. The joint probability density function (jpdf) of the length ℓ and the scalar difference $\Delta\phi$ provides additional information about the local structure of scalar fields that allows us to analyze conditional statistics. The jpdf of $\Delta\phi$ and ℓ is shown in fig. 6 for two different time steps and illustrates the increase of both $\Delta\phi$ and ℓ during decay. Additionally, the conditional mean scalar difference $\langle \Delta\phi | \ell \rangle$ is shown in fig. 6. The conditional average rises with ℓ indicating that large elements have on average a large scalar difference.

In the next step, the temporal evolution of the normalized conditional scalar difference is analyzed. Following the scaling theory of Kolmogorov (1941*a,b*), Obukhov (1949) and Corrsin (1951), statistics in the so-called inertial range should depend only on the mean energy dissipation $\langle \varepsilon \rangle$ and the mean scalar dissipation $\langle \chi \rangle$, but should be independent of molecular effects. This hypothesis requires the conditional scalar difference to scale as

$$\langle \Delta\phi | \ell \rangle = \langle \chi \rangle^{1/2} \langle \varepsilon \rangle^{-1/6} \ell^{1/3}. \quad (12)$$

Figure 7 shows the normalized conditional mean $\langle \Delta\phi | \ell \rangle$ for different time steps. A distinct scaling regime can be observed with a scaling exponent close to 1/3. Departures from the analytical scaling exponents can be explained by a combination of finite Reynolds number effects and internal intermittency, cf. Frisch (1995). Moreover, all time steps collapse to a single curve indicating that self-preservation of dissipation elements can be reached in decaying turbulence by choosing ℓ_m as characteristic length scale.

4 Conclusion

We have studied the self-preservation of scalar turbulence in homogeneous isotropic decaying turbulence with the method of dissipation elements. The analysis was based on a highly resolved direct numerical simulation. It indicates that dissipation elements are self-preserving during decay and that the mean length ℓ_m is the relevant characteristic length scale to normalize statistics of dissipation elements.

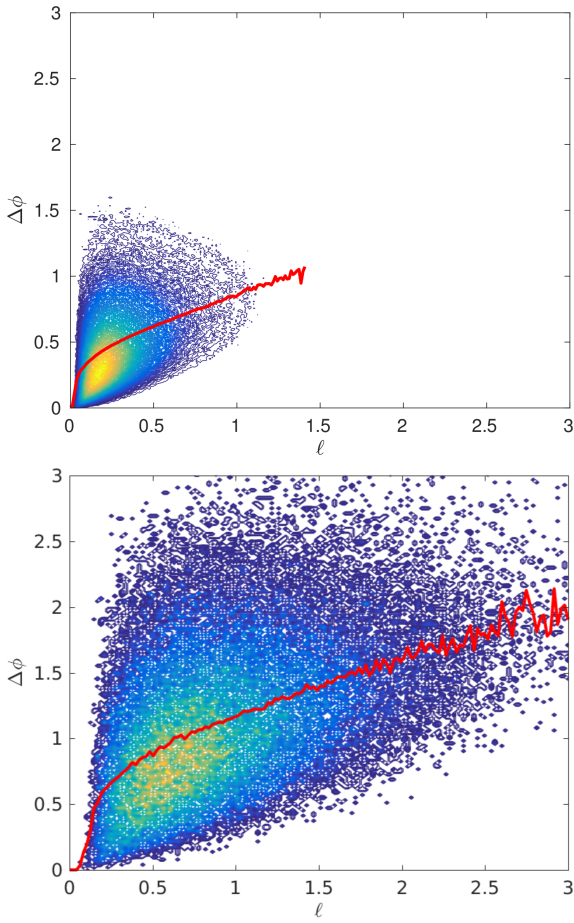


Figure 6: Joint distribution function $P(\Delta\phi, \ell)$ at $t = 1$ (top) and $t = 10$. The red line refers to the conditional average $\langle\Delta\phi|\ell\rangle$.

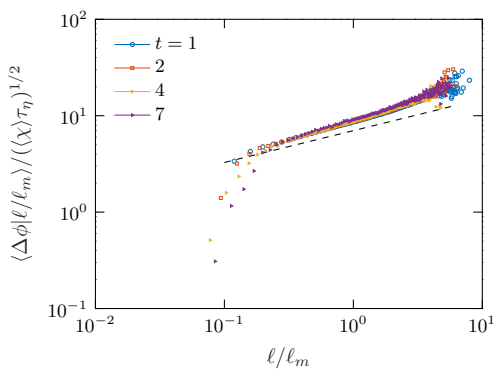


Figure 7: Normalized conditional mean $\langle\Delta\phi|\ell\rangle$ for different time steps. Note that the time step $t = 10$ is omitted from the figure because the number of dissipation elements is not high enough to compute well converged two-point statistics. The black dashed line refers to a scaling exponent of $\ell^{1/3}$.

The main findings are:

1. The length distribution $P(\ell)$ of dissipation elements is strongly non-gaussian and exhibits a strong dependence on time. After normalization with the mean length ℓ_m are curves collapse.
2. The DNS provides a clear support that the mean length ℓ_m is proportional to the Kolmogorov length scale η .
3. A scaling with ℓ_m is not limited to statistics involving the length of dissipation elements. From DNS it was shown that ℓ_m is also the relevant length scale for the conditional mean $\langle\Delta\phi|\ell\rangle$ of the scalar difference.

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