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Short and Long term Effects of Water Markets

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Abstract

We aim at checking from a theoretical viewpoint the claims made about water markets in the literature. We prove that water markets improve farmers' profits and production efficiency but do not necessarily improve total production and do not always encourage private investments. Taking into account the transaction cost of water markets and the social cost due to the inactivity of some farmers that may result from water markets, we prove that water markets do not always improve total surplus.

JEL Classification: Q25, C72, O33.

Key words and phrases: Water markets, Water scarcity, Production efficiency, Total production, Private investment, Social costs, Transaction costs.

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1 Introduction

In several countries having limited water resources, water is being sold cheaply, which encourages water wasting behaviour. The design of an adequate pricing to urge users to take water scarcity into account, implying necessarily a substantial rise of prices, would probably meet resistance from users and would moreover require accurate information on demands. Water markets have been precisely suggested to manage more efficiently water resources while ensuring traditional users' approval, without requiring information on demands. Each user is given a water use right on the basis of his/her historical use at the historical price. An exchange of these rights is allowed, which should lead to a new water price better revealing the water value as it results from a balance of water offer and demand. It should be expected that water markets would result in a more efficient use of water, an improvement of farmers' situation, an improvement of total production thus in an improvement of social welfare. In this paper, we aim to check whether these expectations are always theoretically founded, taking into account the transaction cost of water markets and the social cost due to the inactivity of some farmers that may result from water markets.

In several countries and regions around the world, although water provision requires high costs, water is being attributed through quotas to farmers at very low prices. Some farmers are using water inefficiently (with regard to its real value) while other more efficient farmers are not being satisfied with the quantities received. A water market would result in a balance between offer and demand, thus avoiding the rationing of farmers, and in water transfer from the least to the most efficient farmers (Lahmandi-Ayed and Matoussi, 2003). When the efficiency difference between farmers is sufficiently high, some farmers among the most inefficient ones would stop production activity. This very advantage in terms of water use may have highly negative socio-economic impacts. First inefficient farmers becoming inactive, are very likely to move to cities with all the related families thus increase the number of rural exoders with all their urban, social, political and economic problems mainly in the developing countries. Second they are unemployed persons. The possibility of conversion to another activity is only theoretical for the major part of farmers who are poorly educated thus hardly convertible. Third from the viewpoint of junior farmers, the introduction of a water market results in the introduction of an intermediary in water sale, thus in a rise of water prices. Within a short period, junior farmers will become themselves "senior", and may dislike that a difference remain, which may result in a tenseness in farmers' relationship. Finally people may not think acceptable that a resource involving high public costs, be offered graciously to inefficient farmers that will live on its sale or rent. Hence there is a real cost to put some farmers outside the productive sector which is taken simply into account in this paper.

In the long run water markets are expected to urge farmers to invest in water saving technologies to sell the water saved, which would mitigate at least some of the short term drawbacks. The few empirical observations on the question are contradictory. According to Hearne and Easter (1995), concerning the Chilean water market experience, farmers sell in general a part of their use rights, which allows them to invest in new irrigation techniques that conserve better the resource and results in improving the global production without requiring new mobilizations harmful to the environ-

ment. However, according to Bauer (1997) concerning the same experience, “despite expectations on the contrary, there has been almost no private investment in irrigation technology for the purpose of selling rights to the water saved”. In Most of Chile, the author observes that water use efficiency remains at its traditional level and that flood irrigation remains the dominant practice. Therefore the question of whether water markets foster private investments in water saving technologies, needs to be explored from a theoretical viewpoint.

We consider a simple model involving unequally efficient farmers. We suppose that water resources are being rationed. In the short run farmers have only the possibility of exchanging water. We prove that in the short run water markets improve farmers’ profits thus production efficiency but do not necessarily improve total production. Taking into account the transaction cost of water markets and the social cost due to the inactivity of some farmers which may result from water markets, we prove that water markets do not necessarily improve social welfare. In the long run farmers have the possibility of investing in water saving technologies before exchanging water rights. We prove that water markets do not always foster private investments when compared to the status quo (situation without water market).

The related literature. Water markets look very similar to tradeable pollution permits. Thus it would be natural to compare our results concerning the investment decisions of farmers in the long run with the papers dealing with the innovation in pollution abating technologies within polluting industries, even if a main difference exists compelling to deal separately with each sort of problem: a pollution permit is not an input as it is the case for water rights. Malueg (1989) and Milliman and Prince (1989) prove that the introduction of tradeable pollution permits may actually decrease some firms’ incentives to adopt more effective pollution control technologies. But both papers examine the question at the firm’s level. Jung, Krutilla and Boyd (1996) extend Milliman and Prince’s approach from the firm to the industry level. They prove that auctioned permits provide the most incentive effects to promote the development and adoption of advanced pollution abatement technology. But Jung et al. model the pollution permits price mechanism only roughly while in our model the water price is function of the investment decisions since the demand of each farmer is function of his/her level of investment. The more recent paper of Requate (1998) examines the incentives to innovate (adopt a cleaner technology) within a polluting industry under emission taxes and tradeable permits. Requate proves that “there is no unique ranking between those tools, i.e. neither does one of the two tools provide a higher incentive to adopt a new technology in all cases”. His conclusion is close in spirit to our own one in that tradeable rights do not necessarily provide higher incentives to innovate. His model is richer than the previous ones in that he models more richly the pollution price mechanism and takes the feedback on the output market into account. The weakness of the previous paper lies nevertheless in the hypothesis that only one firm has the possibility of adopting the new cleaner technology, while the possibility of investing in water saving technologies is given in our paper to all inefficient farmers.

The paper is organized as follows. Section 2 describes the model. Section 3 deals with the short term effects of water markets. Section 4 deals with their long term effects mainly in terms of private investment. Section 5 concludes. An appendix contains the proofs.

2 The model

2.1 The general framework

Consider n farmers ($i = 1, \dots, n$) producing a homogenous farm good sold at an exogenous price p . This is a simplifying but reasonable hypothesis as a water market is local while the output trade may be made at a national or international level. The model accounts for the effect of a local switch to water market within a given group of farmers sharing a given water resource, which thus has a negligible effect on the output price.

The limiting factor is supposed to be water thus the produced quantity and costs may be expressed function of the quantity of water used. More precisely, when the quantity of water x_i is applied by farmer i , $f_i(x_i)$ is the quantity of farm good produced, while $C_i(x_i)$ is the total cost stemming from inputs other than water (fertilizers, labor, chemicals...). Functions f_i and C_i are naturally supposed to be increasing. We suppose that they are twice continuously differentiable with $\frac{\partial^2 f_i}{\partial x_i^2} \leq 0$ hence f_i is concave, while $\frac{\partial^2 C_i}{\partial x_i^2} \geq 0$ hence C_i is convex, and that at least one among the two inequalities is strict.

Denote by Ω the total exogenous offer. The authority assigns a given quota ω_i to each farmer i . We have $\Omega = \sum_{i=1}^n \omega_i$.

We refer to *status quo* the situation without water market. In this situation, each farmer may buy a quantity of water up to his/her quota ω_i , at the historical price $p_h = 0$. Hypothesis $p_h = 0$ supposed for simplicity is close to reality in several developing countries as Tunisia or Egypt for instance. Farmer i 's profit in the status quo is given by:

$$\pi_i(x_i) = pf_i(x_i) - C_i(x_i).$$

The solution of the first order condition $\pi'_i(x_i) = pf'_i(x_i) - C'_i(x_i) = 0$ is denoted by \tilde{x}_i^s . The demand of farmer i if he/she were not constrained by his/her quota is: $x_i^s = \max(\tilde{x}_i^s, 0)$.

Taking farmer i 's quota into account, his/her actual demand d_i^s is given by:

$$d_i^s = \begin{cases} x_i^s & \text{if } x_i^s < \omega_i \\ \omega_i & \text{if } x_i^s \geq \omega_i \end{cases}$$

We suppose that

$$\sum_{i=1}^n x_i^s > \Omega. \quad (1)$$

This inequality means that the total desired quantity is not satisfied, which would be natural in a scarcity context. It will also ensure as we will see the existence of a price equilibrium when a water market is introduced. This implies that there exists necessarily some rationed farmers, i.e. such that $x_i^s > \omega_i$.

In the welfare analysis, we focus on the implied group of farmers and the authority who organizes the water market (who is inactive in the status quo). This partial analysis is a first step to have an idea on the effect of water markets on the agricultural sector taking into account their direct costs. The explicit consideration of interests outside the group of farmers and the organizing authority would require at least to model

adequately the output market thus for instance to deal with water market within the other groups of farmers and to have information on the demand side. This would be a more complete treatment of the subject and may be a further step of the present research. However the demand side is implicitly dealt with through the examination of the effect of water market on total production.

The social welfare in the status quo is given by

$$W_s = \sum_{i=1}^n \pi_i(d_i).$$

Suppose now that a water market is introduced. We suppose that this involves a transaction cost T , often mentioned in the literature. The transaction cost T is a global cost corresponding to all the costs incurred by the organizing authority to set up the water market and make it work. In this paper, we suppose that farmers do not pay fees to the organizing authority when they participate in the water market. Cost T thus corresponds somehow to a cost assumed by the remaining of the society in order to promote water markets. Indeed when a new practice is introduced, even if it is expected to do only good, the mere mentioning of a fee to be paid may give rise to suspicion thus may discourage the adhesion of farmers. Thus we think that the transaction cost should be incurred entirely by the organizing authority at least for some period of time if it wants to promote water markets. If farmers were to pay fees, the sharing of the transaction cost is not a simple issue and the conclusions of the paper may change substantially depending on the way those fees are calculated, another issue which can be dealt with in a next work.

Section 3 deals with the short term effects of water markets. Farmers have only the possibility after receiving their quotas to exchange water, which results in a new price p_e . We suppose that they are price-takers.

Under some conditions water markets result in the inactivity of some farmers. We suppose that this inactivity involves a social fixed cost S per each inactive farmer. Cost S may be viewed as the cost necessary to solve all the problems implied by the inactivity of one farmer (rural exodus, conversion to another activity...) ¹.

2.2 A specification of the model for the long run

Section 4 deals with the long term effects of a water market. We suppose that in the long run farmers have the possibility of making investments to “improve their productivities” before water exchange. To deal simply with the long run, we choose a particular specification of the relevant functions.

We suppose that all farmers produce with the same cost function $C_i(x_i) = cx_i^2$ and that the n farmers are divided into two homogeneous groups. The first group involves q “inefficient farmers” ($q \geq 2$), $i = 1, \dots, q$, i.e. that have the same function $f_1(x) = \delta_1 x$, thus the same low “productivity” δ_1 . The second group involves m “efficient farmers”, $i = q + 1, \dots, q + m$, “up-to-date” i.e. that have the best possible productivity δ_2 , implying function $f_2(x) = \delta_2 x$. Parameters δ_1 and δ_2 correspond to the quantity of

¹See the introduction for a more detailed explanation.

output obtained per unit of water used, respectively by inefficient and efficient farmers. The unit of the farm good is chosen such that $\delta_2 = 1$.

In order to check whether the long term effects of water markets mitigate the short term ones, the difference between the two productivities is chosen such that only the efficient farmers would remain productive in the short run if a water exchange were to take place². For simplicity, we consider a specific numerical case: $\frac{c\Omega}{mp} = 1/4$ and $\delta_1 = 1/3$; and we take the case of farmers having the same quota: $\omega_i = \omega$. Hence, the relation $\frac{c\Omega}{mp} = 1/4$ becomes

$$\frac{c\omega}{p} = \frac{m}{4(m+q)}. \quad (2)$$

Moreover we suppose that

$$m \leq 2q. \quad (3)$$

Equality (2) together with Inequality (3) imply that in the status quo a farmer maximizes his/her profit when he/she uses his/her whole quota (See the proof of Proposition 2). This means that all farmers are being rationed in the status quo.

In the long run the inefficient farmers have the possibility of improving their productivities. To move from productivity δ_1 to some productivity μ with $\delta_1 \leq \mu \leq 1$, an inefficient farmer must invest $I(\mu) = a(\mu - \delta_1)$. Parameter $a > 0$ may be called the intensity of investment and measures how intensive an investment must be to move from the initial productivity to some given new one. We suppose that efficient farmers cannot improve their productivities, they have the best available one.

Inefficient farmers are involved in a non-cooperative game in which they choose simultaneously their investment levels (or equivalently the level of their new productivities). In the second step or the “exchange” step, after observing the choices of the first step, with the new productivities, a water exchange occurs between farmers, supposing that farmers have a competitive behaviour.

3 The short run

In this section we compare the status quo with a water market in the short run from the following viewpoints: the profit of each farmer and production efficiency, total production and social welfare. We prove that a water market always improves each farmer’s profit thus production efficiency but does not necessarily improve total production and social welfare.

When a water market is introduced, the profit of farmer i is given by:

$$\pi_i = p_e(\omega_i - x_i) + pf_i(x_i) - C_i(x_i).$$

$p_e(\omega_i - x_i)$ is the part of the profit coming from the exchange (to be further referred to as “exchange profit”) which may be either positive if the farmer is globally a water seller or negative if the farmer is globally a water buyer; while $pf_i(x_i) - C_i(x_i)$ is the profit coming from production (to be referred to as “production profit”).

²This is a consequence of a result obtained by Lahmandi-Ayed and Matoussi (2003) recalled in Proposition 1 later in this paper.

Denote by $x_i(p_e)$ the solution of the first order condition:

$$pf'_i(x_i) - C'_i(x_i) = p_e.$$

The demand of farmer i is given by $d_i(p_e) = \max(x_i(p_e), 0)$.

A price equilibrium p_e^* satisfies:

$$\sum_{i=1}^n d_i(p_e^*) = \Omega.$$

Result 1 (Existence of price equilibrium) *Under the general framework of the model, there exists a unique price equilibrium.*

Result 1 is needed to ensure that the demonstrated properties have cases of application. It holds for general specifications of f_i and C_i under the general hypotheses supposed in the model.

Result 2 (Profits, Production efficiency) *Under the general framework of the model, the water market improves each farmer's profit thus improves production efficiency*

Result 2 is a natural one for, with a water market each farmer has the possibility of applying exactly quantity d_i^s (his/her demand in the status quo), which gives him/her a production profit equal to his/her profit in the status quo, and selling the difference $\omega_i - d_i^s$ at price p_e , which gives him/her an additional exchange profit. Water markets offer indeed more possibilities to farmers by releasing the constraint on demands, which necessarily improves the situation of each one of them. Consequently, if they are given the choice, farmers have always interest to participate in the water market.

Denote by A the difference between the sum of profits with a water market at price equilibrium and the sum of profits in the status quo. Necessarily $A > 0$. Hence in terms of production, a water market improves production efficiency in the sense that it improves the profit of the whole sector as it improves each farmer's profit.

However the improvement of production efficiency is not synonymous of the improvement of total production. This is illustrated through the following simple example.

Result 3 (Total production) *Suppose that there are two farmers $i = 1, 2$. Farmer i has function $f_i(x_i) = \delta_i x_i$, the cost function $C_i(x_i) = c_i x_i^2$ and the quota $\omega_i = \Omega/2$. Let $\delta_1 = 1$, $\delta_2 = 3$, $c_1 = 0.5$, $c_2 = 2.9$ and $\frac{\Omega}{p} = 1$. With these values of the parameters, the switch to water market decreases total production relative to the status quo.*

Note that we consider the case of two farmers for exposition simplicity. But the same type of result may be proved for two groups of homogenous farmers. The reason that total production decreases with the chosen example is that the most productive farmer has also the highest cost. Thus there is a shift to more production by the farmer with lowest productivity and lowest cost³. Obviously the result is not a general

³An anonymous referee is thanked for this remark.

one⁴ but is provided only to say that it is a possible case. As cost C_i corresponds to the costs implied by the inputs other than water (labor, fertilizers...), the quantities to be used with a given amount of water thus the implied cost C_i may depend on several parameters as the land's slope or the soil's nature. Thus all combinations are *a priori* possible (high productivity with high costs, low productivity with low costs, high productivity with low costs...) including the given example.

Although a water market always increases the profit of each farmer thus always increases total profit (which is true in particular with the chosen example), this does in no way imply an increase of total production. In short terms, the example provided by Result 3 together with the general Result 2 show that the sector may end up earning more and producing less after setting up a water market!

Finally, in terms of welfare, if only the welfare of incumbent farmers is considered, it is obvious that a water market improves the social surplus, since it improves the profit of each one of them. But if the transaction and the inactivity costs are considered, a water market has an ambiguous effect on social surplus.

If equilibrium involves h inactive farmers, the surplus difference between both situations

$$W_w - W_s = A - hS - T.$$

The water market improves the social welfare only if:

$$A - hS - T > 0$$

The last inequality allows to see simply the advantages and disadvantages of water markets, as far as only farmers and the organizing authority are concerned. A water market improves the farmers' profit but involves costs and may result in the inactivity of some farmers. A net benefit would result only if the gain outweighs the losses.

But number h is endogenous. The consideration of a specific example allows to calculate that number and derive conclusions on whether a water market improves welfare, depending on exogenous parameters.

An example

Lahmandi-Ayed and Matoussi (2003) studied the case of linear functions $f_i(x_i) = \mu_i x_i$, μ_i being the productivity of farmer i , and a quadratic cost function $C(x_i) = cx_i^2$. Farmers are supposed to have the same cost function and to differ only by their functions f_i . Thus a farmer with a higher productivity is more efficient than a farmer with a lower productivity. Suppose that farmers are ordered as follows: $\mu_1 \leq \mu_2 \cdots \leq \mu_n$. And denote by $(u_i)_{1 \leq i \leq n}$ defined by:

$$u_i = \sum_{j=i+1}^{j=n} (\mu_j - \mu_i)$$

At equilibrium "Productive" farmers are those who produce a positive quantity of the farm good and "non-productive" farmers those who make profit only from water resale. Lahmandi-Ayed and Matoussi (2003) proved Proposition 1 stated below.

⁴In the example provided in Proposition 1 a water market increases total production.

Proposition 1 (Lahmandi-Ayed and Matoussi, 2003) *At the competitive equilibrium,*

- 1) *if $2c\Omega/p > u_1$ then all the farmers are productive (thus $h = 0$).*
- 2) *if $2c\Omega/p \leq u_{n-1}$, then only Farmer n , the most efficient one, is productive (thus $h = n - 1$).*
- 3) *otherwise, there exists some integer $2 \leq i \leq n - 1$ such that $u_i < 2c\Omega/p \leq u_{i-1}$, in which case only farmers $j = i, \dots, n$ are productive (thus $h = i - 1$).*

What is particular here is that the water market improves not only production efficiency as we defined it, but total production, as water is transferred from the least to the most efficient farmers in terms of production. Moreover in the first case where the difference in productivities is small enough, all farmers remain active.

Consider now *the specification of the model adopted for the long run* which is a particular case of the above one. The parameters of the example have been chosen such that only the efficient farmers are active in the short run if a water market is set up (thus $h = q$).

After calculations, the improvement of the sum of profits due to the water market is equal to $A = (5/12)\frac{qp\Omega}{m+q}$. The water market improves the social welfare only if⁵:

$$(5/12)\frac{qp\Omega}{m+q} - T - qS > 0. \quad (4)$$

Denote by $\alpha = (5/12)p\Omega$. Inequation 4 is equivalent to

$$q < \hat{q} = \frac{\alpha - T - mS + [(\alpha - T - mS)^2 + 4STm]^{1/2}}{2S},$$

which may also be written as:

$$S < \frac{1}{q}[(5/12)\frac{qp\Omega}{m+q} - T].$$

Therefore there is an overall benefit in the short run from a water market only if inefficient farmers are not too numerous and/or the cost due to inactivity of a farmer is not too high. Otherwise, the social welfare worsens with a water market relative to the status quo. Indeed in the opposite case, the improvement of the farmers' profit implied by a water market is not enough to outweigh the too high social costs due to those inactive farmers.

4 The long run

We now suppose that farmers have the possibility of investing in water saving technologies as in the specification described in the model for the long run. As in the

⁵Note that Inequality 4 involves now only exogenous parameters.

short run case, it is easy to check that a water market improves each farmer's profit thus production efficiency. Hence each farmer has interest to participate in the water market if he/she is given the choice. However we prove that a water market does not always encourage private investments relative to the status quo.

Result 4 (Long run, farmers' profits and production efficiency) *Under the specification for the long run, a water market improves each farmer's profit thus production efficiency.*

Each farmer has always the possibility within a water market to keep his/her initial productivity, apply the same quantity of water as the status quo and sell the remaining quantity, which gives him/her a profit at least equal to his/her profit in the status quo. The improvement of each farmer's profit naturally implies the improvement of the sector's profit.

It remains now to check whether a water market encourages investments. We consider the investment decisions respectively in the status quo then with a water market. Then the investment decisions in both situations are compared.

Proposition 2 (Investment decisions in the status quo) *Under the specification for the long run, in the status quo,*

- If $p\omega > a$ all inefficient farmers choose $\mu = 1$.
- If $p\omega \leq a$ all inefficient farmers choose $\mu = 1/3$, i.e., no investment is made.

Suppose now that a water market is set up. Proposition 3 provides the investment decisions of farmers in such conditions. Before doing so, we need the following lemma.

Lemma 1 *Under the specification for the long run, an inefficient farmer necessarily chooses productivity $\mu = 1/3$ or productivity $\mu = 1$. Thus after the investment choice, productive farmers have necessarily a productivity equal to 1.*

Lemma 1 states that an inefficient farmer actually makes his/her choice between keeping his/her initial productivity and moving to the best one. He/she never makes intermediary decisions. This is because the investment is linear w.r.t. the difference between productivities. This result allows to simplify the comparison between the water market and the status quo in terms of private investment, as it amounts to the comparison of the number of investing farmers in both situations.

Proposition 3 (Investment decisions with a water market) *Under the specification for the long run, let $(u_t)_{m \leq t \leq m+q-1}$ be the sequence defined by:*

$$u_t = (3/2)mp\omega \left(\frac{1}{2t(t+1)} + \frac{(m+q)}{4(t+1)^2} \right) \quad (5)$$

Three cases are possible:

- $u_m \leq a$, there is a unique Nash equilibrium in which all inefficient farmers keep their productivity at its initial level.
- $a < u_{m+q-1}$, there is a unique Nash equilibrium in which all inefficient farmers move to productivity 1.
- Otherwise, there exists some integer h such that $u_{m+h} \leq a < u_{m+h-1}$, in which case the set of Nash equilibria involves all the q -uples in which there are exactly h investing farmers who move to productivity 1 and $(q - h)$ farmers that keep their initial productivity.

The more a is high, the more investments are intensive. Hence, when a is too high ($a \geq u_m$), it is interesting for no inefficient farmer to invest. When a is sufficiently small ($a < u_{m+q-1}$), all inefficient farmers will move to the best productivity. For intermediary values of a , it is interesting to make the investment only if those who have made so are not so numerous. Indeed if the number of investing productive farmers is too large, the water demand is high, making more interesting to be just a water retailer without production activity thus to make no investment.

We are now ready to compare the investment decisions in both situations. The ranking of $p\omega$, u_m and u_{m+q-1} plays a key role in the comparison of the investment decisions (Remember that the sequence (u_m) is defined in Proposition 3.)

We prove by a simple calculation that we always have:

$$p\omega > u_{m+q-1}.$$

We also prove that $p\omega < u_m$ is equivalent to⁶

$$\frac{5}{3} + \frac{10}{3m} + \frac{2}{3m^2} < \frac{q}{m}.$$

Result 5 (Comparison) *Under the specification for the long run, for the comparison of the incentives for private investment between the status quo and a water market, two cases are possible.*

1. When $\frac{q}{m} < \frac{5}{3} + \frac{10}{3m} + \frac{2}{3m^2}$, the number of investing farmers in the status quo is always greater than or equal to that number with a water market. In this case, a water market provides lower incentives for private investments.
2. When $\frac{5}{3} + \frac{10}{3m} + \frac{2}{3m^2} < \frac{q}{m}$, two subcases have to be distinguished.
 - (a) When $a < p\omega$ the number of investing farmers with a water market is less than or equal to that number in the status quo.
 - (b) When $a \geq p\omega$, the number of investing farmers with a water market is greater than or equal to that number in the status quo.

⁶Note that when m is very large this comes roughly to the comparison of q/m with $5/3$.

From Result 5 the consideration of the effects of water markets in terms of private investments turns out to be more complicated than expected by a rough intuitive approach. In some cases water markets may dissuade farmers from investing while the same farmers would invest if no water market is set up. These findings which could not be easily guessed in advance, may be explained as follows.

With a water market, the profit of an (initially) inefficient farmer comes from production and water resale. The decision of investing in new technologies will be taken only if water resale is not very profitable. Water markets result in two contradictory effects. First a water market better reveals the value of water through the water equilibrium price, thus urges farmers to save water through adequate investments. At the same time, the possibility of water resale may dissuade farmers from production and investment. Without a water market, inefficient farmers make profit only from production. The decision of investment is taken if it is profitable in terms of production only.

In the first case ($\frac{q}{m} < \frac{5}{3} + \frac{10}{3m} + \frac{2}{3m^2}$), the proportion of initially inefficient farmers is relatively low, then the second effect is always the winner. Indeed in this case water being scarce because provided by few farmers, water resale is profitable and inefficient farmers are more willing to rely on water resale than on production, to make profit. The existence of a water market always reduces the number of investing farmers when compared to the status quo.

In the second case ($\frac{q}{m} > \frac{5}{3} + \frac{10}{3m} + \frac{2}{3m^2}$), the number of inefficient farmers is relatively high, the choice between water resale and investment-production becomes more complicated. The two effects have comparable consequences. Hence *a priori* the number of investing farmers with a water market may be greater or smaller than that number in the status quo. For sufficiently intensive investments and/or sufficiently low output or quota ($p\omega < a < u_m$), it appears that the first effect is stronger than the second one. This can be explained in a double way. Considering the production and investment side, a high intensity of investment and/or a low output price have two contradictory effects. Investment and production are not profitable as production is costly w.r.t. entries, reducing the incentive for investment. This direct effect raises water offer, lowers the water price thus making the water resale less profitable and production more attractive! An equilibrium between the two effects occurs when some farmers among the inefficient ones invest and the remaining ones continue to sell their water rights. The explanation is simpler when we view the result in terms of quotas. When the quota is low, the available water is scarce, which makes its resale interesting and lowers the incentives for investment and production within a water market.

5 Concluding Remarks

To conclude, the claims made about the effects of water markets in the short and long run, are not always theoretically founded. Several concluding remarks and perspectives for future research may be driven from the analysis.

Water markets improving each farmer's profit thus the whole sector's profit, ensure traditional farmers' support and are likely to improve the agricultural sector's situation in countries where farmers are among the poorest people, without increasing the water

offer thus without harming the environment. The same amount of water is used more efficiently through water markets. However only traditional farmers are considered in the model. The results may change if “potential farmers” are considered. A water market may even dissuade new farmers from entering in the sector as it results in a rise of water prices, while they would be younger, more highly educated, thus more aware of the modern techniques than traditional farmers. A further interesting step to this research would be to deal with the effects of water markets considering a group of farmers of two types: traditional farmers with historical use rights and potential ones with no use rights.

However the improvement of production efficiency does not necessarily imply an improvement of total production. We indeed prove that a water market may not improve total production, which may be problematic in a scarcity or poverty context or when the considered farm good has a strategic or food security role (for instance some kinds of cereals in Tunisia).

Taking into account the transaction cost of water markets and the social cost due to the inactivity of the least efficient farmers, we prove that a water market does not necessarily improve social surplus. The welfare analysis would be more complete if the price of the farm good is endogenized and the demand side of the farm good properly taken into account. This would be another possible further research step.

Finally, a water market sometimes discourages private investments. This last result, contrary to the roughly intuitive expectations but consistent with some observations on water market experiences, proves that the short term drawbacks of a water market are not always mitigated by its long term effects.

Appendix: Proofs

Proof of Result 1. First note that $x_i^s = d_i(0)$. Inequality (1) is then equivalent to $\sum_{i=1}^n d_i(0) > \Omega$. Equation $pf'_i(x_i) - C'_i(x_i) = p_e$ defines implicitly $\tilde{x}_i(p_e)$ as a continuous decreasing function of p_e . d_i is then a continuous non-increasing function of p_e , and so is $\sum_{i=1}^n d_i$. On the other hand there exists \bar{p}_e such that for all $p_e > \bar{p}_e$, for all $i = 1, \dots, n$, $pf'_i(0) - C'_i(0) - p_e < 0$, then $\pi'_i(x_i) < 0$ for all $x_i \geq 0$, which implies $d_i(p_e) = 0$, so $\sum_{i=1}^n d_i(p_e) = 0$. Consequently, there exists a unique $p_e^* > 0$ such that $\sum_{i=1}^n d_i(p_e^*) = \Omega$. ■

Proof of Result 2. As $d_i(p_e)$ maximizes the profit of farmer i with a water market, it ensures to him/her a better profit than d_i^s his/her demand in the status quo, which is written as

$$pf_i(d_i(p_e)) - C_i(d_i(p_e)) + p_e(\omega_i - d_i(p_e)) \geq pf_i(d_i^s) - C_i(d_i^s) + p_e(\omega_i - d_i^s).$$

As $p_e(\omega_i - d_i^s) \geq 0$ since $d_i^s \leq \omega_i$, we have

$$pf_i(d_i^s) - C_i(d_i^s) + p_e(\omega_i - d_i^s) \geq pf_i(d_i^s) - C_i(d_i^s),$$

implying

$$pf_i(d_i(p_e)) - C_i(d_i(p_e)) + p_e(\omega_i - d_i(p_e)) \geq pf_i(d_i^s) - C_i(d_i^s),$$

which says that the profit of farmer i is better under water market than in the status quo.

Summing these inequalities, together with $\sum_{i=1}^n d_i(p_e) = \sum_{i=1}^n \omega_i$, we prove that a water market improves the profit of the whole sector. ■

Proof of Result 3. With these values, we have first: $x_i^s > \omega_i$, so that in the status quo $d_i^s = \omega_i = \Omega/2$ and total production equals $\mathcal{P}^s = (\delta_1 + \delta_2)\Omega/2$.

With a water market, we have $\delta_2 - \delta_1 < \frac{2c_2\Omega}{p}$, which ensures the activity of both farmers at equilibrium. Demands of farmers are respectively given by the following:

$$d_1 = \frac{p\delta_1 - p\delta_2 + 2c_2\Omega}{2(c_1 + c_2)}$$

and

$$d_2 = \frac{p\delta_2 - p\delta_1 + 2c_1\Omega}{2(c_1 + c_2)}.$$

Total production is then given by

$$\mathcal{P}^w = \delta_1 d_1 + \delta_2 d_2 = \frac{p(\delta_2 - \delta_1)^2 + 2\Omega(c_2\delta_1 + c_1\delta_2)}{2(c_1 + c_2)}.$$

$\mathcal{P}^w < \mathcal{P}^s$ is equivalent to $\delta_2 - \delta_1 < \frac{\Omega(c_2 - c_1)}{p}$, which holds with the chosen values of the parameters. ■

Proof of Result 4. Denote by $d_i(\mu_i, \mu_{-i})$ the farmer i 's demand of water when he/she chooses μ_i and the other farmers choose μ_{-i} . Denote by μ^* the q -uple of productivities chosen by farmers at equilibrium. We have the following:

$$\begin{aligned} pf_i(d_i(\mu_i^*, \mu_{-i}^*)) - C_i(d_i(\mu_i^*, \mu_{-i}^*)) - I(\mu_i^*) + p_e(\omega_i - d_i(\mu_i^*, \mu_{-i}^*)) \\ \geq pf_i(d_i(\delta_1, \mu_{-i}^*)) - C_i(d_i(\delta_1, \mu_{-i}^*)) + p_e(\omega_i - d_i(\delta_1, \mu_{-i}^*)) \end{aligned} \quad (6)$$

$$\geq pf_i(d_i^s) - C_i(d_i^s) + p_e(\omega - d_i^s) \quad (7)$$

$$\geq pf_i(d_i^s) - C_i(d_i^s). \quad (8)$$

Inequality 6 holds as μ_i^* is a best reply to μ_{-i}^* thus is better than the initial productivity δ_1 . Inequality 7 holds as quantity $d_i(\delta_1, \mu_{-i}^*)$ is chosen to maximise farmer i 's profit within a water market thus is better than d_i^s the farmer's demand in the status quo.

This proves that a water market improves each farmer's profit.

Now summing the inequalities

$$pf_i(d_i(\mu_i^*, \mu_{-i}^*)) - C_i(d_i(\mu_i^*, \mu_{-i}^*)) - I(\mu_i^*) + p_e(\omega_i - d_i(\mu_i^*, \mu_{-i}^*)) \geq pf_i(d_i^s) - C_i(d_i^s),$$

we obtain

$$\sum_{i=1}^n pf_i(d_i(\mu_i^*, \mu_{-i}^*)) - C_i(d_i(\mu_i^*, \mu_{-i}^*)) - I(\mu_i^*) \geq \sum_{i=1}^n pf_i(d_i^s) - \sum_{i=1}^n C_i(d_i^s).$$

■

Proof of Proposition 2: Denote by μ the productivity chosen by an inefficient farmer. His/her profit when he allocates quantity x to production, is given by:

$$\pi = p\mu x - cx^2 - a(\mu - 1/3)$$

His profit is maximum at $\bar{x} = \omega$. Indeed, first order condition yields: $x^* = p\mu/2c \geq p/6c$, to be compared to ω . But relation (2) equivalent to $\omega = pm/4c(m+q)$ together with inequation (3), implies $x^* \geq \omega$.

After calculations, the profit is given by:

$$\pi = (p\omega - a)\mu - \frac{mp\omega}{4(m+q)} + \frac{1}{3}a,$$

which implies the result. ■

Proof of Lemma 1: Consider an inefficient farmer. Denote by μ the productivity he/she chooses at the second step.

Suppose that μ is such that he/she remains non-productive in the exchange step, his/her profit is then given by:

$$\pi(\mu) = p_e\omega - a(\mu - 1/3)$$

But p_e is independent of μ , thus on this segment, the profit is a decreasing linear function w.r.t. μ .

Suppose now that μ is such that he/she becomes productive in the exchange step. Let t be the number of productive farmers other than the considered one. Note that t is greater than m for, efficient farmers are productive.

The equilibrium water price is given by:

$$p_e = \left(\frac{1}{t+1}\right)[p(\mu + \alpha) - 2c(t+1)\omega],$$

α being the sum of productivities of all the other productive farmers.

The productive quantity is given by:

$$x = \frac{pt}{2(t+1)c}\mu - \frac{p}{2(t+1)c}\alpha + \omega$$

The total profit of the considered farmer is given by:

$$\pi(\mu) = [p_e(\omega - x) + p\mu x - cx^2] - a(\mu - 1/3)$$

The coefficient of μ^2 has the same sign as $1 - \frac{t+2}{2t+2}$ which is non negative for all $t \geq 0$.

Hence, the profit of the farmer is a continuous function that is linear decreasing then convex. It reaches its maximum at $1/3$ or at 1 . ■

Proof of Proposition 3: Consider an inefficient farmer $i \in \{1, \dots, q\}$. Let t be the number of productive farmers (in the first and the second group other than the considered one). According to Lemma (1), they have necessarily a productivity equal to 1. Farmer i makes his/her choice between $\mu_i = 1/3$ and $\mu_i = 1$.

If $\mu_i = 1/3$, then the equilibrium water price is equal to

$$p_e = \frac{p}{t} - \frac{2c\Omega}{t} = p - \frac{2c\Omega}{t}.$$

The corresponding profit is given by:

$$\pi(1/3) = \left(p - \frac{2c\Omega}{t}\right)\omega$$

using the relation $\frac{c\Omega}{mp} = 1/4$, this profit becomes

$$\pi(1/3) = \left(1 - \frac{m}{2t}\right)p\omega.$$

If $\mu_i = 1$, the equilibrium water price is equal to

$$p_e = p - \frac{2c\Omega}{t+1},$$

and the productive quantity is equal to

$$x_i = \frac{\Omega}{t+1}.$$

The corresponding profit is equal to

$$\pi(1) = (p_e(\omega - x) + px - cx^2) - (2/3)a$$

Using Equality (2), this profit is equal to

$$\pi(1) = p\omega\left(1 - \frac{m}{2(t+1)}\right) + p\Omega\left(\frac{m}{4(t+1)^2}\right) - (2/3)a.$$

The difference is then given by:

$$\pi(1) - \pi(1/3) = mp\frac{\omega}{2t(t+1)} + mp\frac{(m+q)\omega}{4(t+1)^2} - (2/3)a.$$

Farmer i moves to productivity $\mu_i = 1$ if $a < u_t$.

Note that $(u_t)_{m \leq t \leq m+q-1}$ is a decreasing sequence. Three cases are then possible:

If $u_m \leq a$, then for all t , $u_t \leq a$. This implies that whatever the number of other investing farmers, Farmer i has no interest to invest.

If $a < u_{m+q-1}$ then for all t , $a < u_t$. This implies that whatever the number of other investing farmers, Farmer i invests.

Otherwise, there exists some integer h such that $u_h \leq a < u_{h-1}$. In this case the best reply of Farmer i is to move to productivity 1 if the number of other productive farmers is less or equal to $h - 1$, and to keep his/her productivity if this number is greater or equal to h . ■

Proof of Result 5.

1) When $\frac{q}{m} < \frac{5}{3} + \frac{10}{3m} + \frac{2}{3m^2}$, we have: $u_{m+q-1} < u_m < p\omega$.

- When $a < p\omega$ all farmers invest in the status quo. However with a water market all of them invest only when $a < u_{m+q-1}$. The number of investing farmers with a water market is at most equal to $q - 1$ when $u_{m+q-1} < a$.
- When $a > p\omega > u_m > u_{m+q-1}$, whether or not a water market is implemented, no farmer invests.

2) When $\frac{q}{m} > \frac{5}{3} + \frac{10}{3m} + \frac{2}{3m^2}$, we have: $u_{m+q-1} < p\omega < u_m$.

Denote by n_{wm} the number of investing farmers with a water market and by n_s the number of investing farmers in the status quo. Results are summarized in the table below:

a	n_{wm}	Comparison	n_s
$a < p\omega$	$\leq q$	\leq	q
$p\omega < a < u_m$	> 0	$>$	0
$a > u_m$	0	$=$	0

■

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