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The Bell's theorem revisited: a subtle, though crucial, assumption has gone unnoticed, and it might not be justified!

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The Bell's theorem stands as an insuperable roadblock in the path to a very desired intuitive solution of the Einstein-Podolsky-Rosen paradox and, hence, it lies at the core of the current lack of a clear interpretation of the quantum formalism. The theorem states through an experimentally testable inequality that the predictions of quantum mechanics for the Bell's polarization states of two entangled particles cannot be reproduced by any statistical model of hidden variables that shares certain intuitive features. In this paper we show, however, that the proof of the Bell's theorem involves a subtle, though crucial, assumption that is not required by fundamental physical principles and, hence, it is not necessarily fulfilled in the experimental setup that tests the inequality. In fact, this assumption can neither be properly implemented within the standard framework of quantum mechanics. Namely, the proof of the Bell's theorem assumes that there exists an absolute preferred frame of reference, supposedly provided by the lab, which would enable to compare the orientation of the polarization measurement devices for successive realizations of the experiment and, hence, to define jointly their response functions over the space of hypothetical hidden configurations for all their possible alternative settings. The need for this assumption can be readily checked by noticing that the proof of the theorem does not necessarily hold when the orientation of one of the detectors is taken as a reference to define the relative orientation of the second detector, in spite that this is an absolutely legitimate choice according to Galileo's principle of relativity. We solve this puzzle by noticing that only the relative orientation between the two measurement devices in every single realization of the experiment is a properly defined physical degree of freedom, while their global rigid orientation is an spurious unphysical gauge degree of freedom. Therefore, the absolute frame of reference required by the proof of the Bell's theorem does not necessarily exist. In particular, an absolute frame of

reference cannot exist in models in which the gauge symmetry of the experimental setup under global rigid rotations of the two detectors is spontaneously broken by the hidden configurations of the pair of entangled particles and a non-zero geometric phase appears under some cyclic gauge symmetry transformations. Following this observation we build an explicitly local model of hidden variables that reproduces the predictions of quantum mechanics for the Bell's states.

I. INTRODUCTION

The Bell's theorem is one of the fundamental theorems upon which relies the widespread belief that quantum mechanics is the ultimate mathematical framework within which the hypothetical final theory of the fundamental building blocks of Nature and their interactions should be formulated. The theorem states through an experimentally testable inequality (the Bell's inequality) that statistical models of hidden variables that share certain intuitive features cannot reproduce the predictions of quantum mechanics for the entangled polarization states of two particles (Bell's states) [1, 2]. These predictions have been confirmed beyond doubt by very carefully designed experiments [3–12].

In these experiments a source emits pairs of particles whose polarizations are arranged in a Bell's entangled state:

$$|\Psi_\Phi\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle^{(A)} |\downarrow\rangle^{(B)} - e^{i\Phi} |\downarrow\rangle^{(A)} |\uparrow\rangle^{(B)}), \quad (1)$$

where $\{|\uparrow\rangle, |\downarrow\rangle\}^{(A,B)}$ are eigenstates of Pauli operators $\sigma_Z^{(A,B)}$ along locally defined Z-axes for each one of the two particles. The two emitted particles travel off the source in opposite directions towards two widely separated detectors, which test their polarizations. The orientation of each one of the detectors can be freely and independently set along any arbitrary direction in the XY-plane perpendicular to the locally defined Z-axis. Upon detection each particle causes a binary response of its detector, either +1 or -1. Thus, each pair of entangled particles produces an outcome in the space of possible events $\{(-1, -1), (-1, +1), (+1, -1), (+1, +1)\}$. We refer to each detected pair as a single realization of the experiment.

Quantum mechanics predicts that the statistical correlation between the outcomes of the two detectors in a long sequence of realizations of the experiment is given by:

$$E(\Delta, \Phi) = -\cos(\Delta - \Phi), \quad (2)$$

where Δ is the relative angle between the orientations of the two detectors. In particular, when $\Delta - \Phi = 0$ we get that $E = -1$, so that all outcomes in the sequence must be either $(-1, +1)$ or $(+1, -1)$.

The Bell's theorem states that prediction (2) cannot be reproduced by any model of hidden variables that shares certain intuitive features. In particular, the CHSH version

of the theorem states that for the said generic models of hidden variables the following inequality is fulfilled for any set of values $(\Delta_1, \Delta_2, \delta)$ [13]:

$$|E(\Delta_1) + E(\Delta_2) + E(\Delta_1 - \delta) - E(\Delta_2 - \delta)| \leq 2. \quad (3)$$

On the other hand, according to quantum mechanics the magnitude in the left hand side of the inequality reaches a maximum value of $2\sqrt{2}$, known as Tsirelson's bound [14], for certain values of Δ_1 , Δ_2 and δ . For example, $\Delta_1 = -\Delta_2 = \frac{1}{2}\delta = \frac{\pi}{4}$. As it was noted above, carefully designed experiments have confirmed that the CHSH inequality is violated according to the predictions of quantum mechanics and, therefore, have ruled out all the generic models of hidden variables constrained by the Bell's inequality (3).

In this paper we show, however, that the generic models of hidden variables considered by the Bell's theorem all share a subtle crucial feature that is not necessarily fulfilled in the actual experimental tests of the Bell's inequality. Indeed, the considered feature cannot be derived from fundamental physical principles and may even be at odds with the fundamental principle of relativity. Moreover, this feature neither can be properly implemented within the standard framework of quantum mechanics. We follow this observation to explicitly build a local model of hidden variables that does not share the disputed feature and, thus, it is capable to reproduce the predictions of quantum mechanics for the Bell's polarization states of two entangled particles.

Our model puts forward for consideration the possibility that quantum mechanics might not be the ultimate mathematical framework of fundamental physics. In fact, it is interesting to notice that the way how our model solves the apparent 'non-locality' associated to entanglement in the standard quantum formalism is very similar to the way how General Relativity solves the 'non-locality' of Newton's theory of gravitation: in our model quantum entanglement is the result of a curved metric in the space in which the hypothetical hidden variables live.

II. OUTLINE

Any local statistical model of hidden variables that aims to describe the Bell's experiment consists of some space \mathcal{S} of possible hidden configurations for the pair of entangled particles, labelled here as $\lambda \in \mathcal{S}$, together with a well-defined (density of) probability $\rho(\lambda)$ for each one

of them to occur in every single realization. The model must also specify well-defined binary functions $s_{\Omega_A}^{(A)}(\lambda) = \pm 1$, $s_{\Omega_B}^{(B)}(\lambda) = \pm 1$ to describe the outcomes that would be obtained at detectors A and B when the pair of entangled particles occurs in the hidden configuration $\lambda \in \mathcal{S}$ and their polarizations are tested along directions Ω_A and Ω_B , respectively.

The proof of the CHSH inequality (3) involves two well-defined possible orientations Ω_A and Ω'_A for the polarization test of particle A and two well-defined possible orientations Ω_B and Ω'_B for the polarization test of particle B, and assumes that the considered model of hidden variables assigns to each possible hidden configuration $\lambda \in \mathcal{S}$ a 4-tuple of binary values $(s_{\Omega_A}^{(A)}(\lambda), s_{\Omega'_A}^{(A)}(\lambda), s_{\Omega_B}^{(B)}(\lambda), s_{\Omega'_B}^{(B)}(\lambda)) \in \{-1, +1\}^4$ to describe the outcomes that would be obtained in each one of the two detectors in case that it would be set along each one of its two available orientations. Hence, it is straightforward to obtain that for any $\lambda \in \mathcal{S}$,

$$s_{\Omega_A}^{(A)}(\lambda) \cdot (s_{\Omega_B}^{(B)}(\lambda) + s_{\Omega'_B}^{(B)}(\lambda)) + s_{\Omega'_A}^{(A)}(\lambda) \cdot (s_{\Omega_B}^{(B)}(\lambda) - s_{\Omega'_B}^{(B)}(\lambda)) = \pm 2, \quad (4)$$

since the first term is non-zero only when $s_{\Omega_B}^{(B)}(\lambda)$ and $s_{\Omega'_B}^{(B)}(\lambda)$ have the same sign, while the second term is non-zero only when they have opposite signs. The CHSH inequality (3) is then obtained by averaging (4) over the whole space \mathcal{S} of all possible hidden configurations, since

$$\left| \int d\lambda \rho(\lambda) \left\{ s_{\Omega_A}^{(A)}(\lambda) \cdot (s_{\Omega_B}^{(B)}(\lambda) + s_{\Omega'_B}^{(B)}(\lambda)) + s_{\Omega'_A}^{(A)}(\lambda) \cdot (s_{\Omega_B}^{(B)}(\lambda) - s_{\Omega'_B}^{(B)}(\lambda)) \right\} \right| \leq 2, \quad (5)$$

while

$$\int d\lambda \rho(\lambda) s_{\Omega_A}^{(A)}(\lambda) \cdot s_{\Omega_B}^{(B)}(\lambda) = E(\Delta_1), \quad (6)$$

$$\int d\lambda \rho(\lambda) s_{\Omega_A}^{(A)}(\lambda) \cdot s_{\Omega'_B}^{(B)}(\lambda) = E(\Delta_2), \quad (7)$$

$$\int d\lambda \rho(\lambda) s_{\Omega'_A}^{(A)}(\lambda) \cdot s_{\Omega_B}^{(B)}(\lambda) = E(\Delta_1 - \delta), \quad (8)$$

$$\int d\lambda \rho(\lambda) s_{\Omega'_A}^{(A)}(\lambda) \cdot s_{\Omega'_B}^{(B)}(\lambda) = E(\Delta_2 - \delta), \quad (9)$$

In this argument the involved directions Ω_A , Ω'_A , Ω_B and Ω'_B seem to be fixed with respect to some external frame of reference provided by the lab. Nonetheless, the data collected in such an experimental setup could be alternatively analyzed within frames of reference aligned, for example, with the magnetic axis of the Sun or the rotational axis of the Galaxy, with respect to which the orientations of the detectors for different realizations of the experiment are not fixed anymore. Obviously, the conclusions of the analysis must remain

the same, independently of the lab frame chosen. Indeed, the proof of the CHSH inequality actually requires only three well-defined angles, $\Delta_1 \equiv \angle(\Omega_B, \Omega_A)$, $\Delta_2 \equiv \angle(\Omega'_B, \Omega_A)$ and $\delta \equiv \angle(\Omega'_A, \Omega_A)$, which correspond, respectively, to the relative orientations of Ω_B , Ω'_B and Ω'_A with respect to Ω_A , which serves as a reference direction. The reference direction Ω_A serves also to define the hidden configuration $\lambda \in \mathcal{S}$ of the pair of entangled particles in every single realization of the experiment, since the description of a physical state must necessarily be done with respect to a reference frame. Otherwise, the orientation with respect to any external lab frame, either the optical table or the stars in the sky, of this reference direction Ω_A at different single realizations of the Bell's experiment is absolutely irrelevant: it is an spurious gauge degree of freedom, which can be set to zero (see Fig. 1).

The proof of the CHSH inequality seems straightforward and unavoidable. Nonetheless, the main claim of this paper is that this proof, as well as the proofs of all other versions of the Bell's inequality, involve a subtle, though crucial, implicit assumption that cannot be derived from fundamental physical principles and, indeed, it might not be fulfilled in the actual experimental setup that tests the inequality. Namely, in each realization of a Bell's experiment the polarization of each one of the two entangled particles is tested along a single direction. Hence, the relative orientation Δ of the two measurement devices in each single realization of the experiment is a properly defined physical magnitude, which can be set to values Δ_1 , Δ_2 or any other desired value. On the other hand, the definition of the angle δ that appears in the proof of the CHSH inequality requires a comparison of the global orientation of the measurement devices for different realizations of the Bell's experiment and, thus, it requires the existence of an absolute preferred frame of reference with respect to which the orientations of each one of the devices could be defined. Otherwise, we could choose the orientation of, say, detector A as the reference direction for every single realization of the experiment and define the orientation of the other detector with respect to it, in which case the proof of the Bell's theorem does not necessarily hold as we shall show later. Obviously, such an absolute preferred frame of reference would not be needed if the polarization of each one of the two entangled particles could be tested along two different directions at once in every single realization of the experiment, but this is certainly not the case.

The preferred frame of reference needed to prove the Bell's theorem is supposed to be provided by the lab. However, the conditions that a reference frame must fulfill in order to qualify as a preferred absolute frame are far from obvious and, in any case, its existence

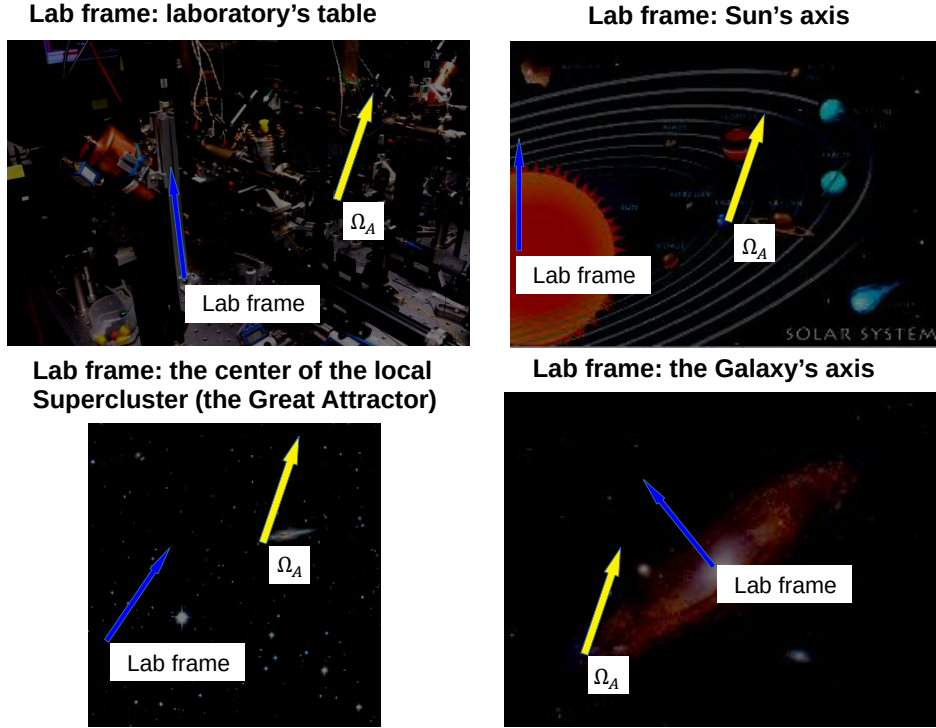


FIG. 1: The orientation of the reference direction Ω_A with respect to the chosen lab frame is a spurious gauge degree of freedom.

is an overbold assumption whose fulfillment has never been explored neither theoretically or experimentally. In fact, the existence of an absolute preferred frame of reference would be clearly at odds with Galileo's principle of relativity. Moreover, it is straightforward to show that this assumption cannot be properly implemented within the standard framework of quantum mechanics either. The argument goes as follows. The Bell's state (1) that describes the pair of entangled particles is defined in terms of the bases $\{|\uparrow\rangle, |\downarrow\rangle\}^{(A,B)}$ of eigenstates of the Pauli operators $\sigma_Z^{(A,B)}$ along locally defined Z-axes for each one of the particles. Since these eigenstates are defined up to a global phase, the phase Φ in expression (1) cannot be properly defined with respect to a lab frame. In order to properly define it we need to choose an arbitrary setting of the two detectors that test the polarizations of the pair of entangled particles as a reference. This reference setting defines *parallel* directions

along the XY-planes at the sites where each one of the two particles are detected. Then, the phase Φ of the entangled state (1) can be properly defined with respect to this reference setting with the help of the measured correlations between the outcomes of the two detectors, $E = -\cos(\Phi)$. Furthermore, we can use this reference setting to properly define a relative rotation Δ of the orientations of the two measurement devices. On the other hand, since we must use an arbitrary setting of the detectors as a reference, their absolute orientation is an unphysical gauge degree of freedom (see Fig. 2). In summary, in order to describe the setting of the measurement devices in a Bell's experiment within the standard framework of quantum mechanics we need to specify both Φ and Δ with respect to some otherwise arbitrary reference setting of the detectors. Nonetheless, only their difference $\Delta - \Phi$ is independent of the chosen reference setting and, hence, the correlation between the outcomes of the two devices can only depend on this difference (2).

In the absence of an absolute preferred frame of reference the global rigid orientation of the two detectors is, as we have already noticed before, an spurious (unphysical) gauge degree of freedom and, hence, the proof of the CHSH inequality (as well as of all other versions of the Bell's inequality) holds only for models in which the considered hidden configurations are symmetrically invariant under a rigid rotation of the two measuring devices. On the other hand, we shall show below that the proof of the inequality does not necessarily hold when this symmetry is (spontaneously) broken by the hidden configuration of the entangled particles, since then a non-zero geometric phase may appear under cyclic gauge transformations. Indeed, the crucial role of the angle δ in the proof of the CHSH inequality is an obvious indication that in order to violate it the gauge symmetry under a rigid rotation of the two detectors must be spontaneously broken.

In fact, it is obvious from (2) that the entanglement of the two particles explicitly breaks the symmetry of the system under a rotation of the relative orientation of the two detectors. Since a reference direction is needed for this symmetry to get broken, the gauge symmetry under a rigid rotation of the two detectors must be also spontaneously broken. From this perspective the phase Φ that appears in the description of the source (1) seems to play the role of a Goldstone mode associated to the spontaneously broken gauge symmetry, that is, the phase Φ appears instead of the spurious gauge degree of freedom δ that would describe the global rigid orientation of the two detectors. Under these circumstances, it is not possible to compare different settings of the detectors with respect to an external lab

frame of reference: they can only be compared with respect to a frame in which they all share the same preferred direction, *e.g.* the reference frame set by the orientation of one of the detectors. This requirement can be explained as follows.

In the proof of the CHSH inequality it is implicitly assumed, as we have already noticed above, that there exists a preferred frame of reference, which defines a set of coordinates $\lambda \in \mathcal{S}$ over the space \mathcal{S} of all possible hidden configurations that can be used to describe the response function of each one of the two detectors in each one of its two available orientations (defined with respect to the said preferred frame). Above we denoted these response functions as $s_{\Omega_A}^{(A)}(\lambda)$, $s_{\Omega'_A}^{(A)}(\lambda)$, $s_{\Omega_B}^{(B)}(\lambda)$, $s_{\Omega'_B}^{(B)}(\lambda)$. Nonetheless, in general, we should allow for each one of the two detectors to define its proper set of coordinates over the space \mathcal{S} . Thus, for a given setting of the detectors we shall denote as λ_A and λ_B the sets of coordinates associated to detector A and detector B, respectively, so that their responses would be given as $s(\lambda_A)$ and $s(\lambda_B)$ by some universal function $s(\cdot)$ of the locally defined coordinate of the hidden configuration. Since these two sets of coordinates parameterize the same space of hidden configurations \mathcal{S} there must exist some invertible transformation that relates them:

$$\lambda_B = -L(\lambda_A; \Delta - \Phi), \quad (10)$$

which may depend parametrically on the relative orientation $\Delta - \Phi$ between the two detectors. This transformation must fulfill the constraint

$$d\lambda_A \rho(\lambda_A) = d\lambda_B \rho(\lambda_B), \quad (11)$$

in order to guarantee that the probability of every hidden configuration to occur remains invariant under a change of coordinates, while the (density of) probability $\rho(\cdot)$ is functionally invariant for both sets of coordinates. However, these constraints do not forbid the possibility that the set of coordinates accumulates a non-zero geometric phase $\alpha \neq 0$ through certain cyclic gauge transformations:

$$(-L_{\bar{\Delta}_2}) \circ (-L_{\bar{\Delta}_2 - \bar{\delta}}) \circ (-L_{\bar{\Delta}_1 - \bar{\delta}}) \circ (-L_{\bar{\Delta}_1}) \neq \mathbb{I}, \quad (12)$$

In such a case there does not exist a single set of coordinates that can be used to define the response functions of each one of the two detectors in its two available orientations (defined

with respect to an external frame), as required by the proof of the inequality (3). Therefore, in order to compare the four different experiments involved in the CHSH inequality we must choose the orientation of one of the detectors as a reference direction, as we do below in (13), so that they all may be described with a common set of coordinates. The appearance of a non-zero geometric phase under a cyclic transformation is a well-known phenomena in physical models involving gauge symmetries [15] and, therefore, we should not rule out the possibility that it also occurs in models of hidden variables for the Bell's states. The Bell's theorem, however, cannot account for such models.

Following these observations we were able to explicitly build a local model of hidden variables that reproduces the predictions of quantum mechanics for the Bell's polarization states. In our model the hidden configurations of the pair of entangled particles are described by a *pointer*, which sets an arbitrarily oriented preferred direction and, thus, spontaneously breaks the symmetry of the setup under rigid rotations of the two detectors. As we have just noticed, and we shall show later on in further detail, in order to compare different realizations of the experiment within the framework of such a model we must choose a common reference direction, which can be either the orientation of the hidden configuration of the pair of entangled particles or, alternatively, the orientation of one of the detectors, say, detector A. Since the former may not be directly experimentally accessible, we are left only with the latter option. Thus, in such a model we only need to specify the binary values for $s(\lambda_A)$, $s(\lambda_B)$, $s(\lambda'_B)$, $s(\lambda''_B)$ and $s(\lambda'''_B)$ for each possible hidden configuration $\lambda_A \in \mathcal{S}$ of the pair of entangled particles, where $\lambda_B = -L(\lambda_A; \Delta_1)$, $\lambda'_B = -L(\lambda_A; \Delta_2)$, $\lambda''_B = -L(\lambda_A; \Delta_1 - \delta)$, $\lambda'''_B = -L(\lambda_A; \Delta_2 - \delta)$. It is then straightforward to notice that the magnitude

$$s(\lambda_A) \cdot (s(\lambda_B) + s(\lambda'_B) + s(\lambda''_B) - s(\lambda'''_B)), \quad (13)$$

which comes instead of (4), can take values out of the interval $[-2, 2]$. Hence, these models are not constrained by the CHSH inequality (3).

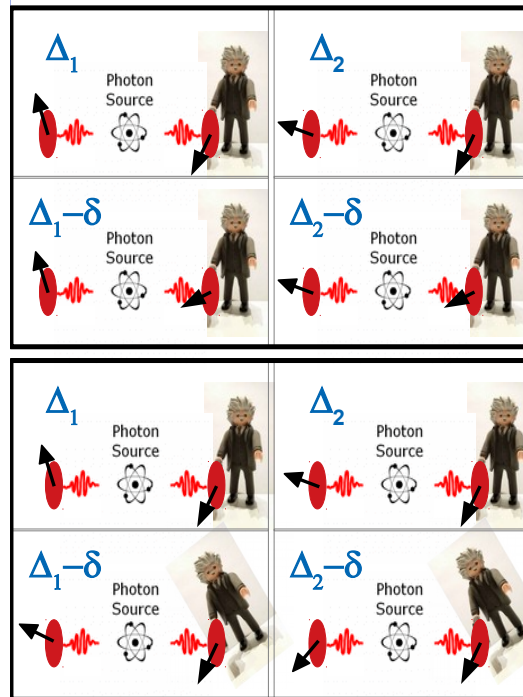


FIG. 2: Two descriptions of the experimental setup required for testing the Bell's inequality. In the description above the lab frame is taken to be fixed, while in the description below the orientation of detector A is taken to be fixed. The relative angle between the two detectors is set at four possible values Δ_1 , Δ_2 , $\Delta_1 - \delta$ and $\Delta_2 - \delta$. When considering models in which the hypothetical hidden configurations of the pairs of entangled particles spontaneously break the symmetry under rigid rotations of the orientations of the two measurement devices, only the latter choice allows to properly compare the four different settings.

Outcome	$a = +1$	$a = +1$	$a = -1$	$a = -1$
Setting	$b = +1$	$b = -1$	$b = +1$	$b = -1$
$A = +1$ $B = +1$	p_1	0	0	$1 - p_1$
$A = +1$ $B = -1$	p_2	0	0	$1 - p_2$
$A = -1$ $B = +1$	p_3	0	0	$1 - p_3$
$A = -1$ $B = -1$	0	p_4	$1 - p_4$	0

Table 1. Conditional probabilities for a toy model with two binary inputs and two binary outcomes that cannot be reproduced by a realistic and local underlying theory [16].

These arguments can be stated in more abstract terms as follows. Quantum predictions for the Bell's experiment are commonly described as a set of conditional probabilities $p(a, b|A, B)$, where $a = \pm 1$ and $b = \pm 1$ are the two possible outcomes at each one of the two detectors and $A = \pm 1$ and $B = \pm 1$ describe two possible choices for the setting of each one of the two detectors. It is then proven that these conditional probabilities cannot be obtained in terms of a local model of hidden variables, defined by its configuration space $\lambda \in \mathcal{S}$, its density of probability $\rho(\lambda)$ and its local response functions $a = f(\lambda, A)$, $b = f(\lambda, B)$ [2].

This statement can be clearly illustrated with the help of the toy model described in Table 1 [16], where conditional probabilities for each one of the four possible results of an experiment with two binary outcomes $a, b = \pm 1$ (columns) are given for each one of four possible settings, defined by two independent binary inputs $A, B = \pm 1$ (rows). For these probabilities to be properly defined we require that $p_1, p_2, p_3, p_4 \in [0, 1]$. It can be readily checked that for each set of input values (rows) the sum of the probabilities for all possible results of the experiment (columns) equals 1. These conditional probabilities, however, cannot be obtained within the framework of an underlying local model of hidden variables:

the conditional probabilities listed in the first three rows would imply $a = b$, that is, the outcomes of the two detectors in any of their four possible settings must have the same sign, which is obviously inconsistent with the conditional probabilities listed in the fourth row.

Nonetheless, it is straightforward to identify in this abstract reformulation of the Bell's theorem the same unjustified implicit assumption that we have noticed above, namely, that there are two well-defined choices for the setting of each one of the detectors. We have noticed above that we can properly define and measure only the conditional probabilities $p(a, b|D)$, where $a = \pm 1$ and $b = \pm 1$ are, as before, the outcomes at each one of the two detectors and $D = 1, 2, 3, 4$ defines four possible relative orientations between them. We did notice also that quantum mechanics as well makes theoretical predictions only for these conditional probabilities $p(a, b|D)$. Under these looser constraints the Bell's theorem does not necessarily hold:

Consider, for example, the toy model described in Table 2. The conditional probabilities are identical to those described in Table 1 for each one of the four possible results of the experiment, but the setting of the measurement devices is now described by a single parameter $D = 1, 2, 3, 4$. Each input value corresponds to a given relative orientation of the two devices. The new model simply states that when the devices are set at $D = 1, 2, 3$ their outcomes must have the same sign, and when they are set at $D = 4$ their outcomes must have opposite signs. Obviously, this latter model is not necessarily in contradiction with an underlying local model of hidden variables.

Outcome	$a = +1$	$a = +1$	$a = -1$	$a = -1$
Setting	$b = +1$	$b = -1$	$b = +1$	$b = -1$
$D = 1$	p_1	0	0	$1 - p_1$
$D = 2$	p_2	0	0	$1 - p_2$
$D = 3$	p_3	0	0	$1 - p_3$
$D = 4$	0	p_4	$1 - p_4$	0

Table 2. Conditional probabilities for a toy model with a single input with four possible values and two binary outcomes. They can be reproduced by an underlying theory.

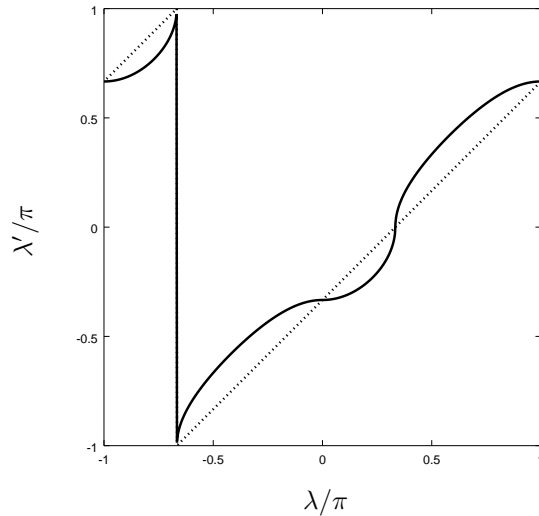


FIG. 3: Plot of the transformation law $\lambda \rightarrow \lambda' = L(\lambda; \Delta)$ for $\Delta = \pi/3$ (solid line), compared to the corresponding linear transformation (dotted line).

III. THE MODEL

We shall now build an explicitly local statistical model of hidden variables that reproduces the predictions of quantum mechanics for the Bell's states (1) and, hence, it is not constrained by the Bell's inequality. Further aspects of the model are discussed in [21]. The crux of the model is, as we have already noticed above, the spontaneous breaking of the gauge symmetry of the experimental setup under global rigid rotations of the orientation of the detectors. The symmetry is broken by the hidden configuration of the pairs of entangled particles. Furthermore, we allow for a non-zero geometric phase to accumulate through cyclic gauge transformations (12). Under these circumstances there does not exist an absolute preferred frame, other than the orientation of one of the detectors, to which we can refer in order to compare different realizations of the experiment.

The gauge symmetry is spontaneously broken because in the considered model the hidden configuration of the pair of entangled particles has a preferred direction randomly oriented over a unit circle \mathcal{S} in the XY-plane. Each one of the two detectors defines over this circle \mathcal{S} a frame of reference with its own set of associated coordinates, which we shall denote as $\lambda_A \in [-\pi, +\pi)$ for detector A and $\lambda_B \in [-\pi, +\pi)$ for detector B. Since the two sets of coordinates parameterize the same space \mathcal{S} they must be related by some transformation

law:

$$\lambda_B = -L(\lambda_A; \Delta - \Phi). \quad (14)$$

This transformation law states that a hidden configuration whose preferred direction is oriented along an angle λ_A with respect to detector A is oriented along an angle λ_B given by (14) with respect to detector B, where Δ is the relative angle between the two detectors and Φ is the phase that characterizes the source of entangled particles as defined above.

The transformation law (14) does not violate neither locality nor causality: it may well be a fundamental law of Nature. Indeed, the notions of locality and causality in special relativity stem from a similar relationship $v' = T(v; V)$ between the velocities v and v' of a point particle with respect to two different inertial frames moving with relative velocity V . Moreover, (14) is only a generalization of the euclidean linear relationship that states that given two *classical* detectors whose orientations form an angle Δ and a pointer oriented along an angle λ with respect to one of them, the said pointer is oriented along an angle $\lambda - \Delta$ with respect to the other detector.

In order to reproduce the predictions of quantum mechanics we define the transformation law (14) as follows:

- If $\bar{\Delta} \in [0, \pi)$,

$$L(\lambda; \bar{\Delta}) = \begin{cases} q(\lambda - \bar{\Delta}) \cdot \arccos(-\cos(\bar{\Delta}) - \cos(\lambda) - 1), & \text{if } -\pi \leq \lambda < \bar{\Delta} - \pi, \\ q(\lambda - \bar{\Delta}) \cdot \arccos(+\cos(\bar{\Delta}) + \cos(\lambda) - 1), & \text{if } \bar{\Delta} - \pi \leq \lambda < 0, \\ q(\lambda - \bar{\Delta}) \cdot \arccos(+\cos(\bar{\Delta}) - \cos(\lambda) + 1), & \text{if } 0 \leq \lambda < \bar{\Delta}, \\ q(\lambda - \bar{\Delta}) \cdot \arccos(-\cos(\bar{\Delta}) + \cos(\lambda) + 1), & \text{if } \bar{\Delta} \leq \lambda < +\pi, \end{cases} \quad (15)$$

- If $\bar{\Delta} \in [-\pi, 0)$,

$$L(\lambda; \bar{\Delta}) = \begin{cases} q(\lambda - \bar{\Delta}) \cdot \arccos(-\cos(\bar{\Delta}) + \cos(\lambda) + 1), & \text{if } -\pi \leq \lambda < \bar{\Delta}, \\ q(\lambda - \bar{\Delta}) \cdot \arccos(+\cos(\bar{\Delta}) - \cos(\lambda) + 1), & \text{if } \bar{\Delta} \leq \lambda < 0, \\ q(\lambda - \bar{\Delta}) \cdot \arccos(+\cos(\bar{\Delta}) + \cos(\lambda) - 1), & \text{if } 0 \leq \lambda < \bar{\Delta} + \pi, \\ q(\lambda - \bar{\Delta}) \cdot \arccos(-\cos(\bar{\Delta}) - \cos(\lambda) - 1), & \text{if } \bar{\Delta} + \pi \leq \lambda < +\pi, \end{cases} \quad (16)$$

where

$$q(\lambda - \bar{\Delta}) = \text{sign}((\lambda - \bar{\Delta}) \bmod ([-\pi, \pi))),$$

and the function $y = \arccos(x)$ is defined in its main branch, such that $y \in [0, \pi]$ while $x \in [-1, +1]$. In Fig. 3 the transformation $L(\lambda; \bar{\Delta})$ is graphically shown for the particular case $\bar{\Delta} = \pi/3$. It is straightforward to check that the transformation law (14) is strictly monotonic and fulfills the differential relationship

$$|d(\cos(\lambda_B))| = d\lambda_B |\sin(\lambda_B)| = d\lambda_A |\sin(\lambda_A)| = |d(\cos(\lambda_A))|, \quad (17)$$

while the parameter $\Delta - \Phi$ plays the role of an the integration constant.

Locality is explicitly enforced in our model by requiring that the outcome of each one of the detectors in reponse to the hidden configuration of the pair of entangled particles depends only on its locally defined orientation, that is, $s(\lambda_A) = \pm 1$ for detector A and $s(\lambda_B) = \pm 1$ for detector B, where λ_B and λ_A are related by relationship (14) and $s(\cdot)$ is the binary response function of the detectors, which for the sake of simplicity we define here as

$$s(l) = \begin{cases} +1, & \text{if } l \in [0, +\pi), \\ -1, & \text{if } l \in [-\pi, 0). \end{cases} \quad (18)$$

In order to complete our statistical model we need to specify also the (density of) probability $\rho(l)$ of each hidden configuration $l \in \mathcal{S}$ over the space \mathcal{S} to occur in every single realization of the pair of entangled particles. By symmetry considerations this density of

probability must be functionally identical from the point of view of both detectors, independently of their relative orientation. Moreover, the condition of 'free-will' demands that the probability of each hidden configuration to occur in any single realization of the experiment cannot depend on the parameterizations of the space \mathcal{S} associated to each one of the two detectors. This condition can be precisely stated as:

$$d\lambda_A \rho(\lambda_A) = d\lambda_B \rho(\lambda_B). \quad (19)$$

It is straightforward to show from (17) that this condition is fulfilled if the probability density $\rho(l)$ is given by:

$$\rho(l) = \frac{1}{4} |\sin(l)|. \quad (20)$$

We can now compute within the framework of this model the statistical correlations expected between the outcomes of the two detectors as a function of their relative orientation. The binary outcomes of each one of the two detectors define a partition of the phase space \mathcal{S} of all the possible hidden configurations into four coarse subsets,

$$\begin{aligned} (s^{(A)} = +1; s^{(B)} = +1) &\iff \lambda_A \in [0, \Delta - \Phi) \\ (s^{(A)} = +1; s^{(B)} = -1) &\iff \lambda_A \in [\Delta - \Phi, \pi) \\ (s^{(A)} = -1; s^{(B)} = +1) &\iff \lambda_A \in [\Delta - \Phi - \pi, 0) \\ (s^{(A)} = -1; s^{(B)} = -1) &\iff \lambda_A \in [-\pi, \Delta - \Phi - \pi), \end{aligned}$$

where we have assumed without any loss of generality that $\Delta - \Phi \in [0, \pi)$. Each one of these four coarse subsets happen with a probability given by:

$$\begin{aligned} p(+1, +1) &= \int_0^{\Delta - \Phi} \rho(\lambda_A) d\lambda_A = \frac{1}{4} (1 - \cos(\Delta - \Phi)), \\ p(+1, -1) &= \int_{\Delta - \Phi}^{\pi} \rho(\lambda_A) d\lambda_A = \frac{1}{4} (1 + \cos(\Delta - \Phi)), \\ p(-1, +1) &= \int_{\Delta - \Phi - \pi}^0 \rho(\lambda_A) d\lambda_A = \frac{1}{4} (1 + \cos(\Delta - \Phi)), \\ p(-1, -1) &= \int_{-\pi}^{\Delta - \Phi - \pi} \rho(\lambda_A) d\lambda_A = \frac{1}{4} (1 - \cos(\Delta - \Phi)). \end{aligned}$$

These conditional probabilities reproduce the predictions of quantum mechanics (2):

$$E(\Delta, \Phi) = p(+1, +1) + p(-1, -1) - p(+1, -1) - p(-1, +1) = -\cos(\Delta - \Phi).$$

Finally, we notice that in spite of the non-trivial transformation law (14) our model comply with the trivial demand that a relative rotation of the measurement apparatus by an angle Δ followed by a second relative rotation by an angle Δ' results into a final rotation by an angle $\Delta + \Delta'$. Consider, for example, an initial reference setting \mathcal{T}_0 in which the outcomes of the two measurement apparatus are correlated by an amount $E = -\cos(\Phi)$. The angular coordinates of the hidden configurations with respect to each one of the two measurement devices, λ_A and λ_B , would be related in this reference setting by the relationship:

$$\lambda_B = -L(\lambda_A; -\Phi). \quad (21)$$

We now define a new measurement setting \mathcal{T}_1 obtained from the initial setting \mathcal{T}_0 by rotating the relative orientation of the two apparatus by an angle Δ . The angular coordinates λ_A and λ'_B defined with respect to this new setting would be related by:

$$\lambda'_B = -L(\lambda_A; \Delta - \Phi). \quad (22)$$

A third measurement setting \mathcal{T}_2 is obtained from the intermediate setting \mathcal{T}_1 by rotating the relative orientation of the two apparatus by an additional angle Δ' . In the intermediate setting \mathcal{T}_1 , which is now taken as reference to define the second rotation, the pair of particles appears to be in a polarization state characterized by a phase $\Phi' = -\Delta + \Phi$. Hence, the angular coordinates λ_A and λ''_B defined with respect to the setting \mathcal{T}_2 would be related by the transformation law:

$$\lambda''_B = -L(\lambda_A; \Delta' - \Phi') = -L(\lambda_A; \Delta' + \Delta - \Phi). \quad (23)$$

By comparison of the transformation law (21) for the initial setting \mathcal{T}_0 and the transformation law (23) for the setting \mathcal{T}_2 , we realize that the latter has been obtained from the initial setting by rotating the apparatus by an angle $\Delta' + \Delta$, as we had demanded.

In order to complete the description of the Bell's experiment we define two new settings \mathcal{T}_3 and \mathcal{T}_4 , which are obtained, respectively, from \mathcal{T}_1 and \mathcal{T}_2 by cancelling the phase Φ in the reference setting \mathcal{T}_0 . Hence, in these settings the angular coordinates of the hidden configurations with respect to the two measurement apparatus are related by the relationships:

$$\lambda'''_B = -L(\lambda_A; \Delta). \quad (24)$$

and

$$\lambda_B'''' = -L(\lambda_A; \Delta' + \Delta), \quad (25)$$

respectively. Thus, we could intuitively think about the four settings of the detectors involved in a Bell's experiment as corresponding to two possible values for the relative angle Δ and two possible values for the phase Φ , while they all four share the orientation of one the two detectors, say detector A, taken as reference.

Finally, let us notice that when we substitute the coherent source of pairs of entangled particles (1) by the incoherent *classical* source (where all the mixed coherent sources are defined with respect to the same arbitrary setting of the two detectors):

$$\hat{\rho} = \int_{2\pi} d\Phi |\Psi_\Phi\rangle\langle\Psi_\Phi| = |\uparrow\rangle\langle\uparrow|^{(A)} \otimes |\downarrow\rangle\langle\downarrow|^{(B)} + |\downarrow\rangle\langle\downarrow|^{(A)} \otimes |\uparrow\rangle\langle\uparrow|^{(B)},$$

the broken rotational symmetries are statistically restored and the outcomes of the two measurement devices become uncorrelated for all settings. Only then, when the rotational symmetries are restored, we can safely define separately the orientations of each one of the measurement devices with respect to some external reference frame and, thus, describe the phase space of its possible settings with the help of these two angles (Ω_A, Ω_B) .

IV. DISCUSSION

The Bell's theorem is one of the pillars upon which relies the widely accepted belief that quantum mechanics is the ultimate mathematical framework within which the hypothetical final theory of the fundamental building blocks of Nature and their interactions must be formulated. The theorem proves through an experimentally testable inequality (the Bell's inequality) that the predictions of quantum mechanics for the Bell's polarization states of two entangled particles cannot be reproduced by any underlying theory of hidden variables that shares certain intuitive features.

In this paper we have shown, however, that these intuitive features include a subtle, but crucial, assumption that is not required by fundamental physical principles and, hence, it is not necessarily fulfilled in the experimental setup that tests the inequality. In fact,

this subtle assumption cannot be implemented within the framework of standard quantum mechanics either.

Namely, the proof of the theorem requires the existence of an absolute preferred frame of reference, supposedly provided by the lab, with respect to which the orientations of each one of the two measurement devices can be independently defined for each single realization of the experiment. This preferred frame is needed in order to compare the orientations of the detectors in sequential realizations of the experiment. Notwithstanding, this requirement is at odds with Galileo's fundamental principle of relativity. Moreover, such an absolute preferred frame may not exist when the hidden configurations of the pair of entangled particles spontaneously break the rotational symmetry of the experimental setup under rigid rotations of the two detectors. In such a case, in order to compare different realizations of the experiment we must pick the orientation of one of the detectors as a common reference direction, with respect to which the relative orientation of the second detector is defined. Thus the Bell's theorem does not necessarily hold (see Table 2).

Following these ideas we explicitly built a model of hidden variables for the Bell's states of two entangled particles that reproduces the predictions of quantum mechanics. Further details of the model are discussed in [21]. In two additional accompanying papers we have used these same ideas to build explicit local models of hidden variables for the GHZ state of three entangled particles [22] and also for the qutrit [23].

These models, thus, open a window to a very desired realistic interpretation of the quantum formalism and, maybe, also to unexplored fundamental physics underlying the principles of quantum mechanics.

We find very intriguing that the solution presented here of the Einstein-Podolsky-Rosen paradox [17] is intrinsically related to the symmetries and metric of space and wonder if this solution could open a window through which quantum mechanics and Einstein's general relativity theory of gravitation could be put together.

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