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The Bell's theorem revisited: a subtle, though crucial, assumption has gone unnoticed, and it might not be justified!

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The Bell's theorem stands as an insuperable roadblock in the path to a very desired intuitive solution of the Einstein-Podolsky-Rosen paradox and, hence, it lies at the core of the current lack of a clear interpretation of the quantum formalism. The theorem states through an experimentally testable inequality that the predictions of quantum mechanics for the Bell's polarization states of two entangled particles cannot be reproduced by any statistical model of hidden variables that shares certain intuitive features. In this paper we show, however, that the proof of the Bell's inequality involves a subtle, though crucial, assumption that is not required by fundamental physical principles and, moreover, it might not be fulfilled in the experimental setup that tests the inequality. In fact, this assumption can neither be properly implemented within the framework of quantum mechanics. Namely, the proof of the Bell's theorem assumes that there exists an absolute preferred frame of reference, supposedly provided by the lab, which enables to compare the orientation of the polarization measurement devices for successive realizations of the experiment. The need for this assumption can be readily checked by noticing that the theorem does not hold when the orientation of one of the detectors is taken as a reference frame to define the relative orientation of the second detector, in spite that this frame is an absolutely legitimate choice according to Galileo's principle of relativity. We further notice that the absolute frame of reference required by the proof of the Bell's theorem cannot exist in models in which the hidden configuration of the pair of entangled particles has a randomly set preferred direction that spontaneously breaks the global rotational symmetry. In fact, following this observation we build an explicit local model of hidden variables that reproduces the predictions of quantum mechanics for the Bell's states.

I. INTRODUCTION

The Bell's theorem is one of the fundamental theorems upon which relies the widespread belief that quantum mechanics is the ultimate mathematical framework within which the hypothetical final theory of the fundamental building blocks of Nature and their interactions should be formulated. The theorem states through an experimentally testable inequality (the Bell's inequality) that local statistical models of hidden variables that share certain intuitive features cannot reproduce the predictions of quantum mechanics for the entangled polarization states of two particles (Bell's states) [1, 2]. These predictions have been confirmed beyond doubt by very carefully designed experiments [3–6].

In these experimental tests a source emits pairs of particles whose polarizations are arranged in a Bell's entangled state:

$$|\Psi_\Phi\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\rangle^{(A)} |\downarrow\rangle^{(B)} - e^{i\Phi} |\downarrow\rangle^{(A)} |\uparrow\rangle^{(B)} \right), \quad (1)$$

where $\{|\uparrow\rangle, |\downarrow\rangle\}^{(A,B)}$ are eigenstates of Pauli operators $\sigma_Z^{(A,B)}$ along locally defined Z-axes for each one of the two particles. The two emitted particles travel off the source in opposite directions towards two widely separated detectors, which test their polarizations. The orientation of each one of the detectors can be freely and independently set along any arbitrary direction in the XY-plane perpendicular to the locally defined Z-axis. Upon detection each particle causes a binary response of its detector, either +1 or -1. Thus, each pair of entangled particles produces an outcome in the space of possible events $\{(-1, -1), (-1, +1), (+1, -1), (+1, +1)\}$. We refer to each detected pair as a single realization of the experiment.

Quantum mechanics predicts that the statistical correlation between the outcomes of the two detectors in a long sequence of realizations of the experiment is given by:

$$E(\Delta, \Phi) = -\cos(\Delta - \Phi), \quad (2)$$

where Δ is the relative angle between the orientations of the two detectors. In particular, when $\Delta - \Phi = 0$ we get that $E = -1$, so that all outcomes in the sequence must be either $(-1, +1)$ or $(+1, -1)$.

The Bell's theorem states that prediction (2) cannot be reproduced by any model of hidden variables that shares certain intuitive features. In particular, the CHSH version of

the theorem states that for said generic models of hidden variables the following inequality is fulfilled for any set of values $(\Delta_1, \Delta_2, \delta)$ [7]:

$$|E(\Delta_1) + E(\Delta_2) + E(\Delta_1 - \delta) - E(\Delta_2 - \delta)| \leq 2. \quad (3)$$

On the other hand, according to quantum mechanics the magnitude in the left hand side of the inequality reaches a maximum value of $2\sqrt{2}$, known as Tsirelson's bound [8], for certain values of Δ_1 , Δ_2 and δ . As it was noted above, carefully designed experiments have confirmed that the CHSH inequality is violated according to the predictions of quantum mechanics and, therefore, have ruled out all said generic local models of hidden variables.

In this paper we show, however, that the generic models of hidden variables considered by the Bell's theorem all share a subtle crucial feature that is not required by fundamental physical principles and, in fact, it is at odds with the fundamental principle of relativity. Hence, this feature might not hold for the experimental setup that tests the Bell's inequality and, indeed, it either cannot be properly implemented within the framework of quantum mechanics. We further show that local models of hidden variables that are not required to share this unjustified feature are not constrained by the Bell's inequality. In fact, following this observation, we explicitly build a local model of hidden variables that reproduces the predictions of quantum mechanics for the Bell's polarization states of two entangled particles.

II. RESULTS

A. Outline

Any local statistical model of hidden variables that aims to describe the Bell's experiment consists of some space \mathcal{S} of possible hidden configurations for the pair of entangled particles labelled as $\lambda \in \mathcal{S}$, together with a well-defined (density of) probability $\rho(\lambda)$ for each one of them to occur in every single realization. The model must also specify well-defined binary values $s_{\Omega_A}^{(A)}(\lambda) = \pm 1$, $s_{\Omega_B}^{(B)}(\lambda) = \pm 1$ to describe the outcomes that would be obtained at detectors A and B when the pair of entangled particles occurs in the hidden configuration $\lambda \in \mathcal{S}$ and their polarizations are tested along directions Ω_A and Ω_B , respectively.

The proof of the CHSH inequality involves two well-defined possible orientations Ω_A and Ω'_A for the polarization test of particle A and two well-defined possible orientations Ω_B

and Ω'_B for the polarization test of particle B, and assumes that the considered model of hidden variables assigns to each possible hidden configuration $\lambda \in \mathcal{S}$ a 4-tuple of binary values $(s_{\Omega_A}^{(A)}(\lambda), s_{\Omega'_A}^{(A)}(\lambda), s_{\Omega_B}^{(B)}(\lambda), s_{\Omega'_B}^{(B)}(\lambda)) \in \{-1, +1\}^4$ to describe the outcomes that would be obtained in each one of the two detectors in case that it would be set along each one of its two available orientations. Hence, it is straightforward to obtain that for any $\lambda \in \mathcal{S}$,

$$s_{\Omega_A}^{(A)}(\lambda) \cdot (s_{\Omega_B}^{(B)}(\lambda) + s_{\Omega'_B}^{(B)}(\lambda)) + s_{\Omega'_A}^{(A)}(\lambda) \cdot (s_{\Omega_B}^{(B)}(\lambda) - s_{\Omega'_B}^{(B)}(\lambda)) = \pm 2, \quad (4)$$

since the first term is non-zero only when $s_{\Omega_B}^{(B)}(\lambda)$ and $s_{\Omega'_B}^{(B)}(\lambda)$ have the same sign, while the second term is non-zero only when they have opposite signs. The CHSH inequality (3) is then obtained by averaging (4) over the whole space \mathcal{S} of all possible hidden configurations.

In this argument the involved directions $\Omega_A, \Omega'_A, \Omega_B$ and Ω'_B seem to be fixed with respect to some external frame of reference provided by the lab. Nonetheless, the data collected in such an experimental setup could be alternatively analyzed with respect to frames of reference aligned, for example, with the magnetic axis of the Sun or the rotational axis of the Galaxy, with respect to which the orientations of the detectors for different realizations of the experiment are not fixed anymore. Obviously, the conclusions of the analysis must remain the same, independently of the lab frame chosen. Indeed, the proof of the CHSH inequality actually requires only three well-defined angles, $\Delta_1 \equiv \angle(\Omega_B, \Omega_A)$, $\Delta_2 \equiv \angle(\Omega'_B, \Omega_A)$ and $\delta \equiv \angle(\Omega'_A, \Omega_A)$, which correspond, respectively, to the relative orientations of Ω_B, Ω'_B and Ω'_A with respect to Ω_A , which serves as a reference direction. The reference direction Ω_A serves also to define the hidden configuration $\lambda \in \mathcal{S}$ of the pair of entangled particles in every single realization of the experiment, since the description of a physical state must necessarily be done with respect to a reference frame. Otherwise, the global orientation of this reference direction for different single realizations of the Bell's experiment with respect to any external lab frame, either the optical table or the stars in the sky, is absolutely irrelevant: it is an spurious gauge degree of freedom, which can be set to zero (see Fig. 1).

The main claim of this paper is that the above proof of the CHSH inequality (as well as of all other versions of the Bell's inequality) involves a subtle, though crucial, implicit assumption that cannot be derived from fundamental physical principles and, indeed, it might not be fulfilled in the actual experimental setup that tests the inequality. Namely, in each realization of a Bell's experiment the polarization of each one of the two entangled particles is tested along a single direction. Hence, the relative orientation Δ of the two measurement

devices in each single realization of the experiment is a properly defined physical magnitude, which can be set to values Δ_1 , Δ_2 or any other desired value. On the contrary, the definition of the angle δ that appears in the proof of the CHSH inequality requires a comparison of the global orientation of the measurement devices for different realizations of the Bell's experiment and, thus, it requires the existence of an absolute preferred frame of reference with respect to which the orientations of each one of the devices could be defined. Otherwise, we could choose the orientation of, say, detector A as the reference direction for every single realization of the experiment and define the orientation of the other detector with respect to it, in which case, as we shall show below, the proof of the Bell's theorem would not hold. Obviously, such an absolute preferred frame of reference would not be needed if the polarization of each one of the two entangled particles could be tested along two different directions at once in every single realization of the experiment, but unfortunately this is certainly not the case.

This absolute preferred frame of reference is supposed to be provided by the lab. However, the conditions that a reference frame must fulfill in order to qualify as the preferred absolute frame are far from obvious. In fact, the existence of an absolute preferred frame of reference is at odds with Galileo's principle of relativity. In any case, the existence of such an absolute frame is an assumption whose fulfillment has never been explored neither theoretically or experimentally. Indeed, it is straightforward to show that this assumption cannot be properly implemented within the framework of quantum mechanics. The Bell's state (1) that describes the pair of entangled particles is defined in terms of the bases $\{|\uparrow\rangle, |\downarrow\rangle\}^{(A,B)}$ of eigenstates of the Pauli operators $\sigma_Z^{(A,B)}$ along locally defined Z-axes for each one of the particles. Since these eigenstates are defined up to a global phase, the phase Φ in (1) would not be properly defined. In order to properly define it we need to choose as a reference an arbitrary setting of the two detectors that test the polarizations of the pair of entangled particles. This reference setting defines *parallel* directions along the XY-planes at the sites where each one of the two particles are detected. Then, the phase Φ of the entangled state (1) can be properly defined with the help of the measured correlations between the outcomes of the two detectors in this reference setting, $E = -\cos(\Phi)$. Furthermore, we can use this reference setting to properly define a relative rotation Δ of the orientations of the two measurement devices. On the other hand, since we are using an arbitrary setting of the detectors as a reference, it is obvious that we cannot properly define their absolute

orientation (see Fig. 1).

In fact, as we shall explicitly show below, such an absolute preferred frame of reference cannot exist if the entanglement of the two particles spontaneously breaks the gauge symmetry of the system under global rigid rotations of the orientation of the two detectors. In particular, this could be the case if the hidden configuration of the pair of entangled particles has a randomly set preferred direction. It is obvious from (2) that the entanglement of the two particles explicitly breaks the symmetry of the system under relative rotations of the orientation of the two detectors. Therefore, we should consider as a feasible possibility that the entanglement of the two particles spontaneously breaks the gauge symmetry of the system under global rigid rotations of the orientation of the two detectors. Indeed, the crucial role of the angle δ in the proof of the CHSH inequality is a strong indication that, in order to violate it, this gauge symmetry must be broken: in any model in which the symmetry is not broken we can choose the global orientation of the detectors so that the proof holds.

A model of hidden variables in which the global gauge symmetry under rigid rotations of the orientation of the two detectors is broken and the orientation of, say, detector A is taken as a gauge-fixing reference direction only needs to specify the binary values for the physical observables $s_{\Omega_A}^{(A)}(\lambda)$, $s_{\Omega_B}^{(B)}(\lambda)$, $s_{\Omega'_B}^{(B)}(\lambda)$, $s_{\Omega''_B}^{(B)}(\lambda)$ and $s_{\Omega'''_B}^{(B)}(\lambda)$ for each possible hidden configuration $\lambda \in \mathcal{S}$ of the pair of entangled particles, where $\angle(\Omega_B, \Omega_A) = \Delta_1$, $\angle(\Omega'_B, \Omega_A) = \Delta_2$, $\angle(\Omega''_B, \Omega_A) = \Delta_1 - \delta$, $\angle(\Omega'''_B, \Omega_A) = \Delta_2 - \delta$ are the relative orientations between the two detectors in the four available settings. The reason is that in order to compare different realizations of the experiment within the framework of such a model we need to choose a common reference direction, which can be either the orientation of the hidden configuration of the pair of entangled particles or, alternatively, the orientation of one of the detectors. Since the former may not be directly experimentally accessible, we are left only with the latter option. It is then straightforward to notice that the magnitude

$$s_{\Omega_A}^{(A)}(\lambda) \cdot \left(s_{\Omega_B}^{(B)}(\lambda) + s_{\Omega'_B}^{(B)}(\lambda) + s_{\Omega''_B}^{(B)}(\lambda) - s_{\Omega'''_B}^{(B)}(\lambda) \right), \quad (5)$$

which comes instead of (4), can take values out of the interval $[-2, 2]$. Hence, these models are not constrained by the CHSH inequality (3).

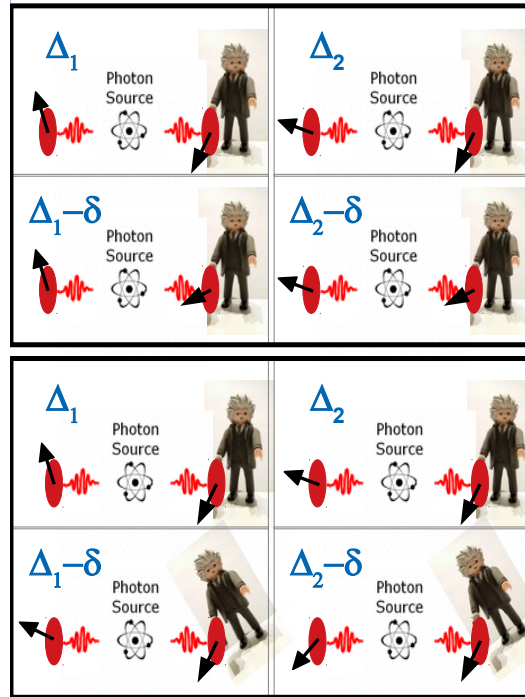


FIG. 1: Two physically undistinguishable descriptions of the experimental setup required for testing the Bell's inequality. In the description above the lab frame is taken to be fixed, while in the description below the orientation of detector A is taken to be fixed. The relative angle between the two detectors is set at four possible values Δ_1 , Δ_2 , $\Delta_1 - \delta$ and $\Delta_2 - \delta$.

Outcome	$a = +1$	$a = +1$	$a = -1$	$a = -1$
Setting	$b = +1$	$b = -1$	$b = +1$	$b = -1$
$A = +1$ $B = +1$	p_1	0	0	$1 - p_1$
$A = +1$ $B = -1$	p_2	0	0	$1 - p_2$
$A = -1$ $B = +1$	p_3	0	0	$1 - p_3$
$A = -1$ $B = -1$	0	p_4	$1 - p_4$	0

Table 1. Conditional probabilities for a toy model with two binary inputs and two binary outcomes that cannot be reproduced by a realistic and local underlying theory [9].

These arguments can be stated in more abstract terms as follows. Quantum predictions for the Bell's experiment are commonly described as a set of conditional probabilities $p(a, b|A, B)$, where $a = \pm 1$ and $b = \pm 1$ are the two possible outcomes at each one of the two detectors and $A = \pm 1$ and $B = \pm 1$ describe two possible choices for the setting of each one of the two detectors. It is then proven that these conditional probabilities cannot be obtained in terms of a local model of hidden variables, defined by its configuration space $\lambda \in \mathcal{S}$, its density of probability $\rho(\lambda)$ and its local response functions $a = f(\lambda, A)$, $b = f(\lambda, B)$ [2].

This statement can clearly be illustrated with the help of the toy model described in Table 1 [9], where conditional probabilities for each one of the four possible results of an experiment with two binary outcomes $a, b = \pm 1$ (columns) are given for each one of four possible settings, defined by two independent binary inputs $A, B = \pm 1$ (rows). For these probabilities to be properly defined we require that $p_1, p_2, p_3, p_4 \in [0, 1]$. It can be readily checked that for each set of input values (rows) the sum of the probabilities for all possible results of the experiment (columns) equals 1. These conditional probabilities, however, cannot be obtained within the framework of an underlying local model of hidden variables: the conditional probabilities listed in the first three rows would imply $a = b$, that is, the outcomes of the two detectors in any of their four possible settings must have the same sign, which is obviously inconsistent

with the conditional probabilities listed in the fourth row.

Nonetheless, it is straightforward to identify in this abstract reformulation of the Bell's theorem the same unjustified implicit assumption that we have noticed above, namely, that there are two well-defined choices for the setting of each one of the detectors. We have noticed above that we can properly define and measure only the conditional probabilities $p(a, b|D)$, where $a = \pm 1$ and $b = \pm 1$ are, as before, the outcomes at each one of the two detectors and $D = 1, 2, 3, 4$ defines four possible relative orientations between them. We did notice also that quantum mechanics as well makes theoretical predictions only for these conditional probabilities $p(a, b|D)$. Under these looser constraints the Bell's theorem does not necessarily hold:

Consider, for example, the toy model described in Table 2. The conditional probabilities are identical to those described in Table 1 for each one of the four possible results of the experiment, but the setting of the measurement devices is now described by a single parameter $D = 1, 2, 3, 4$. Each input value corresponds to a given relative orientation of the two devices. The new model simply states that when the devices are set at $D = 1, 2, 3$ their outcomes must have the same sign, and when they are set at $D = 4$ their outcomes must have opposite signs. Obviously, this latter model is not necessarily in contradiction with an underlying local model of hidden variables.

Outcome	$a = +1$	$a = +1$	$a = -1$	$a = -1$
Setting	$b = +1$	$b = -1$	$b = +1$	$b = -1$
$D = 1$	p_1	0	0	$1 - p_1$
$D = 2$	p_2	0	0	$1 - p_2$
$D = 3$	p_3	0	0	$1 - p_3$
$D = 4$	0	p_4	$1 - p_4$	0

Table 2. Conditional probabilities for a toy model with a single input with four possible values and two binary outcomes. They can be reproduced by an underlying theory.

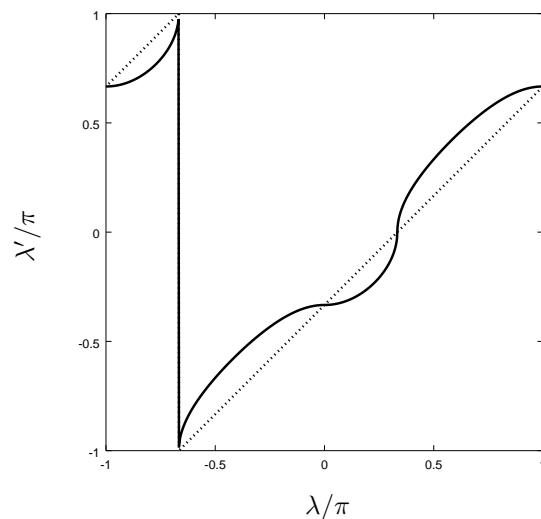


FIG. 2: Plot of the transformation law $\lambda \rightarrow \lambda' = L(\lambda; \Delta)$ for $\Delta = \pi/3$ (solid line), compared to the corresponding linear transformation (dotted line).

B. The statistical model

We shall now build and discuss in detail an explicitly local statistical model of hidden variables that is not constrained by the Bell's inequality. In fact, the model reproduces the predictions of quantum mechanics for the Bell's states (1). Further aspects of the model are discussed in [14]. The crux of the model is the spontaneous breaking by the hidden configurations of the pair of entangled particles of the gauge symmetry of the experimental setting under global rigid rotations of the orientation of the detectors, so that there cannot exist an absolute preferred frame to which we can refer in order to compare different realizations of the experiment other than the orientation of one of the detectors.

The symmetry is spontaneously broken because the hidden configuration of the pair of entangled particles has a preferred direction randomly oriented over a unit circle \mathcal{S} . Each one of the two detectors defines a frame of reference with its own set of associated coordinates over this circle \mathcal{S} , $\lambda \in [-\pi, +\pi)$ for detector A and $\lambda' \in [-\pi, +\pi)$ for detector B. Since the two sets of coordinates parameterize the same space \mathcal{S} they must be related by some transformation law:

$$\lambda' = -L(\lambda; \Delta - \Phi). \quad (6)$$

This transformation law states that a hidden configuration whose preferred direction is oriented along an angle λ with respect to detector A is oriented along an angle λ' with respect to detector B, where Δ is the relative angle between the two detectors and Φ is the phase that characterizes the source of entangled particles as defined above.

The transformation law (6) does not necessarily violate neither locality nor causality: it may well be a fundamental law of Nature. Indeed, the notions of locality and causality in special relativity stem from a similar relationship $v' = T(v; V)$ between the velocities v and v' of a point particle with respect to two different inertial frames moving with relative velocity V . Indeed, (6) is only a generalization of the euclidean linear relationship that states that given two *classical* detectors whose orientations form an angle Δ , then a pointer oriented along an angle λ with respect to one of them is oriented along an angle $\lambda' = \lambda - \Delta$ with respect to the other.

In order to reproduce the predictions of quantum mechanics we define the transformation law (6) as follows:

- If $\Delta \in [0, \pi)$,

$$L(\lambda; \Delta) = \begin{cases} q(\lambda) \cdot \text{acos}(-\cos(\Delta) - \cos(\lambda) - 1), & \text{if } -\pi \leq \lambda < \Delta - \pi, \\ q(\lambda) \cdot \text{acos}(+\cos(\Delta) + \cos(\lambda) - 1), & \text{if } \Delta - \pi \leq \lambda < 0, \\ q(\lambda) \cdot \text{acos}(+\cos(\Delta) - \cos(\lambda) + 1), & \text{if } 0 \leq \lambda < \Delta, \\ q(\lambda) \cdot \text{acos}(-\cos(\Delta) + \cos(\lambda) + 1), & \text{if } \Delta \leq \lambda < +\pi, \end{cases} \quad (7)$$

- If $\Delta \in [-\pi, 0)$,

$$L(\lambda; \Delta) = \begin{cases} q(\lambda) \cdot \operatorname{acos}(-\cos(\Delta) + \cos(\lambda) + 1), & \text{if } -\pi \leq \lambda < \Delta, \\ q(\lambda) \cdot \operatorname{acos}(+\cos(\Delta) - \cos(\lambda) + 1), & \text{if } \Delta \leq \lambda < 0, \\ q(\lambda) \cdot \operatorname{acos}(+\cos(\Delta) + \cos(\lambda) - 1), & \text{if } 0 \leq \lambda < \Delta + \pi, \\ q(\lambda) \cdot \operatorname{acos}(-\cos(\Delta) - \cos(\lambda) - 1), & \text{if } \Delta + \pi \leq \lambda < +\pi, \end{cases} \quad (8)$$

where, for the sake of compactness, we have denoted the difference $\Delta - \Phi$ simply as Δ ,

$$q(\lambda) = \operatorname{sign}((\lambda - \Delta) \bmod ([-\pi, \pi))),$$

and the function $y = \operatorname{acos}(x)$ is defined in its main branch, such that $y \in [0, \pi]$ while $x \in [-1, +1]$. In Fig. 2 the transformation $L(\lambda; \Delta)$ is graphically shown for the particular case $\Delta = \pi/3$. It is straightforward to check that the transformation law (6) fulfills the differential relationship

$$|d(\cos(\lambda'))| = d\lambda' |\sin(\lambda')| = d\lambda |\sin(\lambda)| = |d(\cos(\lambda))|, \quad (9)$$

and the parameter Δ plays the role of an the integration constant.

Locality is explicitly enforced in our model by requiring that the outcome of each one of the detectors in reponse to the hidden configuration of the entangled particles depends only on their relative orientation, that is, $s^{(A)}(\lambda) = \zeta(\lambda) = \pm 1$ for detector A and $s^{(B)}(\lambda) = \zeta(\lambda') = \pm 1$ for detector B, where λ' is related to λ by relationship (6) and $\zeta(l)$ is the binary response function of the detectors, which for the sake of simplicity we define as

$$\zeta(l) = \begin{cases} +1, & \text{if } l \in [0, +\pi) \\ -1, & \text{if } l \in [-\pi, 0). \end{cases} \quad (10)$$

In order to complete our statistical model we need to specify also the (density of) probability $\rho(l)$ of each hidden configuration $l \in \mathcal{S}$ over the space \mathcal{S} to occur in every single realization of the pair of entangled particles. By symmetry considerations this density of

probability must be functionally identical from the point of view of both detectors, independently of their relative orientation. Moreover, the condition of 'free-will' demands that the probability of each hidden configuration to occur in any single realization of the experiment cannot depend on the parameterizations of the space \mathcal{S} associated to each one of the two detectors. This condition can be precisely stated as:

$$d\lambda \rho(\lambda) = d\lambda' \rho(\lambda'). \quad (11)$$

It is straightforward to show from (9) that this condition is fulfilled if the probability density $\rho(l)$ is given by:

$$\rho(l) = \frac{1}{4} |\sin(l)|. \quad (12)$$

We can now compute within the framework of this model the statistical correlations expected between the outcomes of the two detectors as a function of their relative orientation. The binary outcomes of each one of the two detectors define a partition of the phase space of all the possible hidden configurations into four coarse subsets,

$$\begin{aligned} (s^{(A)} = +1; s^{(B)} = +1) &\iff l \in [0, \Delta - \Phi) \\ (s^{(A)} = +1; s^{(B)} = -1) &\iff l \in [\Delta - \Phi, \pi) \\ (s^{(A)} = -1; s^{(B)} = +1) &\iff l \in [\Delta - \Phi - \pi, 0) \\ (s^{(A)} = -1; s^{(B)} = -1) &\iff l \in [-\pi, \Delta - \Phi - \pi), \end{aligned}$$

where we have assumed without any loss of generality that $\Delta - \Phi \in [0, \pi)$. Each one of these four coarse subsets happen with a probability given by:

$$\begin{aligned} p(+1, +1) &= \int_0^{\Delta - \Phi} \rho(l) dl = \frac{1}{4} (1 - \cos(\Delta - \Phi)), \\ p(+1, -1) &= \int_{\Delta - \Phi}^{\pi} \rho(l) dl = \frac{1}{4} (1 + \cos(\Delta - \Phi)), \\ p(-1, +1) &= \int_{\Delta - \Phi - \pi}^0 \rho(l) dl = \frac{1}{4} (1 + \cos(\Delta - \Phi)), \\ p(-1, -1) &= \int_{-\pi}^{\Delta - \Phi - \pi} \rho(l) dl = \frac{1}{4} (1 - \cos(\Delta - \Phi)). \end{aligned}$$

These conditional probabilities reproduce the predictions of quantum mechanics (2):

$$E(\Delta, \Phi) = p(+1, +1) + p(-1, -1) - p(+1, -1) - p(-1, +1) = -\cos(\Delta - \Phi).$$

Finally, we notice that in spite of the non-trivial transformation law (6) our model comply with the trivial demand that a relative rotation of the measurement apparatus by an angle Δ followed by a second rotation by an angle Δ' results into a final rotation by an angle $\Delta + \Delta'$. Consider, for example, an initial reference setting \mathcal{T}_0 in which the outcomes of the two measurement apparatus are correlated by an amount $E = -\cos(\Phi)$. The angular coordinates of the hidden configurations with respect to each one of the two measurement devices, λ and λ' , would be related in this reference setting by the relationship:

$$\lambda' = -L(\lambda; -\Phi). \quad (13)$$

We now define a new measurement setting \mathcal{T}_1 obtained from the initial setting \mathcal{T}_0 by rotating the relative orientation of the two apparatus by an angle Δ . The angular coordinates λ and λ'' defined with respect to this new setting would be related by:

$$\lambda'' = -L(\lambda; \Delta - \Phi). \quad (14)$$

A third measurement setting \mathcal{T}_2 is obtained from the intermediate setting \mathcal{T}_1 by rotating the relative orientation of the two apparatus by an additional angle Δ' . In the intermediate setting \mathcal{T}_1 , which is now taken as reference to define the second rotation, the pair of particles appears to be in a polarization state characterized by a phase $\Phi' = -\Delta + \Phi$. Hence, the angular coordinates λ and λ''' defined with respect to the setting \mathcal{T}_2 would be related by the transformation law:

$$\lambda''' = -L(\lambda; \Delta' - \Phi') = -L(\lambda; \Delta' + \Delta - \Phi). \quad (15)$$

By comparison of the transformation law (13) for the initial setting \mathcal{T}_0 and the transformation law (15) for the setting \mathcal{T}_2 , we realize that the latter has been obtained from the initial setting by rotating the apparatus by an angle $\Delta' + \Delta$, as we had demanded.

In order to complete the description of the Bell's experiment we define two new settings \mathcal{T}_3 and \mathcal{T}_4 , which are obtained, respectively, from \mathcal{T}_1 and \mathcal{T}_2 by cancelling the phase Φ in the reference setting \mathcal{T}_0 . Hence, in these settings the angular coordinates of the hidden configurations with respect to the two measurement apparatus are related by the relationships:

$$\lambda'''' = -L(\lambda; \Delta). \quad (16)$$

and

$$\lambda'''' = -L(\lambda; \Delta' + \Delta), \quad (17)$$

respectively.

The phase Φ , which characterizes the quantum source with respect to the reference setting of the detectors, plays an obvious crucial role in this last argument. Indeed, the argument would not work for a *classical* source without this degree of freedom. Therefore, a few further clarifications about it may be helpful.

First, it is important to notice that the phase Φ is not introduced 'ad hoc' in the statistical model of hidden variables discussed above. This phase describes an actual physical magnitude: for a given reference setting of the two measuring devices there exists a continuous infinite family of different sources of pairs of entangled particles, parameterized by the angle $\Phi \in [0, 2\pi)$, each one of which produces a different correlation $E = -\cos(\Phi)$ between the binary outcomes of the two devices. These different sources are described by the different quantum states $|\Psi_\Phi\rangle$ described by eq. (1). However, as explained in eq. (13,14,15,16,17) and in subsection 2.1, it is crucial to notice that the phase Φ that characterizes the quantum state of the source of entangled particles can only be properly defined with respect to a setting of the measuring devices chosen as reference, and with respect to which any relative rotation of the devices Δ performed later on is referred. In other words, the phase Φ is a physical degree of freedom just like the angle Δ is, and they both can be properly defined only with respect to a reference setting of the measuring devices. In fact, the statistical correlation between the outcomes of the two devices in a Bell's experiment depends only on the angle $\Delta - \Phi$, which is independent of the setting of the devices chosen as reference. Indeed, experimentalists use this freedom in order to fix their set-up: given an actual source of entangled particles they calibrate the orientations of the two measuring devices in order to get full anti-correlation between their outcomes and, with respect to this calibrated reference setting, they set $\Phi = 0$ for the source and, later on, define a relative rotation Δ between the orientation of the devices.

In the model of hidden variables considered in this paper the phase Φ appears as a result of the spontaneous breaking of the symmetry of the system under global rigid rotations of the orientation of the two detectors. The symmetry is spontaneously broken by the hidden configuration of the pair of entangled particles, which sets a preferred direction. In particular, as explained through the paper, when the global rotational symmetry is broken the

orientations of each one of the measuring devices, Ω_A and Ω_B , cannot be defined separately with respect to an external lab frame, but only with respect to the preferred direction set by the hidden configuration of the particles (which, nonetheless, is not directly accessible). Hence, only the relative orientation between the devices can be properly defined. The apparently disappeared degree of freedom actually corresponds to the phase Φ that characterizes the source (which would be the Goldstone degree of freedom associated to the spontaneously broken rotational symmetry). Or said in other words: the phase space $(\Omega_A, \Omega_B, \Phi)$ contains an spurious unphysical gauge degree of freedom that cannot be properly defined either theoretically nor experimentally. The actual physical phase space (Δ, Φ) of possible settings for a Bell's experiment is obtained from the space $(\Omega_A, \Omega_B, \Phi)$ as the quotient set under the equivalence relation defined by the gauge transformation, so that setting the orientation of one of the detectors as a reference direction must be understood as a gauge fixing condition. Thus, the two degrees of freedom (Ω_A, Ω_B) that would be needed to describe an experimental set-up for a *classical* source of pairs of particles correspond to the two degrees of freedom (Δ, Φ) that describe the experimental set-up for a quantum source of entangled particles that break the global rotational symmetry. Therefore, when we substitute the coherent source of pairs of entangled particles (1) by the incoherent *classical* source

$$\hat{\mu} = \int_{2\pi} d\Phi |\Psi_\Phi\rangle\langle\Psi_\Phi| = |\uparrow\rangle\langle\uparrow|^{(A)} \otimes |\downarrow\rangle\langle\downarrow|^{(B)} + |\downarrow\rangle\langle\downarrow|^{(A)} \otimes |\uparrow\rangle\langle\uparrow|^{(B)},$$

the broken symmetries are statistically restored and the outcomes of the two measurement devices become uncorrelated for all settings. Thus, only for the *classical* source we can safely assume the existence of an absolute external frame of reference with respect to which we can define independently the orientations of each one of the two measuring devices and describe the phase space of its possible settings as (Ω_A, Ω_B) . This is not possible for a quantum source of entangled particles, for which the phase space of possible settings must be described by the angles (Δ, Φ) .

III. DISCUSSION

In this paper we have shown that the proof of the Bell's theorem requires the existence of an absolute frame of reference, supposedly provided by the lab, with respect to which the orientations of each one of the two measurement devices can be independently defined for each single realization of the experiment. On the other hand, we have shown that such an absolute frame does not exist if the hidden configurations of the pair of entangled particles spontaneously break the rotational symmetry of the system. In such a case, in order to compare different realizations of the experiment we must pick the orientation of one of the detectors as a common reference direction with respect to which the relative orientation of the second detector is defined and, hence, the Bell's theorem does not necessarily hold (see Table 2).

Following these ideas we have explicitly built a model of hidden variables for the Bell's states of two entangled particles that reproduces the predictions of quantum mechanics. Further details of the model are discussed in [14]. In two additional accompanying papers we have used these same ideas to build explicit local models of hidden variables for the GHZ state of three entangled particles [15] and also for the qutrit [16].

These models, thus, open a window to a very desired realistic interpretation of the quantum formalism and, maybe, also to unexplored fundamental physics underlying the principles of quantum mechanics.

We find very intriguing that the solution presented here of the Einstein-Podolsky-Rosen paradox [10] is intrinsically related to the symmetries and metrics of space and wonder if this solution could open a window through which quantum mechanics and Einstein's general relativity theory of gravitation could be put together.

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- [1] J.S. Bell, "On the Einstein-Podolsky-Rosen paradox", *Physics* **1964**, 1, 195-200.
 - [2] A. Fine, "Hidden variables, joint probability, and the Bell inequalities", *Phys. Rev. Lett.* **1982**, 48, 291.
 - [3] B. Hensen *et al*, "Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres", *Nature* **2015**, 526, 682.
 - [4] A. Aspect, J. Dalibard and G. Roger, "Experimental test of Bell's inequalities using time-

- varying analyzers”, *Phys. Rev. Lett.* **1982**, 49, 1804.
- [5] G. Weihs et al., ”Violation of Bell’s inequality under strict Einstein locality conditions”, *Phys. Rev. Lett.* **1998**, 81, 5039.
- [6] H. Wiseman, ”Quantum physics: Death by experiment for local realism”, *Nature* **2015**, 526, 649.
- [7] J.F. Clauser, M.A. Horne, A. Shimony and R.A. Holt, ”Proposed experiment to test local hidden variables theories”, *Phys. Rev. Lett.* **1969**, 23, 880884. DOI: 10.1103/PhysRevLett.23.880.
- [8] B. S. Cirelson, ”Quantum generalizations of Bell’s inequality”, *Lett. Math. Phys.* **1980**, 4, 93.
- [9] J.E. Avron, private communication.
- [10] A. Einstein, B. Podolsky and N. Rosen, ”Can quantum mechanical description of physical reality be considered complete ?”, *Phys. Rev.* **1935**, 47, 777-780, DOI: 10.1103/Phys.Rev.47.777.
- [11] D. Bohm, *Quantum Theory* **1951**, Prentice-Hall, New York.
- [12] S. Kochen and E.P. Specker, ”The problem of hidden variables in quantum mechanics”, *J. Math. Mech.* **1967**, 17, 59-87.
- [13] J.S. Bell, ”On the problem of hidden variables in quantum mechanics”, *Physics* **1966**, 38, 447-452.
- [14] D.H. Oaknin, ”Solving the EPR paradox: an explicit statistical local model of hidden variables for the singlet state”, **2017**, arXiv:1411.5704.
- [15] D.H. Oaknin, ”Solving the Greenberger-Horne-Zeilinger paradox: an explicit local model of hidden variables for the GHZ state”, **2017**, arXiv:1709.00167.
- [16] D.H. Oaknin, ”Bypassing the Kochen-Specker theorem: and explicit non-contextual model of hidden variables for the qutrit”, arXiv:1805.04935.