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► **To cite this version:**

David H Oaknin. Bell vs. Galileo: the proof of the inequality clashes with the principle of relativity. 2018. hal-01862953v1

HAL Id: hal-01862953

<https://hal.science/hal-01862953v1>

Preprint submitted on 28 Aug 2018 (v1), last revised 22 Apr 2022 (v5)

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Bell vs. Galileo: the proof of the inequality clashes with the principle of relativity

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The Bell's theorem stands as an insuperable roadblock in the path to a very desired intuitive solution of the Einstein-Podolsky-Rosen paradox and, hence, it lies at the core of the current lack of a clear interpretation of the quantum formalism. The theorem states through an experimentally testable inequality that the predictions of quantum mechanics for the Bell's polarization states of two entangled particles cannot be reproduced by any statistical model of hidden variables that shares certain intuitive features. In this paper we show, however, that the proof of the Bell's inequality involves a subtle, though crucial, assumption that is not required by fundamental physical principles and, in fact, it is not fulfilled in the experimental setup that tests the inequality. Indeed, this assumption is at odds with the principle of relativity and, thus, it cannot be properly implemented within the framework of quantum mechanics either. Furthermore, we show that local models of hidden variables that do not comply with this unjustified assumption are not necessarily constrained by the Bell's inequality and can reproduce the predictions of quantum mechanics for the Bell's states.

Bell's theorem is one of the fundamental theorems upon which relies the widespread belief that quantum mechanics is the ultimate mathematical framework within which the hypothetical final theory of the fundamental building blocks of Nature and their interactions should be formulated. The theorem states through an experimentally testable inequality (the Bell's inequality) that local statistical models of hidden variables that share certain intuitive features cannot reproduce the predictions of quantum mechanics for the entangled polarization states of two particles (Bell's states) [1, 2]. These predictions have been confirmed beyond doubt by very carefully designed experiments [3–6].

In these experimental tests a source emits pairs of particles whose polarizations are arranged in a Bell's entangled state:

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\rangle^{(A)} |\downarrow\rangle^{(B)} - e^{i\Phi} |\downarrow\rangle^{(A)} |\uparrow\rangle^{(B)} \right), \quad (1)$$

where $\{|\uparrow\rangle, |\downarrow\rangle\}^{(A,B)}$ are eigenstates of Pauli operators $\sigma_Z^{(A,B)}$ along locally defined Z-axes for each one of the two particles. The two emitted particles travel off the source in opposite directions towards two widely separated detectors, which test their polarizations. The orientation of each one of the detectors can be freely and independently set along any arbitrary direction in the XY-plane perpendicular to the locally defined Z-axis. Upon detection each particle causes a binary response of its detector, either +1 or -1. Thus, each pair of entangled particles produces an outcome in the space of possible events $\{(-1, -1), (-1, +1), (+1, -1), (+1, +1)\}$. We refer to each detected pair as a single realization of the experiment.

Quantum mechanics predicts that the statistical correlation between the outcomes of the two detectors in a long sequence of realizations of the experiment is given by:

$$E(\Delta, \Phi) = -\cos(\Delta - \Phi), \quad (2)$$

where Δ is the relative angle between the orientations of the two detectors. In particular, when $\Delta - \Phi = 0$ we get that $E = -1$, so that all outcomes in the sequence must be either $(-1, +1)$ or $(+1, -1)$.

The Bell's theorem states that prediction (2) cannot be reproduced by any model of hidden variables that shares certain intuitive features. In particular, the CHSH version of the theorem states that for all these generic models of hidden variables the following inequality is fulfilled for any set of values $(\Delta_1, \Delta_2, \Delta)$ [7]:

$$|E(\Delta_1) + E(\Delta_2) + E(\Delta_1 - \Delta) - E(\Delta_2 - \Delta)| \leq 2. \quad (3)$$

On the other hand, according to quantum mechanics the magnitude in the left hand side of the inequality reaches a maximum value of $2\sqrt{2}$, known as Tsirelson's bound [8]. As it was noted above, carefully designed experiments have confirmed that the CHSH inequality is violated according to the predictions of quantum mechanics and, therefore, have ruled out all said generic local models of hidden variables.

Any local statistical model of hidden variables that aims to describe the Bell's experiment consists of some space \mathcal{S} of possible hidden configurations $\lambda \in \mathcal{S}$ for the pair of entangled particles, together with a well-defined (density of) probability $\rho(\lambda)$ for each one of them to occur in every single realization. The model must also specify well-defined binary values $s_{\Omega_A}^{(A)}(\lambda) = \pm 1$, $s_{\Omega_B}^{(B)}(\lambda) = \pm 1$ to describe the outcomes that would be obtained at detectors A and B when the pair of entangled particles occurs in the hidden configuration $\lambda \in \mathcal{S}$ and their polarizations are tested along directions Ω_A and Ω_B , respectively.

The proof of the CHSH inequality involves two well-defined possible orientations Ω_A and Ω'_A for the polarization test of particle A and two well-defined possible orientations Ω_B and Ω'_B for the polarization test of particle B, and assumes that the model of hidden variables assigns to each possible hidden configuration $\lambda \in \mathcal{S}$ a 4-tuple

of binary values $\left(s_{\Omega_A}^{(A)}(\lambda), s_{\Omega'_A}^{(A)}(\lambda), s_{\Omega_B}^{(B)}(\lambda), s_{\Omega'_B}^{(B)}(\lambda)\right) \in \{-1, +1\}^4$ to describe the outcomes that would be obtained in each one of the two detectors in case that it would be set along each one of its two available orientations. Hence, it is straightforward to obtain that for any $\lambda \in \mathcal{S}$,

$$s_{\Omega_A}^{(A)}(\lambda) \cdot \left(s_{\Omega_B}^{(B)}(\lambda) + s_{\Omega'_B}^{(B)}(\lambda)\right) + s_{\Omega'_A}^{(A)}(\lambda) \cdot \left(s_{\Omega_B}^{(B)}(\lambda) - s_{\Omega'_B}^{(B)}(\lambda)\right) = \pm 2, \quad (4)$$

since the first term is non-zero only when $s_{\Omega_B}^{(B)}(\lambda)$ and $s_{\Omega'_B}^{(B)}(\lambda)$ have the same sign, while the second term is non-zero only when they have opposite signs. The CHSH inequality (3) is then obtained by averaging (4) over the whole space \mathcal{S} of all possible hidden configurations.

The proof of the CHSH inequality, thus, requires three well-defined angles, $\Delta_1 \equiv \angle(\Omega_B, \Omega_A)$, $\Delta_2 \equiv \angle(\Omega'_B, \Omega_A)$ and $\delta \equiv \angle(\Omega'_A, \Omega_A)$, which correspond, respectively, to the relative orientations of Ω_B , Ω'_B and Ω'_A with respect to Ω_A , which serves as a reference direction. This reference direction serves also to define the hidden configuration $\lambda \in \mathcal{S}$ of the pair of entangled particles, since the description of a physical state must necessarily be done with respect to a reference frame. The orientation of this reference direction is an spurious/irrelevant gauge degree of freedom, which according to the principle of relativity can be set to zero for every single realization of the Bell's experiment.

In order to stress the meaning of this last observation let us consider an experimental setup in which the four directions Ω_A , Ω'_A , Ω_B and Ω'_B are fixed with respect to, say, the optical table on which we perform the experiments. The collected experimental data can be analyzed with respect to this lab frame or with respect to lab frames located at, for example, the center of the Sun or the center of the Galaxy or the center of the local supercluster, in which the four possible orientations of the two detectors are not fixed anymore for different single realizations of the experiment. Obviously, the proof of the Bell's theorem remains valid for any choice of the lab frame: it only requires that the three relative angles Δ_1 , Δ_2 and δ will be well-defined, while the global orientation of the reference direction Ω_A in the lab frame is absolutely irrelevant.

This apparently trivial observation leads us directly to the main claim of this paper, namely, that the proof of the CHSH inequality relies on a subtle, though crucial, unjustified assumption, which is not required by fundamental physical principles and, in fact, it is not fulfilled in the actual experimental setup designed to test the inequality. Indeed, the disputed assumption is at odds with the principle of relativity. Unfortunately, this assumption appears in the proof of all versions of the Bell's theorem.

In the actual setup of a Bell's experiment the angles Δ_1 and Δ_2 , which describe two different relative orientations of detectors A and B, can always be properly defined. But how can we properly define the angle δ between two possible orientations of detector A, if we are using it as a reference direction? As we have noted above, according to the principle of relativity we can choose the orientation of detector A as a reference direction for every single realization of the experiment and, hence, arbitrarily set it to zero.

We could properly define the angle δ if the experimental setup would enable us to test the polarization of each one of the particles of every pair along two different directions at once. Notwithstanding, this is not the case: each particle of every pair can be tested along a single direction. Therefore, in order to properly define the angle δ there should exist an absolute preferred frame of reference with respect to which we could define the global orientation of the detectors. Notwithstanding, this is not the case either: such an absolute preferred frame would clash with the principle of relativity. In other words, while the relative orientation between the two detectors is a properly defined physical feature of the experimental setup, their global orientation is an spurious gauge degree of freedom (see Fig. 1).

It is straightforward to show that the global orientation of the experimental setup cannot be properly defined within the standard framework of quantum mechanics either. The Bell's state (1) that describes the pair of entangled particles is defined in terms of the bases $\{| \uparrow \rangle, | \downarrow \rangle\}^{(A,B)}$ of eigenstates of the Pauli operators $\sigma_Z^{(A,B)}$. Since these eigenstates are defined up to a global phase, the phase Φ in (1) would not be properly defined. In order to properly define it we need to choose an arbitrary setting of the two detectors as a reference. This setting defines *parallel* directions at the sites of the two particles. Then, the phase Φ of the entangled state (1) can be properly defined with the help of the measured correlations between the outcomes of the two detectors $E = -\cos(\Phi)$. Furthermore, we can use this reference setting to properly define a relative rotation Δ of the orientations of the two apparatus (see Fig. 2). Nevertheless, we cannot define their absolute orientation δ without referring to an inexistent preferred absolute frame of reference.

We can take advantage of this gauge symmetry in order to build an explicitly local statistical model of hidden variables that is not constrained by the Bell's inequality and, indeed, reproduces the predictions of quantum mechanics and the experimental data. The details of the model are discussed in [9]. Here we briefly outline its most relevant features.

We notice that a statistical model of hidden variables that aims to describe the Bell's experiment, in which the orientation of, say, detector A is taken as a gauge-fixing reference direction, only needs to specify the binary val-

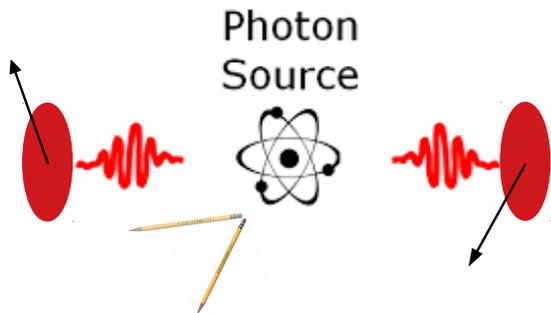


FIG. 1. According to the principle of relativity a global rotation of the detectors while keeping the pencil/lab-frame fixed cannot be distinguished in any sense from a rotation of the pencil/lab-frame while keeping the global orientation of the detectors fixed, which is obviously irrelevant for the outcome of the Bell's experiment, unless we imagine an inexistent absolute preferred frame of reference.

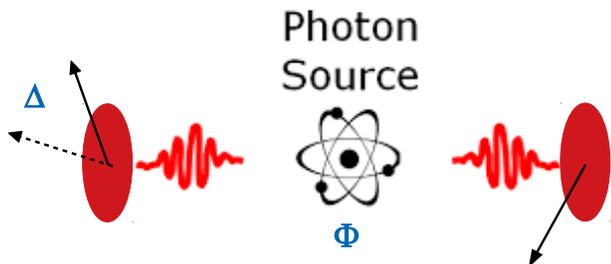


FIG. 2. Only with respect to a reference setting of the two detectors (solid arrows), which serves as a definition of *parallel* directions, it is possible to properly define the phase Φ that characterizes the source and rotations of the relative orientation Δ between the two detectors (dashed arrow).

ues for the physical observables $s_{\Omega_A}^{(A)}(\lambda)$, $s_{\Omega_B}^{(B)}(\lambda)$, $s_{\Omega'_B}^{(B)}(\lambda)$, $s_{\Omega''_B}^{(B)}(\lambda)$ and $s_{\Omega'''_B}^{(B)}(\lambda)$ for each possible hidden configuration $\lambda \in \mathcal{S}$, where $\angle(\Omega_B, \Omega_A) = \Delta_1$, $\angle(\Omega'_B, \Omega_A) = \Delta_2$, $\angle(\Omega''_B, \Omega_A) = \Delta_1 - \delta$, $\angle(\Omega'''_B, \Omega_A) = \Delta_2 - \delta$ are the relative orientations between the two detectors in four available settings. It is then straightforward to notice that the magnitude

$$s_{\Omega_A}^{(A)}(\lambda) \cdot \left(s_{\Omega_B}^{(B)}(\lambda) + s_{\Omega'_B}^{(B)}(\lambda) + s_{\Omega''_B}^{(B)}(\lambda) - s_{\Omega'''_B}^{(B)}(\lambda) \right), \quad (5)$$

can take values out of the interval $[-2, 2]$ and, therefore, these models are not constrained by the CHSH inequality (3). That is, the proof of the inequality (4) does not hold for these gauge models because it involves adding terms defined under different gauge-fixing conditions [13].

The notion of locality can be precisely defined within this framework in a way that is explicitly consistent with the principle of relativity. First, we notice that each one of the two detectors defines a frame of reference with its own set of associated coordinates, $\lambda \in \mathcal{S}$ for detector A and $\lambda' \in \mathcal{S}$ for detector B, over the space \mathcal{S} of all possible hidden configurations. Since the two sets of coordinates parameterize the same space \mathcal{S} they must be related by some transformation law:

$$\lambda' = L(\lambda; \Delta - \Phi). \quad (6)$$

This transformation law states that a hidden configuration oriented along direction λ with respect to detector A is oriented along direction λ' with respect to detector B, when the two detectors are related by a relative angle Δ and the source by a phase Φ . This transformation law does not violate locality or causality: it may well be a fundamental law of Nature. Indeed, the notion of locality and causality in special relativity stems from a similar relationship $v' = T(v; V)$ between the velocities v and v' of a point particle with respect to two different inertial frames moving with relative velocity V .

Locality simply demands that the outcome of each one of the detectors in response to a hidden configuration of the entangled particles shall depend only on their relative orientation, that is, $s^{(A)} = \zeta(\lambda) = \pm 1$ for detector A and $s^{(B)} = \zeta(\lambda') = \pm 1$ for detector B, where $\zeta(\cdot)$ is the response function of the detector.

Moreover, the condition of 'free-will' demands that the probability of each hidden configuration to happen in a single realization of the experiment cannot depend on the chosen parameterization of the space \mathcal{S} . This condition can be precisely stated as:

$$d\lambda \rho(\lambda) = d\lambda' \rho(\lambda'), \quad (7)$$

where $\rho(\cdot)$ is the density of probability over the space \mathcal{S} , which by symmetry considerations must be functionally identical from the point of view of all detectors. This density of probability fixes, through condition (7), the transformation law (6) that relates the two sets of coordinates, which in turn fixes, together with the response function, the statistical correlation between the outcomes of the two detectors.

We have used these same ideas to build explicit local models of hidden variables for the Bell's states of two

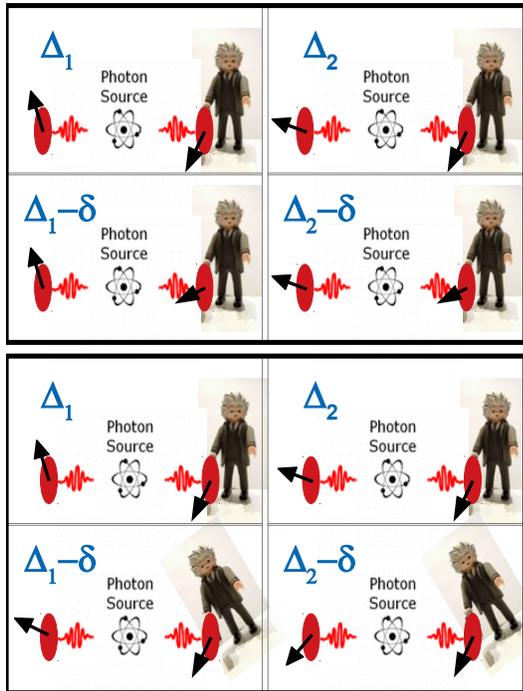


FIG. 3. Two physically undistinguishable descriptions of the experimental setup required for testing the Bell's inequality. In the description above the lab frame is taken to be fixed, while in the description below the orientation of detector A is taken to be fixed. The relative angle between the two detectors is set at four possible values Δ_1 , Δ_2 , $\Delta_1 - \delta$ and $\Delta_2 - \delta$.

entangled particles [9], the GHZ state of three entangled particles [10] and also for the qutrit [11]. We find very intriguing that the solution presented here of the Einstein-Podolsky-Rosen paradox [12] is intrinsically related to the symmetries and metrics of space and wonder if this solution opens the window to bringing together quantum mechanics and Einstein's general relativity theory of gravitation.

In summary, we have shown in this paper that the proof of the Bell's theorem involves an unjustified assumption, which is not required by fundamental physical principles. In fact, this assumption is not fulfilled in the experimental setup designed to test the Bell's inequality and neither it can be properly implemented within the framework of quantum mechanics. Indeed, the said assumption does violate the principle of relativity since it implies the existence of an absolute preferred frame of reference with respect to which the orientations of the two detectors, as well as the orientation of the possible hidden configurations of the entangled particles, are defined.

I wish to thank Dr. Paz London for very useful discussions on the ideas contained in this paper and for his valuable help to make the presentation clearer.

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