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Constraints on $2\mathcal{L}2q$ operators from $\mu \leftrightarrow e$ flavor-changing meson decays

Sacha Davidson¹ and Albert Saporta²

¹*LUPM, CNRS, Université Montpellier, Place Eugène Bataillon, F-34095 Montpellier, Cedex 5, France*

²*Université de Lyon, France; Université Lyon 1 CNRS/IN2P3 IPNL, 69622 Villeurbanne cedex, France*



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We study lepton-flavor-violating two- and three-body decays of pseudoscalar mesons in effective field theory. We give analytic formulas for the decay rates in the presence of a complete basis of QED \times QCD-invariant operators. The constraints are obtained at the experimental scale, then translated to the weak scale via one-loop renormalization group equations. The large renormalization-group mixing between tensor and (pseudo)scalar operators weakens the constraints on scalar and pseudoscalar operators at the weak scale.

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I. INTRODUCTION

The discovery of neutrino oscillations [1,2] established nonzero neutrino masses and mixing angles [3]. If neutrinos are taken massless in the Standard Model (SM), then new physics (NP) is required to explain the oscillation data. There are several possibilities to search for NP signatures, such as looking for new particles at the LHC [4,5]. Another possibility is to look for new processes among known SM particles, such as charged lepton flavor violation (CLFV) [6,7], which we define to be a contact interaction that changes the flavor of charged leptons. If neutrinos have renormalizable masses via the Higgs mechanism, then their contribution to CLFV rates is suppressed via the Glashow-Iliopoulos-Maiani mechanism by a factor $\propto (m_\nu/M_W)^4 \sim 10^{-48}$. However, various extensions of the Standard Model that contain heavy new particles (see e.g., Refs. [6–9] and references therein), can predict CLFV rates comparable to the current experimental sensitivities. Indeed, low-energy precision experiments searching for forbidden SM modes, are sensitive to NP scales \gg TeV [6]. Many experiments search for CLFV processes; for example, the $\mu \leftrightarrow e$ flavor change can be probed in the decays $\mu \rightarrow e\gamma$ [10] and $\mu \rightarrow 3e$ [11,12], in $\mu \rightarrow e$ conversions on nuclei [13–15] and also in meson decays such as $K, D, B \rightarrow \bar{\mu}e$ [3,16–22].

In this paper, we focus on leptonic and semileptonic pseudoscalar meson decays with a $\mu^\pm e^\mp$ in the final

state [3]. We assume that these decays could be mediated by two-lepton, two-quark contact interactions, induced by heavy new particles at the scale $\Lambda_{NP} > m_W$. The contact interactions are included in a bottom-up effective field theory (EFT) [23–25] approach, as a complete set of dimension-six, QED \times QCD-invariant operators [6], containing a muon, an electron and one of the quark-flavor-changing combinations ds, bs, bd or cu .

Many studies on related topics can be found in the literature. The experimental sensitivity to the coefficients of four-fermion operators (sometimes referred to as one-operator-at-a-time bounds), evaluated at the experimental scale, has been compiled by various authors [26–28]. Reference [29] compared the sensitivities of the LHC vs low-energy processes, to quark flavor-diagonal scalar operators. The constraints on combinations of lepton-flavor-changing operator coefficients, which can be obtained from the decays of same-flavor mesons, were studied in Ref. [30], and the radiative decays of B, D and K mesons were discussed in Ref. [31]. Lepton flavor-conserving, but quark flavor-changing meson decays (which occur in the Standard Model), are widely studied [32]. In particular, B decays attract much current interest, due to the observed anomalies [33–37] which suggest lepton universality violation [38–44]. Lepton flavor changes have been widely studied in various models (see e.g., references in Refs. [6,7,45]). More model-independent studies, that take into account loop corrections (or equivalently, renormalization group running) have also been performed for the $\mu \leftrightarrow e$ flavor change [46,47]. Finally, with respect to the calculations in this manuscript, the leptonic branching ratio of pseudoscalar mesons is well known, and can be found in Refs. [26,28,48,49] and semi-leptonic branching ratios in various scenarios can be found in Refs. [50–58].

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The aim of this paper is to obtain constraints on the operator coefficients describing meson decays at the experimental scale, and then transport the bounds to the weak scale [59]. The four-fermion operators that could induce the meson decays are listed in Sec. II. Section III gives the branching ratios for the leptonic and semileptonic decays as a function of the operator coefficients. In Sec. IV, we then use the available bounds to constrain the coefficients at the experimental scale ($\Lambda_{\text{exp}} \sim 2$ GeV) by computing a covariance matrix, which allows us to take into account the interferences among the operators. The bounds are then evolved from the experimental scale to the weak scale ($\Lambda_W \sim m_W$) in Sec. V, using the renormalization group equations (RGEs) of QED and QCD for four-fermion operators [46,47]. As discussed in the final section, these equations give a significant mixing of tensor operators to the (pseudo)scalars between Λ_{exp} and Λ_W , which significantly weakens the bounds on (pseudo)scalar coefficients at Λ_W .

II. A BASIS OF μ - e INTERACTIONS AT LOW ENERGY

We are interested in four-fermion operators involving an electron, a muon and two quarks of different flavors, which are constructed with chiral fermions, because the lepton masses are frequently neglected, and it simplifies the matching at the weak scale onto SU(2)-invariant operators. The operators are added to the Lagrangian as

$$\delta\mathcal{L} = +2\sqrt{2}G_F \sum_O \sum_{\zeta} C_O^{\zeta} \mathcal{O}_O^{\zeta} + \text{H.c.} \quad (1)$$

where the subscript O identifies the Lorentz structure, the superscript $\zeta = l_1 l_2 q_i q_j$ gives the flavor indices, and both run over the possibilities in the lists below, extrapolated from Refs. [6,60]:

$$\begin{aligned} \mathcal{O}_{V,YY}^{\mu\mu c} &= (\bar{e}\gamma^{\alpha}P_Y\mu)(\bar{u}\gamma_{\alpha}P_Yc), & \mathcal{O}_{V,YX}^{\mu\mu c} &= (\bar{e}\gamma^{\alpha}P_Y\mu)(\bar{u}\gamma_{\alpha}P_Xc), \\ \mathcal{O}_{V,YY}^{\mu\mu u} &= (\bar{e}\gamma^{\alpha}P_Y\mu)(\bar{c}\gamma_{\alpha}P_Yu), & \mathcal{O}_{V,YX}^{\mu\mu u} &= (\bar{e}\gamma^{\alpha}P_Y\mu)(\bar{c}\gamma_{\alpha}P_Xu), \\ \mathcal{O}_{S,YY}^{\mu\mu c} &= (\bar{e}P_Y\mu)(\bar{u}P_Yc), & \mathcal{O}_{S,YX}^{\mu\mu c} &= (\bar{e}P_Y\mu)(\bar{u}P_Xc), \\ \mathcal{O}_{S,YY}^{\mu\mu u} &= (\bar{e}P_Y\mu)(\bar{c}P_Yu), & \mathcal{O}_{S,YX}^{\mu\mu u} &= (\bar{e}P_Y\mu)(\bar{c}P_Xu), \\ \mathcal{O}_{T,YY}^{\mu\mu c} &= (\bar{e}\sigma P_Y\mu)(\bar{u}\sigma P_Yc), \\ \mathcal{O}_{T,YY}^{\mu\mu u} &= (\bar{e}\sigma P_Y\mu)(\bar{c}\sigma P_Yu), \\ \mathcal{O}_{V,YY}^{\mu ds} &= (\bar{e}\gamma^{\alpha}P_Y\mu)(\bar{d}\gamma_{\alpha}P_Ys), & \mathcal{O}_{V,YX}^{\mu ds} &= (\bar{e}\gamma^{\alpha}P_Y\mu)(\bar{d}\gamma_{\alpha}P_Xs), \\ \mathcal{O}_{V,YY}^{\mu sd} &= (\bar{e}\gamma^{\alpha}P_Y\mu)(\bar{s}\gamma_{\alpha}P_Yd), & \mathcal{O}_{V,YX}^{\mu sd} &= (\bar{e}\gamma^{\alpha}P_Y\mu)(\bar{s}\gamma_{\alpha}P_Xd), \\ \mathcal{O}_{S,YY}^{\mu ds} &= (\bar{e}P_Y\mu)(\bar{d}P_Ys), & \mathcal{O}_{S,YX}^{\mu ds} &= (\bar{e}P_Y\mu)(\bar{d}P_Xs), \\ \mathcal{O}_{S,YY}^{\mu sd} &= (\bar{e}P_Y\mu)(\bar{s}P_Yd), & \mathcal{O}_{S,YX}^{\mu sd} &= (\bar{e}P_Y\mu)(\bar{s}P_Xd), \\ \mathcal{O}_{T,YY}^{\mu ds} &= (\bar{e}\sigma P_Y\mu)(\bar{d}\sigma P_Ys), \\ \mathcal{O}_{T,YY}^{\mu sd} &= (\bar{e}\sigma P_Y\mu)(\bar{s}\sigma P_Yd), \end{aligned} \quad (2)$$

$$\mathcal{O}_{T,YY}^{\mu ds} = (\bar{e}\sigma P_Y\mu)(\bar{d}\sigma P_Ys), \quad \mathcal{O}_{T,YY}^{\mu sd} = (\bar{e}\sigma P_Y\mu)(\bar{s}\sigma P_Yd), \quad (3)$$

where $YY \in \{LL, RR\}$, $XY \in \{LR, RL\}$, and the list is given explicitly for the kaon and D meson operators. The lists for the B_d and B_s are obtained from Eq. (3) by replacing $ds \rightarrow db, sb$. The operators are normalized such that the Feynman rule will be $+iC/\Lambda^2$. The operators in the lists (2) and (3) transform a muon into an electron; the $e \rightarrow \mu$ operators arise in the +H.c. term of Eq. (1). So in these conventions, the lepton flavor indices are always $e\mu$, and do not need to be given. In the following sections, we give the decay rates of pseudoscalar mesons, composed of constituent quarks \bar{q}_i and q_j , into $e^+\mu^-$ or $e^-\mu^+$. Then we obtain constraints on the operator coefficients by comparing to the experimental upper bounds on the branching ratios, e.g., $\text{BR}(P_1 \rightarrow e^{\pm}\mu^{\mp}) = \text{BR}(P_1 \rightarrow e^+\mu^-) + \text{BR}(P_1 \rightarrow e^-\mu^+) < \dots$ which we assume apply independently to both decays.

This gives independent and identical bounds on $e^{\mu q_i q_j}$ and $e^{\mu q_j q_i}$.

In this work, we choose an operator basis with nonchiral quark currents, which is convenient for the nonchiral hadronic matrix elements involved in meson decays. Thus, the operators describing the contact interactions that can mediate leptonic ($\bar{q}_i q_j \rightarrow \bar{\mu}e$) and semileptonic ($q_i \rightarrow q_j \bar{\mu}e$) CLFV pseudoscalar meson decays at a scale $\Lambda_{\text{exp}} \sim 2$ GeV ($\Lambda_{\text{exp}} \sim m_b \simeq 4.2$ GeV for bs and bd) are written as

$$\begin{aligned} \mathcal{O}_{S,X}^{\mu q_i q_j} &= (\bar{e}P_X\mu)(\bar{q}_i q_j), \\ \mathcal{O}_{P,X}^{\mu q_i q_j} &= (\bar{e}P_X\mu)(\bar{q}_i \gamma^5 q_j), \\ \mathcal{O}_{V,X}^{\mu q_i q_j} &= (\bar{e}\gamma^{\alpha}P_X\mu)(\bar{q}_i \gamma_{\alpha} q_j), \\ \mathcal{O}_{A,X}^{\mu q_i q_j} &= (\bar{e}\gamma^{\alpha}P_X\mu)(\bar{q}_i \gamma_{\alpha} \gamma^5 q_j), \\ \mathcal{O}_{T,X}^{\mu q_i q_j} &= (\bar{e}\sigma^{\alpha\beta}P_X\mu)(\bar{q}_i \sigma_{\alpha\beta} P_X q_j), \end{aligned} \quad (4)$$

where $q_{i,j} \in \{u, d, s, c, b\}$, $P_X = P_{R,L} = \frac{1 \pm \gamma_5}{2}$ and $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^{\mu}, \gamma^{\nu}]$.

In this case, the coefficients ϵ of the operators in Eq. (4) are

$$\begin{aligned} \epsilon_{S,X}^{\mu q_i q_j} &= \frac{1}{2}(C_{S,XR}^{\mu q_i q_j} + C_{S,XL}^{\mu q_i q_j}), & \epsilon_{P,X}^{\mu q_i q_j} &= \frac{1}{2}(C_{S,XR}^{\mu q_i q_j} - C_{S,XL}^{\mu q_i q_j}), \\ \epsilon_{V,X}^{\mu q_i q_j} &= \frac{1}{2}(C_{V,XR}^{\mu q_i q_j} + C_{V,XL}^{\mu q_i q_j}), & \epsilon_{A,X}^{\mu q_i q_j} &= \frac{1}{2}(C_{V,XR}^{\mu q_i q_j} - C_{V,XL}^{\mu q_i q_j}), \\ \epsilon_{T,X}^{\mu q_i q_j} &= C_{T,XX}^{\mu q_i q_j}. \end{aligned} \quad (5)$$

In the next section, we compute the branching ratio for the (semi)leptonic decays as a function of the coefficients of Eq. (5).

III. LEPTONIC AND SEMILEPTONIC PSEUDOSCALAR MESON DECAYS

There are a multitude of bounds on rare meson decays coming from precision experiments [3,28]. The aim of this paper is to use these bounds to constrain the coefficients of Eq. (5). Thus, in this section, we compute the leptonic

and semileptonic pseudoscalar meson decay branching ratio as a function of these coefficients.

A. Leptonic decay branching ratio

We are interested in decays such as $P_1 \rightarrow l_1 \bar{l}_2$ where $\{l_1, l_2\}$ are leptons of mass m_1, m_2 and P_1 is a pseudoscalar meson of mass M ($P_1 \in \{K_L^0(\frac{ds+\bar{s}d}{\sqrt{2}}), D^0(\bar{u}c), B^0(\bar{b}d)\}$). In the presence of new physics, the leptonic decay branching ratio of a pseudoscalar meson P_1 of mass M is written as [26,28,49]

$$\begin{aligned} & \frac{\text{BR}(P_1 \rightarrow l_1 \bar{l}_2)}{C_{2\text{body}}} \\ &= (|\epsilon_{P,L}|^2 + |\epsilon_{P,R}|^2) \tilde{P}'^2 (M^2 - m_1^2 - m_2^2) \\ &+ (|\epsilon_{A,L}|^2 + |\epsilon_{A,R}|^2) \tilde{A}'^2 [(M^2 - m_1^2 - m_2^2)(m_1^2 + m_2^2) \\ &+ 4m_1^2 m_2^2] - 2(\epsilon_{P,L} \epsilon_{A,R} + \epsilon_{P,R} \epsilon_{A,L}) \tilde{P}' \tilde{A}' m_2 (M^2 + m_1^2 - m_2^2) \\ &+ 2(\epsilon_{P,L} \epsilon_{A,L} + \epsilon_{P,R} \epsilon_{A,R}) \tilde{P}' \tilde{A}' m_1 (M^2 + m_2^2 - m_1^2) \\ &- 4\epsilon_{P,L} \epsilon_{P,R} \tilde{P}'^2 m_1 m_2 - 4\epsilon_{A,L} \epsilon_{A,R} \tilde{A}'^2 M^2 m_1 m_2 \end{aligned} \quad (6)$$

where $C_{2\text{body}} = \frac{\tau r^* G_F^2}{\pi M^2}$, $r^* = \frac{1}{2M} \sqrt{(M^2 - (m_1 + m_2)^2)(M^2 - (m_1 - m_2)^2)}$, $m_{1,2}$ are the masses of the leptons and τ is the lifetime of P_1 . For simplicity, we dropped the flavor superscript ($\zeta = l_1 l_2 q_i q_j$) of the coefficients.

The expectation values of the quark current for a pseudoscalar meson are written as [28,49]

$$\tilde{P}' = \frac{1}{2} \langle 0 | \bar{q}_i \gamma^5 q_j | P_1 \rangle = \frac{f_{P_1} M^2}{2(m_i + m_j)}, \quad \tilde{A}' k^\mu = \frac{1}{2} \langle 0 | \bar{q}_i \gamma^\mu \gamma^5 q_j | P_1 \rangle = \frac{f_{P_1} k^\mu}{2} \quad (7)$$

where $m_{i,j}$ are the masses of the quarks, f_{P_1} is the decay constant of the meson and k^μ is the momentum of the meson. These formulas are used for pions, kaons, and D and B mesons. The values of the constants are given in Appendix A. Note that tensor operators do not contribute to the leptonic decay, because the trace of the product of the Dirac matrices contained in the tensor operator vanishes in this case.

B. Semileptonic decay branching ratio

We are interested in decays such as $P_1 \rightarrow l_1 \bar{l}_2 P_2$ where $\{l_1, l_2\}$ are leptons of mass m_1, m_2 and $\{P_1, P_2\}$ are pseudoscalar mesons of mass M, m_3 [$P_1 \in \{K^+(u\bar{s}), D^+(c\bar{d}), B^+(u\bar{b}), B_s^+(s\bar{b})\}$ and $P_2 \in \{\pi^+(u\bar{d}), K^+(u\bar{s})\}$]. The semileptonic decay branching ratio is written as [61]

$$\begin{aligned} & \text{BR}(P_1 \rightarrow l_1 \bar{l}_2 P_2) \\ &= \frac{\tau}{512\pi^3 M^3} \frac{1}{2J+1} \int_{(m_1+m_2)^2}^{(M-m_3)^2} dq^2 \\ &\times \int_{-1}^1 d\cos\theta \frac{|\mathcal{M}|^2 \sqrt{\lambda(M^2, m_3^2, q^2)} \sqrt{\lambda(q^2, m_1^2, m_2^2)}}{q^2} \end{aligned} \quad (8)$$

where $q = (p_1 + p_2)$ is the transferred momentum, θ is the angle between the direction of propagation of the lighter meson (P_2) and the antilepton (l_2) in the lepton's reference frame, τ and J are the lifetime and the spin of P_1 and $|\mathcal{M}|^2$ is the matrix element of the semileptonic decay. The Källén function is defined as $\lambda(x, y, z) = (x - y - z)^2 - 4yz$.

In the presence of new physics, the matrix element in the semileptonic decay branching ratio of Eq. (8) is written as

$$\begin{aligned} \frac{|\mathcal{M}|^2}{8G_F^2} &= 2(|\epsilon_{S,L}|^2 + |\epsilon_{S,R}|^2) \tilde{S}^2(p_1 \cdot p_2) \\ &+ \frac{1}{4} (|\epsilon_{V,L}|^2 + |\epsilon_{V,R}|^2) [f_+^2 (4(p_1 \cdot P)(p_2 \cdot P) - 2P^2(p_1 \cdot p_2)) + f_-^2 (4(p_1 \cdot q)(p_2 \cdot q) - 2q^2(p_1 \cdot p_2)) \\ &+ 4f_+ f_- ((p_1 \cdot q)(p_2 \cdot P) + (p_1 \cdot P)(p_2 \cdot q) - (p_1 \cdot p_2)(P \cdot q))] \\ &+ 4(|\epsilon_{T_R}|^2 + |\epsilon_{T_L}|^2) \tilde{T}'^2 [4(p_1 \cdot q)(p_2 \cdot P)(P \cdot q) + 4(p_1 \cdot P)(p_2 \cdot q)(P \cdot q) - 2(p_1 \cdot p_2)(P \cdot q)^2 \\ &+ 2P^2 q^2(p_1 \cdot p_2) - 4P^2(p_1 \cdot q)(p_2 \cdot q) - 4q^2(p_1 \cdot P)(p_2 \cdot P)] \\ &- 2(\epsilon_{S,L} \epsilon_{V,R} + \epsilon_{S,R} \epsilon_{V,L}) \tilde{S} m_2 [(f_+(p_1 \cdot P) + f_-(p_1 \cdot q))] \\ &+ 2(\epsilon_{S,L} \epsilon_{V,L} + \epsilon_{S,R} \epsilon_{V,R}) \tilde{S} m_1 [(f_+(p_2 \cdot P) + f_-(p_2 \cdot q))] \\ &+ 8(\epsilon_{S,R} \epsilon_{T_R} + \epsilon_{S,L} \epsilon_{T_L}) \tilde{S} \tilde{T}' [(p_1 \cdot P)(p_2 \cdot q) - (p_1 \cdot q)(p_2 \cdot P)] \end{aligned}$$

$$\begin{aligned}
& -4\epsilon_{S,L}\epsilon_{S,R}\tilde{S}^2 m_1 m_2 - \epsilon_{V,L}\epsilon_{V,R} m_1 m_2 [f_-^2 q^2 + f_+^2 P^2 + 2f_+ f_- (P \cdot q)] \\
& + 4(\epsilon_{V,L}\epsilon_{T_R} + \epsilon_{V,R}\epsilon_{T_L})\tilde{T}' m_2 [f_+ ((p_1 \cdot q)p^2 - (P \cdot p_1)(P \cdot q)) + f_- ((p_1 \cdot q)(P \cdot q) - (p_1 \cdot P)q^2)] \\
& + 4(\epsilon_{V,R}\epsilon_{T_R} + \epsilon_{V,L}\epsilon_{T_L})\tilde{T}' m_1 [(f_+ ((P^2)(p_2 \cdot q) - (p_2 \cdot P)(P \cdot q)) + f_- ((p_2 \cdot q)(P \cdot q) - q^2(p_2 \cdot P)))] \\
& + 16\epsilon_{T_R}\epsilon_{T_L}\tilde{T}'^2 m_1 m_2 [(P \cdot q)^2 - P^2 q^2]
\end{aligned} \tag{9}$$

where p_1 , p_2 , k , p_3 are respectively the 4-momenta of leptons 1 and 2, and the 4-momenta of P_1 and P_2 , $P = k + p_3$ and the hadronic matrix elements are written as [28,49–52]

$$\begin{aligned}
\tilde{V}^\mu &= \frac{1}{2} \langle P_2 | \bar{q}_i \gamma^\mu q_j | P_1 \rangle = \frac{1}{2} (P^\mu f_+^{P_1 P_2}(q^2) + q^\mu f_-^{P_1 P_2}(q^2)), \\
\tilde{S} &= \frac{1}{2} \langle P_2 | \bar{q}_i q_j | P_1 \rangle = \frac{1}{2} \frac{(M^2 - m_3^2)}{(m_{q_i} - m_{q_j})} f_0^{P_1 P_2}(q^2), \\
\tilde{T}^{\mu\nu} &= \frac{1}{2} \langle P_2 | \bar{q}_i \sigma^{\mu\nu} q_j | P_1 \rangle \\
&= -\frac{i}{2} \frac{(f_+^{P_1 P_2}(q^2) - f_-^{P_1 P_2}(q^2))}{M^*} (P^\mu q^\nu - P^\nu q^\mu), \\
\tilde{T}' &= \frac{1}{2} \frac{(f_+^{P_1 P_2}(q^2) - f_-^{P_1 P_2}(q^2))}{M^*}.
\end{aligned} \tag{10}$$

For simplicity, we suppressed the q^2 dependence of the form factors $f_{+,-,0}$ in Eq. (9), and the flavor superscript ($\zeta = l_1 l_2 q_i q_j$) of the coefficients. Notice that there is no interference between $\epsilon_{S,L}$ ($\epsilon_{S,R}$) and ϵ_{T_R} (ϵ_{T_L}) because the trace of the product of Dirac matrices involved in tensor and scalar operators of different chirality vanishes. The form factors and the scalar product in Eq. (9) are given in Appendix B.

For simplicity, we do not give the analytic expression of the integrated semileptonic decay branching ratio, but only perform the integrals numerically.

IV. COVARIANCE MATRIX

In this section, we use the branching ratios (BRs) of Eqs. (6) and (8) to compute a covariance matrix, that will

give constraints on the coefficients that account for possible interferences. We note that BR_2^{exp} [BR_3^{exp}] is the experimental upper limit on the leptonic decay $P_1 \rightarrow \bar{l}_1 l_2$ [semileptonic decay $P_1 \rightarrow P_2 \bar{l}_1 l_2$] branching ratio and M_2 [M_3] is the associated covariance matrix.

We can write the decay branching ratio of Eqs. (6) and (8) in the form

$$\vec{\epsilon}^T M^{-1} \vec{\epsilon} = 1 \tag{11}$$

where $\vec{\epsilon}^T$ ($\vec{\epsilon}$) is a row (column) vector of coefficients, and M^{-1} is the inverse of the covariance matrix.

The explicit forms of the 4×4 and 6×6 matrices are given in Appendix D. The diagonal elements of the covariance matrix M represents the squared bounds on our coefficients, and the off-diagonal elements represent the correlations between coefficients.

A. Bounds on the coefficients

In this section, we give constraints on the coefficients for the kaon, D and B meson leptonic and semileptonic decays. As explained in Sec. III, tensor operators do not contribute to the leptonic decays of mesons. Thus, the available upper limits on leptonic [semileptonic] pseudoscalar meson branching ratios will give constrains on the $\epsilon_{P,X}$ and $\epsilon_{A,X}$ [$\epsilon_{S,X}$, $\epsilon_{V,X}$ and $\epsilon_{T,X}$] coefficients. Indeed, hadronic matrix elements with scalar, vector or tensor quark current structure vanish in the leptonic case, while hadronic matrix elements with pseudoscalar or axial structure vanish in the semileptonic case. We consider the CLFV decays with the associated experimental upper limits given in Table I [3].

TABLE I. Experimental bounds on leptonic and semileptonic decays.

Decay	Leptonic	Semileptonic
K	$BR_2^{\text{exp}}(K_L^0 \rightarrow \mu^\pm e^\mp) < 4.7 \times 10^{-12}$ [16]	$BR_3^{\text{exp}}(K^+ \rightarrow \pi^+ \bar{\mu} e) < 1.3 \times 10^{-11}$
	...	$BR_3^{\text{exp}}(K^+ \rightarrow \pi^+ \bar{e} \mu) < 5.2 \times 10^{-10}$ [19]
D	$BR_2^{\text{exp}}(D^0 \rightarrow \mu^\pm e^\mp) < 1.3 \times 10^{-8}$ [17]	$BR_3^{\text{exp}}(D^+ \rightarrow \pi^+ \bar{\mu} e) < 3.6 \times 10^{-6}$
	...	$BR_3^{\text{exp}}(D^+ \rightarrow \pi^+ \bar{e} \mu) < 2.9 \times 10^{-6}$ [20]
D_s	...	$BR_3^{\text{exp}}(D_S^+ \rightarrow K^+ \bar{\mu} e) < 9.7 \times 10^{-6}$
	...	$BR_3^{\text{exp}}(D_S^+ \rightarrow K^+ \bar{e} \mu) < 1.4 \times 10^{-5}$ [20]
B	$BR_2^{\text{exp}}(B^0 \rightarrow \mu^\pm e^\mp) < 2.8 \times 10^{-9}$ [18]	$BR_3^{\text{exp}}(B^+ \rightarrow \pi^+ \mu^\pm e^\mp) < 1.7 \times 10^{-7}$ [21]
	...	$BR_3^{\text{exp}}(B^+ \rightarrow K^+ \mu^\pm e^\mp) < 9.1 \times 10^{-8}$ [22]
B_s	$BR_2^{\text{exp}}(B_S^0 \rightarrow \mu^\pm e^\mp) < 1.1 \times 10^{-8}$ [18]	–

The bounds in Table I will be used to constrain the coefficients at Λ_{exp} and at Λ_W after the RGE evolution of the coefficients (see Sec. V). The covariance matrices at Λ_{exp} for the (semi)leptonic meson decays are given in Appendix E, and the bounds on coefficients are summarized in Tables II–IV.

TABLE II. Constraints on the dimensionless four-fermion coefficients $\epsilon_{P,X}^{l_1 l_2 q_i q_j}$ and $\epsilon_{S,X}^{l_1 l_2 q_i q_j}$ at the experimental (Λ_{exp} for K and D meson decays and Λ_{m_b} for B meson decays) and weak (Λ_W) scale after the RGE evolution. The last two columns are the sensitivities, or SO-at-a-time bounds; see Sec. VD. All bounds apply under permutations of the lepton and/or quark indices.

$\epsilon_{P,X}^{l_1 l_2 q_i q_j}$	Λ_{exp}	Λ_W	SO, Λ_{exp}	SO, Λ_W
$\epsilon_{P,X}^{euds}$	2.32×10^{-7}	4.06×10^{-7}	1.28×10^{-8}	7.82×10^{-9}
$\epsilon_{P,X}^{eucu}$	1.75×10^{-3}	1.08×10^{-3}	7.92×10^{-5}	4.84×10^{-5}
$\epsilon_{P,X}^{eubd}$	2.35×10^{-4}	1.66×10^{-4}	5.13×10^{-6}	3.61×10^{-6}
$\epsilon_{P,X}^{eubs}$	1.75×10^{-4}	1.23×10^{-4}	8.27×10^{-6}	5.83×10^{-6}

$\epsilon_{S,X}^{l_1 l_2 q_i q_j}$	Λ_{exp}	Λ_W	SO, Λ_{exp}	SO, Λ_W
$\epsilon_{S,X}^{euds}$	1.05×10^{-6}	5.68×10^{-7}	7.67×10^{-7}	4.68×10^{-7}
$\epsilon_{S,X}^{eucu}$	1.34×10^{-3}	8.25×10^{-4}	1.33×10^{-3}	8.1×10^{-4}
$\epsilon_{S,X}^{eubd}$	1.44×10^{-5}	1.01×10^{-5}	1.44×10^{-5}	1.01×10^{-5}
$\epsilon_{S,X}^{eubs}$	2.25×10^{-5}	1.59×10^{-5}	2.24×10^{-5}	1.58×10^{-5}

TABLE III. Constraints on the dimensionless four-fermion coefficients $\epsilon_{A,X}^{l_1 l_2 q_i q_j}$ and $\epsilon_{V,X}^{l_1 l_2 q_i q_j}$ at the experimental (Λ_{exp} for K and D meson decays and Λ_{m_b} for B meson decays) and weak (Λ_W) scale after the RGE evolution. The last two columns are the sensitivities, SO-at-a-time bounds; see Sec. VD. All bounds apply under permutations of the lepton and/or quark indices.

$\epsilon_{A,X}^{l_1 l_2 q_i q_j}$	Λ_{exp}	Λ_W	SO, Λ_{exp}	SO, Λ_W
$\epsilon_{A,X}^{euds}$	5.45×10^{-6}	5.45×10^{-6}	3.01×10^{-7}	3.01×10^{-7}
$\epsilon_{A,X}^{eucu}$	4.51×10^{-2}	4.52×10^{-2}	2.04×10^{-3}	2.04×10^{-3}
$\epsilon_{A,X}^{eubd}$	1.48×10^{-2}	1.48×10^{-2}	3.23×10^{-4}	3.23×10^{-4}
$\epsilon_{A,X}^{eubs}$	1.11×10^{-2}	1.11×10^{-2}	5.27×10^{-4}	5.27×10^{-4}

$\epsilon_{V,X}^{l_1 l_2 q_i q_j}$	Λ_{exp}	Λ_W	SO, Λ_{exp}	SO, Λ_W
$\epsilon_{V,X}^{euds}$	4.94×10^{-6}	4.94×10^{-6}	2.93×10^{-6}	2.93×10^{-6}
$\epsilon_{V,X}^{eucu}$	1.45×10^{-3}	1.64×10^{-3}	1.39×10^{-3}	1.39×10^{-3}
$\epsilon_{V,X}^{eubd}$	1.49×10^{-5}	1.03×10^{-4}	1.48×10^{-5}	1.48×10^{-5}
$\epsilon_{V,X}^{eubs}$	2.56×10^{-5}	8.05×10^{-5}	2.54×10^{-5}	2.54×10^{-5}

TABLE IV. Constraints on the dimensionless four-fermion coefficients $\epsilon_{T_X}^{l_1 l_2 q_i q_j}$ at the experimental (Λ_{exp} for K and D meson decays and Λ_{m_b} for B meson decays) and weak (Λ_W) scale after the RGE evolution. The last two columns are the sensitivities, or SO-at-a-time bounds; see Sec. VD. All bounds apply under permutations of the lepton and/or quark indices.

$\epsilon_{T_X}^{l_1 l_2 q_i q_j}$	Λ_{exp}	Λ_W	SO, Λ_{exp}	SO, Λ_W
$\epsilon_{T_X}^{euds}$	1.23×10^{-5}	1.45×10^{-5}	8.76×10^{-6}	1.03×10^{-5}
$\epsilon_{T_X}^{eucu}$	2.01×10^{-3}	2.37×10^{-3}	1.93×10^{-3}	2.28×10^{-3}
$\epsilon_{T_X}^{eubd}$	2.01×10^{-5}	2.26×10^{-5}	2×10^{-5}	2.25×10^{-5}
$\epsilon_{T_X}^{eubs}$	3.89×10^{-5}	4.37×10^{-5}	3.87×10^{-5}	4.35×10^{-5}

V. RENORMALIZATION GROUP EQUATIONS

In this section, we review the evolution of operator coefficients from the experimental scale ($\Lambda_{\text{exp}} \sim 2$ GeV) up to the weak scale ($\Lambda_W \sim 80$ GeV) via the one-loop RGEs of QED and QCD [46,47]. We only consider the QED \times QCD-invariant operators of Eq. (4). The matching onto the SMEFT basis [62] and the running above m_W [63] will be studied at a later date.

A. Anomalous dimensions for meson decays

Figure 1 illustrates some of the one-loop diagrams that renormalize our operators below the weak scale. Operator mixing is induced by photon loops, whereas the QCD corrections only rescale the S , P and T operator coefficients. After including one-loop corrections in the $\overline{\text{MS}}$ scheme, the operator coefficients will run with scale μ according to [46]

$$\mu \frac{\partial}{\partial \mu} \vec{\epsilon} = \frac{\alpha_e}{4\pi} \vec{\epsilon} \Gamma^e + \frac{\alpha_s}{4\pi} \vec{\epsilon} \Gamma^s \quad (12)$$

where Γ^e and Γ^s are the QED and QCD anomalous dimension matrices and $\vec{\epsilon}$ is a row vector that contains the operator coefficients of Eq. (5). In this work, we use the approximate analytic solution [64] of Eq. (12) to compute the running and mixing of the coefficients between Λ_{exp} and Λ_W :

$$\epsilon_I(\Lambda_{\text{exp}}) = \epsilon_J(\Lambda_W) \lambda^{a_J} \left(\delta_{JI} - \frac{\alpha_e \tilde{\Gamma}_{JI}^e}{4\pi} \log \frac{\Lambda_W}{\Lambda_{\text{exp}}} \right) \quad (13)$$

where I and J represent the super- and subscripts which label operator coefficients, λ encodes the QCD corrections, and $\tilde{\Gamma}_{JI}^e$ is the ‘‘QCD-corrected’’ one-loop, anomalous dimension matrix for QED [65,66]. The elements of $\tilde{\Gamma}_{JI}^e$ are defined as

$$\tilde{\Gamma}_{JI}^e = \Gamma_{JI}^e f_{JI}, \quad f_{JI} = \frac{1}{1 + a_J - a_I} \frac{\lambda^{a_I - a_J} - \lambda}{1 - \lambda},$$

$$\Gamma^e = \begin{bmatrix} \Gamma_{\text{SPT}} & 0 \\ 0 & \Gamma_{VA} \end{bmatrix} \quad (14)$$

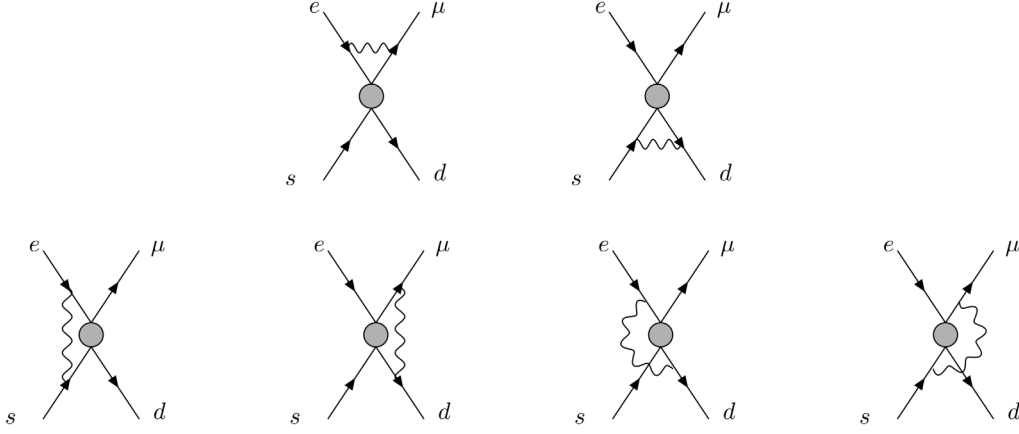


FIG. 1. Examples of one-loop gauge vertex corrections to four-fermion operators. The wave-function renormalization diagrams are missing.

where there is no sum on I, J , $\lambda = \frac{\alpha_s(\Lambda_W)}{\alpha_s(\Lambda_{\text{exp}})}$, and $a_J = \frac{\Gamma_J^*}{2\beta_0} = \{-\frac{12}{23}, -\frac{12}{23}, \frac{4}{23}\}$ for $J \in \{S, P, T\}$. The QED anomalous dimensions are

$$\Gamma_{\text{SPT}} = \begin{bmatrix} l_1 l_2 q_i q_j & 0 & l_1 l_2 q_i q_j \\ \gamma_{PP} & 0 & \gamma_{PT} \\ 0 & \gamma_{SS} & \gamma_{ST} \\ l_1 l_2 q_i q_j & \gamma_{TS} & l_1 l_2 q_i q_j \\ \gamma_{TP} & \gamma_{TS} & \gamma_{TT} \end{bmatrix},$$

$$\Gamma_{\text{VA}} = \begin{bmatrix} l_1 l_2 q_i q_j & l_1 l_2 q_i q_j \\ \gamma_{AA} & \gamma_{AV} \\ l_1 l_2 q_i q_j & l_1 l_2 q_i q_j \\ \gamma_{VA} & \gamma_{VV} \end{bmatrix} \quad (15)$$

where the matrix elements in Γ_{SPT} and Γ_{VA} are defined in Sec. V.

Combining the first and second diagrams of Fig. 1 with the wave function diagrams renormalize the scalars and pseudoscalars, while the last four diagrams mix the tensors to the scalars and pseudoscalars:

$$\gamma_{SS}^{q,q} = \begin{array}{c|cc} & \epsilon_{S,L}^{qq} & \epsilon_{S,R}^{qq} \\ \hline \epsilon_{S,L}^{qq} & -6(1+Q_q^2) & 0 \\ \epsilon_{S,R}^{qq} & 0 & -6(1+Q_q^2) \end{array}$$

$$\gamma_{TS}^{q,q} = \begin{array}{c|cc} & \epsilon_{S,L}^{qq} & \epsilon_{S,R}^{qq} \\ \hline \epsilon_{T,L}^{qq} & 48Q_q & 0 \\ \epsilon_{T,R}^{qq} & 0 & 48Q_q \end{array} \quad (16)$$

$$\gamma_{PP}^{q,q} = \begin{array}{c|cc} & \epsilon_{P,L}^{qq} & \epsilon_{P,R}^{qq} \\ \hline \epsilon_{P,L}^{qq} & -6(1+Q_q^2) & 0 \\ \epsilon_{P,R}^{qq} & 0 & -6(1+Q_q^2) \end{array}$$

$$\gamma_{TP}^{q,q} = \begin{array}{c|cc} & \epsilon_{P,L}^{qq} & \epsilon_{P,R}^{qq} \\ \hline \epsilon_{T,L}^{qq} & -48Q_q & 0 \\ \epsilon_{T,R}^{qq} & 0 & 48Q_q \end{array} \quad (17)$$

Similarly, the last four diagrams mix the (pseudo)scalars into the tensors. Only the wave-function diagrams renormalize the tensors, because for the first and second diagrams $\gamma^\mu \sigma \gamma_\mu = 0$. We obtain

$$\gamma_{TT}^{q,q} = \begin{array}{c|cc} & \epsilon_{T,L}^{qq} & \epsilon_{T,R}^{qq} \\ \hline \epsilon_{T,L}^{qq} & 2(1+Q_q^2) & 0 \\ \epsilon_{T,R}^{qq} & 0 & 2(1+Q_q^2) \end{array}$$

$$\gamma_{S(P)T}^{q,q} = \begin{array}{c|cc} & \epsilon_{T,L}^{qq} & \epsilon_{T,R}^{qq} \\ \hline \epsilon_{S(P),L}^{qq} & (-)2Q_q & 0 \\ \epsilon_{S(P),R}^{qq} & 0 & 2Q_q \end{array} \quad (18)$$

Finally, for the vectors and axial vectors, there is no running, but the last four diagrams contribute to the mixing of vector and axial coefficients

$$\gamma_{AV}^{q,q} = \begin{array}{c|cc} & \epsilon_{V,L}^{qq} & \epsilon_{V,R}^{qq} \\ \hline \epsilon_{A,L}^{qq} & 12Q_q & 0 \\ \epsilon_{A,R}^{qq} & 0 & -12Q_q \end{array}$$

$$\gamma_{VA}^{q,q} = \begin{array}{c|cc} & \epsilon_{A,L}^{qq} & \epsilon_{A,R}^{qq} \\ \hline \epsilon_{V,L}^{qq} & 12Q_q & 0 \\ \epsilon_{V,R}^{qq} & 0 & -12Q_q \end{array} \quad (19)$$

B. RGEs of operator coefficients

In this section we compute the evolution of the bounds from Λ_{exp} to Λ_W . In the previous section, we found a mixing between pseudoscalar and tensor coefficients, and between vector and axial coefficients. Thus, the coefficients that contributed only to the leptonic (semileptonic) decays at Λ_{exp} will also contribute to the semileptonic (leptonic) decays at Λ_W via the mixing.

The matrices describing the evolution of the coefficients from Λ_{exp} to Λ_W for all the decays were obtained with Eq. (13) and are given in Appendix C.

C. Evolution of the bounds

In order to constrain the coefficients at Λ_W , the constraints needs to be expressed in terms of coefficients at Λ_W . However, the mixing of the pseudoscalar (axial) into the tensor (vector), and vice versa, implies that leptonic and semileptonic branching ratios can both depend on any of the ten coefficients, which we arrange in a vector as $\vec{\epsilon}' = (\epsilon_{P,L}, \epsilon_{A,L}, \epsilon_{P,R}, \epsilon_{A,R}, \epsilon_{S,L}, \epsilon_{V,L}, \epsilon_{T,L}, \epsilon_{S,R}, \epsilon_{V,R}, \epsilon_{T,R})_{\Lambda_W}$. The 10×10 matrix we need to invert to compute the bounds at Λ_W is now written as

$$(M')^{-1} = \mathcal{R}^T \begin{pmatrix} M_2^{-1} & 0_{4 \times 6} \\ 0_{6 \times 4} & M_3^{-1} \end{pmatrix} \mathcal{R} \quad (20)$$

where M_2^{-1} and M_3^{-1} are the 4×4 and 6×6 matrices defined in Appendix D that we inverted to obtain the bounds at Λ_{exp} (see Sec. IV) and \mathcal{R} has the form of the matrices defined in Eqs. (C1), (C2) and (C3). Finally, Eq. (11) is written in the new basis as

$$\vec{\epsilon}'^T (M')^{-1} \vec{\epsilon}' = 1 \quad (21)$$

where $\vec{\epsilon}'$ is the vector of coefficients at Λ_W , $(M')^{-1}$ is the matrix in Eq. (20) and the superscript T means matrix transposition. All the covariance matrices at Λ_W can be found in Appendix E. In Tables II–IV we summarize all the bounds on the coefficients at Λ_{exp} and Λ_W .

In the leptonic decays, the evolution of the bounds on the pseudoscalar coefficients between Λ_{exp} and Λ_W is the most important effect of the RGEs as shown in the first two columns of the left panel of Table II. As can be seen in Eqs. (C1), (C2) or (C3), the running of the (pseudo)scalar coefficients is $\sim 1.6(1.4)$, which means that if we neglect the mixing of the tensor into (pseudo)scalar coefficients, the bounds on ϵ_S and ϵ_P will be better at Λ_W for all the decays we considered. However, the large mixing of the tensor coefficients into the (pseudo)scalar ones [see Eqs. (16), (17) and (C1)–(C3)] weaken the bounds on pseudoscalar coefficients at Λ_W for the kaon decay. This is due to the fact that the bounds on ϵ_T^{euds} (see the first two columns of Table IV) are much weaker than the bounds on ϵ_P^{euds} at Λ_{exp} (see the first two columns of the left panel of Table II). Thus, the mixing of ϵ_T into ϵ_P will lead to weaker bounds on ϵ_P at Λ_W for the kaon decay.

For the D , B and B_s meson decays, the bounds on ϵ_P are a bit closer to the bound on ϵ_T at Λ_{exp} . Even with the large mixing of the tensor into the pseudoscalar coefficients, the bounds on $\epsilon_P^{e\mu cu}$, $\epsilon_P^{e\mu bd}$ and $\epsilon_P^{e\mu bs}$ will be slightly better at Λ_W because the running will be larger than the mixing.

In the semileptonic decays, there is also a mixing between scalar and tensor coefficients, but the bounds on scalar coefficients at Λ_W increase a bit because, similarly to $\epsilon_P^{e\mu cu}$, $\epsilon_P^{e\mu bd}$ and $\epsilon_P^{e\mu bs}$, the bounds on all the scalar coefficients (first two columns of the right panel of Table II) are close to the bounds on the tensor coefficients at Λ_{exp} . The running of the scalars will be stronger than the mixing of the tensors into the scalars, and thus, the bounds on ϵ_S are better at Λ_W for all the decays.

For the axial and vector coefficients, there is no running and the mixing is small. The bounds on ϵ_A^{euds} and ϵ_V^{euds} at Λ_{exp} are very close (see Table III); this explains why there is no evolution of these bounds at Λ_W . However, for the D , B and B_s decays, the bounds on ϵ_A are much weaker than the bounds on ϵ_V at Λ_{exp} , especially for the B and B_s decays. Thus, the bounds on $\epsilon_A^{e\mu cu}$, $\epsilon_A^{e\mu bd}$ and $\epsilon_A^{e\mu bs}$ do not evolve significantly at Λ_W , but the mixing of the axial into vector coefficients will lead to weaker bounds on $\epsilon_V^{e\mu cu}$, $\epsilon_V^{e\mu bd}$ and $\epsilon_V^{e\mu bs}$ at Λ_W as shown in the first two columns of the two panels of Table III.

Finally, the running of tensor coefficients is tiny, and the mixing of the (pseudo)scalar coefficients into the tensor ones is small. Thus, the evolution of the bounds is small for the tensor coefficients (first two columns of Table IV) similarly to the bounds on vector and axial coefficients in the kaon decay (first two columns of Table III). Finally, the matching at Λ_W along with the evolution of the bounds between Λ_W and Λ_{NP} will be given in a future publication [67].

D. Single operator approximation

We also computed the sensitivities of the various decays to the coefficients at Λ_{exp} , and these are given in the third columns of Tables II to IV. The sensitivity is the value of the coefficient below which it could not have been observed, and is calculated as a “single operator” (SO)-at-a-time bound, that is by allowing only one nonzero coefficient at a time in the branching ratio [see Eqs. (6) and (9)]. This is different from setting bounds on coefficients (first two columns of Tables II to IV), which are obtained with all coefficients nonzero, and exclude the parameter space outside the allowed range. It is clear that the sensitivities are sometimes an excellent approximation to the bounds, and sometimes differ by orders of magnitude.

To compute the evolution of the sensitivities of the decays to the coefficients at Λ_W (given in the last columns of Tables II–IV), we still kept only one nonzero coefficient at Λ_{exp} and considered only the running of the coefficients [the diagonal terms in Eqs. (C1)–(C3)]. For example, computing the sensitivity of the leptonic kaon decay to a pseudoscalar coefficient at Λ_W in the SO approximation requires multiplying the first term in Eq. (D3) by the first (or third) diagonal term squared in Eq. (C1). Then,

inverting the product and taking the square root will give the sensitivity of the decay to the coefficient at Λ_W .

E. Updating the bounds

In future years, the experimental data on LFV meson decays could improve, so in this section, we consider how to update our bounds, without inverting large matrices.

The bounds on coefficients at Λ_{exp} obtained in this work are of the form $|\epsilon| < \sqrt{\text{BR}^{\text{exp}}} \times \text{constant}$. Thus, all the bounds at Λ_{exp} given in Tables II–IV can be updated by rescaling by $\sqrt{(\text{BR}_{\text{new}}^{\text{exp}})/(\text{BR}_{\text{old}}^{\text{exp}})}$ when the data improves. However, in principle, the 10×10 matrix of Eq. (20) must then be inverted to obtain the bounds at Λ_W . So we now describe approximations that allow to obtain the bounds at Λ_W with manageable matrices.

For the semileptonic decay, the bounds at Λ_{exp} can be obtained by neglecting all the interference terms between the scalar, vector and tensor coefficients of either chirality [see Eq. (9)]. The 6×6 matrix in Eq. (D2) then becomes diagonal and easy to invert. This approximation will give bounds at Λ_{exp} on $\epsilon_{S,X}$, $\epsilon_{V,X}$ and $\epsilon_{T,X}$ close to those obtained in the first columns of Tables II–IV (which include the interference terms).

In the leptonic decay [Eq. (6)], a reasonable approximation for the bounds at Λ_{exp} is to keep the interference between axial and pseudoscalar coefficients of opposite chirality [with $m_2 = m_\mu$ in Eq. (6)]. The other interference terms, proportional to $m_1 = m_e$, can be neglected. Thus, bounds on ϵ_A and ϵ_P at Λ_{exp} , which are a reasonable approximation to the first columns of Tables II and III, can be obtained by inverting a 2×2 matrix in the basis $(\epsilon_{P,X}, \epsilon_{A,Y})$ where $X \in L, R$ and $Y \in R, L$, instead of the 4×4 matrix in Eq. (D1).

To obtain bounds at Λ_W , it is necessary to keep the mixing between ϵ_S , ϵ_P , ϵ_T , and between ϵ_V and ϵ_A . Then, the bounds on ϵ_S , ϵ_P , ϵ_T , ϵ_V and ϵ_A at Λ_W can be obtained by considering M^{-1} in Eq. (20) as a product of 5×5 matrices in the basis $(\epsilon_{P,X}, \epsilon_{S,X}, \epsilon_{T,X}, \epsilon_{V,Y}, \epsilon_{A,Y})$ where X and Y are the chiralities. However, ϵ_S , ϵ_P and ϵ_T must have the same chirality, but different from the chirality of ϵ_V and ϵ_A in order to take into account the mixing induced by the RGEs, that occurs only for coefficients of the same chirality [see Eqs. (13) and (C1)–(C3)]. This is due to the fact that it is necessary to keep the interference between axial and pseudoscalar coefficients of different chiralities to compute the bounds on $\epsilon_{P,X}$ and $\epsilon_{A,Y}$.

VI. CONCLUSION

In this paper, we considered operators which simultaneously change lepton and quark flavor, and obtain constraints on the coefficients using available data on (semi)leptonic pseudoscalar meson decays. Section II listed the dimension-six, two lepton–two quark operators and their associated coefficients at the experimental scale

Λ_{exp} . Scalar, pseudoscalar, vector, axial and tensor operators were included. The leptonic and semileptonic branching ratios of pseudoscalar mesons, as a function of the operator coefficients, were given in Sec. III. We found that tensor operators do not contribute to the leptonic decays but only to the semileptonic decays, in which the interference between $\epsilon_{S,L}$ ($\epsilon_{S,R}$) and ϵ_{T_R} (ϵ_{T_L}) vanishes. The constraints on operator coefficients, evaluated at the experimental scale, are given in Tables II–IV and discussed in Sec. IV. The bounds are obtained via the appropriate covariance matrices, which allows to take into account the interferences among operators [see Eqs. (6), (9), (D1) and (D2)]. The matrices are given in Appendix B. Section V gave the renormalization group evolution of the coefficients from the experimental to the weak scale Λ_W , and the formalism used to compute the covariances matrices at Λ_W . The weak-scale constraints on the coefficients are given in Tables II–IV. The large mixing of tensor coefficients into (pseudo)scalar coefficients has important consequences on the evolution of the bounds on scalar and pseudoscalar coefficients. Indeed, in the case of the kaon decay, the experimental-scale bounds on tensor coefficients are significantly weaker than those on pseudoscalars. As a result, the pseudoscalar bounds are weaker at Λ_W , compared to the bounds at Λ_{exp} . The bounds on scalar coefficients at Λ_W are slightly stronger than at Λ_{exp} . There is no running for the vector and axial coefficients, due to the fact that we considered quark-flavor-changing operators, and the mixing is small, but the bounds on axial coefficients are much weaker than the bounds on vector coefficients for the D , B and B_s decays. This leads to much weaker bounds on vector coefficients at Λ_W . Similarly, the running and mixing of tensor coefficients are small. As a result, the bounds on the axial and tensor coefficients do not evolve significantly between the experimental and weak scales.

We conclude by noting the importance of including interferences among operators in computing the bounds on their coefficients. As shown in Sec. VD, the sensitivities of the decays to ϵ_P and ϵ_A obtained at Λ_{exp} and to ϵ_P , ϵ_A and ϵ_V at Λ_W in the single-operator approximation are better by several orders of magnitude compared to the bounds obtained by keeping the interferences among operators. We found that the renormalization group running between the experimental and weak scales has an important effect on the evolution of the bounds, especially the large mixing of the tensor (axial) into the pseudoscalar (vector), which lead to weaker bounds on pseudoscalar (vector) coefficients at Λ_W for the kaon (D , B and B_s) decay.

APPENDIX A: CONSTANTS

In this appendix, we give all the constants used in our calculations:

P_1	K_L^0	K^+	D^0	D^+	D_S^+	B^0	B_S^0	B^+
f_{P_1} (MeV)	155.72 [68,69]	155.6 [68,69]	211.5 [68,70]	212.6 [68,70]	249.8 [70]	190.9 [68]	230.7 [70]	187.1 [68]
$f_+^{P_1\pi}(0)$	0.966 [69]	0.966 [69]	0.666 [69]	0.666 [69]	0.666 [69]	0.25 [71]	0.25 [71]	0.25 [71]
$f_+^{P_1K}(0)$	0.747 [69]	0.747 [69]	0.747 [69]	0.31 [71]	0.31 [71]	0.31 [71]
λ_+	2.82×10^{-2} [3]	2.97×10^{-2} [3]
λ_0	1.8×10^{-2} [3]	1.95×10^{-2} [3]

All the masses and lifetimes can be found in Ref. [3].

APPENDIX B: KINEMATICS AND FORM FACTORS FOR SEMILEPTONIC DECAYS

In this appendix, we give the form factor and the detailed scalar product of Eq. (9).

The q^2 dependence of the form factors for the kaon is given by [50]

$$f_{+,0}^{K\pi}(q^2) = f_+^{K\pi}(0) \left(1 + \lambda_{+,0} \frac{q^2}{M_\pi^2} \right), \quad f_-^{K\pi}(q^2) = f_+^{K\pi}(0) (\lambda_0 - \lambda_+) \frac{M_{K^+}^2 - M_{\pi^+}^2}{M_{\pi^+}^2} \quad (\text{B1})$$

and for the D and B mesons they are given by [51,52]

$$f_+(q^2) = \frac{f_+(0)}{1 - q^2/m_{J^P}^2}, \quad f_0(q^2) = \frac{f_0(0)}{1 - q^2/m_{0^+}^2}, \quad f_-(q^2) = (f_0(q^2) - f_+(q^2)) \frac{M^2 - m_3^2}{q^2} \quad (\text{B2})$$

where $\lambda_{+,0}$ are constants, and m_{J^P} is the mass of the lightest resonance with the right quantum numbers to mediate the transition (D_S^+ and D_S^{*+} for example). We took $q^2 = q_{\max}^2 = (M - m_3)^2$ to compute the form factors f_+ , f_- and f_0 . All these values can be found in Appendix A.

Finally, the scalar products in Eq. (9) can be written as functions of the two kinematical variables q^2 and $\cos\theta$ [3,61] in the phase space integrals of Eq. (8):

$$p_1 \cdot p_2 = \frac{q^2 - m_1^2 - m_2^2}{2}, \quad p_1 \cdot q = \frac{q^2 + m_1^2 - m_2^2}{2}, \quad p_2 \cdot q = \frac{q^2 + m_2^2 - m_1^2}{2}, \quad (\text{B3})$$

$$p_3 \cdot q = \frac{M^2 - m_3^2 - q^2}{2}, \quad p_1 \cdot p_3 = p_3 \cdot q - p_2 \cdot p_3, \quad p_1 \cdot P = p_1 \cdot q + 2p_1 \cdot p_3, \quad p_2 \cdot P = p_2 \cdot q + 2p_2 \cdot p_3, \quad (\text{B4})$$

$$p_2 \cdot p_3 = \frac{1}{4q^2} (M^2 - m_3^2 - q^2)(q^2 + m_2^2 - m_1^2) + \frac{1}{4q^2} \sqrt{\lambda(M^2, m_3^2, q^2)} \sqrt{\lambda(q^2, m_1^2, m_2^2)} \cos\theta, \quad (\text{B5})$$

$$k \cdot p_3 = \frac{M^2 + m_3^2 - q^2}{2}, \quad P \cdot q = M^2 - m_3^2, \quad P^2 = 2M^2 + 2m_3 - q^2. \quad (\text{B6})$$

APPENDIX C: RGEs

In this appendix, we give the 10×10 matrices obtained with Eq. (13) we used to obtain the bounds at Λ_W [with Eq. (20)].

For the decay of light quarks (kaon and D meson decays), the experimental scale is taken as 2 GeV because most of the time, it is the renormalization scale chosen to obtain the lattice form factors.

The evolution of the coefficients ($e^{e\mu ds}$) involved in the kaon decays is given by

$$\begin{pmatrix} \epsilon_{P,L} \\ \epsilon_{A,L} \\ \epsilon_{P,R} \\ \epsilon_{A,R} \\ \epsilon_{S,L} \\ \epsilon_{V,L} \\ \epsilon_{T_L} \\ \epsilon_{S,R} \\ \epsilon_{V,R} \\ \epsilon_{T_R} \end{pmatrix}_{\Lambda_{\text{exp}}} = \begin{pmatrix} 1.64 & 0 & 0 & 0 & 0 & 0 & -0.0429 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0.00857 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.64 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0429 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -0.00857 & 0 \\ 0 & 0 & 0 & 0 & 1.64 & 0 & 0.0429 & 0 & 0 & 0 \\ 0 & 0.00857 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -0.00162 & 0 & 0 & 0 & 0.00162 & 0 & 0.849 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.64 & 0 & 0.0429 \\ 0 & 0 & 0 & -0.00857 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0.00162 & 0 & 0 & 0 & 0 & 0.00162 & 0 & 0.849 \end{pmatrix} \times \begin{pmatrix} \epsilon_{P,L} \\ \epsilon_{A,L} \\ \epsilon_{P,R} \\ \epsilon_{A,R} \\ \epsilon_{S,L} \\ \epsilon_{V,L} \\ \epsilon_{T_L} \\ \epsilon_{S,R} \\ \epsilon_{V,R} \\ \epsilon_{T_R} \end{pmatrix}_{\Lambda_W}. \quad (\text{C1})$$

For the D meson decays, the evolution of the coefficients ($e^{\mu cu}$) is given by

$$\begin{pmatrix} \epsilon_{P,L} \\ \epsilon_{A,L} \\ \epsilon_{P,R} \\ \epsilon_{A,R} \\ \epsilon_{S,L} \\ \epsilon_{V,L} \\ \epsilon_{T_L} \\ \epsilon_{S,R} \\ \epsilon_{V,R} \\ \epsilon_{T_R} \end{pmatrix}_{\Lambda_{\text{exp}}} = \begin{pmatrix} 1.64 & 0 & 0 & 0 & 0 & 0 & 0.0857 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -0.0171 & 0 & 0 & 0 \\ 0 & 0 & 1.64 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0857 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0.0171 & 0 \\ 0 & 0 & 0 & 0 & 1.64 & 0 & -0.0857 & 0 & 0 & 0 \\ 0 & -0.0171 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0.00325 & 0 & 0 & 0 & -0.00325 & 0 & 0.847 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.64 & 0 & -0.0857 \\ 0 & 0 & 0 & 0.0171 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -0.00325 & 0 & 0 & 0 & 0 & -0.00325 & 0 & 0.847 \end{pmatrix} \times \begin{pmatrix} \epsilon_{P,L} \\ \epsilon_{A,L} \\ \epsilon_{P,R} \\ \epsilon_{A,R} \\ \epsilon_{S,L} \\ \epsilon_{V,L} \\ \epsilon_{T_L} \\ \epsilon_{S,R} \\ \epsilon_{V,R} \\ \epsilon_{T_R} \end{pmatrix}_{\Lambda_W}. \quad (\text{C2})$$

In the B and B_s meson decays, the reference scale is the b quark mass ($\Lambda_{m_b} \sim 4.18$ GeV). Thus, the evolution of the coefficients ($e^{\epsilon^{\mu bd}}$ and $e^{\epsilon^{\mu bs}}$) is slightly smaller.

In fact, in Eq. (13), the part with the anomalous dimension that gives the matrix element in Eq. (C1) is multiplied by a factor $\log(\frac{\Lambda_w}{\Lambda_{m_b}})/\log(\frac{\Lambda_w}{\Lambda_{\text{exp}}}) \sim 0.8$. Moreover, the strong coupling constant at Λ_{m_b} will also be smaller [$\alpha_s(\Lambda_{m_b}) \sim 0.23$ and $\alpha_s(\Lambda_{\text{exp}}) \sim 0.3$]. Thus, for the B and B_s meson decays, the evolution of the coefficients ($e^{\epsilon^{\mu bd}}$ and $e^{\epsilon^{\mu bs}}$) is given by

$$\begin{aligned}
 & \begin{pmatrix} \epsilon_{P,L} \\ \epsilon_{A,L} \\ \epsilon_{P,R} \\ \epsilon_{A,R} \\ \epsilon_{S,L} \\ \epsilon_{V,L} \\ \epsilon_{T_L} \\ \epsilon_{S,R} \\ \epsilon_{V,R} \\ \epsilon_{T_R} \end{pmatrix}_{\Lambda_{\text{exp}}} = \begin{pmatrix} 1.42 & 0 & 0 & 0 & 0 & 0 & -0.0317 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0.00686 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.42 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0317 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -0.00686 & 0 \\ 0 & 0 & 0 & 0 & 1.42 & 0 & 0.0317 & 0 & 0 & 0 \\ 0 & 0.00686 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -0.00126 & 0 & 0 & 0 & 0.00126 & 0 & 0.890 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.42 & 0 & 0.0317 \\ 0 & 0 & 0 & -0.00686 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0.00126 & 0 & 0 & 0 & 0 & 0.00126 & 0 & 0.890 \end{pmatrix} \\
 & \times \begin{pmatrix} \epsilon_{P,L} \\ \epsilon_{A,L} \\ \epsilon_{P,R} \\ \epsilon_{A,R} \\ \epsilon_{S,L} \\ \epsilon_{V,L} \\ \epsilon_{T_L} \\ \epsilon_{S,R} \\ \epsilon_{V,R} \\ \epsilon_{T_R} \end{pmatrix}_{\Lambda_w}. \tag{C3}
 \end{aligned}$$

APPENDIX D: COVARIANCE MATRIX

In this appendix, we give details of the formalism introduced in Eq. (11) of Sec. IV. The matrices in the basis $(\epsilon_{P,L}, \epsilon_{A,L}, \epsilon_{P,R}, \epsilon_{A,R})$ and $(\epsilon_{S,L}, \epsilon_{V,L}, \epsilon_{T_L}, \epsilon_{S,R}, \epsilon_{V,R}, \epsilon_{T_R})$ are written as

$$M_2^{-1} = \frac{1}{\text{BR}_2^{\text{exp}}} \begin{bmatrix} SP'_+ & \frac{1}{2}SP_+VA'_+ & \frac{1}{2}SP_+SP'_- & \frac{1}{2}SP_+VA'_- \\ \frac{1}{2}SP_+VA'_+ & VA'_- & \frac{1}{2}SP_-VA'_+ & \frac{1}{2}VA_+VA'_- \\ \frac{1}{2}SP_+SP'_- & \frac{1}{2}SP_-VA'_+ & SP'_- & \frac{1}{2}SP_-VA'_- \\ \frac{1}{2}SP_+VA'_- & \frac{1}{2}VA_+VA'_- & \frac{1}{2}SP_-VA'_- & VA'_+ \end{bmatrix}, \tag{D1}$$

$$M_3^{-1} = \frac{1}{\text{BR}_3^{\text{exp}}} \begin{bmatrix} SP_+ & \frac{1}{2}SP_+VA_- & \frac{1}{2}SP_+T_+ & \frac{1}{2}SP_+SP_- & \frac{1}{2}SP_+VA_+ & \frac{1}{2}SP_+T_- \\ \frac{1}{2}SP_+VA_- & VA_- & \frac{1}{2}VA_-T_+ & \frac{1}{2}SP_-VA_- & \frac{1}{2}VA_+VA_- & \frac{1}{2}VA_-T_- \\ \frac{1}{2}SP_+T_+ & \frac{1}{2}VA_-T_+ & T_+ & \frac{1}{2}SP_-T_+ & \frac{1}{2}VA_+T_+ & \frac{1}{2}T_+T_- \\ \frac{1}{2}SP_+SP_- & \frac{1}{2}SP_-VA_- & \frac{1}{2}SP_-T_+ & SP_- & \frac{1}{2}SP_-VA_+ & \frac{1}{2}SP_-T_- \\ \frac{1}{2}SP_+VA_+ & \frac{1}{2}VA_+VA_- & \frac{1}{2}VA_+T_+ & \frac{1}{2}SP_-VA_+ & VA_+ & \frac{1}{2}VA_+T_- \\ \frac{1}{2}SP_+T_- & \frac{1}{2}VA_-T_- & \frac{1}{2}T_+T_- & \frac{1}{2}SP_-T_- & \frac{1}{2}VA_+T_- & T_- \end{bmatrix}. \tag{D2}$$

Inverting M_2^{-1} (M_3^{-1}) will give the bounds on the coefficients involved in the leptonic (semileptonic) decays. Finally, note that for semileptonic kaon and D meson decays, the experimental upper limits are not the same for μ^+e^- and μ^-e^+ in the final state. In this case, we sum the M_3^{-1} for each bound and then invert it to obtain the covariance matrix of Sec. IV. The matrix elements of Eq. (D1) are written as

$$\begin{aligned}
SP'_+ &= SP'_- = C_{2\text{body}} \tilde{P}'^2 (P_1^2 - m_i^2 - m_j^2), \\
VA'_- &= VA'_+ = C_{2\text{body}} \tilde{A}'^2 [(P_1^2 - m_i^2 - m_j^2)(m_i^2 + m_j^2) + 4m_i^2 m_j^2], \\
SP_+ VA'_- &= SP_- VA'_+ = -2C_{2\text{body}} \tilde{P}' \tilde{A}' m_j (P_1^2 + m_i^2 - m_j^2), \\
SP_+ VA'_+ &= SP_- VA'_- = 2C_{2\text{body}} \tilde{P}' \tilde{A}' m_i (P_1^2 + m_j^2 - m_i^2), \\
SP_+ SP'_- &= -4C_{2\text{body}} \tilde{P}'^2 m_j m_i, \\
VA_+ VA'_- &= -4C_{2\text{body}} \tilde{A}'^2 P_1^2 m_j m_i, \\
C_{2\text{body}} &= \frac{\tau_{P_1} r^* G_F^2}{\pi P_1^2}.
\end{aligned} \tag{D3}$$

For simplicity we note that $d\phi = \int_{(m_1+m_2)^2}^{(M-m_3)^2} dq^2 \int_{-1}^1 d\cos\theta \frac{\sqrt{\lambda(M^2, m_3^2, q^2)} \sqrt{\lambda(q^2, m_1^2, m_2^2)}}{q^2}$ and the matrix elements of Eq. (D2) are written as

$$\begin{aligned}
SP_+ &= SP_- = 2C_{3\text{body}} \tilde{S}^2 (p_1 \cdot p_2) d\phi, \\
VA_+ &= VA_- = \frac{1}{4} C_{3\text{body}} [f_+^2 (4(p_1 \cdot P)(p_2 \cdot P) - 2P^2(p_1 \cdot p_2)) + f_-^2 (4(p_1 \cdot q)(p_2 \cdot q) - 2q^2(p_1 \cdot p_2)) \\
&\quad + 4f_+ f_- ((p_1 \cdot q)(p_2 \cdot P) + (p_1 \cdot P)(p_2 \cdot q) - (p_1 \cdot p_2)(P \cdot q))] d\phi, \\
T_+ &= T_- = 4C_{3\text{body}} \tilde{T}'^2 [4(p_1 \cdot q)(p_2 \cdot P)(P \cdot q) + 4(p_1 \cdot P)(p_2 \cdot q)(P \cdot q) - 2(p_1 \cdot p_2)(P \cdot q)^2 \\
&\quad + 2P^2 q^2 (p_1 \cdot p_2) - 4P^2 (p_1 \cdot q)(p_2 \cdot q) - 4q^2 (p_1 \cdot P)(p_2 \cdot P)] d\phi, \\
SP_+ VA_- &= SP_- VA_+ = -2C_{3\text{body}} \tilde{S} m_2 [(f_+(p_1 \cdot P) + f_-(p_1 \cdot q))] d\phi, \\
SP_+ VA_+ &= SP_- VA_- = 2C_{3\text{body}} \tilde{S} m_1 [(f_+(p_2 \cdot P) + f_-(p_2 \cdot q))] d\phi, \\
SP_+ SP_- &= -4C_{3\text{body}} \tilde{S}^2 m_1 m_2 d\phi, \\
VA_+ VA_- &= -C_{3\text{body}} m_1 m_2 [f_-^2 q^2 + f_+^2 P^2 + 2f_+ f_- (P \cdot q)] d\phi, \\
T_+ T_- &= 16C_{3\text{body}} \tilde{T}'^2 m_1 m_2 [(P \cdot q)^2 - P^2 q^2] d\phi, \\
SP_+ T_+ &= SP_- T_- = 8C_{3\text{body}} \tilde{S} \tilde{T}' [(p_1 \cdot P)(p_2 \cdot q) - (p_1 \cdot q)(p_2 \cdot P)] d\phi, \\
SP_+ T_- &= SP_- T_+ = 0, \\
VA_+ T_- &= VA_- T_+ = 4C_{3\text{body}} \tilde{T}' m_2 [f_+ ((p_1 \cdot q)p^2 - (P \cdot p_1)(P \cdot q)) + f_- ((p_1 \cdot q)(P \cdot q) - (p_1 \cdot P)q^2)] d\phi, \\
VA_+ T_+ &= VA_- T_- = 4C_{3\text{body}} \tilde{T}' m_1 [f_+ ((P^2)(p_2 \cdot q) - (p_2 \cdot P)(P \cdot q)) + f_- ((p_2 \cdot q)(P \cdot q) - (q^2)(p_2 \cdot P))] d\phi, \\
C_{3\text{body}} &= \frac{\tau_{P_1} 8G_F^2}{\pi^3 512M^3}.
\end{aligned} \tag{D4}$$

APPENDIX E: COVARIANCE MATRICES AT Λ_{exp} AND Λ_W

In this appendix, we give the covariance matrices at Λ_{exp} and Λ_W , after the RGE evolution.

1. Kaon decays

Using the upper limit of Table I, for the leptonic kaon decays, we compute the associated covariance matrix in the basis $(\epsilon_{P,L}^{e\mu ds}, \epsilon_{A,L}^{e\mu ds}, \epsilon_{P,R}^{e\mu ds}, \epsilon_{A,R}^{e\mu ds})$:

$$\begin{pmatrix} 5.38 \times 10^{-14} & -2.33 \times 10^{-14} & -1.25 \times 10^{-15} & 1.26 \times 10^{-12} \\ -2.33 \times 10^{-14} & 2.97 \times 10^{-11} & 1.26 \times 10^{-12} & -4.03 \times 10^{-13} \\ -1.25 \times 10^{-15} & 1.26 \times 10^{-12} & 5.38 \times 10^{-14} & -2.33 \times 10^{-14} \\ 1.26 \times 10^{-12} & -4.03 \times 10^{-13} & -2.33 \times 10^{-14} & 2.97 \times 10^{-11} \end{pmatrix}. \quad (\text{E1})$$

Then we use the bounds on the semileptonic kaon decay to compute the covariance matrix for the semileptonic decays in the basis $(\epsilon_{S,L}^{euds}, \epsilon_{V,L}^{euds}, \epsilon_{T,L}^{euds}, \epsilon_{S,R}^{euds}, \epsilon_{V,R}^{euds}, \epsilon_{T,R}^{euds})$:

$$\begin{pmatrix} 1.09 \times 10^{-12} & 3.51 \times 10^{-12} & 6.11 \times 10^{-12} & 1.39 \times 10^{-14} & 1.96 \times 10^{-13} & 7.49 \times 10^{-13} \\ 3.51 \times 10^{-12} & 2.44 \times 10^{-11} & 4.26 \times 10^{-11} & 1.96 \times 10^{-13} & 2.10 \times 10^{-12} & 6.50 \times 10^{-12} \\ 6.11 \times 10^{-12} & 4.26 \times 10^{-11} & 1.51 \times 10^{-10} & 7.49 \times 10^{-13} & 6.50 \times 10^{-12} & 1.58 \times 10^{-11} \\ 1.39 \times 10^{-14} & 1.96 \times 10^{-13} & 7.49 \times 10^{-13} & 1.09 \times 10^{-12} & 3.51 \times 10^{-12} & 6.11 \times 10^{-12} \\ 1.96 \times 10^{-13} & 2.10 \times 10^{-12} & 6.50 \times 10^{-12} & 3.51 \times 10^{-12} & 2.44 \times 10^{-11} & 4.26 \times 10^{-11} \\ 7.49 \times 10^{-13} & 6.50 \times 10^{-12} & 1.58 \times 10^{-11} & 6.11 \times 10^{-12} & 4.26 \times 10^{-11} & 1.51 \times 10^{-10} \end{pmatrix}. \quad (\text{E2})$$

The diagonal elements give the bounds on $|e|^2$. The bounds on the coefficients are the square roots of the diagonal elements. For instance, $\epsilon_{S,L}^{euds}$ is excluded above $\sqrt{1.09 \times 10^{-12}}$.

The covariance matrix in the basis $(\epsilon_{P,L}^{euds}, \epsilon_{A,L}^{euds}, \epsilon_{P,R}^{euds}, \epsilon_{A,R}^{euds}, \epsilon_{S,L}^{euds}, \epsilon_{V,L}^{euds}, \epsilon_{T,L}^{euds}, \epsilon_{S,R}^{euds}, \epsilon_{V,R}^{euds}, \epsilon_{T,R}^{euds})_{\Lambda_W}$ is

$$\begin{pmatrix} 1.64 \times 10^{-13} & -2.55 \times 10^{-14} & -1.55 \times 10^{-14} & 7.73 \times 10^{-13} & -2.91 \times 10^{-14} & 1.31 \times 10^{-12} & 5.51 \times 10^{-12} & -9.15 \times 10^{-16} & 2.07 \times 10^{-13} & 5.75 \times 10^{-13} \\ -2.55 \times 10^{-14} & 2.97 \times 10^{-11} & 7.73 \times 10^{-13} & -4.03 \times 10^{-13} & -7.10 \times 10^{-15} & -4.64 \times 10^{-13} & -4.30 \times 10^{-13} & 7.35 \times 10^{-16} & -2.15 \times 10^{-14} & -6.72 \times 10^{-14} \\ -1.55 \times 10^{-14} & 7.73 \times 10^{-13} & 1.64 \times 10^{-13} & -2.55 \times 10^{-14} & 9.15 \times 10^{-16} & -2.07 \times 10^{-13} & -5.75 \times 10^{-13} & 2.91 \times 10^{-14} & -1.31 \times 10^{-12} & -5.51 \times 10^{-12} \\ 7.73 \times 10^{-13} & -4.03 \times 10^{-13} & -2.55 \times 10^{-14} & 2.97 \times 10^{-11} & -7.35 \times 10^{-16} & 2.15 \times 10^{-14} & 6.72 \times 10^{-14} & 7.10 \times 10^{-15} & 4.64 \times 10^{-13} & 4.30 \times 10^{-13} \\ -2.91 \times 10^{-14} & -7.10 \times 10^{-15} & 9.15 \times 10^{-16} & -7.35 \times 10^{-16} & 3.22 \times 10^{-13} & 8.29 \times 10^{-13} & -1.11 \times 10^{-12} & -8.03 \times 10^{-15} & -8.12 \times 10^{-14} & -3.49 \times 10^{-14} \\ 1.31 \times 10^{-12} & -4.64 \times 10^{-13} & -2.07 \times 10^{-13} & 2.15 \times 10^{-14} & 8.29 \times 10^{-13} & 2.44 \times 10^{-11} & 5.02 \times 10^{-11} & -8.12 \times 10^{-14} & 2.10 \times 10^{-12} & 7.66 \times 10^{-12} \\ 5.51 \times 10^{-12} & -4.30 \times 10^{-13} & -5.75 \times 10^{-13} & 6.72 \times 10^{-14} & -1.11 \times 10^{-12} & 5.02 \times 10^{-11} & 2.10 \times 10^{-10} & -3.49 \times 10^{-14} & 7.66 \times 10^{-12} & 2.19 \times 10^{-11} \\ -9.15 \times 10^{-16} & 7.35 \times 10^{-16} & 2.91 \times 10^{-14} & 7.10 \times 10^{-15} & -8.03 \times 10^{-15} & -8.12 \times 10^{-14} & -3.49 \times 10^{-14} & 3.22 \times 10^{-13} & 8.29 \times 10^{-13} & -1.11 \times 10^{-12} \\ 2.07 \times 10^{-13} & -2.15 \times 10^{-14} & -1.31 \times 10^{-12} & 4.64 \times 10^{-13} & -8.12 \times 10^{-14} & 2.10 \times 10^{-12} & 7.66 \times 10^{-12} & 8.29 \times 10^{-13} & 2.44 \times 10^{-11} & 5.02 \times 10^{-11} \\ 5.75 \times 10^{-13} & -6.72 \times 10^{-14} & -5.51 \times 10^{-12} & 4.30 \times 10^{-13} & -3.49 \times 10^{-14} & 7.66 \times 10^{-12} & 2.19 \times 10^{-11} & -1.11 \times 10^{-12} & 5.02 \times 10^{-11} & 2.10 \times 10^{-10} \end{pmatrix}. \quad (\text{E3})$$

2. D meson decays

The bounds of Table I on leptonic D meson decays give the following covariance matrix in the basis $(\epsilon_{P,L}^{e\mu c u}, \epsilon_{A,L}^{e\mu c u}, \epsilon_{P,R}^{e\mu c u}, \epsilon_{A,R}^{e\mu c u})$:

$$\begin{pmatrix} 3.07 \times 10^{-6} & -3.55 \times 10^{-7} & -2.86 \times 10^{-8} & 7.91 \times 10^{-5} \\ -3.55 \times 10^{-7} & 2.04 \times 10^{-3} & 7.91 \times 10^{-5} & 7.30 \times 10^{-7} \\ -2.86 \times 10^{-8} & 7.91 \times 10^{-5} & 3.07 \times 10^{-6} & -3.55 \times 10^{-7} \\ 7.91 \times 10^{-5} & 7.30 \times 10^{-7} & -3.55 \times 10^{-7} & 2.04 \times 10^{-3} \end{pmatrix}. \quad (\text{E4})$$

Using bounds on the semileptonic decays of D and D_s mesons in the basis $(\epsilon_{S,L}^{e\mu c u}, \epsilon_{V,L}^{e\mu c u}, \epsilon_{T,L}^{e\mu c u}, \epsilon_{S,R}^{e\mu c u}, \epsilon_{V,R}^{e\mu c u}, \epsilon_{T,R}^{e\mu c u})$ gives

$$\begin{pmatrix} 1.80 \times 10^{-6} & 1.32 \times 10^{-7} & -3.19 \times 10^{-8} & -2.10 \times 10^{-8} & -1.61 \times 10^{-7} & 1.79 \times 10^{-8} \\ 1.32 \times 10^{-7} & 2.10 \times 10^{-6} & 3.65 \times 10^{-7} & -1.61 \times 10^{-7} & 9.7 \times 10^{-8} & 7.06 \times 10^{-7} \\ -3.19 \times 10^{-8} & 3.65 \times 10^{-7} & 4.03 \times 10^{-6} & 1.79 \times 10^{-8} & 7.06 \times 10^{-7} & 2.30 \times 10^{-7} \\ -2.10 \times 10^{-8} & -1.61 \times 10^{-7} & 1.79 \times 10^{-8} & 1.80 \times 10^{-6} & 1.32 \times 10^{-7} & -3.19 \times 10^{-8} \\ -1.61 \times 10^{-7} & 9.7 \times 10^{-8} & 7.06 \times 10^{-7} & 1.32 \times 10^{-7} & 2.10 \times 10^{-6} & 3.65 \times 10^{-7} \\ 1.79 \times 10^{-8} & 7.06 \times 10^{-7} & 2.30 \times 10^{-7} & -3.19 \times 10^{-8} & 3.65 \times 10^{-7} & 4.03 \times 10^{-6} \end{pmatrix}. \quad (\text{E5})$$

The covariance matrix in the basis $(\epsilon_{P,L}^{\mu\sigma i}, \epsilon_{A,L}^{\mu\sigma i}, \epsilon_{P,R}^{\mu\sigma i}, \epsilon_{A,R}^{\mu\sigma i}, \epsilon_{S,L}^{\mu\sigma i}, \epsilon_{V,L}^{\mu\sigma i}, \epsilon_{T_L}^{\mu\sigma i}, \epsilon_{S,R}^{\mu\sigma i}, \epsilon_{V,R}^{\mu\sigma i}, \epsilon_{T_R}^{\mu\sigma i})_{\Lambda_W}$ is

$$\begin{pmatrix} 1.15 \times 10^{-6} & -2.16 \times 10^{-7} & -1.15 \times 10^{-8} & 4.81 \times 10^{-5} & -1.45 \times 10^{-8} & -2.62 \times 10^{-8} & -2.97 \times 10^{-7} & -1.55 \times 10^{-9} & -8.69 \times 10^{-7} & -1.68 \times 10^{-8} \\ -2.16 \times 10^{-7} & 2.04 \times 10^{-3} & 4.81 \times 10^{-5} & 7.31 \times 10^{-7} & 1.81 \times 10^{-9} & 3.50 \times 10^{-5} & 8.22 \times 10^{-9} & 8.70 \times 10^{-9} & -1.09 \times 10^{-8} & 1.99 \times 10^{-7} \\ -1.15 \times 10^{-8} & 4.81 \times 10^{-5} & 1.15 \times 10^{-6} & -2.16 \times 10^{-7} & 1.55 \times 10^{-9} & 8.69 \times 10^{-7} & 1.68 \times 10^{-8} & 1.45 \times 10^{-8} & 2.62 \times 10^{-8} & 2.97 \times 10^{-7} \\ 4.81 \times 10^{-5} & 7.31 \times 10^{-7} & -2.16 \times 10^{-7} & 2.04 \times 10^{-3} & -8.70 \times 10^{-9} & 1.09 \times 10^{-8} & -1.99 \times 10^{-7} & -1.81 \times 10^{-9} & -3.50 \times 10^{-5} & -8.22 \times 10^{-9} \\ -1.45 \times 10^{-8} & 1.81 \times 10^{-9} & 1.55 \times 10^{-9} & -8.70 \times 10^{-9} & 6.80 \times 10^{-7} & 1.03 \times 10^{-7} & 2.73 \times 10^{-7} & -5.58 \times 10^{-9} & -5.42 \times 10^{-8} & 2.96 \times 10^{-8} \\ -2.62 \times 10^{-8} & 3.50 \times 10^{-5} & 8.69 \times 10^{-7} & 1.09 \times 10^{-8} & 1.03 \times 10^{-7} & 2.70 \times 10^{-6} & 4.31 \times 10^{-7} & -5.42 \times 10^{-8} & 9.66 \times 10^{-8} & 8.36 \times 10^{-7} \\ -2.97 \times 10^{-7} & 8.22 \times 10^{-9} & 1.68 \times 10^{-8} & -1.99 \times 10^{-7} & 2.73 \times 10^{-7} & 4.31 \times 10^{-7} & 5.62 \times 10^{-6} & 2.96 \times 10^{-8} & 8.36 \times 10^{-7} & 3.21 \times 10^{-7} \\ -1.55 \times 10^{-9} & 8.70 \times 10^{-9} & 1.45 \times 10^{-8} & -1.81 \times 10^{-9} & -5.58 \times 10^{-9} & -5.42 \times 10^{-8} & 2.96 \times 10^{-8} & 6.80 \times 10^{-7} & 1.03 \times 10^{-7} & 2.73 \times 10^{-7} \\ -8.69 \times 10^{-7} & -1.09 \times 10^{-8} & 2.62 \times 10^{-8} & -3.50 \times 10^{-5} & -5.42 \times 10^{-8} & 9.66 \times 10^{-8} & 8.36 \times 10^{-7} & 1.03 \times 10^{-7} & 2.70 \times 10^{-6} & 4.31 \times 10^{-7} \\ -1.68 \times 10^{-8} & 1.99 \times 10^{-7} & 2.97 \times 10^{-7} & -8.22 \times 10^{-9} & 2.96 \times 10^{-8} & 8.36 \times 10^{-7} & 3.21 \times 10^{-7} & 2.73 \times 10^{-7} & 4.31 \times 10^{-7} & 5.62 \times 10^{-6} \end{pmatrix}. \quad (\text{E6})$$

3. B meson decays

The bound on the leptonic decay of the B meson (see Table I) gives the following covariance matrix in the basis $(\epsilon_{P,L}^{e\mu bd}, \epsilon_{A,L}^{e\mu bd}, \epsilon_{P,R}^{e\mu bd}, \epsilon_{A,R}^{e\mu bd})$:

$$\begin{pmatrix} 5.53 \times 10^{-8} & 9.23 \times 10^{-8} & 1.20 \times 10^{-9} & 3.48 \times 10^{-6} \\ 9.23 \times 10^{-8} & 2.20 \times 10^{-4} & 3.48 \times 10^{-6} & 6.89 \times 10^{-6} \\ 1.20 \times 10^{-9} & 3.48 \times 10^{-6} & 5.53 \times 10^{-8} & 9.23 \times 10^{-8} \\ 3.48 \times 10^{-6} & 6.89 \times 10^{-6} & 9.23 \times 10^{-8} & 2.20 \times 10^{-4} \end{pmatrix}. \quad (\text{E7})$$

The covariance matrix in the basis $(\epsilon_{S,L}^{e\mu bd}, \epsilon_{V,L}^{e\mu bd}, \epsilon_{T_L}^{e\mu bd}, \epsilon_{S,R}^{e\mu bd}, \epsilon_{V,R}^{e\mu bd}, \epsilon_{T_R}^{e\mu bd})$ is

$$\begin{pmatrix} 2.07 \times 10^{-10} & 1.21 \times 10^{-11} & 1.52 \times 10^{-12} & -3.90 \times 10^{-15} & -5.74 \times 10^{-14} & 5.18 \times 10^{-15} \\ 1.21 \times 10^{-11} & 2.23 \times 10^{-10} & 2.81 \times 10^{-11} & -5.74 \times 10^{-14} & 2.87 \times 10^{-14} & 2.32 \times 10^{-13} \\ 1.52 \times 10^{-12} & 2.81 \times 10^{-11} & 4.03 \times 10^{-10} & 5.18 \times 10^{-15} & 2.32 \times 10^{-13} & 3.50 \times 10^{-14} \\ -3.90 \times 10^{-15} & -5.74 \times 10^{-14} & 5.18 \times 10^{-15} & 2.07 \times 10^{-10} & 1.21 \times 10^{-11} & 1.52 \times 10^{-12} \\ -5.74 \times 10^{-14} & 2.87 \times 10^{-14} & 2.32 \times 10^{-13} & 1.21 \times 10^{-11} & 2.23 \times 10^{-10} & 2.81 \times 10^{-11} \\ 5.18 \times 10^{-15} & 2.32 \times 10^{-13} & 3.50 \times 10^{-14} & 1.52 \times 10^{-12} & 2.81 \times 10^{-11} & 4.03 \times 10^{-10} \end{pmatrix}. \quad (\text{E8})$$

The covariance matrix in the basis $(\epsilon_{P,L}^{eubd}, \epsilon_{A,L}^{eubd}, \epsilon_{P,R}^{eubd}, \epsilon_{A,R}^{eubd}, \epsilon_{S,L}^{eubd}, \epsilon_{V,L}^{eubd}, \epsilon_{T_L}^{eubd}, \epsilon_{S,R}^{eubd}, \epsilon_{V,R}^{eubd}, \epsilon_{T_R}^{eubd})_{\Lambda_W}$ is

$$\begin{pmatrix} 2.74 \times 10^{-8} & 6.51 \times 10^{-8} & 5.94 \times 10^{-10} & 2.45 \times 10^{-6} & -1.10 \times 10^{-12} & -4.46 \times 10^{-10} & 5.02 \times 10^{-11} & 1.89 \times 10^{-14} & 1.68 \times 10^{-8} & -8.41 \times 10^{-13} \\ 6.51 \times 10^{-8} & 2.20 \times 10^{-4} & 2.45 \times 10^{-6} & 6.89 \times 10^{-6} & -2.11 \times 10^{-12} & -1.51 \times 10^{-6} & 9.19 \times 10^{-11} & 7.76 \times 10^{-11} & 4.73 \times 10^{-8} & -3.47 \times 10^{-9} \\ 5.94 \times 10^{-10} & 2.45 \times 10^{-6} & 2.74 \times 10^{-8} & 6.51 \times 10^{-8} & -1.89 \times 10^{-14} & -1.68 \times 10^{-8} & 8.41 \times 10^{-13} & 1.10 \times 10^{-12} & 4.46 \times 10^{-10} & -5.02 \times 10^{-11} \\ 2.45 \times 10^{-6} & 6.89 \times 10^{-6} & 6.51 \times 10^{-8} & 2.20 \times 10^{-4} & -7.76 \times 10^{-11} & -4.73 \times 10^{-8} & 3.47 \times 10^{-9} & 2.11 \times 10^{-12} & 1.51 \times 10^{-6} & -9.19 \times 10^{-11} \\ -1.10 \times 10^{-12} & -2.11 \times 10^{-12} & -1.89 \times 10^{-14} & -7.76 \times 10^{-11} & 1.03 \times 10^{-10} & 7.83 \times 10^{-12} & -1.03 \times 10^{-11} & -2.10 \times 10^{-15} & -5.78 \times 10^{-13} & 3.15 \times 10^{-15} \\ -4.46 \times 10^{-10} & -1.51 \times 10^{-6} & -1.68 \times 10^{-8} & -4.73 \times 10^{-8} & 7.83 \times 10^{-12} & 1.06 \times 10^{-8} & 3.09 \times 10^{-11} & -5.78 \times 10^{-13} & -3.24 \times 10^{-10} & 2.41 \times 10^{-11} \\ 5.02 \times 10^{-11} & 9.19 \times 10^{-11} & 8.41 \times 10^{-13} & 3.47 \times 10^{-9} & -1.03 \times 10^{-11} & 3.09 \times 10^{-11} & 5.10 \times 10^{-10} & 3.15 \times 10^{-15} & 2.41 \times 10^{-11} & 4.30 \times 10^{-14} \\ 1.89 \times 10^{-14} & 7.76 \times 10^{-11} & 1.10 \times 10^{-12} & 2.11 \times 10^{-12} & -2.10 \times 10^{-15} & -5.78 \times 10^{-13} & 3.15 \times 10^{-15} & 1.03 \times 10^{-10} & 7.83 \times 10^{-12} & -1.03 \times 10^{-11} \\ 1.68 \times 10^{-8} & 4.73 \times 10^{-8} & 4.46 \times 10^{-10} & 1.51 \times 10^{-6} & -5.78 \times 10^{-13} & -3.24 \times 10^{-10} & 2.41 \times 10^{-11} & 7.83 \times 10^{-12} & 1.06 \times 10^{-8} & 3.09 \times 10^{-11} \\ -8.41 \times 10^{-13} & -3.47 \times 10^{-9} & -5.02 \times 10^{-11} & -9.19 \times 10^{-11} & 3.15 \times 10^{-15} & 2.41 \times 10^{-11} & 4.30 \times 10^{-14} & -1.03 \times 10^{-11} & 3.09 \times 10^{-11} & 5.10 \times 10^{-10} \end{pmatrix}. \quad (E9)$$

4. B_s meson decays

The bound on the leptonic decay of the B_s meson in the basis $(\epsilon_{P,L}^{eubs}, \epsilon_{A,L}^{eubs}, \epsilon_{P,R}^{eubs}, \epsilon_{A,R}^{eubs})$ gives

$$\begin{pmatrix} 3.06 \times 10^{-8} & -1.22 \times 10^{-8} & -3.40 \times 10^{-10} & 1.94 \times 10^{-6} \\ -1.22 \times 10^{-8} & 1.24 \times 10^{-4} & 1.94 \times 10^{-6} & -1.80 \times 10^{-7} \\ -3.40 \times 10^{-10} & 1.94 \times 10^{-6} & 3.06 \times 10^{-8} & -1.22 \times 10^{-8} \\ 1.94 \times 10^{-6} & -1.80 \times 10^{-7} & -1.22 \times 10^{-8} & 1.24 \times 10^{-4} \end{pmatrix}. \quad (E10)$$

The bound on the B_s meson decaying into a kaon (Table I) in the basis $(\epsilon_{S,L}^{eubs}, \epsilon_{V,L}^{eubs}, \epsilon_{T_L}^{eubs}, \epsilon_{S,R}^{eubs}, \epsilon_{V,R}^{eubs}, \epsilon_{T_R}^{eubs})$ gives

$$\begin{pmatrix} 5.05 \times 10^{-10} & 3.47 \times 10^{-11} & 5.07 \times 10^{-12} & -1.13 \times 10^{-14} & -1.65 \times 10^{-13} & 1.73 \times 10^{-14} \\ 3.47 \times 10^{-11} & 6.53 \times 10^{-10} & 9.54 \times 10^{-11} & -1.65 \times 10^{-13} & 8.78 \times 10^{-14} & 7.90 \times 10^{-13} \\ 5.07 \times 10^{-12} & 9.54 \times 10^{-11} & 1.51 \times 10^{-9} & 1.73 \times 10^{-14} & 7.90 \times 10^{-13} & 1.38 \times 10^{-13} \\ -1.13 \times 10^{-14} & -1.65 \times 10^{-13} & 1.73 \times 10^{-14} & 5.05 \times 10^{-10} & 3.47 \times 10^{-11} & 5.07 \times 10^{-12} \\ -1.65 \times 10^{-13} & 8.78 \times 10^{-14} & 7.90 \times 10^{-13} & 3.47 \times 10^{-11} & 6.53 \times 10^{-10} & 9.54 \times 10^{-11} \\ 1.73 \times 10^{-14} & 7.90 \times 10^{-13} & 1.38 \times 10^{-13} & 5.07 \times 10^{-12} & 9.54 \times 10^{-11} & 1.51 \times 10^{-9} \end{pmatrix}. \quad (E11)$$

The covariance matrix in the basis $(\epsilon_{P,L}^{eubs}, \epsilon_{A,L}^{eubs}, \epsilon_{P,R}^{eubs}, \epsilon_{A,R}^{eubs}, \epsilon_{S,L}^{eubs}, \epsilon_{V,L}^{eubs}, \epsilon_{T_L}^{eubs}, \epsilon_{S,R}^{eubs}, \epsilon_{V,R}^{eubs}, \epsilon_{T_R}^{eubs})_{\Lambda_W}$ is

$$\begin{pmatrix} 1.52 \times 10^{-8} & -8.62 \times 10^{-9} & -1.69 \times 10^{-10} & 1.37 \times 10^{-6} & -1.35 \times 10^{-12} & 6.16 \times 10^{-11} & 6.41 \times 10^{-11} & -5.11 \times 10^{-15} & 9.39 \times 10^{-9} & 2.42 \times 10^{-13} \\ -8.62 \times 10^{-9} & 1.24 \times 10^{-4} & 1.37 \times 10^{-6} & -1.80 \times 10^{-7} & 1.21 \times 10^{-13} & -8.51 \times 10^{-7} & -1.29 \times 10^{-11} & 4.33 \times 10^{-11} & -1.24 \times 10^{-9} & -1.94 \times 10^{-9} \\ -1.69 \times 10^{-10} & 1.37 \times 10^{-6} & 1.52 \times 10^{-8} & -8.62 \times 10^{-9} & 5.11 \times 10^{-15} & -9.39 \times 10^{-9} & -2.42 \times 10^{-13} & 1.35 \times 10^{-12} & -6.16 \times 10^{-11} & -6.41 \times 10^{-11} \\ 1.37 \times 10^{-6} & -1.80 \times 10^{-7} & -8.62 \times 10^{-9} & 1.24 \times 10^{-4} & -4.33 \times 10^{-11} & 1.24 \times 10^{-9} & 1.94 \times 10^{-9} & -1.21 \times 10^{-13} & 8.51 \times 10^{-7} & 1.29 \times 10^{-11} \\ -1.35 \times 10^{-12} & 1.21 \times 10^{-13} & 5.11 \times 10^{-15} & -4.33 \times 10^{-11} & 2.51 \times 10^{-10} & 2.21 \times 10^{-11} & -3.90 \times 10^{-11} & -6.11 \times 10^{-15} & -4.33 \times 10^{-13} & 9.78 \times 10^{-15} \\ 6.16 \times 10^{-11} & -8.51 \times 10^{-7} & -9.39 \times 10^{-9} & 1.24 \times 10^{-9} & 2.21 \times 10^{-11} & 6.49 \times 10^{-9} & 1.07 \times 10^{-10} & -4.33 \times 10^{-13} & 8.57 \times 10^{-12} & 1.42 \times 10^{-11} \\ 6.41 \times 10^{-11} & -1.29 \times 10^{-11} & -2.42 \times 10^{-13} & 1.94 \times 10^{-9} & -3.90 \times 10^{-11} & 1.07 \times 10^{-10} & 1.91 \times 10^{-9} & 9.78 \times 10^{-15} & 1.42 \times 10^{-11} & 1.74 \times 10^{-13} \\ -5.11 \times 10^{-15} & 4.33 \times 10^{-11} & 1.35 \times 10^{-12} & -1.21 \times 10^{-13} & -6.11 \times 10^{-15} & -4.33 \times 10^{-13} & 9.78 \times 10^{-15} & 2.51 \times 10^{-10} & 2.21 \times 10^{-11} & -3.90 \times 10^{-11} \\ 9.39 \times 10^{-9} & -1.24 \times 10^{-9} & -6.16 \times 10^{-11} & 8.51 \times 10^{-7} & -4.33 \times 10^{-13} & 8.57 \times 10^{-12} & 1.42 \times 10^{-11} & 2.21 \times 10^{-11} & 6.49 \times 10^{-9} & 1.07 \times 10^{-10} \\ 2.42 \times 10^{-13} & -1.94 \times 10^{-9} & -6.41 \times 10^{-11} & 1.29 \times 10^{-11} & 9.78 \times 10^{-15} & 1.42 \times 10^{-11} & 1.74 \times 10^{-13} & -3.90 \times 10^{-11} & 1.07 \times 10^{-10} & 1.91 \times 10^{-9} \end{pmatrix}. \quad (E12)$$

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