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A hybrid L-shaped method to solve a bi-objective stochastic transshipment-enabled inventory routing problem

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Abstract

Recently, ‘greenness’ has become a very much needed condition in the transportation industry. In this study we develop a ‘green’, transshipment-enabled model for the Inventory Routing Problem (IRP), in a many-to-one distribution network where demand for each product is realistically assumed to be uncertain. The proposed framework is a bi-objective stochastic programming model. The first objective function aims to minimize the expected value of the supply chain costs including inevitable shortage costs. The second objective function aims to minimize the total quantity of the greenhouse gas (GHG) emission produced by the vehicles and disposed products. We introduce a very practical innovative application of transshipment option to control transportation cost, reduce GHG emissions and absorb the uncertainty. In order to solve the proposed model an efficient hybrid algorithm combining L-shaped method (a sort of decomposition approach for stochastic optimization) and compromise programming (a well-known approach for multi-objective optimization) is proposed. The results show that how companies can make a reasonable tradeoff between the cost and environmental concerns and emphasize the role of transshipment option as a lever to improve both economic and environmental performance and absorb the demand fluctuations.

Keywords: Inventory routing problem; stochastic programming; transshipment; green supply chain; uncertain demand; hybrid L-shaped method.

1. Introduction

The inventory routing problem has attracted considerable attention in the academic literature because it integrates two well-studied problems in supply chain management (SCM): inventory control and vehicle routing. These two issues have been traditionally dealt with independently, but their integration may have an impressive impact on the overall system performance ([Campbell and Savelsbergh 2004](#)).

Logistics and supply chain is now a widely-recognized source of added value for the same companies that are also under pressure to improve their environmental track record. Freight transport, storage, handling of materials, and the inventory management all correspond to areas where companies can optimize their service, beat off competition, and boost profits. However, this can potentially be attained at an environmental footprint which, in the eyes of

customers looking for a greener, more responsible service, could ultimately lead to a financial loss. Despite the importance of "*green*" concept in logistics and supply chain, relatively few studies have looked at the integration of this concept in inventory routing problems ([Alkawaleet et al. 2014](#), [Soysal et al. 2016](#)). We investigate where there is a green will, there is a profitable way by reassessing the policies with which companies transport, store and deliver supplies to their many clients with stochastic demands from a large number of points. We examine this precise dilemma, seeking to identify the happiest possible medium between smooth business operations and minimal pollution levels by the help of transshipment policy.

The remainder of the paper is organized as follows: in Section 2, we review the related studies on IRP in the literature and explain the main contributions of this research, in Section 3, we provide the description of the particular configuration that we consider in our work. The mathematical formulation of the problem is provided in Section 4. In section 5, the solving procedure is described and followed by a numerical study that is performed in Section 6 where we also discuss the managerial insights of this study. The conclusions of our work are presented in Section 7 along with some natural avenues for further research in this area.

2. Literature review and motivation

Recent reviews on the IRP found fewer than a hundred papers addressing the combined VRP-inventory management problem ([Baitaa et al. 1998](#), [Cordeau et al. 2007](#), [Andersson et al. 2010](#)). IRP can be broadly categorized according to the criteria such as single or multi products, single or multi-vehicles, finite or infinite planning horizon, single or multi-periods, single or multiple customers, single or multi-objective, whether the demand is deterministic or stochastic, whether the shortage is ignored or taken to account, whether the environmental consideration is considered or neglected and whether the IRP is transshipment-enabled or not. An overview of the related studies on IRP is presented in [Table 1](#). Despite the fact that the

concept of "green logistics" has recently attracted the attention of the researchers, few studies have addressed the environmental concerns in their IRP models (Al Shamsi et al. 2014, Soysal et al. 2015 and 2016). Although Soysal et al. (2015) and (2016) have considered CO₂ emissions of the vehicles (as a function of load and speed) but they assumed a homogenous fleet, converse to the heterogeneous fleet in our model where the various vehicle types with different capacity, environmental index, and fuel consumption rates are available and therefore an appropriate vehicle-type assignment should be also taken into account. Additionally, very few studies on IRP has been devoted to the transshipment policy (Mirzapour al-e-hashem and Rekik 2012, Coelho et al. 2012, Jemai et al. 2012), and none of these none-traditional approaches has applied the multi-objective framework to elaborate the inherent conflict between economic aspects and environmental concerns in IRP decisions. Also, in all variants of IRP, we couldn't find any effort to study the impact of the transshipment on GHG emissions except Mirzapour al-e-hashem and Rekik (2012) in which the demand is assumed to be deterministic unlike to our problem. Therefore, the question that how the transshipment-enabled IRP (TRIP) can be implemented in an uncertain environment has been never studied. Our problem is also classified as a many-to-one structure. There are very few studies in this category (cf. Soysal et al. 2016) under which a set of suppliers serve a single customer (or manufacturer). The other variants of supply networks are one-to-many and many-to-many. In the former structure a single supplier serves a set of customers (e.g. Jia et al. 2014), and in the latter structure (e.g. Ramkumar et al. 2012) several suppliers serve a set of customers.

According to Table 1, Several other variants of IRP can also be found depending on the other assumptions in the models such as IRP with direct deliveries (Mishra and Raghunathan 2004), the IRP with heterogeneous fleet (Persson and Gothe-Lundgren 2005, Mirzapour al-e-hashem and Rekik, 2014), the IRP with time windows (Liu and Lee 2011, Iassinovskaia et al.

2016), the IRP for perishable products (Coelho and Laporte 2014, Hauge et al. 2014, Mirzaei & Seifi 2015, Soysal et al. 2015) and cyclic IRPs (Chitsaz et al. 2016). The reader is referred to Andersson et al. 2010 and Coelho et al. 2014 for an excellent and comprehensive overview about IRPs.

Our paper contributes to the existing investigations by considering the transshipment impact on both the economic and the environmental performance of a stochastic TIRP in a many-to-one supply chain. As mentioned before, in this structure, a set of suppliers serves a single assembly plant. In a stochastic transshipment-enabled IRP, a vehicle may provide a specific product for the assembly plant, either directly from the main supplier who is the original manufacturer of that product or from a temporary storage of some other suppliers where this product may be available (temporarily stored there in previous trips (Nonas and Jornsten 2005, 2007, Jemai et al. 2012)). As Table 1 shows, transshipment option is rarely integrated within the context of inventory routing problems (Coelho et al. 2012, Mirzapour al-e-hashem and Rekik, 2014, Chrysochoou and Ziliaskopoulos, 2015). It is even more scarce when both economic and environmental aspects of transshipment is analyzed (Mirzapour al-e-hashem and Rekik, 2014). For instance, Mirzapour al-e-hashem and Rekik (2014) emphasized that all these investigations considered only the economic performance of the IRP without looking after the implication on the environmental footprint of the joint inventory and distribution solution. Transshipment option can theoretically result in improvement of the performance of the supply chain through reductions of the lead times, saving some routes (Economic effects), as well as greenhouse gas emissions (environmental effects). However, this policy can be limited in practice, when the suppliers are not coordinated very well or compete together. On the other hand, double handling maybe creates some operational issues. In this paper, we investigate the important role of transshipment as an option dealing with uncertain demand. It is discovered that the transshipment could absorb

a lion share of demand's perturbations and suggests appropriate solutions to both economic and environmental concerns. We will show cases where transshipment could bring conflicting situations to the decision maker and cases where transshipment is aligning both the economic and environmental objectives in the presence of uncertainty. Besides, our bi-objective modeling of the problem and the associated numerical results will give the freedom to the decision maker to judgmentally choose between many sets of solutions reflecting his/her own preference on the ecological footprint.

Based on above discussion, the green considerations, the option of transshipments and the assumption of demand uncertainty constitute collectively a significant departure from the current state of knowledge in this area.

Table 1. Overview of the related literature on IRP

	Shortage	Uncertainty	Greenness	Transshipment	Multi-Objective	Multi-period	Fleet	Modeling approach	Solution procedure	Other features
Bell et al. (1983)	-	-	-	-	-		Homogeneous	MIP	Lagrange + B&B	
Federgruen and Zipkin 1984	√	Demand	-	-	-		Homogeneous	MIP	Bender's Decomposition	
Chien et al. 1989	-	-	-	-	-		Homogeneous	MIP	Lagrange + Heuristic	
Speranza and Ukovich 1994	-	-	-	-	-		Homogeneous	MIP	MPSX solver	
Bertazzi and Speranza 2002	-	-	-	-	-	√	Homogeneous	MIP	Relaxation	
Kleywegt et al. 2002	√	Demand	-	-	-	√	Homogeneous	Markov	DP + Approximation	direct deliveries
Campbell and Savelsbergh 2004	-	-	-	-	-	√	Homogeneous	IP	Decomposition + Heuristic	
Kleywegt et al. 2004	√	Demand	-	-	-	√	Homogeneous	Markov	DP + Approximation	
Mishra and Raghunathan 2004	√	Demand	-	-	-	√	Single	MIP	Exact	product substitution
Sindhuchao et al. 2005	-	-	-	-	-	-	Homogeneous	MIP	B&P + neighborhood search heuristic	
Persson and Gothe-Lundgren 2005	-	-	-	-	-	√	Heterogeneous	MIP	Column generation	
Abdelmaguid and Dessouky 2006	√	-	-	-	-	√	Homogeneous	MIP	GA	
Archetti et al. 2007	-	-	-	-	-	√	Homogeneous	MIP	B&C	
Yu et al., 2008	-	-	-	-	-	√	Homogeneous	MIP	Lagrange + sub-gradient	split delivery
Abdelmaguid et al. 2009	√	-	-	-	-	√	Homogeneous	MIP	Heuristic	
Hvattum and Løkketangen 2009	√	Demand	-	-	-	-	Homogeneous	Markov	Scenario-tree - heuristic	
Huang and Lin 2010	√	Demand	-	-	-	-	Homogeneous	MIP	ACO	vehicle failure
Shen et al. 2011	√	-	-	√	-	√	Homogeneous	MIP	Lagrangian	

									Relaxation	
Bertazzi et al. 2011	√	Demand	-	-	-	√	Single	DP	B&C + Rollout	
Moin et al. 2011	-	-	-	-	-	√	Homogeneous	ILP	Hybrid GA	
Liu and Lee 2011	√	-	-	-	-	-	Homogeneous	MIP	Tabu search	Time window
Mirzapour al-e-hashem and Rekik 2012	√	-	√	√	-	√	Heterogeneous	MIP	B&B	
Jemai et al. 2012	√	Demand	-	√	-		Homogeneous	(R,s,S)	Exact	
Coelho et al. 2012	-	-	-	√	-		Single	MIP	Neighborhood search heuristic	
Solyali et al. 2012	√	Demand	-	-	-	√	Homogeneous	Robust + MIP	B&C	
Shukla et al. 2013	-	Demand	-	-	-	√	Homogeneous	NLMIP	GA	
Coelho and Laporte 2014	-	-	-	-	-	√	Homogeneous	MIP	B&C	Perishability
Hauge et al. 2014	-	-	-	-	-	-	Homogeneous	IP	Column generation	Waste collection
Al Shamsi et al. 2014	-	-	√	-	-	√	Homogeneous	MIP	Simulation	Perishability
Alkawaleet et al. 2014	-	-	√	-	-	-	Heterogeneous	IP	GAMS	
Mirzaei & Seifi 2015	Lost Sale	-	-	-	-	√	Homogeneous	MIP	Simulated annealing	Perishability
Soysal et al. 2015	√	Demand	√	-	-	√	Homogeneous	Chance constraint	Simulation	Perishability
Soysal et al. 2016	√	Demand	√	-	-	√	Homogeneous	Chance constraint	CPLEX	Perishability
Cheng et al. 2016	-	-	√	-	-	√	Homogeneous	MIP	Hybrid GA	
Chitsaz et al. 2016	-	-	-	-	-	√	Homogeneous	MIP	Decomposition	Cyclic IRP
Iassinovskaia et al. 2016	-	-	-	-	-	√	Homogeneous	MIP	B&C + heuristics	Closed loop
Soysal 2016b	√	Demand	√	-	-	√	Homogeneous	MIP	CPLEX	Closed loop
This Study	Backorder & Lost sale	Demand	√	√	√	√	Heterogeneous	MIP	L-shaped Method	Disposal

3. Problem description

Assume a many-to-one supply network consists of one assembly plant and a set of suppliers each one providing one product type to the plant. The planning horizon is not limited but for modeling purpose it is set to two periods. The demand of the assembly plant for each product in each period is uncertain. The company needs to decide on the order quantities before the realization of the uncertain demand.

The company is assumed to have an internal contract with a rental truck company (Depot) that ships the forecasted requirements from the suppliers to the assembly plant (without being aware of the real demand at the start of each period). This rental truck company has a heterogeneous fleet including several types of trucks. Each one is characterized by its own capacity, fixed and variable transportation cost and also its GHG emission index (without loss of generality we can also consider electrical vehicles or other low GHG emission vehicles). At the end of the period, the real demand of the assembly plant is unfolded and the inventory or shortage levels in the assembly plant will be specified accordingly. At the start of the next period, and according to which scenario has already been occurred, the next requirement quantities must be forecasted and consequently the appropriate vehicle routes and vehicle types must be selected. The network of this many-to-one supply chain discussed above is graphically depicted in [Figure 1](#).

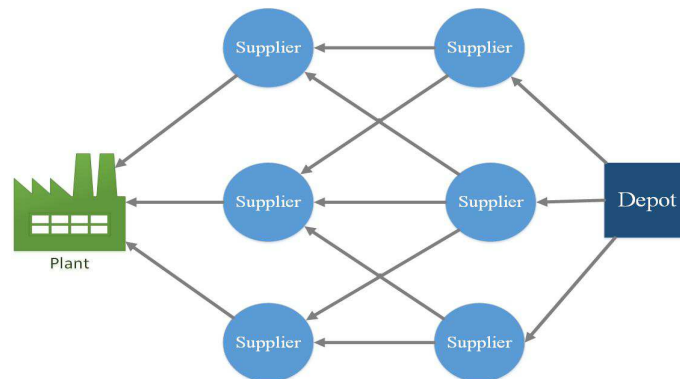


Figure 1. General schema of many-to-one supply network.

When a truck visits a supplier, it picks up some parts based on the forecasted demand of assembly plant and its own container capacity. It is also assumed a transshipment option which allows the vehicle to unload his cargo in supplier store (at least for one period) prior to pick up new cargo. Note that the transshipment option is impossible when the supplier has no empty places in its warehouse. This transshipment option has two advantages; first it enables the vehicle to empty his container partially and copes with his container capacity limitation to pick up the new necessary cargo from the supplier. Second, it can help the trucks to save few routes, since the cargo was unloaded in the supplier warehouse can be either used by other trucks in current period (this state is not an optimal plan as shown in [Moein et al. 2011](#)), or in the succeeding periods to probably fulfill the future demands of the assembly plant. It should be noted that the products type unloaded by the trucks in a supplier store is different from the products type produced by that supplier. Keeping these kinds of products at supplier storehouse obviously imposes a holding cost to that supplier. In this case assembly plant undertakes the payment of this extra holding cost. We will show how the transshipment option especially in uncertain environment plays as a low-cost lever to increase the performance of the inventory routing problem and simultaneously decrease the GHG emissions.

We propose a bi-objective stochastic programming model which aims to find the best configuration of the vehicles types and routes, and the ideal quantities of pickups, deliveries and transshipments in each period in the presence of the demand uncertainty, and in order to meet the two following conflicting criteria, simultaneously; minimizing the weighted sum of the expected value of total costs and minimizing the expected GHG emissions. Total costs of the supply chain include the inventory holding, shortage, transportation costs and costs of disposal process. Also the GHG emission is twofold; the quantity of the GHG level produced by the vehicles during transportation and that produced by products during the disposal

process.

4. Mathematical formulation

In order to solve the problem discussed in the previous section, a stochastic optimization approach is proposed.

A standard single objective formulation of the two-stage stochastic program is written as follows (Kall and Wallace 1994, Sahinidis 2004):

$$\begin{aligned} P_1 : \quad & \min_x \quad c^T x + E_{s \in \xi} [Q(x, \xi(s))] \\ & s.t. \quad Ax = b, \\ & \quad \quad x \geq 0 \end{aligned}$$

where $Q(x, \xi(s))$ is the optimal value of the second stage problem:

$$\begin{aligned} P_2 : \quad & \min_y \quad q^T y \\ & s.t. \quad Tx + Wy = h, \\ & \quad \quad y \geq 0 \end{aligned}$$

where x and y are vectors of the first and second stage decision variables, respectively. The second stage problem depends on the data $\xi = (q, h, T, W)$ where any or all elements can be random. The expectation in P_1 is with respect to the probability distribution of $\xi(s)$ where s is a random variable. Matrices T and W are called technological and recourse matrices, respectively. The second stage problem P_2 can be considered as a penalty for the violation of the constraint $Tx = h$.

P_1 with variables x also known as design variables constitutes the first stage which needs to be decided prior to the realization of the uncertain parameters. P_2 with variables y constitutes the second stage also known as control variables are those made after the uncertainty associated with the random variables is unveiled. Under the assumption of discrete distributions of the uncertain parameters, the problem can be equivalently formulated as a large-scale linear program which can be solved using standard linear programming techniques which is definitely inefficient for large scale problems, so we apply the L-shaped

method to solve the problem in a very reasonable time. Since the proposed model is a bi-objective programming, an iterative framework taking advantage of both L-shape method and compromise programming technique is developed.

The proposed bi-objective stochastic and transshipment-enabled IRP framework uses the following notations:

Sets

$\Omega = \{0, 1, \dots, N+1\}$	set of all nodes
$\omega = \{1, 2, \dots, N\}$	set of suppliers
$O = \{0\}$	depot
$F = \{N + 1\}$	assembly plant
$\phi = \{1, \dots, P\}$	set of product types
$\xi = \{1, \dots, S\}$	set of scenarios
$\eta = \{1, \dots, K\}$	set of vehicle types

Parameters

DF_{ps}	demand of product type p ($1, 2, \dots, P$) at the first period under scenario s .
DS_{ps}	demand of product type p ($1, 2, \dots, P$) at the second period under scenario s .
v_k	variable transportation cost per unit distance for vehicle type k ($1, 2, \dots, K$).
u_k	fixed transportation cost for vehicle type k per trip.
NT_k	the number of available vehicles type k .
cap_k	capacity of vehicle type k .
$icap_i$	extra capacity of supplier i which allows vehicles to store the products of other suppliers
h_{ip}	inventory holding cost in node i for product type p per product unit per period.
π_p	unit shortage cost (backorder) for product type p at the first period in the assembly plant.
π'_p	unit shortage cost (lost sale) for product type p at the second period in the assembly plant.
SC_{ip}	disposal cost in node i for product type p per product unit at the end of the second period.
c_{ij}	length of arc (i, j) .
G_k	GHG emission quantity produced by vehicle type k per traveling distance.
G'_p	GHG emission quantity produced by disposing product type p at the end of the second period.
ρ_s	occurrence probability of scenario s ($1, 2, \dots, S$).
λ	relative weight of the expected total cost.

Decision variables

IF_{ps}	the inventory level of product type p in the assembly plant at the end of the first period under scenario s .
IS_{ips}	the inventory level of product type p in supplier i or in the assembly plant at the end of the second period under scenario s .
BF_{ps}	the shortage level of product type p in the assembly plant at the end of the first period

	under scenario s .
BS_{ps}	the shortage level of product type p in the assembly plant at the end of the second period under scenario s .
XF_{ijk}	a binary variable that determines whether arc (i, j) is visited by vehicle type k at the first period.
XS_{ijks}	a binary variable that determines whether arc (i, j) is visited by vehicle type k at the second period under scenario s .
YF_{ik}	a binary variable that determines whether supplier i is visited by vehicle type k at the first period.
YS_{iks}	a binary variable that determines whether supplier i is visited by vehicle type k at the second period under scenario s .
QF_{ijpk}	the quantity of product type p transported by vehicle type k through arc (i, j) in the first period.
QS_{ijps}	the quantity of product type p transported by vehicle type k through arc (i, j) in the second period under scenario s .
PF_{ip}	the quantity of product type p picked from supplier i at the first period.
PS_{ips}	the quantity of product type p picked from supplier i at the second period under scenario s .
TF_{ip}	the quantity of product type p transshipped to supplier i at the first period.
FSC	first stage costs.
SSC_s	second stage costs under scenario s .
FSG	total GHGs generated in the first stage.
SSG_s	total GHGs generated in the second stage under scenario s .

Minimizing the expected total cost, Z_1 as well as minimizing the expected GHGs, Z_2 , while respecting the different constraints could be written as follows:

$$\text{Min } Z_1 = \underbrace{\sum_{i,j \in \Omega} \sum_k v_k c_{ij} XF_{ijk}}_{(1_1)} + \underbrace{\sum_{i \in \omega, k} u_k XF_{0ik} + \sum_{i \in \omega, p} h_{ip} TF_{ip}}_{(1_2)} + \lambda \times \underbrace{\sum_s \rho_s SSC_s}_{(1_2)} \quad (1)$$

$$\text{Min } Z_2 = \underbrace{\sum_{i,j \in \Omega} \sum_k G_k c_{ij} XF_{ijk}}_{(2_1)} + \sum_s \rho_s \left(\underbrace{\sum_{i,j \in \Omega} \sum_k G_k c_{ij} XS_{ijks} + \sum_{i \in \omega \cup F} \sum_p G'_p IS_{ips}}_{(2_2)} \right) \quad (2)$$

Where:

$$SSC_s = \sum_{i,j \in \Omega, k} v_k c_{ij} XS_{ijks} + \sum_{i \in \omega, k} u_k XS_{0iks} + \sum_{i \in \omega \cup F} \sum_p sc_{ip} IS_{ips} + \sum_p (h_{(N+1)p} IF_{ps} + \pi_p BF_{ps} + \pi'_p BS_{ps}) \quad (3)$$

subject to:

$$IF_{ps} - BF_{ps} = \sum_{i \in \omega, k} QF_{i(N+1)pk} - DF_{ps} \quad \forall p, s \quad (4)$$

$$IS_{ips} = TF_{ip} - PS_{ips} \quad \forall i \in \omega, p \neq i, s \quad (5)$$

$$IS_{(N+1)ps} - BS_{ps} = IF_{ps} - BF_{ps} + \sum_{i \in \omega, k} QS_{i(N+1)ps} - DS_{ps} \quad \forall p, s \quad (6)$$

$$\sum_{j \in \Omega} XF_{ijk} = \sum_{j \in \Omega} XF_{jik} = YF_{ik} \quad \forall i \in \omega, k \quad (7)$$

$$\sum_k YF_{ik} \leq 1, \forall i \in \omega \quad (8)$$

$$\sum_{j \in \Omega} XS_{ijks} = \sum_{j \in \Omega} XS_{jiks} = YS_{iks} \quad \forall i \in \omega, k, s \quad (9)$$

$$\sum_k YS_{iks} \leq 1, \forall i \in \omega, s \quad (10)$$

$$\sum_{j \in \omega \cup O, k} QF_{jipk} + PF_{ip} - TF_{ip} = \sum_{j \in \omega \cup F, k} QF_{ijpk}, \forall i \in \omega, p \quad (11)$$

$$\sum_{j \in \omega \cup O, k} QS_{jipks} + PS_{ips} = \sum_{j \in \omega \cup F, k} QS_{ijpks} \quad \forall i \in \omega, p, s \quad (12)$$

$$\sum_p QF_{ijpk} \leq cap_k XF_{ijk} \quad \forall i, j \in \Omega, k \quad (13)$$

$$\sum_p QS_{ijpks} \leq cap_k XS_{ijks} \quad \forall i, j \in \Omega, k, s \quad (14)$$

$$PS_{ips} \leq TF_{ip} \quad \forall i \in \omega, p \neq i, s \quad (15)$$

$$\sum_p TF_{ip} \leq icap_i \quad \forall i \in \omega \quad (16)$$

$$PF_{ip} = 0 \quad \forall i \in \omega, p \neq i \quad (17)$$

$$TF_{ip} = 0 \quad \forall i \in \omega, p = i \quad (18)$$

$$\sum_{i \in \omega} XF_{oik} \leq NT_k \quad \forall k \quad (19)$$

$$\sum_{i \in \omega} XS_{oiks} \leq NT_k \quad \forall k, s \quad (20)$$

$$\sum_{i \in \omega, k} XF_{oik} \geq 1 \quad (21)$$

$$\sum_{i \in \omega, k} XS_{oiks} \geq 1 \quad \forall s \quad (22)$$

$$\sum_{i \in \omega, k} XF_{i(N+1)k} \geq 1 \quad (23)$$

$$\sum_{i \in \omega, k} XS_{i(N+1)ks} \geq 1 \quad \forall s \quad (24)$$

$$XF_{i0k} = 0 \quad \forall i \in \omega, k \quad (25)$$

$$XS_{i0ks} = 0 \quad \forall i \in \omega, k, s \quad (26)$$

$$XF_{(N+1)ik} = 0 \quad \forall i \in \omega, k \quad (27)$$

$$XS_{(N+1)iks} = 0 \quad \forall i \in \omega, k, s \quad (28)$$

$$XF_{iik} = 0 \quad \forall i \in \Omega, k \quad (29)$$

$$XS_{iiks} = 0 \quad \forall i \in \Omega, k, s \quad (30)$$

$$XF_{0(N+1)k} = 0 \quad \forall k \quad (31)$$

$$XS_{0(N+1)ks} = 0 \quad \forall k, s \quad (32)$$

$$QF_{0ipk} = 0 \quad \forall i \in \omega, p, k \quad (33)$$

$$QS_{0ipks} = 0 \quad \forall i \in \omega, p, k, s \quad (34)$$

$$QF_{ijpk}, QS_{ijpks}, PF_{ipk}, PS_{ipks}, TF_{ipk} \geq 0, \text{ integer} \quad (35)$$

$$YF_{ik}, YS_{iks}, XF_{ijk}, XS_{ijks} \in \{0, 1\} \quad \forall i, j \in \Omega, p, k, s$$

4.1. Objective function components

Equation (1) is the first objective function of the proposed model which has three components; the first component (1_1) is the fixed and variable transportation cost at the first period plus the inventory holding cost of the transshipped products, which is not subject to uncertainty (first stage decision variables); this component is hereafter called as “first stage costs” (*FSC*). The second component of the first objective function, Eq. (1_2), is the expected value of the second stage costs ($E[SSCs]$), consisting of inventory holding costs, shortage costs, scrap and transportation costs, where SSC_s is defined in Eq. (3). Therefore the first objective function of the proposed model is summarized as $MinZ_1 = FSC + \lambda \times E[SSC]$.

Equation (2) is the second objective function of the proposed model and it relates to the greenhouse gas emission levels produced by the combustion of fuels in vehicles and the scraping process of products at the end of the second period. This objective function is also composed of the two following components; the first part of this objective function is the total quantity of GHG emission level produced by the vehicles at the first period and therefore is not subject to uncertainty. In other words, this part of the objective function measures the GHG level produced by vehicles when the real demand is unknown. We refer to this part as the first stage GHGs (*FSG*).

$$FSG = \sum_{i,j \in \Omega} \sum_k G_k C_{ij} XF_{ijk} \quad (36)$$

The second part of the second objective function is the expected value of the total amount of GHG emissions produced by the vehicles and scraped products under all possible

scenarios. Note that the products which remain unused at the end of the second period should be scrapped, and this process produces additional amounts of GHGs.

We refer to this term as the second stage GHGs (SSG) and rewrite it as follows:

$$SSG = \sum_s \rho_s (SSG_s) = E[SSG_s] \quad (37)$$

Therefore the second objective function of the proposed model is summarized as $Min Z_2 = FSG + \lambda \times E[SSG]$.

4.2. Constraints

Constraints (4) are the inventory balance equations for the assembly plant at the first period, and determine that for each scenario, the inventory or shortage level for product type p is equal to the quantity of that product transported to the assembly plant, minus its demand. Constraints (5) are the inventory balance equations at the suppliers' side at the second period and determine the inventory level for product type p at supplier i ($i \neq p$) is equal to the quantity of that product transshipped to this supplier in the previous period, minus the quantity picked up by the vehicles in the current period.

Constraints (6) are an inventory balance equation in the assembly plant at the second period, and implies that for each scenario, the inventory or shortage level for product type p in the current period is equal to its previous level in addition to the total quantity delivered by the vehicles, minus its demand in the current period. Constraints (7-8) guarantee that each supplier should not be visited by the vehicles more than once at the first period. Constraints (9-10) are similar to constraints (7-8) but they refer to the second stage variables under scenario s . Constraints (11) are the inventory balance equations for supplier i visited during the first period and insure that the quantity of product type p shipped from supplier i is equal to the quantity of that product shipped to this supplier, plus the quantity of that product picked up by the vehicles, minus the quantity transshipped to this supplier.

Constraints (12) are the same as constraints (11) for the second period and specify for each scenario that the quantity of product type p shipped from supplier i is equal to the quantity of that product shipped to this supplier, plus the quantity of that product picked up by the vehicles. Constraints (13-14) guarantee for the first and second period that the vehicle's capacity should not be exceeded, and also implies that the vehicles could visit arc (i, j) only once that the variables XF and XS take positive values.

Constraints (15) insure for each scenario that the vehicles could not pick up products from suppliers which are not the producer of that product, in a quantity greater than that transshipped to them in previous period. Constraints (16) specify that the quantity of products transshipped to supplier i cannot exceed the supplier's warehouse capacity which has been specifically assigned to transshipment. Constraints (17) imply that the vehicles in the first period could pick up each product merely from its own supplier. Constraints (18) state that in the case of transshipment, a product must be transported from its producer to the other suppliers. Constraints (19-20) limit the number of vehicles type k available in each period to a given quantity. Constraints (21-24) are introduced to prevent the formation of sub-tours and closed cycles that are not part of the routes between the depot and assembly plant. In other words, this set of constraints ensures that a trip should start at depot (node 0) and end at assembly plant (node $N+1$). Constraints (25-32) determine the impossible arcs in each period. Constraints (33-34) specify for the first and second periods that the vehicles should not ship back any quantity to depot (node 0). Finally, constraints (35) define the variable types.

5. Solving Procedure

When the uncertainty is assumed to take values in a finite set of small cardinality, the two-stage problem is tractable, and even for larger cardinality (but still finite) uncertainty sets (scenarios), large-scale linear programming techniques, such as Bender's decomposition, can be employed to obtain a tractable formulation. The core challenge in solving two-stage

stochastic problems arises from the expectation evaluation of the inner recourse problem. For a scenario-based representation of demand uncertainty this is achieved by explicitly associating a second-stage variable with each demand scenario and then solving an equivalent large-scale extensive formulation of the model. A similar methodology is also applied when the uncertainty is described with a probability distribution by discretizing the demand distribution using techniques such as Monte Carlo sampling and Gaussian quadrature (Gupta and Maranas 2003). Since the proposed model has a bi-objective formulation, it is necessary to select an appropriate approach from the well-recognized multi objective decision making (MODM) techniques. There are three main categories to deal with multi-objective optimization problems: Under the *a priori* methods, the *a posteriori* method and the *interactive* methods. In the *a priori* method, the decision maker states his/her priorities before the solution process and the multi-objective optimization problem is transformed into a single objective problem. The *a posteriori* methods rely upon the simultaneous optimization of all the objective functions. First the efficient solutions of the problem (Pareto set) are generated. Next, at the end of the search process, the decision maker is involved, in order to select the most preferred solution among the Pareto set. Under the last methodology (the interactive approaches), the decision maker successively drives the search with his/her answers towards the most preferred solution (Bozorgi-Amiri et al. 2011, Mirzapour Al-e-Hashem et al. 2012). Due to the multi-objective nature of the proposed framework, we use a compromising technique to solve the model. In this method the decision maker is questioned to express his priorities about the two objectives before the solution process. Then according to his answers the weights of each objective (θ) are determined. Therefore, the proposed multi-objective optimization problem is transformed into a normalized (dimensionless) single objective problem and summarized as follows:

$$\text{Min } Z = \left[\theta \times \left(\frac{Z_1 - Z_{1\min}}{Z_{1\max} - Z_{1\min}} \right)^p + (1 - \theta) \times \left(\frac{Z_2 - Z_{2\min}}{Z_{2\max} - Z_{2\min}} \right)^p \right]^{1/p} \quad (44)$$

On the other hand, the proposed problem is formulated under stochastic programming, so we apply an efficient variant of decomposition approach called L-shaped method (Louveaux and Birge, 2009). We merge the proposed compromise programming with L-shaped method to cope with multi-objective stochastic model's complexity. In inner loops of the L-shaped method, the compromise programming is called. It first solves the model separately with single objective functions, Z_1, Z_2 (setting $\theta=0$ and 1) to obtain $Z_{i\min}$ and $Z_{i\max}$ ($i=1, 2$). Then it builds the compromising objective function (Z). The essential steps of the hybrid L-shaped method are depicted in Figure 2 and described as follows:

5.1. Hybrid L-shaped Algorithm

- 1- Set $s=0, k_1=0, k_2=0$, and $\phi = -\infty$;
 - 2- Solve master problem, and get first stage variables (XF, TF, QF and YF);
 - 3- Set first stage variables as parameters in Feasibility Problem (FP) (cf. Appendix);
 - 4- If $s=S$ go to step 7 otherwise set $s=s+1$ and solve FP under scenario s ;
 - 5- If objective function of FP equals to zero go to step 4 otherwise go to step 6
 - 6- Set $k_1 = k_1 + 1, s = 0$, and calculate parameters for feasibility cut k_1 (cf. Appendix) and return to step 2.
 - 7- Set $s = 0$, and set first stage variables as parameters in Optimality Problem (OP);
 - 8- If $s = S$, go to step 9 otherwise set $s=s+1$, and solve OP under scenario s and calculate $Z_{opt}(s)$;
 - 9- If $\sum_{s \in \xi} \rho_s \times Z_{opt}(s) = \phi$, the obtained solution is optimal, otherwise proceed; (cf. Appendix)
 - 10- Set $k_2 = k_2 + 1, s=0$, calculate parameters for Optimality Cut k_2 (applying Optimality Cut) and return to step 2.
- Where ϕ is the auxiliary variable for Master Problem (Representing second stage part of objective function), and $\mathcal{K}_1 = \{1, \dots, K_1\}$, $\mathcal{K}_2 = \{1, \dots, K_2\}$ are set of all feasibility and Optimality cuts respectively.

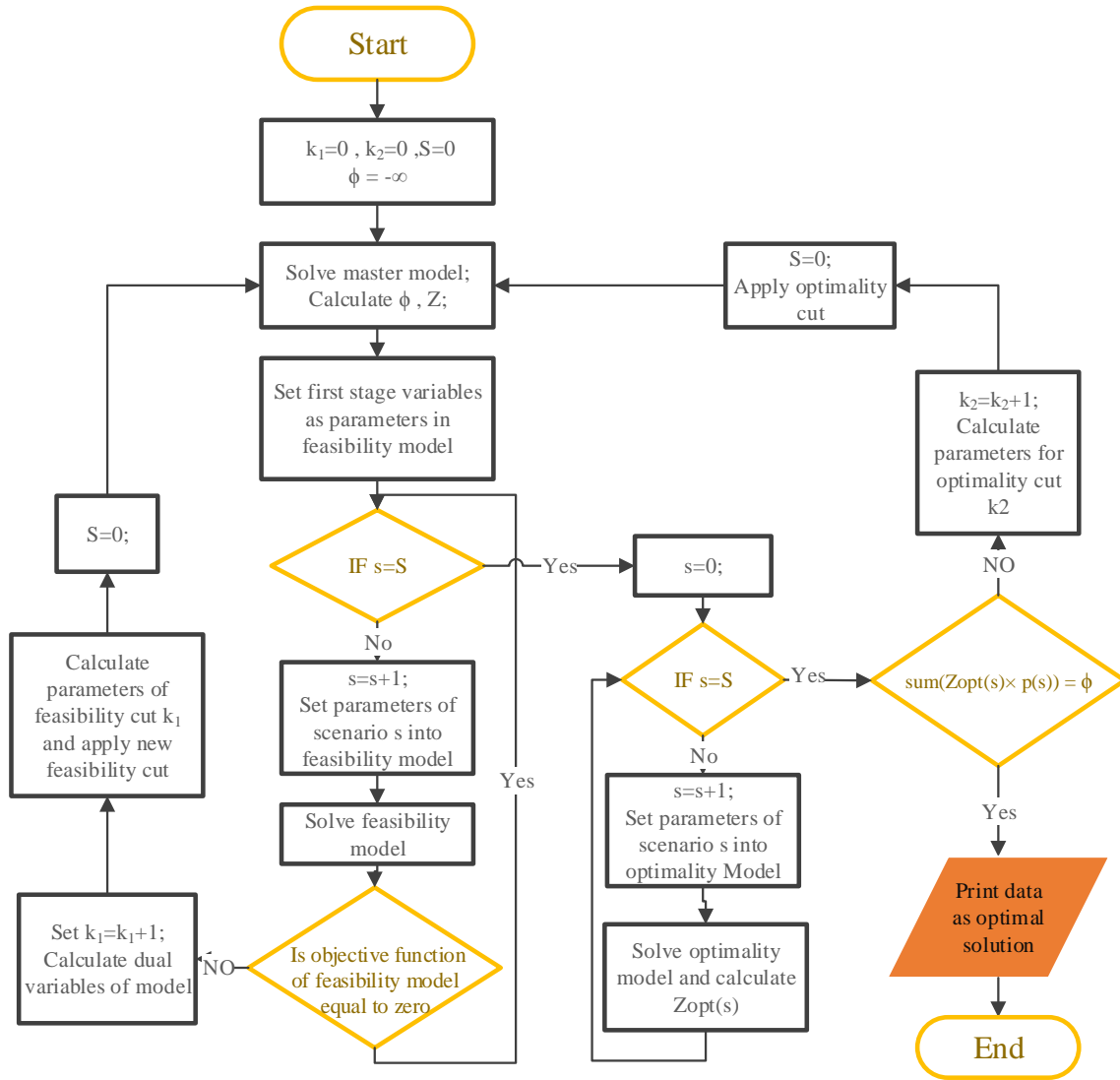


Figure 2. Flowchart of the proposed L-shaped method.

6. Numerical study and managerial insights

The aims of this section is fourfold: 1) to demonstrate the straightforward application of the model discussed in the previous section; 2) to analyze the impact of transshipment on economic and ecological aspects, and study its role in presence of uncertainty, 3) to conduct sensitivity analyses for different parameters and derive managerial insights on the model's application and 4) to show the efficiency of the proposed hybrid L-shaped method.

We consider the case of a typical hospital which is assumed to be supplied by eight drugstores (S_1, S_2, \dots, S_8) that provide five different medicines. A single fleet (O) with a triple

type of special trucks ships the medicines from the drugstores (suppliers) to the hospital in each period. Since the demand for this kind of products are highly variable, the forecasted demand is merely reliable for few periods. The planning time horizon is therefore assumed to be 2 periods and demand follows a pre-specified distribution function. The products are assumed highly perishable with certain expiration dates (two periods). Therefore, the products at the end of the second period should be disposed. For some chemical products such as medicines the disposal procedure (incineration) itself creates GHGs. All computations were run by using GAMS software on a workstation with 3.2 GHz i5 and 4 GB RAM under Win 7.

The information about the capacity and cost rates for the truck types, as well as the rest of the data are summarized in Table 2. The variable transportation costs are estimated based on fuel consumption rates and fuel price. The fixed transportation costs are mainly function of the truck capacity. Since in this study a scenario-based approach is used to represent demand uncertainty, we randomly generate finite scenarios.

Table 2. Vehicles Data.

Vehicle type k	V_k (monetary unit/km)	u_k (monetary unit)	NT_k	cap_k (product unit)	GHG _k (kg/km)
1	8.0	320	4	150	5
2	7.5	700	4	250	12
3	7.0	2000	4	450	30

The travel distances are provided in Table 3. We also assume that the unit inventory holding cost per period is the same for all drugstores, equal to 5, and we assume a unit holding cost of 20, for the hospital. As previously discussed, the medicines may not be stored at the depot. The unit backorder and lost sale shortage costs π , π' and the GHG emission quantity produced by disposed products are provided in Table 4. The GHG emissions for different vehicle types can be determined based on standard measures as presented in Figure 3.

Vehicle type	Shipment weight [ton]	Distance [km]	Transport work [tkm]	CO ₂ [kg]	NO _x [g]	HC [g]	CO [g]	PM [g]
Van petrol	5.0	120,00	600,00	319,20	408,00	114,00	3294,00	<0.01
Van diesel	5.0	120,00	600,00	621,00	1638,00	102,00	570,00	120,00
Small truck	5.0	120,00	600,00	151,20	1728,00	114,00	498,00	42,00
Medium truck	5.0	120,00	600,00	106,20	834,00	36,00	162,00	18,00
Heavy truck	5.0	120,00	600,00	74,40	582,00	24,00	108,00	12,00
Tractor + 'city trailer'	5.0	120,00	600,00	73,80	606,00	24,00	120,00	12,00
Truck + trailer	5.0	120,00	600,00	44,40	366,00	12,00	72,00	6,00
Tractor + semitrailer	5.0	120,00	600,00	37,80	306,00	12,00	60,00	6,00
Tractor + megatrailer	5.0	120,00	600,00	34,80	282,00	12,00	78,00	<0.01
Truck + semitrailer	5.0	120,00	600,00	34,20	276,00	12,00	78,00	6,00
Pick-up petrol	5.0	120,00	600,00	1197,60	930,00	318,00	5748,00	<0.01
Pick-up diesel	5.0	120,00	600,00	1275,60	2838,00	294,00	918,00	342,00

Figure 3. GHG emission levels for different vehicle types (Mirzapour Al-e-hashem et al. (2013)).

Table 5 shows the demand for each medicine, each period and each scenario. 5 different scenarios are generated based on a normal distribution function with parameters $\mu=50, 60, 70, 80, 90$, $\sigma=15, 17, 20, 25, 27$, and occurrence probability of 0.15, 0.20, 0.30, 0.20 and 0.15, respectively.

Table 3. Travel distances (c_{ij}) between nodes (km).

	Depot	S ₁	S ₂	S ₃	S ₄	S ₅	S ₆	S ₇	S ₈	Main store
Depot	0	306	228	162	129	222	249	418	319	322
S ₁	306	0	136	168	203	143	285	114	118	100
S ₂	228	136	0	66	182	189	322	219	92	217
S ₃	162	168	66	0	129	165	280	269	157	224
S ₄	129	203	182	129	0	93	155	317	257	197
S ₅	222	143	189	165	93	0	144	250	234	106
S ₆	249	285	322	280	155	144	0	384	378	218
S ₇	418	114	219	269	317	250	384	0	150	170
S ₈	319	118	92	157	257	234	378	150	0	218
Main store	322	100	217	224	197	106	218	170	218	0

Table 4. Shortage costs and GHG emission quantity produced by disposed product

product type p	1	2	3	4	5	6	7	8
π_p	10	10	20	20	15	10	10	15
π'_p	20	20	40	40	30	20	20	30
G'_p	3	4	5	5	4	3	5	6

Table 5. Demand under different scenarios.

P / Period	Scenario									
	1		2		3		4		5	
	1	2	1	2	1	2	1	2	1	2
1	65	63	91	45	78	71	70	88	76	78
2	35	86	80	130	68	62	61	77	89	81
3	54	120	86	74	68	82	113	76	84	41
4	50	140	65	85	79	103	78	122	95	98
5	135	176	53	133	102	136	105	135	84	121
6	88	80	65	54	79	103	78	102	95	88
7	91	106	53	103	92	106	92	94	80	101
8	88	52	65	45	79	71	78	88	95	78

As shown in Table 6, the test problem is solved for θ , P equal to 1, and the obtained results are reported for the first and second stages costs as well as GHGs. $\theta=1$ is equivalent to the single objective version of the proposed model; that is, the decision maker is not concerned with the environmental challenges.

Table 6. Objective function components for $\theta=1$.

$\theta=1$	First stage		Second stage					E[SSC _s]
	FSC		SSC _s					
$Z_1^* = 23518.6$			1	2	3	4	5	
	10152.8	13881.3	12055.9	13565.4	13815.4	13598.1		13365.8
$Z_2 = 51691.3$	FSG		SSG _s					E[SSG _s]
	24453.4	21590.0	20823.0	32163.1	32163.1	25021.4		27237.9

As shown, the optimum value for the first objective function (Z_1^*) is equal to 23518.6. The worst value for GHGs (Z_2) is also obtained in this state (51691.3). The value for the second stage costs varies based on which scenario takes place, for example if scenario 2 is occurred the second stage costs (SSC_2) that will be incurred is equal to 12055.9, and the corresponding GHGs are therefore equal to 20823.0.

The optimal configuration of the vehicle routes as well as the vehicles types for the first and second stage of the planning horizon are reported in Table 7. For example, $\{2\} 0 < 2 < 4 < 3 < 9$ means that a vehicle type 2 must start from the depot (0) and visit the suppliers 2, 4 and 3, then it ends the trip by going to the main store (9).

Table 7. Solution configuration for $\theta=1$.

First stage	{2} 0 ▶ 4(81 ₄ ↑) ▶ 5(135 ₅ ↑) ▶ 9 {3} 0 ▶ 3(86 ₃ ↑) ▶ 2(74 ₂ ↑) ▶ 8(106 ₈ ↑) ▶ 7 (184 ₇ ↑)▶ 1(91 ₁ ↑,12 ₈ *↓,79 ₇ *↓) ▶ 9	
Second stage	scenario	1 {2} 0 ▶ 3(88 ₃ ↑) ▶ 2(34 ₂ ↑) ▶ 1(37 ₁ ↑,12 ₈ ↑,79 ₇ ↑) ▶ 9 {3} 0 ▶ 4(109 ₄ ↑) ▶ 6(165 ₆ ↑) ▶ 5(165 ₅ ↑) ▶ 9
		2 {3} 0 ▶ 4(69 ₄ ↑) ▶ 5(51 ₅ ↑) ▶ 3(74 ₃ ↑) ▶ 2(136 ₂ ↑) ▶ 1(45 ₁ ↑, 12 ₈ ↑,51 ₇ ↑) ▶ 9
		3 {3} 0 ▶ 4(101 ₄ ↑) ▶ 6(182 ₆ ↑) ▶ 5(103 ₅ ↑) ▶ 9 {3} 0 ▶ 3(64 ₃ ↑) ▶ 2(56 ₃ ↑) ▶ 8(44 ₈ ↑) ▶ 1(58 ₁ ↑, 12 ₈ ↑,79 ₇ ↑) ▶ 9
		4 {3} 0 ▶ 4(119 ₄ ↑) ▶ 6(180 ₆ ↑) ▶ 5(105 ₅ ↑) ▶ 9 {3} 0 ▶ 3(103 ₃ ↑) ▶ 2(64 ₂ ↑) ▶ 8(60 ₈ ↑) ▶ 1(67 ₁ ↑, 12 ₈ ↑,79 ₇ ↑) ▶ 9
		5 {3} 0 ▶ 4(112 ₄ ↑) ▶ 3(39 ₂ ↑) ▶ 2(81 ₂ ↑) ▶ 8(67 ₈ ↑) ▶ 1(63 ₁ ↑, 12 ₈ ↑,76 ₇ ↑) ▶ 9
<p>$\{k\} i(m_p\uparrow) \blacktriangleright j$ means that a truck type k picks up m units of product type p from node i and goes to node j.</p> <p>In case $i \neq p$ ($i(m_p\uparrow) \blacktriangleright j$), it also implies that, this m units is previously transshipped to node i, from node p.</p> <p>$i(m_p^*\downarrow)$ means that m units of product type p is transshipped (unloaded) to node i by the vehicle.</p>		

As shown in Table 7, at the first stage a vehicle type 3 (the largest one) starts its trip from node 0 and continues to suppliers 3, 2, 8, 7 and 1 and picks up 86, 74, 106, 184 and 91 units of their products, respectively. Moreover, the truck transships 12 units of product 8 and 79 units of product 7 to supplier 1 (shown by stars). The trip ends when the truck goes to the main store (node 9) and delivers the pickups. Also a vehicle type 2 visits the suppliers 4 and 5 and picks up 81 and 135 units of the related products.

At the second stage depends on what scenario is occurred, the trips, the vehicle types and the pickups vary accordingly. For instance, in scenario 2 a vehicle type 3 visits the suppliers 4, 5, 3, 2 and 1 and picks up 69, 51, 74, 136 and 45 units of their products, respectively. In addition, the truck picks up 12 and 51 units of products types 8 and 7 from the supplier 1, where these quantities previously stored (transshipped) there during the first period. Therefore, it can be shown that the transshipment is an effective strategy to reduce the total supply chain cost and could be adopted in an uncertain environment to absorb the demand fluctuations.

To show how considering the second objective Z_2 can positively impact the environmental footprint of the solution, let us resolve the test problem by setting $\theta=0.5$. That is, the decision

maker is now concerned with GHG_s as much as the costs. The results are presented in [Tables 8 and 9](#).

Table 8. Objective function components for $\theta=0.5$.

$\theta = 0.5$		Second stage					E[SSC _s]
First stage		SSC _s					
	FSC	1	2	3	4	5	
$Z_1=30189.4$	7915.1	22915.6	19238.9	22904.6	23410.3	22904.6	22274.2
	FSG	SSG _s					E[SSG _s]
$Z_2=13192.7$	10392.8	3936.9	2169.0	1640.4	5187.1	1640.4	2799.9

According to [Table 8](#), as expected, the first objective function, Z_1 , in this state ($\theta = 0.5$) in comparison with that of $\theta = 1$ increases ($Z_1: 23518.6 \rightarrow 30189.4$) while the GHG emission level is decreased ($Z_2: 51691.3 \rightarrow 13192.7$). The GHG emission level at the first stage (FSG) is decreased to 10392.8, while for $\theta=1$ it was 24453.4. This reduction is a result of appropriate selection of vehicle types as well as the environmentally oriented pickups and trips configuration.

As seen in [Table 9](#), in the first stage of current solution ($\theta=0.5$), the bigger but not very fuel-efficient vehicle (vehicle type 3) and vehicle type 1 were used in previous solution ($\theta=1$) to handle distribution issues are replaced by two more fuel-efficient vehicle types (2). The transportation cost therefore remains unchanged because the number of vehicles is the same. Due to the fact that the both new vehicle types are less spacious than type 3, transshipment strategy cannot be useful as much as before. In other words, to take the advantage of transshipment strategy as an option to reduce the total travel distance and GHGs through merging the trips, the truck capacity is an important factor which impacts the transportation's fixed and variable costs. Thus the model tries to make a tradeoff between the amounts of GHGs which could be saved by transshipment option (more practical for bigger vehicles) and the amount saved by using less spacious (and more expensive) vehicles. Besides, more transshipment will lead to more holding costs too. On the other hand, the model attempts to make a tradeoff between the GHGs could be saved by partially cancelling some orders which

entails a high risk of unfulfilled demand and the extra charges must be paid for consequential shortage costs.

Table 9. Solution configuration for $\theta=0.5$.

First stage			{2} 0 ▶ 4(152 ₄ ↑) ▶ 5(105 ₅ ↑, 7 ₄ ↓*) ▶ 9
			{2} 0 ▶ 3(86 ₃ ↑) ▶ 2(20 ₂ ↑) ▶ 8(79 ₈ ↑) ▶ 1(65 ₁ ↑) ▶ 9
Second stage	#scenario	1	{2} 0 ▶ 4(38 ₄ ↑) ▶ 5(205 ₅ ↑, 7 ₄ ↑) ▶ 9
		2	{1} 0 ▶ 3(74 ₃ ↑) ▶ 5(71 ₅ ↑, 5 ₄ ↑) ▶ 9
		3	{2} 0 ▶ 4(30 ₄ ↑) ▶ 5(113 ₅ ↑, 7 ₄ ↑) ▶ 9
		4	{2} 0 ▶ 3(103 ₃ ↑) ▶ 5(135 ₅ ↑, 7 ₄ ↑) ▶ 9
		5	{1} 0 ▶ 4(41 ₄ ↑) ▶ 5(100 ₂ ↑, 7 ₄ ↑) ▶ 9

$\{k\} i(m_p\uparrow) \blacktriangleright j$ means that a truck type k picks up m units of product type p from node i and goes to node j .

In case $i \neq p$ ($i(m_p\uparrow) \blacktriangleright j$), it also implies that, this m unit is previously transshipped to node i , from node p .

$i(m_p^*\downarrow)$ means that m units of product type p is transshipped (unloaded) to node i by the vehicle.

According to Table 9, in the first period, a vehicle type 2 begins its trip from depot (node 0), then visits the supplier 4 and picks up 152 units of product type 4, afterward continues to supplier 5 and picks up 105 units of product type 5 and also takes down (transshipped) 7 units of the product type 4 to store temporary there for one period which will be carried to the main store in succeeding period. On the other hand, another truck type 2 starts its trip from depot and goes to suppliers 3, 2, 8 and 1 and picks up 86, 20, 79 and 65 units of their products, respectively, and finish the trip by going to main store. At the second stage, depends on what scenario takes place, an appropriate solution is prepared. For example, in scenario 4 only one truck of type 2 is arranged to handle transportation issues.

In order to elaborate upon the difference between the solutions under $\theta=1$ and $\theta=0.5$, the problem is solved for a lost sale cost $\hat{\pi} = 100$ and the configurations obtained for the first and second stage (Scenario 3) are illustrated in Figure 4.

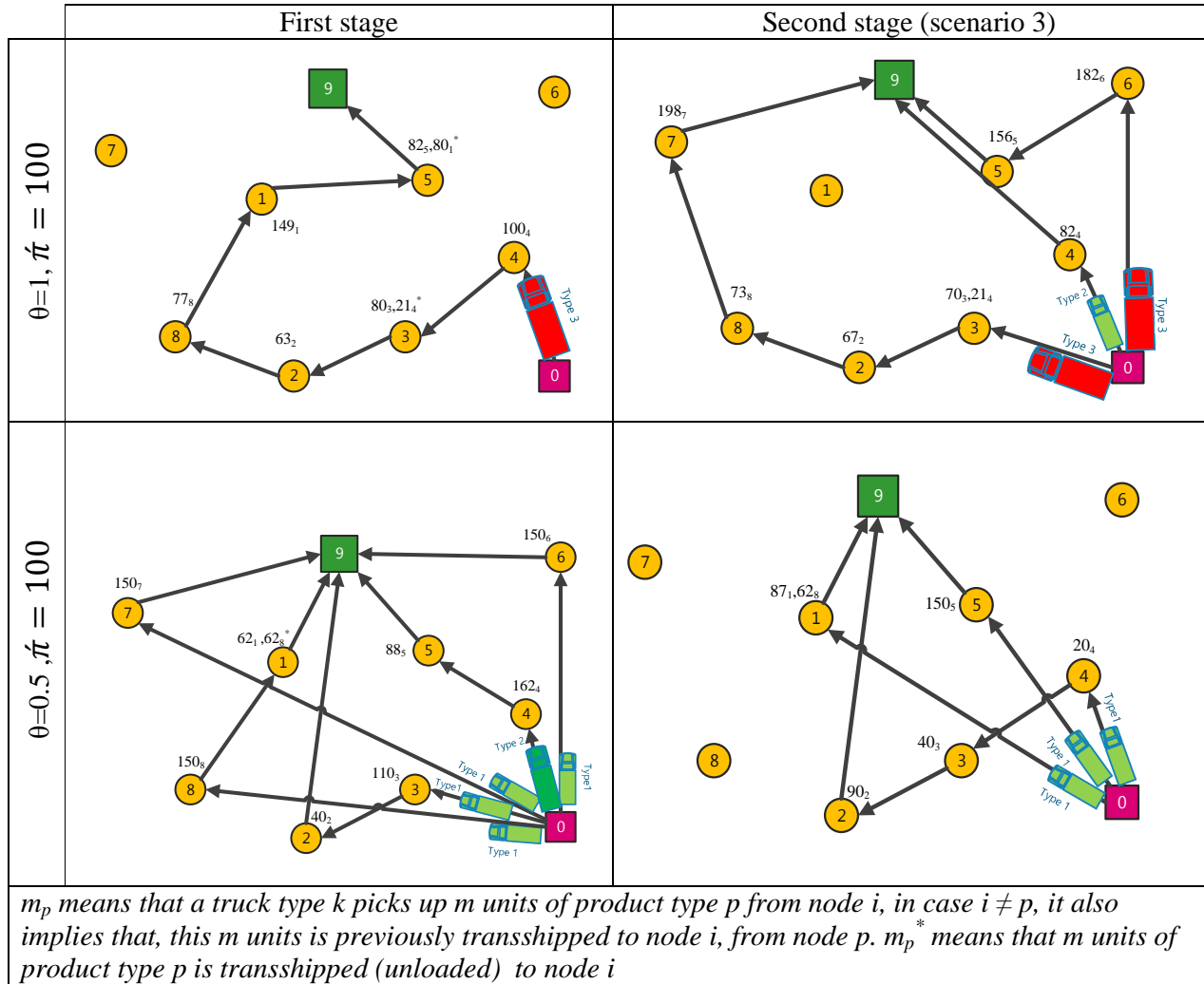


Figure 4. Comparison between the solutions obtained for $\theta=1$ vs. $\theta=0.5$.

As shown in Figure 4, in case of $\theta=1$ the model takes full advantage of the transshipment to cause a considerable saving in trips at the second period where the truck goes directly from node 4 to node 9 and never visit node 1. But as mentioned before since truck type 3 has not a desire GHG index, this solution is not very suitable from the environmental aspect. In case of $\theta=0.5$, the model takes advantage of both transshipment and appropriate vehicle selection, to not only save some trips but also to make use of more fuel-efficient vehicles (vehicles type 1 is more fuel-efficient than types 2 and 3) to reduce the GHGs as well as the total costs.

6.1. Sensitivity analysis

In this section several sensitivity analyses are performed to study how the proposed model makes a trade-off between the greenhouse gas emission levels and the total cost of supply chain especially in the presence of uncertainty; and to investigate when the transshipment option can play its significant role in the supply chain performance while different parameters are varying, including holding cost, shortage cost, disposal cost and even truck's capacity.

We solve the problem by changing the relative weight (θ) from zero to one. The results are depicted in Figure 5. As can be seen in Figure 5, when θ increases, the expected value of the total supply chain cost decreases, and conversely, the expected value of the GHG emission level (produced by the vehicles and scraped products) increases. In spite of the fact that considering "green logistics" ($\theta < 1$) makes a shift up in the total cost of supply chain, a significant reduction takes place in the GHG emission level. In other words, the slope of reduction in GHG level approximately fit the exponential trend while the slope of increasing in the total cost is almost linear.

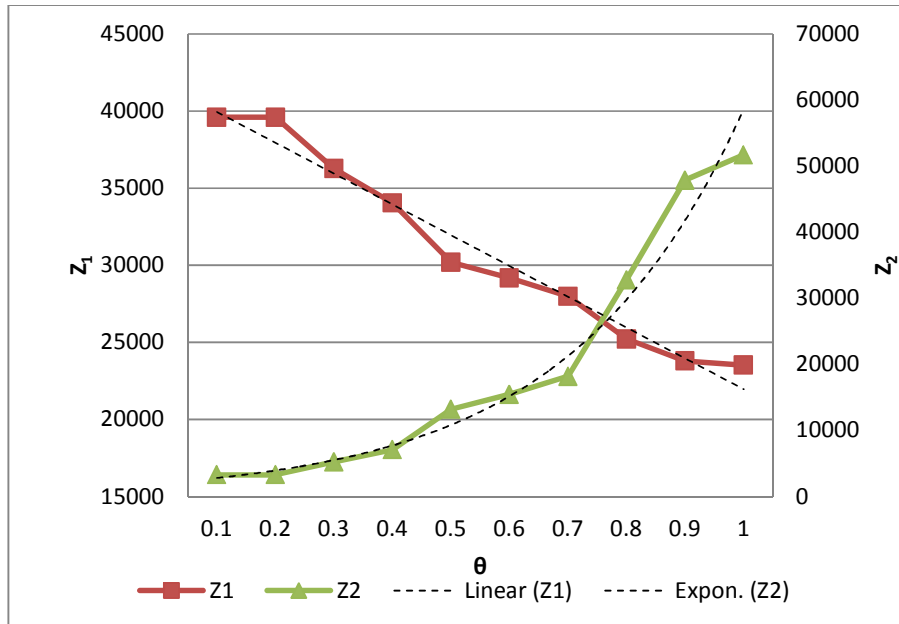


Figure 5. GHG emission level (Z_1) against total cost (Z_2).

This Figure could also be interpreted as a Pareto set where the decision maker could select

the most preferred solutions according to his/her preferences.

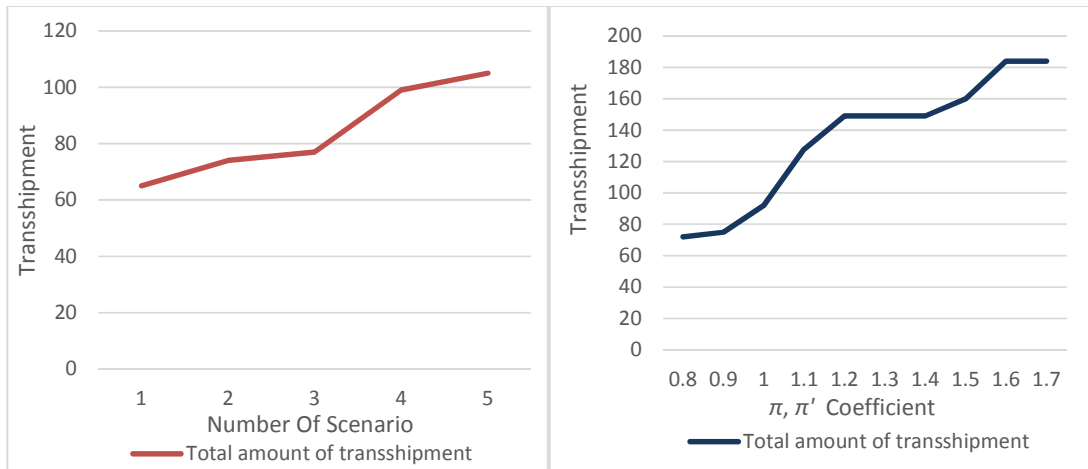
In order to shed more light on the importance of the transshipment option in the presence of uncertain demand, we compare the solutions with and without transshipment option to demonstrate that the transshipment is not only beneficial for the first objective function (total cost) through saving in routes and shortages but also it enables the model to reduce the environmental concerns by creating environmentally-friendly solutions. Thus several numerical examples are generated and solved by using the proposed hybrid L-shaped method to study the impact of transshipment option on the optimal solution in the presence of uncertainty. We generate 10 test problems and solve them twice; with transshipment option ($Z_i^*_{TIRP}$) and without transshipment option ($Z_i^*_{IRP}$). The optimal solutions for the problems with and without transshipment option are then reported for each scenario and the gap between them is calculated by $\Delta i = \frac{Z_i^*_{IRP} - Z_i^*_{TIRP}}{Z_i^*_{IRP}}$. As seen in [Table 10](#), the average, minimum and maximum of Δ_1 are 0.051, 0.011 and 0.088, respectively. With no exception, the transshipment option leads to better solutions with regards the total cost. Similarly, the average, minimum and maximum of Δ_2 are 0.071, -0.03 and 0.173, respectively. Except problem number 8, in other cases the transshipment option has a positive effect on the environmental footprint. For example, in problem 4 the transshipment option causes 5% saving in total cost, and the environmental objective is reduced about 5.2%.

Table 10. *The overall impact of transshipment on objective functions*

#Problem	Transshipment-enabled (TIRP)		Without Transshipment (IRP)		Δ_1	Δ_2
	Z_1	Z_2	Z_1	Z_2		
1	30256.9	13208.1	33166.2	15961.9	0.088	0.173
2	27791.2	24426.6	29036.9	24574.5	0.043	0.006
3	28648.1	12284.5	29507.6	12889.4	0.029	0.047
4	33409.3	29861.9	35152.3	31483.9	0.050	0.052
5	45952.7	33968.8	47700.3	38917.3	0.037	0.127
6	33466.0	38287.6	35936.5	38324.7	0.069	0.001
7	30673.7	21031.8	31983.1	25113.7	0.041	0.163
8	30971.1	27388.4	33858.8	26585.8	0.085	-0.03

9	28761.8	15195.7	29067.7	17614.9	0.011	0.137
10	43409.3	39861.9	46152.3	41483.9	0.059	0.039
average					0.051	0.071

Figure 6-a shows the effect of number of scenarios on total transshipments occurred. In this figure, 5 test problems with different number of scenarios are studied. As seen in Figure 6-a, the total amount of transshipment increases by increasing the number of scenarios. It implies that the transshipment can act as an absorber of uncertainty. The horizontal axis is the number of scenarios and its lowest value is one which represents the deterministic condition with merely one scenario. Moreover, the occurrence probability of all scenarios are assumed the same, for instance in a problem with 5 scenarios the occurrence probability of each scenario is 0.2.



(a) (b)
Figure 6. Transshipment versus the number of scenarios(a) and shortage costs(b)

We then study the importance of transshipment when the shortage is allowed. The unit backorder cost (π) and the unit lost sale cost (π') are both multiplied by a coefficient (that we vary from 0.8 to 1.7) to analyze the transshipment significance for an increasing shortage cost. This coefficient is shown in the horizontal axis of Figure 6-b. As observed in this figure, for increasing shortage penalties, the total transshipped amount increases. In other words, the

model uses transshipment in order to prevent shortage occasions. The trend of increase in transshipment is non-decreasing, and when the shortage cost is nearly doubled, the transshipped quantity is no longer increasing. It implies that the transshipment's benefit is limited by the truck capacity.

We investigate the latter observation and perform a sensitivity analysis to show that the more the capacity of trucks is, the more the use of transshipment policy will be. We multiply the trucks' capacity (Cap_k) by a coefficient illustrated in the horizontal axis of Figure 7. As seen in Figure 7, the truck capacity is an important factor to successfully apply the transshipment policy. We observe that when the trucks' capacity increases, the total amount of transshipment increases until reaching a certain threshold before decreasing. To understand the slope change, it should be noticed that the fixed cost of a truck is directly linked to its capacity. Therefore, the transshipment option is better off with more a capacitated truck until the cost of the latter starts to negatively impact the total cost function. A tradeoff exists between the transshipment opportunity offered by more capacitated trucks and their associated fixed costs.

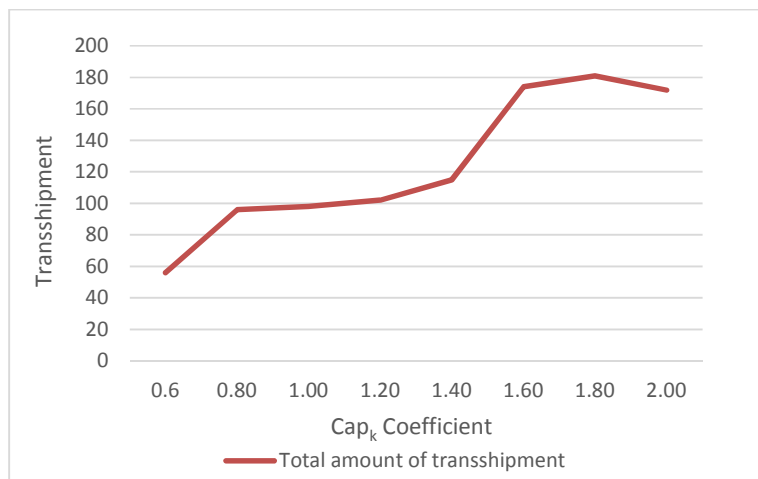


Figure 7. Transshipment versus the trucks' capacity

In Figure 8-a, we analyze the attractiveness of the transshipment option as a function of the

suppliers' holding cost when compared to the factory holding cost. The transshipment can be considered as a key factor to increase the performance of the supply network particularly when the holding cost values are significantly different from one location to another. The factory's unit holding cost is fixed at 10 and the suppliers' unit holding cost increases from 2 to 26 and the ratio of the latter to the former is illustrated in the horizontal axis of [Figure 8-a](#). As seen in [Figure 8-a](#), the results show that by increasing this ratio, the total transshipped quantity decreases up to finally reaching zero at point 2.6. We can therefore conclude that for a supply network where the holding costs in the supplier side are very high when compared to the factory one, the transshipment is not an attractive option.

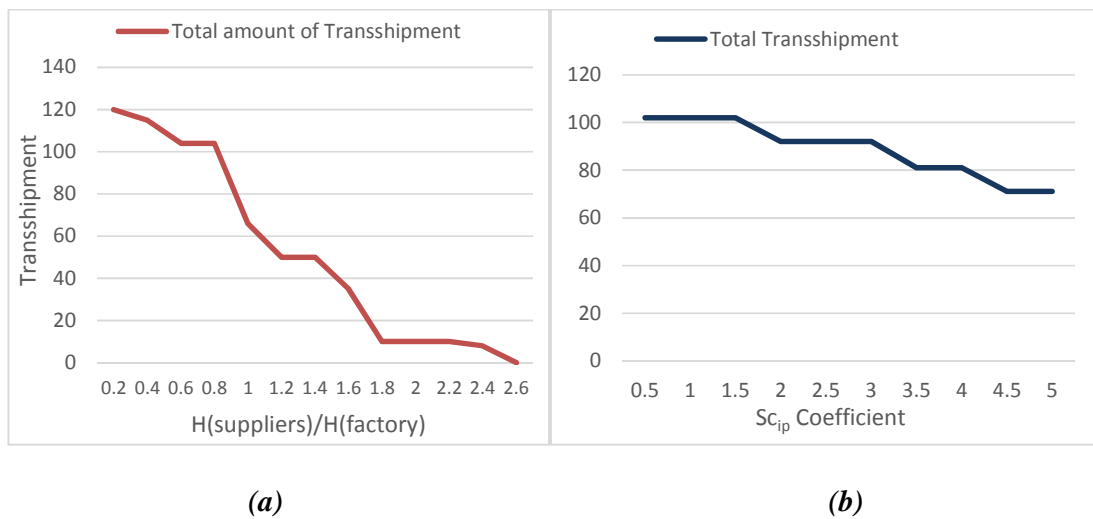


Figure 8. *Transshipment versus the disposal and holding costs*

We finally perform a sensitivity analysis on the disposal cost (Sc_{ip}), to show the attractiveness of the transshipment option when the disposal cost increases. We multiplied the unit disposal cost by a coefficient ranges from 0.5 to 5 as illustrated in the horizontal axis of [Figure 8-b](#). As intuitively expected, when the disposal cost increases, the total transshipment decreases very smoothly, and therefore transshipment is not very sensitive to the disposal cost.

6.2. Consideration of other scenarios

In order to analysis the proposed model in larger scales, ten more test problems with different dimensions are solved. [Table 11](#) reports the optimal solutions obtained for $\theta=1$ and $\theta=0.5$. The first column represents the problem number. The second column represents the dimension of the problem in terms of number of periods, vehicle types, scenarios and nodes, respectively. The next columns compare the two objective functions' components for $\theta=1$ and $\theta=0.5$. Finally, the last two columns compare these two solutions based on economic (Δ_{Z1}) and environmental (Δ_{Z2}) criteria. Δ_{Z1} measures the growing ratio of the expected total costs after applying green concept, and Δ_{Z1} explains the rate of saving in expected total GHG emissions produced over the planning horizon, after taking into account the green concept.

Table 11. Objective function components for $\theta=1$ versus $\theta=0.5$ in different problems.

#P	Dimension T×K×S×N	$\theta=1$		$\theta=0.5$				Δ_{Z1}	$-\Delta_{Z2}$		
		$Z_{1(w=1)}^*$		$Z_{2(w=1)}^*$		$Z_{1(w=0.5)}^*$				$Z_{2(w=0.5)}^*$	
		FSC	E(SSC)	FSG	E(SSG)	FSC	E(SSC)			FSG	E(SSG)
1	2×3×5×5	3125	3305	4800	1668	3440	3823	1550	1164	0.12	0.58
2	2×3×5×7	4645	5021	7341	2794	5411	5586	3100	2230	0.15	0.47
3	2×3×5×9	7673	9909	15306	18691	11104	10631	9481	5237	0.21	0.57
4	2×3×5×11	12178	11154	25362	14632	12253	15428	11086	8116	0.21	0.52
5	2×3×7×13	13335	13467	30333	23887	13789	17987	14667	12443	0.20	0.50
6	2×5×7×15	16005	15552	36118	24445	18270	18957	17990	13998	0.18	0.47
7*	2×5×9×15	15882	16531	48456	26723	18888	19034	19992	19418	0.19	0.48
8*	3**×5×11×15	23456	24987	78891	39044	28761	26273	34957	29898	0.15	0.45
9*	3**×7×11×15	27631	25245	91918	45711	33372	30593	47105	32694	0.21	0.42
10*	5**×7×11×15	45171	41072	155620	74827	50545	50031	88821	41749	0.17	0.43

* the best solution obtained after one and half hour is reported.

** a rolling horizon planning policy is used to solve the problem

T: #periods, K: #vehicles, S: #scenarios, N: #nodes

$$\Delta Z_1 = \frac{Z_1^{\theta=1} - Z_1^{\theta=0.5}}{Z_1^{\theta=1}}$$

$$\Delta Z_2 = \frac{Z_2^{w=\theta} - Z_2^{w=0,\theta}}{Z_2^{w=\theta}}$$

Problems 7, 8, 9 and 10 in [Table 11](#) are the samples for multi-period cases that the proposed framework is applicable by using rolling horizon approach. As [Table 11](#) shows, if companies allow a reduction in the system's profits, they enable to improve their GHG criteria significantly. For example, in problem 7, the supply chain needs to tolerate at least 19% reduction in expected total profits, in order to achieve a 48% reduction in expected total GHGs. This reduction, as discussed in previous section, might be a result of appropriate selection of fuel-efficient vehicles, optimal routes and the ideal levels of inventories,

shortages and transshipments under different scenarios.

As seen in [Table 11](#) the applicability of the proposed model is not limited merely to problems comprising just two periods. That is, it may also be applied to problems referring to more periods using a rolling horizon planning process. Rolling horizon decision making is a common business practice for making decisions in a dynamic stochastic environment ([Sethi and Sorger 1991](#)). There are two principal reasons why rolling horizon might be appropriate for decision-making in uncertain environments. 1) The forecasts for the remote future tend to be unreliable and are, therefore, of inadequate usefulness, 2) for practical reasons, the decisions must be made based on limited information about the future ([Baker 1977](#)). Under this policy, at the end of second period, the demand for the succeeding period would be forecasted. The model therefore could be run by including the new period's data and excluding the unveiled first period one. This procedure repeats every period justifying the term rolling horizon method ([Sazvar et al. 2014](#)). So, we can produce the optimal plan even for multi-period cases. It should also be noted that when the rolling horizon approach is applied, the disposal and the lost-sale shortage may only be considered for the last period, and for internal periods, the inventory is treated to meet the demand of the next period.

7. Conclusions and future research

In this study a novel bi-objective two-stage stochastic programming model is proposed for a stochastic transshipment-enabled IRP. The first objective of the model attempts to find the best configuration of the vehicles, routes, pickups, deliveries and transshipment in order to minimize the total cost of the supply network. The other objective function considers the green logistics and attempts to find the best solution in order to minimize the total GHGs.

We studied that how transshipment can act as a low-cost lever to increase the performance of the supply chain in both economic and environmental aspects. Results showed that when the demand fluctuations are considerable, transshipment option plays a significant role to absorb

the perturbations and help the company to control the environmental concerns by taking advantage of the suppliers' warehouses. Although extra holding costs and double truck loading are incurred when we use transshipment policy, but these extra costs are compensated, to some extent, by saving in travelling distances, and consequently the total associated GHGs produced by the vehicles decrease substantially. Therefore, using transshipment in practice especially when the demand is highly variable is strongly recommended on the ground that the suppliers have a relative degree of harmonization. This policy helps them to achieve coordination in the long term. Then, we introduced a hybrid algorithm composed of an L-shaped method and compromise programming, and finally provided the numerical and sensitivity analyses permitting to show the applicability of the proposed model.

Applying the proposed model to other supply chain structures like many-to-many supply networks, considering other sources of uncertainty such as lead-time and developing other meta-heuristics to efficiently solve the model are some promising areas for further research.

Appendix

In this section the formulations of the master, optimality and feasibility sub-problems of the hybrid L-shaped method are presented.

$$\begin{aligned}
Min Z &= \theta \times \frac{Z_1 - Z_{1min}}{Z_{1max} - Z_{1min}} + (1 - \theta) \times \frac{Z_2 - Z_{2min}}{Z_{2max} - Z_{2min}} \\
Min Z &= \\
&\underbrace{\frac{\theta}{Z_{1max} - Z_{1min}} \times \left(\sum_{i,j \in \Omega} \sum_k v_k c_{ij} XF_{ijk} + \sum_{i \in \omega, k} u_k XF_{oik} + \sum_{i \in \omega, p} h_{ip} TF_{ip} - Z_{1min} \right) + \frac{(1 - \theta)}{Z_{2max} - Z_{2min}} \times \left(\sum_{i,j \in \Omega} \sum_k G_k c_{ij} XF_{ijk} - Z_{2min} \right)}_{\text{First Stage part}} \\
&+ \underbrace{\frac{\theta}{Z_{1max} - Z_{1min}} \times \sum_s \rho_s SSC_s + \frac{(1 - \theta)}{Z_{2max} - Z_{2min}} \times \sum_s \rho_s \left(\sum_{i,j \in \Omega} \sum_k G_k c_{ij} XS_{ijks} + \sum_{i \in \omega \cup F} \sum_p G'_p IS_{ips} \right)}_{\text{Second Stage Part}}
\end{aligned}$$

In order to incorporate feasibility and optimality cuts into the master problem, we need to declare further notations, such as coefficient for first stage variables and right hand sides in

feasibility and optimality cuts. According to the main model formulation, first stage variables which impact the second stage part of the model are QF and TF, therefore in construction of optimality and feasibility cuts these variables would take coefficients, and coefficients for other first stage variables would be zero. In the formulation of the feasibility problem, auxiliary variables would be also needed only in constraints that contain first stage variables (i.e. QF and TF).

L-Shaped Notations

Ψ	Auxiliary variable for Master Problem (Representing second stage part of objective function)
$OC TF_{ip}^{k_2}$	Coefficient of variable TF_{ip} in optimality cut k_2 .
OCR_{k_2}	Right hand side of optimality cut k_2 .
$OC QF_{ijpk}^{k_2}$	Coefficient of variable QF_{ijpk} in optimality cut k_2 .
$FCTF_{ip}^{k_1}$	Coefficient of variable TF_{ip} in feasibility cut k_1 .
FCR_{k_1}	Right hand side of feasibility cut k_1 .
$FC QF_{ijpk}^{k_1}$	Coefficient of variable QF_{ijpk} in feasibility cut k_1 .
TF_{ip}^*	value of variable TF_{ip} as parameter in optimality and feasibility problems
QF_{ijpk}^*	value of variable QF_{ijpk} as parameter in optimality and feasibility problems
$\mu_{k_1}^{(F-\#)}$	Dual value of constraint (F-#) in feasibility problem in constructing feasibility cut k_1 # = {1,2,8}
$\mu_{k_2}^{(O-\#)}$	Dual value of constraint (O-#) in optimality problem in constructing optimality cut k_2 . # = {2,3,9}
$P1_{ps}, N1_{ps}$	
$P2_{ips}, N2_{ips}$	Auxiliary variables for feasibility problem.
$N3_{ips}$	

1- Master Problem

$$\begin{aligned} \text{Min } Z_{\text{master}} &= \frac{\theta}{Z_{1\max} - Z_{1\min}} \times \left(\sum_{i,j \in \Omega} \sum_k v_k c_{ij} X F_{ijk} + \sum_{i \in \omega, k} u_k X F_{oik} + \sum_{i \in \omega, p} h_{ip} T F_{ip} - Z_{1\min} \right) \\ &+ \frac{(1-\theta)}{Z_{2\max} - Z_{2\min}} \times \left(\sum_{i,j \in \Omega} \sum_k G_k c_{ij} X F_{ijk} - Z_{2\min} \right) + \varphi \end{aligned}$$

Subject to:

$$\sum_{i \in \omega} \sum_{j \in \omega} \sum_k \sum_p FCQF_{ijpk}^{k_1} \times QF_{ijpk} + \sum_{i \in \omega, p} FCTF_{ip}^{k_1} \times TF_{ip} \geq FCR_{k_1} \quad \forall k_1 \in \kappa_1 \quad (M-1)$$

$$\sum_{i \in \omega} \sum_{j \in \omega} \sum_k \sum_p OCQF_{ijpk}^{k_2} \times QF_{ijpk} + \sum_{i \in \omega, p} OCTF_{ip}^{k_2} \times TF_{ip} + \varphi \geq OCR_{k_2} \quad \forall k_2 \in \kappa_2 \quad (M-2)$$

$$\sum_{j \in \Omega} XF_{ijk} = \sum_{j \in \Omega} XF_{jik} = YF_{ik} \quad \forall i \in \omega, k \quad (M-3)$$

$$\sum_k YF_{ik} \leq 1, \forall i \in \omega \quad (M-4)$$

$$\sum_p QF_{ijpk} \leq cap_k XF_{ijk} \quad \forall i, j \in \Omega, k \quad (M-5)$$

$$\sum_p TF_{ip} \leq icap_i \quad \forall i \in \omega \quad (M-6)$$

$$PF_{ip} = 0 \quad \forall i \in \omega, p \neq i \quad (M-7)$$

$$TF_{ip} = 0 \quad \forall i \in \omega, p = i \quad (M-8)$$

$$\sum_{i \in \omega} XF_{0ik} \leq NT_k \quad \forall k \quad (M-9)$$

$$\sum_{i \in \omega, k} XF_{0ik} \geq 1 \quad (M-10)$$

$$\sum_{i \in \omega, k} XF_{i(N+1)k} \geq 1 \quad (M-11)$$

$$XF_{i0k} = 0 \quad \forall i \in \omega, k \quad (M-12)$$

$$XF_{(N+1)ik} = 0 \quad \forall i \in \omega, k \quad (M-13)$$

$$XF_{ik} = 0 \quad \forall i \in \Omega, k \quad (M-14)$$

$$XF_{0(N+1)k} = 0 \quad \forall k \quad (M-15)$$

$$QF_{0ipk} = 0 \quad \forall i \in \omega, p, k \quad (M-16)$$

$$QF_{ijpk}, PF_{ipk}, TF_{ipk} \geq 0, \text{integer} \quad (M-17)$$

$$XF_{ijk}, YF_{ik} \in \{0, 1\} \quad \forall i, j \in \Omega, p, k$$

2- Feasibility Problem

$$\text{Min } Zfe(s) = \sum_{p \in \Phi} \sum_{s \in \xi} (P1_{ps} + N1_{ps}) + \sum_{i \in \omega} \sum_{p \in \Phi} \sum_{s \in \xi} (P2_{ips} + N2_{ips} + N3_{ips})$$

Subject to:

$$IF_{ps} - BF_{ps} = \sum_{i \in \omega, k} QF_{i(N+1)pk}^* - DF_{ps} + P1_{ps} - N1_{ps} \quad \forall p, s \quad (F-1)$$

$$IS_{ips} = TF_{ip}^* - PS_{ips} + P2_{ips} - N2_{ips} \quad \forall i \in \omega, p \neq i, s \quad (F-2)$$

$$IS_{(N+1)ps} - BS_{ps} = IF_{ps} - BF_{ps} + \sum_{i \in \omega, k} QS_{i(N+1)pk} - DS_{ps} \quad \forall p, s \quad (F-3)$$

$$\sum_{j \in \Omega} XS_{ijks} = \sum_{j \in \Omega} XS_{jiks} = YS_{iks} \quad \forall i \in \omega, k, s \quad (F-4)$$

$$\sum_k YS_{iks} \leq 1, \forall i \in \omega, s \quad (\text{F-5})$$

$$\sum_{j \in \omega \cup O, k} QS_{jipks} + PS_{ips} = \sum_{j \in \omega \cup F, k} QS_{jipks} \quad \forall i \in \omega, p, s \quad (\text{F-6})$$

$$\sum_p QS_{jipks} \leq cap_k XS_{ijks} \quad \forall i, j \in \Omega, k, s \quad (\text{F-7})$$

$$PS_{ips} \leq TF_{ip}^* - N\mathcal{B}_{ips} \quad \forall i \in \omega, p \neq i, s \quad (\text{F-8})$$

$$\sum_{i \in \omega} XS_{0iks} \leq NT_k \quad \forall k, s \quad (\text{F-9})$$

$$\sum_{i \in \omega, k} XS_{0iks} \geq 1 \quad \forall s \quad (\text{F-10})$$

$$\sum_{i \in \omega, k} XS_{i(N+1)ks} \geq 1 \quad \forall s \quad (\text{F-11})$$

$$XS_{i0ks} = 0 \quad \forall i \in \omega, k, s \quad (\text{F-12})$$

$$XS_{(N+1)ik} = 0 \quad \forall i \in \omega, k, s \quad (\text{F-13})$$

$$XS_{iiks} = 0 \quad \forall i \in \Omega, k, s \quad (\text{F-14})$$

$$XS_{0(N+1)ks} = 0 \quad \forall k, s \quad (\text{F-15})$$

$$QS_{0ipks} = 0 \quad \forall i \in \omega, p, k, s \quad (\text{F-16})$$

$$QS_{jipks}, PS_{ipks} \geq 0, \text{integer} \quad (\text{F-17})$$

3- Optimality Problem

$$\text{Min } Z_{opt}(s) = \frac{\theta}{Z_{1\max} - Z_{1\min}} \times \sum_s SSC_s + \frac{(1-\theta)}{Z_{2\max} - Z_{2\min}} \times \sum_s \left(\sum_{i,j \in \Omega} \sum_k G_k c_{ij} XS_{ijks} + \sum_{i \in \omega \cup F} \sum_p G'_p IS_{ips} \right)$$

Where:

$$\begin{aligned} SSC_s = & \sum_{i,j \in \Omega} \sum_k v_k c_{ij} XS_{ijks} + \sum_{i \in \omega, k} u_k XS_{0iks} + \sum_p h_{(N+1)p} IF_{ps} + \\ & \sum_{i \in \omega \cup F} \sum_p (sc_{ip} IS_{ips} + \pi_p BF_{ps} + \pi'_p BS_{ps}) \end{aligned} \quad (\text{O-1})$$

Subject to:

$$IF_{ps} - BF_{ps} = \sum_{i \in \omega, k} QF_{i(N+1)pk}^* - DF_{ps} \quad \forall p, s \quad (\text{O-2})$$

$$IS_{ips} = TF_{ip}^* - PS_{ips} \quad \forall i \in \omega, p \neq i, s \quad (\text{O-3})$$

$$IS_{(N+1)ps} - BS_{ps} = IF_{ps} - BF_{ps} + \sum_{i \in \omega, k} QS_{i(N+1)pk} - DS_{ps} \quad \forall p, s \quad (O-4)$$

$$\sum_{j \in \Omega} XS_{ijks} = \sum_{j \in \Omega} XS_{jiks} = YS_{iks} \quad \forall i \in \omega, k, s \quad (O-5)$$

$$\sum_k YS_{iks} \leq 1, \forall i \in \omega, s \quad (O-6)$$

$$\sum_{j \in \omega \cup O, k} QS_{ijpk} + PS_{ips} = \sum_{j \in \omega \cup F, k} QS_{ijpk} \quad \forall i \in \omega, p, s \quad (O-7)$$

$$\sum_p QS_{ijpk} \leq cap_k XS_{ijks} \quad \forall i, j \in \Omega, k, s \quad (O-8)$$

$$PS_{ips} \leq TF_{ip}^* \quad \forall i \in \omega, p \neq i, s \quad (O-9)$$

$$\sum_{i \in \omega} XS_{oiks} \leq NT_k \quad \forall k, s \quad (O-10)$$

$$\sum_{i \in \omega, k} XS_{oiks} \geq 1 \quad \forall s \quad (O-11)$$

$$\sum_{i \in \omega, k} XS_{i(N+1)ks} \geq 1 \quad \forall s \quad (O-12)$$

$$XS_{ioks} = 0 \quad \forall i \in \omega, k, s \quad (O-13)$$

$$XS_{(N+1)ik} = 0 \quad \forall i \in \omega, k, s \quad (O-14)$$

$$XS_{iiks} = 0 \quad \forall i \in \Omega, k, s \quad (O-15)$$

$$XS_{0(N+1)ks} = 0 \quad \forall k, s \quad (O-16)$$

$$QS_{oipks} = 0 \quad \forall i \in \omega, p, k, s \quad (O-17)$$

$$QS_{ijpk}, PS_{ipks} \geq 0, \text{integer} \quad (O-18)$$

$$YS_{iks}, XF_{ijk}, XS_{ijks} \in \{0, 1\} \quad \forall i, j \in \Omega, p, k, s$$

4- Feasibility and Optimality Cuts

In order to calculate parameters of feasibility cut, k_1 , we use the following equations;

$$FCQF_{ijpk}^{k_1} = \mu_{k_1}^{(F-1)} \quad (C-1)$$

$$FCTF_{ip}^{k_1} = (\mu_{k_1}^{(F-2)} + \mu_{k_1}^{(F-8)}) \quad (C-2)$$

$$FCR_{k_1} = Zfe^{k_1} - \sum_{i, j \in \omega, k, p} FCQF_{ijpk}^{k_1} \times QF_{ijpk}^{*k_1} - \sum_{i \in \omega, p} FCTF_{ip}^{k_1} \times TF_{ip}^{*k_1} \quad (C-3)$$

Similarly, for parameters of optimality cut, k_2 , we have following equations:

$$OCQF_{ijpk}^{k_2} = \sum_{s \in S} p_s \times \mu_{k_2}^{(O-2)} \quad (C-4)$$

$$OCTF_{ip}^{k_2} = \sum_{s \in S} p_s \times (\mu_{k_2}^{(O-3)} + \mu_{k_2}^{(O-9)}) \quad (C-5)$$

$$OCR_{k_2} = \sum_{s \in S} p_s \times Zopt_s^{k_2} - \sum_{i,j \in \omega, k, p} OCQF_{ijpk}^{k_2} \times QF_{ijpk}^{*k_2} - \sum_{i \in \omega, p} OCTF_{ip}^{k_2} \times TF_{ip}^{*k_2} \quad (C-6)$$

References

- Abdelmaguid, T.F., and Dessouky, M.M. (2006). A genetic algorithm approach to the integrated inventory-distribution problem. *International Journal of Production Research*, 44(21), 4445-4464.
- Abdelmaguid, T.F., Dessouky, M.M., & Ordóñez, F. (2009). Heuristic approaches for the inventory-routing problem with backlogging. *Computers & Industrial Engineering*, 56(4), 1519-1534.
- Al Shamsi, A., Al Raisi, A., Aftab, M., (2014). Pollution-inventory routing problem with perishable goods, in: Golinska, P. (Ed.), *Logistics Operations, Supply Chain Management and Sustainability*. Springer International Publishing. EcoProduction, pp. 585-596.
- Alkawaleet, N., Hsieh, Y.F., Wang, Y., (2014). Inventory routing problem with CO2 emissions consideration, in: Golinska, P. (Ed.), *Logistics Operations, Supply Chain Management and Sustainability*. Springer International Publishing. EcoProduction, pp. 611-619.
- Andersson, H., Ho, A., Christiansen, M., Hasle, G., & Likketangen, A. (2010). Industrial aspects and literature survey: Combined inventory management and routing. *Computers & Operations Research*, 37(9), 1515 -1536.
- Archetti, C., Bertazzi, L., Laporte, G., & Speranza, M.G. (2007). A branch-and-cut algorithm for a vendor-managed inventory-routing problem. *Transportation Science*, 41(3), 382-391.
- Archetti, C., Speranza, M.G., & Hertz A. (2006). Complexity a tabu search algorithm for the split delivery vehicle routing problem. *Transportation Science*, 40, 64-73.
- Baitaa, F., Ukovicha, W., Pesentib, R., & Favarettoc, D. (1998). Dynamic routing-and-inventory problems: a review. *Transportation Research Part A: Policy and Practice*, 32(8), 585-598.
- Baker, K.R. (1977). An experimental study of rolling schedules in production planning. *Decision Science*, 8, 19-27.
- Bell, W.J., Dalberto, L.M., Fisher, M.L., et al. (1983). Improving the distribution of industrial gases with an on-line computerized routing and scheduling optimizer. *Interfaces*, 13(6), 4-23.
- Bertazzi, L., Bosco, A., Guerriero, F., & Laganà, D. (2011). A stochastic inventory routing problem with stock-out. *Transportation Research Part C*, doi:10.1016/j.trc.2011.06.003.
- Bertazzi, L., & Speranza, M.G. (2002). Continuous and discrete shipping strategies for the single link problem. *Transportation Science*, 36(3), 314-325.
- Bozorgi-Amiri, A., Jabalameli, M. S., & Mirzapour Al-e-Hashem, S. M. (2011). A multi-objective robust stochastic programming model for disaster relief logistics under uncertainty. *OR Spectrum*, 1-29.
- Campbell, A. M., & Savelsbergh, M. W. (2004). A decomposition approach for the inventory-routing problem. *Transportation Science*, 38(4), 488-502.
- Cheng, C., Qi, M., Wang, X., & Zhang, Y. (2016). Multi-period inventory routing problem under carbon emission regulations. *International Journal of Production Economics*, 182, 263-275.
- Chien, T. W., Balakrishnan, A. & Wong, R. T. (1989). An integrated inventory allocation and vehicle routing problem. *Transportation Science*, 23(2) 67-76.

- Chitsaz, M., Divsalar, A., & Vansteenwegen, P. (2016). A two-phase algorithm for the cyclic inventory routing problem. *European Journal of Operational Research*, 254 (2), 410-426.
- Coelho, L. C., Cordeau, J. F., & Laporte, G. (2012). The inventory-routing problem with transshipment. *Computers & Operations Research*, 39(11), 2537–2548.
- Coelho, L. C., Cordeau, J. F., & Laporte, G. (2014). Thirty years of Inventory Routing. *Transportation Science*, 48(1), 1-19.
- Coelho, & Laporte, G. (2014). Optimal joint replenishment, delivery and inventory management policies for perishable products. *Computers & Operations Research*, 47, 42–52.
- Cordeau, J.F., Laporte, G., Savelsbergh, M.W.P., & Vigo, D. (2007). Vehicle routing. In C Barnhart and G Laporte, editors, *Transportation*, North-Holland, Amsterdam. 367-428.
- Chrysochoou, E. & Ziliaskopoulos, A. (2015). Stochastic Inventory Routing Problem with Transshipment Recourse Action. 10th Conference on Stochastic Models of Manufacturing and Service Operations.
- Gupta, A., Maranas, C.D. (2003). Managing demand uncertainty in supply chain planning. *Computers and Chemical Engineering*, 27, 1219-1227.
- Federgruen, A., & Zipkin, P. (1984). A combined vehicle routing and inventory allocation problem. *Operations Research*, 32(5) 1019–1037.
- Hauge, K., Larsen, J., Lusby, R. M., & Krapper, E. (2014). A hybrid column generation approach for an industrial waste collection routing problem. *Computers & Industrial Engineering*, 71, 10–20.
- Huang, S.H., & Lin, P.C. (2010). A modified ant colony optimization algorithm for multi-item inventory routing problems with demand uncertainty. *Transportation Research Part E*, 46, 598–611.
- Hvattum, L. M., Løkketangen, A., & Laporte, G. (2009). Scenario tree-based heuristics for stochastic inventory-routing problems. *INFORMS Journal on Computing*, 21(2), 268-285.
- Iassinovskaia, G., Limbourg, S., & Riane, F. (2017). The inventory-routing problem of returnable transport items with time windows and simultaneous pickup and delivery in closed-loop supply chains. *International Journal of Production Economics*, 183 (Part B), 570-582.
- Jemai, Z., Rekik, Y., & Kalai, R. (2012). Inventory routing problems in a context of vendor-managed inventory system with consignment stock and transshipment. *Production Planning & Control: The Management of Operations*, DOI:10.1080/09537287.2012.666844.
- Kall, P., & Wallace, S. W. (1994). Stochastic programming. New York, NY: Wiley.
- Kleywegt, A.J., Nori, V.S., & Savelsbergh, M.W.P. (2002). The stochastic inventory routing problem with direct deliveries. *Transportation Science*, 36(1), 94-118.
- Kleywegt, A.J., Nori, V.S., & Savelsbergh, M.W.P. (2004). Dynamic programming approximations for a stochastic inventory routing problem. *Transportation Science*; 38(1), 42-70.
- Liu, S.C., & Lee, W.T. (2011). A heuristic method for the inventory routing problem with time windows. *Expert Systems with Applications*, 38, 13223–13231.
- Louveaux, F., Birge, J. R., (2009). L-shaped Method for Two-stage Stochastic Programs with Recourse. *Encyclopedia of Optimization*. Second Edition, 1943-1945.
- Mirzapour Al-e-Hashem, S.M.J., Aryanezhad, M.B., & Sadjadi, S.J. (2012). An efficient algorithm to

- solve a multi-objective robust aggregate production planning in an uncertain environment. *Int. J. Advanced Manufacturing Technology*, 58(5-8), 765-782.
- Mirzapour Al-e-hashem, S.M.J., Baboli, A., Sazvar, Z. (2013). A stochastic aggregate production planning model in a green supply chain: Considering flexible lead times, nonlinear purchase and shortage cost functions. *European Journal of Operational Research*, 230(1), 26-41.
- Mirzapour Al-e-hashem, S.M.J., Malekli, H., & Aryanezhad, M.B. (2011). A multi objective robust aggregate production planning in a supply chain under uncertainty. *Int. J. Production Economic*, 134 (1), 28–42.
- Mirzapour Al-e-hashem, S. M. J., & Rekik, Y. (2014). Multi-product multi-period Inventory Routing Problem with a transshipment option: A green approach. *Int. J. Production Economic*, 157, 80–88.
- Mirzaei, S., & Seifi, A. (2015). Considering lost sale in inventory routing problems for perishable goods. *Computers & Industrial Engineering*, 87, 213–227.
- Mishra, B.K., & Raghunathan, S. (2004). Retailer- vs. vendor-managed inventory and brand competition. *Management Science*, 50(4), 445-457.
- Moin, N.H., Salhi, S., & Aziz, N.A.B. (2011). An efficient hybrid genetic algorithm for the multi-product multi-period inventory routing problem. *Int. J. Production Economics*, 133, 334–343.
- Nonas, L.M., & Jornsten, K. (2005). Heuristics in the multi-location inventory system with transshipments. In H Kotzab, S Seuring, M Muller, & G Reiner, editors, *Research Methodologies in Supply Chains Management*, Physica-Verlag, Heidelberg. 509-524.
- Nonas, L.M., & Jornsten, K. (2007). Optimal solution in the multi-location inventory system with transshipments. *Journal of Mathematical Modelling and Algorithms*, 6(1), 47-75.
- Persson, J.A., & Gothe-Lundgren, M. (2005). Shipment planning at oil refineries using column generation and valid inequalities. *European Journal of Operational Research*, 163(3), 631-652.
- Sahinidis, N.V. (2004). Optimization under uncertainty: state-of-the-art and opportunities. *Computers and Chemical Engineering*, 28(6-7), 971-983.
- Sazvar, Z., Mirzapour Al-e-hashem, S.M.J., Baboli, A., & Akbari-Jokar, M.R. (2014). A bi-objective stochastic programming model for a centralized green supply chain with deteriorating products. *International Journal of Production Economics*, 150, 140-154.
- Sethi, S., & Sorger, G. (1991). A theory of rolling horizon decision making. *Annals of Operations Research*, 29(1), 387–415.
- Shen, Q., Chu, F., & Chen, H. (2011). A Lagrangian relaxation approach for a multi-mode inventory routing problem with transshipment in crude oil transportation. *Computers and Chemical Engineering*, 35, 2113– 2123.
- Shukla, N., Tiwari, M. K., & Ceglarek, D. (2013). Genetic-algorithms-based algorithm portfolio for inventory routing problem with stochastic demand. *International Journal of Production Research*, 51(1), 118-137.
- Sindhuchao, S., Romeijn, H.E., Akcali, E., & Boondiskulchok, R. (2005). An integrated inventory-routing system for multi-item joint replenishment with limited vehicle capacity. *Journal of Global*

- Optimization*, 32, 93-118.
- Solyali, O., Cordeau, J. F., & Laporte, G. (2012). Robust inventory routing under demand uncertainty. *Transportation Science*, 46(3), 327-340.
- Soysal, M., Bloemhof-Ruwaard, J. M., Haijema, R., & van der Vorst, J. G. A. J. (2015). Modeling an Inventory Routing Problem for perishable products with environmental considerations and demand uncertainty. *International Journal of Production Economics*, 164, 118–133.
- Soysal, M., Bloemhof-Ruwaard, J. M., Haijema, R., & van der Vorst, J. G. (2016). Modeling a green inventory routing problem for perishable products with horizontal collaboration. *Computers & Operations Research*, in press.
- Soysal, M. (2016). Closed-loop Inventory Routing Problem for returnable transport items. *Transportation Research Part D: Transport and Environment*, 48, 31-45.
- Speranza, M.G., & Ukovich, W. (1994). Minimizing transportation and inventory costs for several products on a single link. *Operations Research*, 42 (5), 879-894.
- Yu, Y., Chen, H., & Chu, F. (2008). A new model and hybrid approach for large scale inventory routing problems. *European Journal of Operational Research*, 189(3), 1022-1040.