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Learning about Inventory Inaccuracies: a Judgment-based Inspection and Inventory Strategy

Selma KHADER¹, Yacine REKIK^{2,a}, Valérie BOTTA-GENOULAZ^{2,b}, Jean-Pierre CAMPAGNE^{2,b}

¹. Aix Marseille Univ, CRET-LOG, 13621 Aix-en-Provence, France

^{2,a}. EM Lyon Business school, DISP EA 4570, 69134 Ecully, France

^{2,b}. Université de Lyon, INSA-Lyon, DISP EA 4570, 69621 Villeurbanne, France

(selma.khader@univ-amu.fr, rekik@em-lyon.com, valerie.botta@insa-lyon.fr, jean-pierre.campagne@orange.fr)

Abstract. It is commonly believed that the information flow always accurately reflects the physical flow of stock. In reality, a discrepancy between the quantity shown by the information system and the physical stock actually available for sales may exist. The paper extends the inventory inaccuracy literature by providing a mechanism permitting to learn about the errors and to judgmentally decide if an inspection of the inventory system is needed to align the physical and information stock levels. Such a judgment-based strategy where the inventory manager decides to perform an inspection or not is compared with a free error strategy. The latter assumes that an advanced identification technology such as the RFID technology is deployed and provides visibility on both the physical and information flows. We deduce analytical results for each strategy and then cross-compare them, which affords us managerial insights on the way to learn about error and the most cost-effective way to tackle the inventory inaccuracy issue.

Keyword: inventory inaccuracy, learning, inspection policy, RFID technology

1. Introduction

In common belief, both the physical flow of products that pass through an inventory system and the associated information flow are free of defects. However, various factors may create a difference between the two flows, and disrupt their synchronized evolution and thus the supply chain performance. Such differences, also known as inventory inaccuracy, may be a major obstacle to performance improvements for the supply chain [1]. The aim of the paper is to extend the classical way to tackle the inventory inaccuracy issue. The vast majority of investigations assumes the scenario where errors distributions are given and then integrated in the profit function leading to inventory policies depending on the errors parameters [2, 3, 4, 5, 6, 1, 7]. Another stream of research investigations proposes an inspection based inventory policy where periodically an inspection is performed in order to align the PH (physical) and IS (information system) stock levels [8, 9]. A last stream of research considers the use of advanced identification technologies (as RFID) to decrease or eliminate the inaccuracy issue [10, 11, 12].

Our contribution to the mature inventory inaccuracy literature concerns, learning about errors and judgmentally decide whether a physical inventory is needed or not. The learning contribution is capital in managing a stock system subject to inaccuracies, particularly because the latter are generally hidden and are accumulate over time. Despite the fact that replenishment decisions may often be subject to judgmental intervention, the effect of judgmentally adjusting replenishment quantities and auditing decision has not attracted much attention in the academic literature, either in modelling or in empirical terms [13].

For this purpose, we develop and compare two strategies. Under the first strategy, we assume that the inventory manager decides to inspect or not, before committing quantities to the customers, based on the demand he receives. Such a strategy will be referred as the Wait & See policy since the inspection decision is postponed until the demand realization becomes known. In the second strategy, the inventory manager could tackle the inaccuracy issue by deploying RFID, which could provide visibility on physical and information flows throughout the entire supply chain.

We develop analytically the optimal inventory policies associated with the two strategies described previously under a single period setting for an e-retailer. We extend them to the multi-period setting in order to derive managerial insights about the judgment in the presence of inaccuracies and the RFID contribution, and to provide a mechanism for learning about error distributions. Learning about errors is a capital task to manage a system subject to inaccuracies. To the best of our knowledge, only Pergamalis

[14] proposed a statistical mechanism to measure the inaccuracy issue. With the Wait & See policy, we furthermore provide an easy to deploy, trustful mechanism to learn about IS and PH errors. The paper is organized as follows: in Section 2, we describe the sequence of events assumed under the single- and the multi-period settings. Section 3 is dedicated to the analytical analysis of the two strategies to cope with the inaccuracy issue. Section 4 extends the analysis to the multi-period setting with a focus on the RFID contribution and on the mechanism used to learn about the error distributions. Finally, Section 5 concludes the paper.

2. Sequence of events and notations

Assuming that the initial IS inventory level is I , we consider the following events during each selling period:

1. Let Y be the order-up-to level decided, based on an estimation of the random demand.
2. After receiving goods from the supply system, the inventory manager updates the information system by scanning products, and then stores them in the warehouse. Because of errors, $Y_{PH} = \gamma_{PH}Y$, i.e. what is physically available in the warehouse, may not be equal to $Y_{IS} = \gamma_{IS}Y$, i.e. what the information system shows as being available for sales where γ_j is a random variable characterizing errors $j = \{IS, PH\}$.
3. During the selling period, the inventory manager receives the cumulative online orders from the final customers. D denotes the demand received remotely from the customers. The inventory manager compares D with the IS inventory record Y_{IS} in order to accept or decline orders. If D is lower than Y_{IS} , he accepts all the orders. Otherwise, he accepts only orders amounting to the IS inventory record. Because of the e-retailing / wholesaling context, demand satisfaction is achieved based on the IS level, and a commitment, $Commitment = Min(D, Y_{IS})$, is performed during the selling period and before shipping the products to the customers.
4. At the end of the selling period, products are delivered to the customers by a collective shipment. All the orders that the e-retailer has committed to should in principle be honored. However, this may not always be the case, due to inventory inaccuracies. Such a situation occurs when the demand is higher than Y_{IS} and Y_{IS} is higher than Y_{PH} . The quantity delivered to the customer is written as $Sales = Min(Commitment, Y_{PH})$

The following notations will be used in developing the two strategies to tackle the inaccuracy issue:

- D_i : the random variable representing demand in period i
- $f_{D_i}(F_{D_i})$: the PDF (CDF respectively) characterizing D_i
- μ_{D_i} : the average of D_i
- σ_{D_i} : the standard deviation of D_i
- γ_{ISi} (resp. γ_{PHi}): the random variable representing IS (PH) errors in period i
- μ_{ISi} (resp. μ_{PHi}): the average of γ_{ISi} (resp. γ_{PHi})
- σ_{ISi} (resp. σ_{PHi}): the standard deviation of γ_{ISi} (resp. γ_{PHi})
- f_{ISi} (resp. F_{ISi}): the PDF (CDF respectively) characterizing γ_{ISi}
- f_{PHi} (resp. F_{PHi}): the PDF (CDF respectively) characterizing γ_{PHi}
- I_i : the inventory level before ordering at the beginning of period i
- r : the unit selling price
- c : the unit purchase cost
- s : the unit salvage cost
- $h = (c - s)$: the unit overstock cost

- $u_1 = (r - c)$: the unit type 1 shortage cost
 K : the unit cost paid for a non-satisfied commitment
 $u_2 = (r - c) + K$: the unit type 2 shortage cost
 $Insp$: the fixed inspection cost
 Y_i : the ordering up quantity in period i
 t : the unit RFID tag cost

3. The single-period analysis

The aim of this section is to analyze the single-period setting. The optimal ordering policy for the two strategies will be derived and interesting mathematical properties concerning the Wait & See policy will be deduced to permit the extension to the multi-period setting. For a better mathematical representation of the single period results, the index i associated with the period number will be removed in this section.

3.1 Analysis of a judgement-based strategy: a “Wait & See” policy

The Wait & See policy consists in postponing the decision to make a commitment or to inspect the inventory level when the information about the customers’ demands is collected. In other words, after the reception of all the customers’ demands, the inventory manager has to decide whether an inspection would be optimal, or it would be better to take the risk of commitment non-satisfaction. If the demand realization (denoted by D_R) becomes known, the profit achieved if no inspection is performed is written as follows (1):

$$\begin{aligned}
 Profit_{s1,DR} &= (r - c)D_R + cI \\
 &- (r - c)[D_R - Y_{IS}]^+ \\
 &- (r - c + K) \left\{ \begin{array}{l} (Y_{IS} - Y_{PH}) \\ -Min[(Y_{IS} - D_R)^+, (Y_{IS} - Y_{PH})] \end{array} \right\} \\
 &+ (c - s)Min[(Y_{IS} - D_R)^+, (Y_{IS} - Y_{PH})] \\
 &- (c - s)[Y_{IS} - D_R]^+
 \end{aligned} \tag{1}$$

The first term of equation (1) represents the sales, the second represents the cost paid when the IS level is not enough to satisfy the real demand, the third represents the cost paid when the PH level is not enough to satisfy the commitment, and the fourth and fifth represent the overstock cost.

By applying the expectation, Equation (1) could be written as Equation (2). In the case where an inspection is decided once the demand realization becomes known, the profit function is written as Equation (3), which could be rewritten as Equation (4) by applying the expectation while respecting the PH error random variable. As mentioned earlier, the inventory manager waits to get the information about the demand realization before deciding about the inspection. The action against errors is decided before committing: inspection or no inspection, depending on the demand value. In such a case, his ordering up quantity decided at the beginning of the selling period does not take into consideration the inaccuracy issue since this issue is tackled at the end of the selling period. In such a case, the inventory manager applies the error-free – i.e. the newsvendor – optimal ordering up quantity as a decision

$$\text{variable: } Y_0 = F_D^{-1} \left[\frac{u_1}{u_1 + h} \right].$$

$$\begin{aligned}
 Profit_{S1_{DR}}(Y, I) &= u_1 D_R \left(1 - \left(F_{IS} \left(\frac{D_R}{Y} \right) - F_{IS}(0) \right) \right) + h D_R \left(1 - F_{IS} \left(\frac{D_R}{Y} \right) \right) \\
 &+ u_1 Y \left(\mu_{IS} \left(F_{IS} \left(\frac{D_R}{Y} \right) - F_{IS}(0) \right) - \sigma_{IS}^2 \left(f_{IS} \left(\frac{D_R}{Y} \right) - f_{IS}(0) \right) \right) \\
 &- h Y \left(\mu_{IS} \left(1 - F_{IS} \left(\frac{D_R}{Y} \right) \right) + \sigma_{IS}^2 f_{IS} \left(\frac{D_R}{Y} \right) \right) \\
 &- u_2 Y (\mu_{IS} - \mu_{PH}) + cI \\
 &+ (u_2 + h) \left(\int_{\gamma_{PH}=0}^{+\infty} \left(\begin{aligned} &(\mu_{IS} - \gamma_{PH})(F_{IS}(\gamma_{PH}) - F_{IS}(0)) \\ & - \sigma_{IS}^2 (f_{IS}(\gamma_{PH}) - f_{IS}(0)) \end{aligned} \right) Y f_{PH}(\gamma_{PH}) d\gamma_{PH} \right. \\
 &+ \int_{\gamma_{IS}=\frac{D_R}{Y}}^{+\infty} \left(\begin{aligned} &(\gamma_{IS} - \mu_{PH}) Y \left(F_{PH}(\gamma_{IS}) - F_{PH} \left(\frac{D_R}{Y} \right) \right) \\ & + \sigma_{PH}^2 Y \left(f_{PH}(\gamma_{IS}) - f_{PH} \left(\frac{D_R}{Y} \right) \right) \end{aligned} \right) f_{IS}(\gamma_{IS}) d\gamma_{IS} \\
 &\left. + \left(F_{PH} \left(\frac{D_R}{Y} \right) - F_{PH}(0) \right) \left(\begin{aligned} &(\mu_{IS} Y - D_R) \left(1 - F_{IS} \left(\frac{D_R}{Y} \right) \right) \\ & + \sigma_{IS}^2 Y f_{IS} \left(\frac{D_R}{Y} \right) \end{aligned} \right) \right)
 \end{aligned} \tag{2}$$

$$Profit_{S2_{DR}} = (r-s) D_R - (c-s) Y_{PH} - (r-s) [D_R - Y_{PH}]^+ - Insp + cI \tag{3}$$

$$Profit_{S2_{DR}}(Y, I) = (u_1 + h) D_R - h \mu_{PH} Y - Insp - (u_1 + h) \int_{\gamma_{PH}=0}^{\frac{D_R}{Y}} (D_R - \gamma_{PH} Y) f_{PH}(\gamma_{PH}) d\gamma_{PH} + cI \tag{4}$$

Then, based on the demand value, he acts against errors by inspecting or by taking the risk of a commitment non-satisfaction. So, in order to decide between inspection or commitment non-satisfaction, the inventory manager has to compare $Profit_{S1_{DR}}(Y_0)$ and $Profit_{S2_{DR}}(Y_0)$. In this case, a critical value of the demand realization, a threshold denoted by D_s , may exist and makes possible to choose the best action against errors (inspecting or taking the risk of a commitment non-satisfaction). The expression of D_s which solves $Profit_{S1_{DR}}(Y_0) = Profit_{S2_{DR}}(Y_0)$ has to satisfy the following Equation (5) if it exists:

$$\left(\begin{aligned} &u_1 D_s \left(1 - \left(F_{IS} \left(\frac{D_s}{Y_0} \right) - F_{IS}(0) \right) \right) + h D_s \left(1 - F_{IS} \left(\frac{D_s}{Y_0} \right) \right) \\ &+ u_1 Y_0 \left(\mu_{IS} \left(F_{IS} \left(\frac{D_s}{Y_0} \right) - F_{IS}(0) \right) - \sigma_{IS}^2 \left(f_{IS} \left(\frac{D_s}{Y_0} \right) - f_{IS}(0) \right) \right) \\ &- h Y_0 \left(\mu_{IS} \left(1 - F_{IS} \left(\frac{D_s}{Y_0} \right) \right) + \sigma_{IS}^2 f_{IS} \left(\frac{D_s}{Y_0} \right) \right) \\ &- u_2 Y_0 (\mu_{IS} - \mu_{PH}) \\ &+ (u_2 + h) \left(\int_{\gamma_{PH}=0}^{+\infty} \left(\begin{aligned} &(\mu_{IS} - \gamma_{PH})(F_{IS}(\gamma_{PH}) - F_{IS}(0)) \\ & - \sigma_{IS}^2 (f_{IS}(\gamma_{PH}) - f_{IS}(0)) \end{aligned} \right) Y_0 f_{PH}(\gamma_{PH}) d\gamma_{PH} \right. \\ &+ \int_{\gamma_{IS}=\frac{D_s}{Y_0}}^{+\infty} \left(\begin{aligned} &(\gamma_{IS} - \mu_{PH}) Y_0 \left(F_{PH}(\gamma_{IS}) - F_{PH} \left(\frac{D_s}{Y_0} \right) \right) \\ & + \sigma_{PH}^2 Y_0 \left(f_{PH}(\gamma_{IS}) - f_{PH} \left(\frac{D_s}{Y_0} \right) \right) \end{aligned} \right) f_{IS}(\gamma_{IS}) d\gamma_{IS} \\ &\left. + \left(F_{PH} \left(\frac{D_s}{Y_0} \right) - F_{PH}(0) \right) \left(\begin{aligned} &(\mu_{IS} Y_0 - D_s) \left(1 - F_{IS} \left(\frac{D_s}{Y_0} \right) \right) \\ & + \sigma_{IS}^2 Y_0 f_{IS} \left(\frac{D_s}{Y_0} \right) \end{aligned} \right) \right) \\ &- (u_1 + h) D_s + h \mu_{PH} Y_0 + Insp + (u_1 + h) \int_{\gamma_{PH}=0}^{\frac{D_s}{Y_0}} (D_s - \gamma_{PH} Y_0) f_{PH}(\gamma_{PH}) d\gamma_{PH} \end{aligned} \right) = 0 \tag{5}$$

To study Equation (5) in-depth, we define the function $X_2(D)$, Equation (6):

$$\begin{aligned}
 X_2(D) = & u_1 D \left(1 - \left(F_{IS} \left(\frac{D}{Y_0} \right) - F_{IS}(0) \right) \right) + h D \left(1 - F_{IS} \left(\frac{D}{Y_0} \right) \right) \\
 & + u_1 Y_0 \left(\mu_{IS} \left(F_{IS} \left(\frac{D}{Y_0} \right) - F_{IS}(0) \right) - \sigma_{IS}^2 \left(f_{IS} \left(\frac{D}{Y_0} \right) - f_{IS}(0) \right) \right) \\
 & - h Y_0 \left(\mu_{IS} \left(1 - F_{IS} \left(\frac{D}{Y_0} \right) \right) + \sigma_{IS}^2 f_{IS} \left(\frac{D}{Y_0} \right) \right) \\
 & - u_2 Y_0 (\mu_{IS} - \mu_{PH}) \\
 & + (u_2 + h) \left(\int_{\gamma_{PH}=0}^{+\infty} \left(\begin{aligned} & (\mu_{IS} - \gamma_{PH})(F_{IS}(\gamma_{PH}) - F_{IS}(0)) \\ & - \sigma_{IS}^2 (f_{IS}(\gamma_{PH}) - f_{IS}(0)) \end{aligned} \right) Y_0 f_{PH}(\gamma_{PH}) d\gamma_{PH} \right. \\
 & \left. + \int_{\gamma_{IS}=\frac{D}{Y_0}}^{+\infty} \left(\begin{aligned} & (\gamma_{IS} - \mu_{PH}) Y_0 \left(F_{PH}(\gamma_{IS}) - F_{PH} \left(\frac{D}{Y_0} \right) \right) \\ & + \sigma_{PH}^2 Y_0 \left(f_{PH}(\gamma_{IS}) - f_{PH} \left(\frac{D}{Y_0} \right) \right) \end{aligned} \right) f_{IS}(\gamma_{IS}) d\gamma_{IS} \right. \\
 & \left. + \left(F_{PH} \left(\frac{D}{Y_0} \right) - F_{PH}(0) \right) \left(\begin{aligned} & (\mu_{IS} Y_0 - D) \left(1 - F_{IS} \left(\frac{D}{Y_0} \right) \right) \\ & + \sigma_{IS}^2 Y_0 f_{IS} \left(\frac{D}{Y_0} \right) \end{aligned} \right) \right) \\
 & - (u_1 + h) D + h \mu_{PH} Y_0 + Insp + (u_1 + h) \int_{\gamma_{PH}=0}^{\frac{D}{Y_0}} (D - \gamma_{PH} Y_0) f_{PH}(\gamma_{PH}) d\gamma_{PH}
 \end{aligned} \tag{6}$$

The first derivative of $X_2(D)$ is written as follows:

$$\begin{aligned}
 \frac{\partial X_2}{\partial D}(D) = & -u_1 \left(F_{IS} \left(\frac{D}{Y_0} \right) - F_{IS}(0) \right) - h F_{IS} \left(\frac{D}{Y_0} \right) \\
 & + \left(F_{PH} \left(\frac{D}{Y_0} \right) - F_{PH}(0) \right) \left(\begin{aligned} & (u_2 + h) \left(F_{IS} \left(\frac{D}{Y_0} \right) - 1 \right) \\ & + (u_1 + h) \end{aligned} \right)
 \end{aligned} \tag{7}$$

A comprehensive numerical study of the last equation performed for different values of the unit costs and the errors' characteristics (cf. Table 1) permits us to verify that $\frac{\partial X_2}{\partial D}(D) \leq 0$ and consequently to assume the property that $X_2(D)$ is a decreasing function.

Table 1: Usual values of the input parameters where $\frac{\partial X_2}{\partial D}(D) \leq 0$ is validated.

h	u_1	u_2	μ_{IS}	μ_{PH}	σ_{IS}	σ_{PH}
[1, 10]	[0.1, 20]	$[u_1, 2u_1]$	[0.9, 1.7]	[0.9, 1.7]	[0.02, 0.17]	[0.02, 0.17]

The limits of $X_2(D)$ when D goes to zero and to infinity are given as follows (8) and (9):

$$\begin{aligned}
 Cws_0 = \lim_{D \rightarrow 0} X_2(D) = & -(u_2 + h) Y_0 (\mu_{IS} - \mu_{PH}) + Insp \\
 & + (u_2 + h) \int_{\gamma_{PH}=0}^{+\infty} \left(\begin{aligned} & (\mu_{IS} - \gamma_{PH})(F_{IS}(\gamma_{PH}) - F_{IS}(0)) \\ & - \sigma_{IS}^2 (f_{IS}(\gamma_{PH}) - f_{IS}(0)) \end{aligned} \right) Y_0 f_{PH}(\gamma_{PH}) d\gamma_{PH} \\
 & + (u_2 + h) \int_{\gamma_{IS}=0}^{+\infty} \left(\begin{aligned} & (\gamma_{IS} - \mu_{PH}) Y_0 (F_{PH}(\gamma_{IS}) - F_{PH}(0)) \\ & + \sigma_{PH}^2 Y_0 (f_{PH}(\gamma_{IS}) - f_{PH}(0)) \end{aligned} \right) f_{IS}(\gamma_{IS}) d\gamma_{IS}
 \end{aligned} \tag{8}$$

$$Cws_{+\infty} = \lim_{D \rightarrow +\infty} X_2(D) = (u_1 - u_2)Y_0(\mu_{IS} - \mu_{PH}) + Insp + (u_2 + h) \int_{\gamma_{PH}=0}^{+\infty} \left(\begin{array}{l} (\mu_{IS} - \gamma_{PH})(F_{IS}(\gamma_{PH}) - F_{IS}(0)) \\ -\sigma_{IS}^2 (f_{IS}(\gamma_{PH}) - f_{IS}(0)) \end{array} \right) Y_0 f_{PH}(\gamma_{PH}) d\gamma_{PH} \quad (9)$$

Depending on the sign of $Cws_0 * Cws_{+\infty}$, one could derive results about the existence of the critical D_S value:

- If $Cws_0 * Cws_{+\infty} \leq 0$, D_S exists and its value is unique.
- If $Cws_0 * Cws_{+\infty} \geq 0$, it means that $Profit_{S1_{D_R}}(Y_0)$ is always higher than $Profit_{S2_{D_R}}(Y_0)$ or vice versa.

Numerical analysis: In order to provide a comprehensive numerical study, we consider two cost structures for the rest of the paper: products with a high (respectively low) margin, assuming that the shortage penalties are greater (respectively lower) than the overstock unit cost. Table 2 provides the values of the different costs:

Table 2: High margin and low margin product cost structures.

High Margin	Low Margin
$c = 2$	$c = 2$
$s = 1$	$s = 1$
$r = 20$	$r = 2.5$

For a normally distributed demand with an average $\mu_D = 20$ and a standard deviation $\sigma_D = 4$, we illustrate the behavior of $Profit_{S1_{D_R}} - Profit_{S2_{D_R}}(Y_0)$ as a function of the realized demand D_R under the high and the low margin structures (see Figure 1).

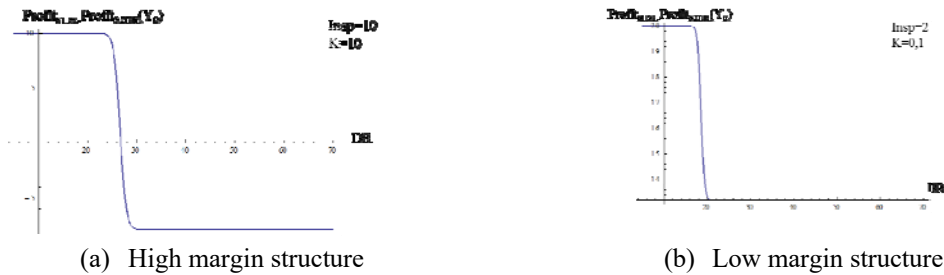


Figure 1. $Profit_{S1_{D_R}} - Profit_{S2_{D_R}}(Y_0)$ as a function of D_R

As illustrated in Figure 1, the value of D_S which solves “ $Profit_{S1_{D_R}} - Profit_{S2_{D_R}}(Y_0) = 0$ ” equal to zero, depends on the values of the inspection and commitment non-satisfaction costs. In the case of a low margin (Figure 1. b), $Profit_{S1_{D_R}}(Y_0)$ is always higher than $Profit_{S2_{D_R}}(Y_0)$. Therefore, the manager should make the commitment without inspection because the cost of a non-satisfied commitment is low compared to that of inspection. However, for the high margin structure, when the realized demand is high, the inventory manager has to be careful about errors and has to perform an inspection because the cost of a non-satisfied commitment is important.

3.2 Analysis of Strategy 2: the RFID-enabled strategy

The aim of Strategy 2 is to analyze the case where an advanced identification technology, such as RFID, is applied. In this regard, the RFID technology can have two direct impacts on the inventory inaccuracy issue. i) It offers a visibility in real time on the actual PH inventory level. ii) It could prevent, if not

eliminate, certain sources of inaccuracy such as theft, deterioration or misplacement. In this case, the RFID technology affects positively the distribution of the random variables describing errors, by decreasing their averages and variability.

Deploying the RFID technology is accompanied by additional fixed and variable costs: we take into consideration only the variable costs, in which only the cost of the RFID tag denoted by t is included. The fixed cost integration could be done thanks to additional Return On Investment studies. We also assume an optimistic RFID deployment scenario where errors are fully eliminated thanks to the RFID tag. When the errors are eliminated, the inventory levels are set to $Y_{IS} = Y_{PH} = Y_{S_3}$, and the expected profit is as follows (10):

$$\begin{aligned} Profit_{S_3} &= rMin[D, Y_{S_3}] + s[Y_{S_3} - D]^+ - (c+t)(Y_{S_3} - I) \\ &= u_1' \mu_D + (u_1 + h) \left[(\mu_D - Y_{S_3}) F_D(Y_{S_3}) - \sigma_D^2 f_D(Y_{S_3}) \right] - u_1' (\mu_D - Y_{S_3}) \end{aligned} \quad (10)$$

with $u_1' = u_1 - t$

The optimal ordering-up quantity corresponds to the optimal solution of the classic model (without error):

$$Y_{S_3}^* = F_D^{-1} \left(\frac{u_1'}{u_1 + h} \right) \quad (11)$$

4. The multi-period analysis

The aim of this section is twofold. i) To extend both the Wait & See and the RFID policies to the multi-period setting, and to compare them throughout a numerical analysis in order to derive the conditions under which the RFID deployment is cost effective. ii) To provide a learning mechanism enabling the inventory manager to learn about both IS and PH errors, and to consider them within the Wait & See policy.

The extension from the single to the multi-period setting is a straightforward exercise since the design of the Wait & See and the RFID policies ensures that a myopic policy is an optimal one for both of them. In fact, under the Wait & See policy, the inventory manager sets the order-up-to level equal to the error-free order-up-to level, and decides either to perform an inspection or to take the risk of a u_2 penalty (i.e. in case of no inspection). His decision to inspect the stock or not is based on his observation of the actual demand he receives. Consequently, the order-up-to level is logically assumed always higher than the end of each period inventory level if the demand is positive. If the final stock is higher than the starting one during a period, the inventory manager could conclude that a transaction problem occurred, and consequently should check his information system. A myopic policy where the inventory manager replenishes up to the error-free order-up-to level, and decides later to perform an inspection or not, is consequently an optimal ordering strategy for the Wait & See multi-period setting.

If we denote by I_i the inventory level just before ordering at the beginning of period i , the inventory system dynamic could be described as follows: the inventory level at the beginning of period i depends on the action decided in period $i-1$

$$I_i = \begin{cases} \left[\gamma_{IS(i-1)} Y - D(i-1)_R \right]^+ & \text{if } D(i-1)_R \leq D(i-1)_S : \\ \text{No inspection was done in period } i-1 & \\ \left[\gamma_{PH(i-1)} Y - D(i-1)_R \right]^+ & \text{if } D(i-1)_R \geq D(i-1)_S : \\ \text{An inspection was done in period } i-1 & \end{cases}$$

In the case of no-inspection in period $i-1$, the inventory level shown in the IS corresponds to the IS level during period $i-1$ (which is subject to IS errors) minus the actual sales of period $i-1$. In the case where an inspection is performed in period $i-1$, the inventory level shown at the beginning of period i , corresponds to the actual physical stock during period $i-1$ minus the sales of this period. The order-up-to level Y is known since it is equal to the error-free order-up-to level under the Wait & See policy. The demand received in period $i-1$ is also known. Therefore, by contrasting the values of the order-up-to level and the demand $D(i-1)_R$ with the value of the stock level, I_i , shown in the inventory record (IS) at

the beginning of period i , the inventory manager is able to learn about the IS error realization during period $i-1$, $\gamma_{IS(i-1)}$. Similarly, when an inspection is performed, the inventory manager is able to learn about the PH error distribution, since a value about $\gamma_{PH(i-1)}$ could be collected. In other words, the inventory manager is able to build the distribution of both IS and PH errors under a multi-period setting if the actual IS and PH inventory levels are contrasted with the values expected to be found. His estimation of both errors is improved over time when more data are collected. The IS error distribution could be built quickly since data can be collected in each period. The learning process about PH errors is slower since error data can be collected only if an inspection is decided.

4.1 Comparison between the Wait & See and the RFID-enabled policies

We assume a numerical situation where the e-retailer faces a normally distributed weekly demand with an average equal to $\mu_{Di} = 20$ and a standard deviation of $\sigma_{Di} = 4$. We also set the parameters of the IS and PH errors which are assumed to be normally distributed with the following parameters ($\mu_{ISi} = 1$, $\sigma_{ISi} = 0.02$, $\mu_{PHi} = 1$ and $\sigma_{PHi} = 0.02$). Regarding the unit cost values, we assume that the unit purchase cost and the unit overstock cost equal $c=2$ and $h=1$ respectively. In order to provide reliable managerial insights, we simulate 22 scenarios by changing the parameters each time: the two shortage costs, the inspection cost and the RFID tag cost. Table 3 represents the values used in our numerical study:

Table 3: Different values of input values scenarios.

Scenario	u_1	u_2	<i>Insp</i>	t
Scenario1	5	10	5	0.1
Scenario2	5	10	10	0.3
Scenario3	5	10	20	0.5
Scenario4	5	20	5	0.1
Scenario5	5	20	10	0.3
Scenario6	5	20	20	0.5
Scenario7	5	30	5	0.1
Scenario8	5	30	10	0.3
Scenario9	5	30	20	0.5
Scenario10	10	20	5	0.1
Scenario11	10	20	10	0.3

Scenario	u_1	u_2	<i>Insp</i>	t
Scenario12	10	20	20	0.5
Scenario13	10	30	5	0.05
Scenario14	10	30	5	0.1
Scenario15	10	30	10	0.3
Scenario16	10	30	20	0.5
Scenario17	20	60	30	0.05
Scenario18	20	40	30	0.05
Scenario19	20	50	20	0.05
Scenario20	20	60	60	0.05
Scenario21	0.5	0.8	1	0.1
Scenario22	0.5	0.8	3	0.1

To be statistically significant, each scenario is studied with a set of 30 generated values of the vector $(\gamma_{IS(i)}, \gamma_{PH(i)}, D_{(i)})$ for $1 \leq i \leq 52$ (number of periods - weeks). The average profit (over the 30 simulations) is then used. Table 4 presents the results of our simulation:

Table 4: Different values of profit under scenarios.

Scenario	Wait & See policy: total profit	RFID deployment: total profit
enario1	5248.4	5165.29
Scenario2	5248.4	4904.46
Scenario3	5248.4	4643.85
Scenario4	5242.38	5165.29
Scenario5	5193.33	4904.46
Scenario6	5193.33	4643.85
Scenario7	5229.86	5165.29
Scenario8	5138.26	4904.46
Scenario9	5138.26	4643.85
Scenario10	10544.	10457.3
Scenario11	10531.6	10205.9

Scenario	Wait & See policy: total profit	RFID deployment: total profit
Scenario12	10531.6	9955.61
Scenario13	10500.7	10520.4
Scenario14	10540.3	10457.3
Scenario15	10500.7	10205.9
Scenario16	10500.7	9955.61
Scenario17	20968.2	20998.9
Scenario18	20996.3	20998.9
Scenario19	20982.3	20998.9
Scenario20	20968.2	20998.9
Scenario21	219.699	60.432
Scenario22	219.699	60.432

As intuitively expected, the parameter inputs heavily influence the choice of deploying the RFID (Table 4). For instance, we note that for a product with a low margin (scenarios 21 and 22), the RFID strategy is not profitable because $h \geq u_1 \text{ and } u_2$. It is better to manage the errors with the Wait & See policy than by investing in a new technology that increases the purchase cost of the product. In contrast, for a product with a high margin, the RFID deployment is profitable when the shortage costs are very high (scenarios 13, 17, 18 and 19). Under these scenarios, a non-satisfied commitment is highly penalizing and consequently the RFID alternative becomes an interesting solution to tackle the inaccuracy issue. Concerning the Wait & See policy, we note that the type 2 shortage and the inspection costs strongly affect its adoption.

4.2 A learning mechanism about the IS and PH errors

An important issue when dealing with inaccuracy errors is the way the inventory manager estimates their distributions. The Wait & See policy enables one to collect data to build the errors' distribution. In each period, the IS errors could be derived by contrasting the actual and the assumed IS levels:

$$\gamma_{IS(i)} = \frac{I_{(i+1)} + D(i)_R}{Y_0} \text{ in case of no inspection.}$$

Similarly, when an inspection is performed, contrasting the actual and the assumed PH levels serves to build the PH error distribution.

In the following, we propose to test the performance of this learning process on IS and PH errors, through a managerial study. For each period, we set the actual error parameters such as

$$\mu_{IS(i)} = \mu_{PH(i)} = 1.5 \text{ and } \sigma_{IS(i)} = \sigma_{PH(i)} = 0.02. \text{ Regarding the unit cost values, we set } u_1 = 10, u_2 = 40 \text{ and } Insp = 3$$

At the end of each period, the inventory manager collects a new estimated value of the IS error to update the average and the standard deviation of the IS distribution. Similarly, each inspection is followed by an update of the average and the standard deviation of the PH distribution. These updated values of the IS and PH errors are used in each period to optimize the Wait & See policy. In other words, the threshold demand level $D_{S(i)}$, permitting one to choose between inspection and no-inspection, is dynamically recalculated based on the new estimates about the error averages and standard deviations.

Starting in period 1 with error-free estimates $\mu_{IS(0)} = \mu_{PH(0)} = 1 \text{ and } \sigma_{IS(0)} = \sigma_{PH(0)} = 0$, Table 5 illustrates the values collected by the inventory manager and the update of the error parameters over periods (Not obs = Not observed).

Based on the Table 5, Figures 2 and 3 illustrate the relative deviation of the average and standard deviation (in percentage) of the estimated values from the actual values. It can be verified that the learning process is faster for the IS error since an update is done in each period. The estimated values converge toward the actual ones with neglected deviation starting from period 10 approximately. The learning process could be an efficient and reliable way to estimate the error distributions.

3. Conclusion

In this paper we studied the impact of the IS and the PH errors on the performance of an inventory management system. We considered the wholesaling / e-retailing context where demand satisfaction is mainly based on the inventory position shown in the information system, which could be subject to errors and might not represent the actual stock physically available. We proposed a judgment-based strategy where the decision to perform the inspection is postponed until the customers' demands are known. An error free strategy thanks to RFID was also provided. By comparing the two strategies, it was shown that the Wait & See policy could improve the performance considerably before the adoption of the RFID investment could be adopted. Besides, we contributed by studying the error learning process, through which the inventory manager could estimate error distributions in order to consider them in his inventory optimization.

Table 5: The learning process about the errors' distributions.

<i>period</i> <i>i</i>	Collected value of γ_{ISi}	inspection Yes/No	collected value of γ_{PHi}	Updated value of: μ_{ISi}	Update value of: μ_{PHi}	Updated value of: σ_{ISi}	Update value of: σ_{PHi}
0	1		1	1	1	0	0
1	1.52296	No	Not obs	1.52296	1	0	0
2	1.50306	Yes	1.50306	1.51301	1.50306	0.009953	0
3	1.49846	No	Not obs	1.50816	1.50306	0.0106335	0
4	1.49799	Yes	1.49799	1.50562	1.50053	0.0102077	0.00253396
5	1.52325	Yes	1.52325	1.50915	1.5081	0.0115377	0.0109128
6	1.51876	No	Not obs	1.51075	1.5081	0.0111249	0.0109128
7	1.47413	No	Not obs	1.50552	1.5081	0.0164412	0.0109128
8	1.4716	Yes	1.4716	1.50128	1.49898	0.0190354	0.0184159
9	1.50239	Yes	1.50239	1.5014	1.49966	0.0179502	0.0165283
10	1.52421	Yes	1.52421	1.50368	1.50375	0.0183521	0.0176451
11	1.46834	No	Not obs	1.50047	1.50375	0.0202342	0.0176451
12	1.475	Yes	1.475	1.49835	1.49964	0.0206121	0.0191857
13	1.47296	Yes	1.47296	1.49639	1.49631	0.0209273	0.0199996
14	1.52784	No	Not obs	1.49864	1.49631	0.0217314	0.0199996
15	1.54012	No	Not obs	1.5014	1.49631	0.0234061	0.0199996
16	1.49929	No	Not obs	1.50127	1.49631	0.0226686	0.0199996
17	1.50281	No	Not obs	1.50136	1.49631	0.0219948	0.0199996
18	1.50468	No	Not obs	1.50155	1.49631	0.0213886	0.0199996
19	1.49388	No	Not obs	1.50114	1.49631	0.0208885	0.0199996
20	1.49626	Yes	1.49626	1.5009	1.4963	0.0203874	0.0188558
21	1.53107	Yes	1.53107	1.50234	1.49978	0.0209075	0.0207063
22	1.49477	No	Not obs	1.50199	1.49978	0.0204876	0.0207063
23	1.47079	Yes	1.47079	1.50064	1.49714	0.0210234	0.0214297
24	1.47419	Yes	1.47419	1.49953	1.49523	0.0212482	0.0214756
25	1.5002	Yes	1.5002	1.49956	1.49561	0.0208194	0.020675
26	1.50953	Yes	1.50953	1.49994	1.49661	0.0205048	0.0202431
27	1.47904	Yes	1.47904	1.4991	1.49544	0.0205052	0.0200416
28	1.51226	Yes	1.51226	1.49964	1.49649	0.0202817	0.0198279
29	1.48665	No	Not obs	1.49919	1.49649	0.0200693	0.0198279
30	1.50666	No	Not obs	1.49944	1.49649	0.0197775	0.0198279
31	1.50189	No	Not obs	1.49952	1.49649	0.0194607	0.0198279
32	1.50953	Yes	1.53003	1.50047	1.49846	0.0198766	0.0207924
33	1.47904	Yes	1.52374	1.50118	1.49987	0.0199755	0.0210201
34	1.52615	No	Not obs	1.50191	1.49987	0.020127	0.0210201
35	1.47028	No	Not obs	1.5010	1.49987	0.0205255	0.0210201
36	1.4958	Yes	1.4958	1.50086	1.49965	0.0202566	0.0204796
37	1.50034	Yes	1.50034	1.50085	1.49969	0.0199811	0.0199616
38	1.50385	No	Not obs	1.50093	1.49969	0.0195241	0.0199616
39	1.51029	No	Not obs	1.50117	1.49969	0.0195241	0.0199616
40	1.52458	No	Not obs	1.50175	1.49969	0.0196218	0.0199616
41	1.51595	No	Not obs	1.5021	1.49969	0.0195044	0.0199616
42	1.48976	Yes	1.48976	1.5018	1.49921	0.0193624	0.0195949
43	1.50761	No	Not obs	1.50194	1.49921	0.0191559	0.0195949
44	1.52116	Yes	1.52116	1.50238	1.50021	0.0191525	0.0196828
45	1.49559	No	Not obs	1.50223	1.50021	0.0189649	0.0196828
46	1.50199	Yes	1.50199	1.50222	1.50029	0.0187576	0.0192536
47	1.48378	No	Not obs	1.50183	1.50029	0.0187468	0.0192536
48	1.48223	Yes	1.48223	1.50142	1.49954	0.0187605	0.019190
49	1.477	Yes	1.477	1.50092	1.49863	0.0188865	0.0193146
50	1.49326	No	Not obs	1.50077	1.49863	0.0187275	0.0193146
51	1.49024	Yes	1.49024	1.50056	1.49831	0.0186003	0.0190082
52	1.5086	Yes	1.5086	1.50072	1.49869	0.0184537	0.0187538

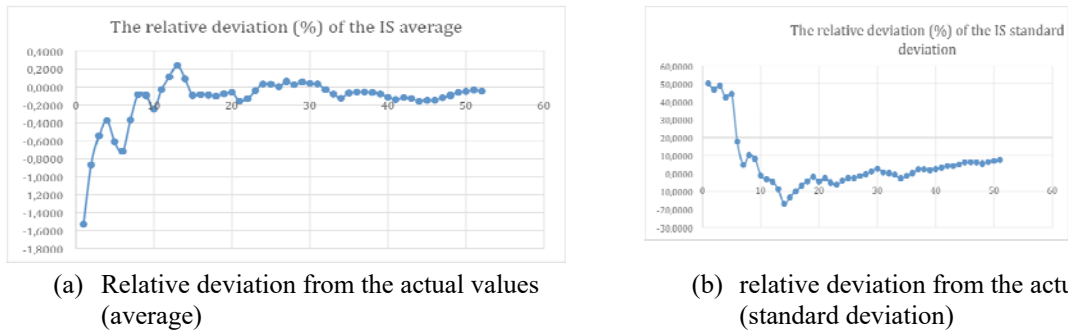


Figure 2. The learning process about the IS errors

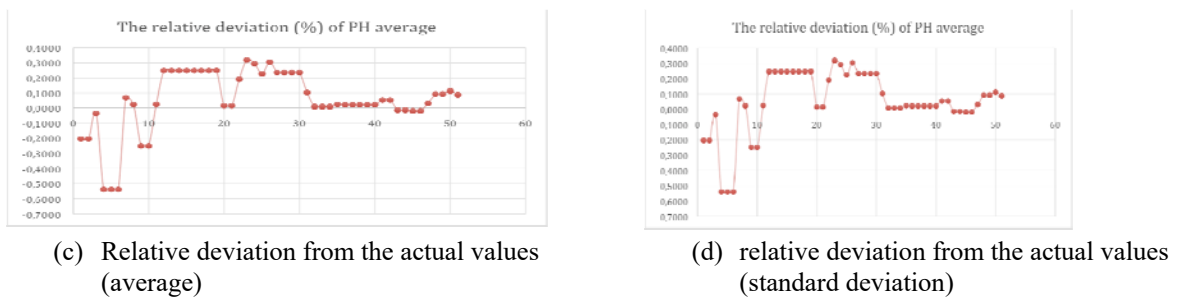


Figure 3. The learning process about the PH errors

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