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The *Liber Restauracionis*: a Newly Discovered Copy of a Mediæval Algebra in Florence

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Abstract

The main purpose of this article is to establish a new critical edition of the anonymous *Liber restauracionis et opposicionis* [Book of Restoration and Opposition] taking into account three manuscripts. Two of them are well-known from earlier historical writing, especially the work of Wolfgang Kaunzner who made the first edition in 1986. The third one is a newly-found manuscript in the Biblioteca Medicea Laurenziana in Florence. We present this copy in detail here. To introduce our edition, we come back to the main arguments which lead us to make the hypothesis that this text is an original Latin adaptation, rather than a possible Arabic-Latin translation of a version/adaptation of al-Khwārizmī's *Algebra*.

Keywords: Algebra, Mediæval Studies, Translation, Critical Edition, Manuscript, Algebraic Symbolism
01A30, 01A35, 01A75

1. Introduction

Our paper concerns the appropriation of the sciences of Islamic countries by European scholars. It focuses on mediæval algebra through a single text that I call the *Liber restauracionis* [Book of Restoration]. After our thorough study of the text in the *Revue d'histoire des mathématiques* (Moyon 2017), we propose here a critical edition based on three copies from the fourteenth century (at the latest). Indeed, when I was correcting the proofs of my

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1. Introduction

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First, I come back to the first two copies that we knew previously in order to present in detail the third one. I then summarize the main results of my study of the *Liber restauracionis*. The whole argument is published in (Moyon 2017) with a French translation of the text. Finally, I give a critical

edition taking into account the Florentine copy, with a more complete critical apparatus than that proposed in (Kaunzner 1986).

2. The manuscripts

2.1. *Biblioteca Apostolica Vaticana, Vat. Lat. 4606* : (V)

According to (Van Egmond 1980, 220–221), this paper *codex* (125 fol.) is from the middle or the second half of the fourteenth century. The *Liber restauracionis* (fol.72r-77r) is the third text of the *codex* including six mathematical texts. Five of them (including ours) are on computation and one of them deals with practical geometry. We should note that the algebraic parts of the Fibonacci's *Liber abbaci* is well-represented with copies of the fourteenth chapter and extracts from the fifteenth chapter.

Our text is entirely written in thirty-two to thirty-eight long lines, on each page, by a single scribe, in gothic hand. Geometrical diagrams (with letters) and tables are in the margins. Many marginal notes – references to Euclid's *Elements* – appear written by another hand, later than the principal scribe (in a cursive hand). Sections are clearly divided thanks to *litterae notabiliores* or minor initials (most of them are flourished initials in the margin). Titles of sections are added in marginalia. When the scribe does not write numbers in their literal form, he uses Arabic numerals in their classical mediæval western forms (Beaujouan 1948; Burnett 2002). Particularly noteworthy is the writing of a fraction with an horizontal bar as in Maghrebian mathematics (Djebbar 1992; Moyon and Spiesser 2015).

This was the first copy of the *Liber restauracionis* known in the historiography. It was used, in the middle of the nineteenth century, by Baldassarre Boncompagni, who edited the text (Boncompagni 1851), and after him by Franz Woepcke (Woepcke 1854, 356-357). It has a title, written in red at the top of fol.72, which announces:

Here begins the book, which is entitled *algebra* and *almucabala*, following the Arabs, and *Book of restoration* according to us. And it was translated by Master Gerard of Cremona in Toledo from Arabic into Latin.¹

¹ *Incipit liber, qui secundum arabes vocatur algebra et almucabala, et apud nos liber restauracionis nominatur, et fuit translatus a magistro Giurardo cremonense in toleto de arabico in latinum.* (ms. Vat. Lat. fol.72r)

This *title* is the reason why Boncompagni, whose “publications were mainly devoted to reconstructing the channels of the transmission of mathematical knowledge from the Arab world to Christian Europe and to its chronology” (Fiocca 2017, 2), edited the text in his paper dedicated to the life and works of Gerard of Cremona (Boncompagni 1851, 412–435):

A translation from Arabic to Latin, made in Toledo by Gerard of Cremona of an important algebraic treatise, was found in the Vatican n°4606 from the recto of the fol.72 to the fol.76. This version being unpublished, it appears to me useful to reproduce it here, with the translation into algebraic language of some rules, and the operations that are indicated in it.²

But, today and since at least Björnbo’s works (Björnbo 1905, 241), we have to consider it as a pseudepigraphon.³

Finally, the manuscript (V) is significant because it was probably the original source⁴ of a vernacular translation – named here (U) – made in Naples, at the end of the fourteenth or the beginning of the fifteenth century: the Biblioteca Apostolica Vaticana, Urb. Lat. 291 (fol.34r-41v).⁵ Its early date (less than 50 years later than the original) suggests a strong interest in the *Liber restauracionis* in the early fifteenth century Italian. This version was studied and edited in (Franci 2003).

2.2. University of Oxford, Bodleian Library, Lyell 52: (L)

This parchment *codex* of 75 folios was written in Italy, in four hands (De la Mare 1971, 143-146). It formerly belonged to the Benedictine Abbey of Admont (shelfmark 612). It is slightly older than the previous one, and

² *Una traduzione dall’ arabo in latino, fatta in Toledo da Gherardo cremonese d’ un importante trattato d’ algebra, trovasi monoscritta nel codice Vaticano n° 4606 dal recto della carta 72 al rovesco della carta 76. Questa versione essendo inedita, parmi utile di riportarla qui appresso colla traduzione in linguaggio algebrico d’ alcune regole, ed operazioni che in essa sono indicate.* (Boncompagni 1851, 27-28)

³ Charles Burnett does not mention it in (Burnett 2001).

⁴ “La maggior parte del contenuto di questo codice, fatta eccezione per il contenuto delle carte dalla 81 alla 102, è una traduzione in volgare di parti del manoscritto latino Vat. lat. 4606 (sec. XIV) della Biblioteca Apostolica Vaticana.” (Franci 2003, 31)

⁵ Description of the manuscript in (Van Egmond 1980, 215-217). Warren Van Egmond specifies in particular that “this ms. is largely an Italian trans. of the Latin text in Vat. Lat. 4606, which in turn is heavily based on Leonardo Pisano’s *Liber abbaci*”.

seems to be from the early fourteenth century (Kaunzner 1985; Kaunzner 1987, 132; Hissette 2003, 139-140). It contains several mathematical texts, above all treatises on *algorismus*, namely texts describing how to use Indian (commonly called Arabic) numerals in arithmetical calculations. In particular, in addition to an *Algorismus de integris abbreviatus* (fol.35r–41v), it includes two versions of al-Khwārizmī’s *Kitāb fī ḥisāb al-Hind* [Book on Indian reckoning]: the *Liber pulueris* (fol.1r–20r) and the *Liber Ysagogarum* (fol.21r–34v) (Allard 1992, xxxvii, xli). Several independent astronomical texts (tables/Almanac) complete this *codex*. Last but not the least, it is interesting to note that the major part of this *codex* is from Arabic astronomical or mathematical writings.

The *Liber restauracionis*, copied by a single gothic hand, occupies eight folios of twenty-eight lines ruled in hard point (one column), from fol.42r to 49v. This is the second copy that we know. It was used by Wolfgang Kaunzner in his critical edition (Kaunzner 1986). There is no short or running titles, nor are there marginal notes. Numbers are written in their Arabic form as in (V). Except in tables of numbers (fol.44v–45r), fractions are represented literally. All geometric diagrams are executed in the margins (fol.43v, 44r, 46r) and they are labelled with letters. The tables of numbers are in the text. We note only three *litterae notabiliores*, including the pen flourished initial U of the *incipit* (fol.42r) and two plain initials (fol.44v, 46r). As a result, the different sections and problems are not well-separated. There are no marginal notes.

Finally, we want to point out that, after a rough analysis,⁶ Wolfgang Kaunzner draws our attention to a significant feature of the manuscript (L) in the history of Latin algebraic texts issued from al-Khwārizmī’s *Algebra*.

Thus, the algebra of L – probably early 14th century – could be the oldest of all listed. Even though the numbers are, for the most part, already ciphered in L and V, the way the fractions are written points to an early date of composition.⁷

⁶ In Kaunzner’s list, two French manuscripts (Paris, BnF 7377A and BnF 9335) are dated from the fourteenth century while Hughes dates them from the thirteenth century (Hughes 1986, 224–226). In addition, he does not seem to know the Florence manuscript, also from the thirteenth century, referenced by Hughes (Biblioteca Nazionale Conv. Soppr. J.V.18, fol.80r–86v) and already known by Björnbo (Björnbo 1911/12, 221). For the textual tradition of Latin versions, see (Hughes 1986, 221–228; Hughes 1989, 11–13).

⁷ *Demnach könnte die Algebra aus L – wahrscheinlich frühes 14. Jahrhundert – die*

So, (L) is certainly not “the oldest of all listed”, nevertheless we can consider it as one of the older ones.⁸ Only three manuscripts of Gerard’s translation, at least, are older: for Barnabas Hughes, they are from the thirteenth century (Hughes 1986, 221–228).

2.3. *Biblioteca Nazionale Centrale of Florence, Conv. Soppr. 414*: (F)

This is the newly discovered copy from the thirteenth/fourteenth century:

The years 1263 and 1304 are given as the ‘present year’ in notes added by another hand on f. 60r and 63v. (Juste 2017a)

Even if the year 1263 is uncertain (see below), the year 1304, confirmed by two different glosses, seems to be relevant. David Juste mentions:

The date 1304 is congruent with the script and is also confirmed by another gloss in the same hand and on the same page, which speaks of 1185 years elapsed since Ptolemy’s epoch of 118 (!) A.D. (118+1185=1303). (Juste 2017b)

Thus (F) is certainly the oldest known manuscript for the *Liber restorationis*, which is not contradicted by palaeographic arguments (in particular for numerals). It could even be contemporary with the older *codices* of Gerard’s version.

Before belonging to the Biblioteca Nazionale Centrale of Florence, the *codex* was in the Basilica della Santissima Annunziata (Crociani et al. 1983, 117–118). The parchment *codex*, consisting of 63 folios, was apparently written by a single hand. It consists of a mathematical and astronomical miscellany. One of the most important texts is undoubtedly the *Almagestum parvum* (fol.1r–45r)⁹ which is a “compendium of the six first books of the



älteste von allen aufgeführten sein. Die Zahlen sind hier, in L und V, zwar größtenteils schon verziffert, aber die Art, wie Brüche geschrieben werden, weist mit auf die frühe Entstehungszeit hin. (Kaunzner 1986, 84) I thank Andrea Bréard for confirming my understanding of this quotation.

⁸ It is also interesting to note that only one known Arabic copy of the al-Khwārizmī’s *Mukhtaṣar* would be older than (L). It is from Medina, thirteenth century (Rashed 2007, 85; Rashed 2009, 85).

⁹ In this respect, the *codex* is recorded in the database *Ptolemaeus Arabus et Latinus. Manuscripts*, see (Juste 2017a).

Almagest [of Ptolemy] with additional material from al-Battānī and others” following Lorch (1992, 407).

Our text consists of the last four folios (fol.60va-63vb) and is presented in two pre-ruled columns of roughly equal size, between forty-one and forty-five lines. The scribe wrote in gothic textura, used Arabic numerals and to a lesser extent a literal form (especially to designate all fractions).¹⁰ No fraction is written using the horizontal bar (as in (V)).¹¹ Geometrical diagrams (with letters) and other tables are in the margins. The beginning of a paragraph is marked with colored pilcrows or *littera notabilior* (all are simple plain initials).¹² This copy does not have short or running titles, nor an *incipit* as in (V).

There are four *marginalia* in those folios, probably written by other hands (at least for three of them) at the beginning (fol.60va/60vb) and at the end (fol.63vb) of our text. All four are quite difficult to read for palaeographical reasons, or due to a poor quality ink.¹³ First of all, in the header of the fol.60v, the sentence (that hardly makes sense): “*Nota quod radix est numerus multiplicatus in se ipsum census est*” clearly refers to the definitions of the objects of algebra: *radix* [root for the Arabic term *jidhr*] and *census* [for the Arabic *māl*, often badly translated in English by *square*] (Oaks and Alkhateeb 2005, 401-402). The second one is longer and more difficult to interpret, in particular because I do not know if it is written by the main scribe or another hand. Even if a distinctive sign, , in the text refers explicitly to this gloss,¹⁴ it refers itself to another text or another gloss apparently written in another *codex*. Another sign, , is used that does not correspond to any other passage in (F). The third one consists in a few words on the left margin, in the fold of the *codex*; it is impossible to read. At last, the fourth one is a longer marginal note at the end of fol.63v: 24 lines in the second column followed by 4 full lines. Nevertheless, this important marginal gloss, due to a later scribe, is an astronomical note without any relation to our text

¹⁰ Note that the scribe used the Roman numeral twice: *x* for 10, *iiii* for 4.

¹¹ An exception in fol.60v with a $2\frac{1}{2}$ in the margin. I conjecture that it is from another hand.

¹² One single initial is not inserted (fol.62v); we can conclude this from the reserved space provided for this purpose and the tiny letter in marginalia.

¹³ I sincerely thank the historians of sciences David Juste and Henry Zepeda and the palaeographer Marc Smith for their help in understanding these glosses.

¹⁴ For this reason, I will give it in its entirety in the edition.

but rather to the previous Ptolemaic texts.¹⁵ Note that, as we mentioned above, this is the gloss that mentions the year 1263 “as the present year” but it seems likely that it is a scholium copied from another manuscript.¹⁶

2.4. An attempt to establish a stemma

The relationships among the manuscripts (L), (F) and (V) may be described using several factors that take into account only the text and not, in a first step, the geometrical diagrams. Indeed, we know that the diagrams are executed separately from the text and often by another hand. I think that we have enough philological arguments not to take the risk of making uncertain hypotheses about the diagrams.

First, as we saw, (V) has a principal title and sectional titles all of which are missing in both (F) and (L). But, as most of those titles are interlinear or marginal, this could be an isolated decision of the scribe of (V). Nevertheless, we have to notice some key features about the texts which I consider as the most striking differences between the manuscripts. In particular, we have identified several discriminating passages, some of which could be omitted because of homoeoteleuton. (F) does not show, at least, two passages found in (L) and (V) (notes 79, 173) but, *a contrario*, shows a passage not found in both (L) and (V) (note 557). Now, (L) lacks three lengthy phrases (more than four words) found in both (F) and (V) (notes 24, 240 et 473). Finally, (V) lacks, at least, four long passages found in (F) and (L) (notes 4.2, 318, 395 et 510). Hence, we can be certain that none is a copy of any of the others.

Secondly, we will argue that both (F) and (L) could have a common ancestor (α), which is not the exemplar (β) of (V). I must begin by stating that numerical data is very stable in the three manuscripts, except for a passage where (V) stands out. The problem in (F) and (L) is $5x^2 = 80$ whereas it is $5x^2 = 40$ in (V) (note 74). Then, more generally, I examined the case of readings common to two of the three manuscripts¹⁷ when considering only the terminological changes (omission or modification of at least one word). I did not take into account the interchange of the same word or

¹⁵ As David Juste confirmed this understanding, I will not propose here an edition of this note.

¹⁶ David Juste confirms this hypothesis in a personal communication.

¹⁷ We thus have the differences of one manuscript with the other two.

words in a sentence,¹⁸ differences due to orthography, grammatical inflection or conjugation¹⁹ or even differences of words that could easily be explained by errors in the reading of the medieval abbreviations (mine or scribe’s).²⁰ I also naturally excluded from this study the three or four passages that present reading difficulties. As a result, we have the following Table 1.

Pair of Manuscripts	(F) & (L)	(F) & (V)	(L) & (V)
Number of Common Readings	53	30	25

Table 1: Table of common readings: quantitative approach

We can conclude that more often than not (F) and (L) agree in a reading against (V), regardless of which reading is correct. Now, if we observe geometric diagrams and tables of numbers (Table 2), this idea is not contradicted.

figures	observations in manuscripts	Mss. differing from the others
Fig. 2	(F)≠(L)≠(V)	(F), (L) and (V)
Fig. 3	(F)≠(L)≠(V)	(F), (L) and (V)
Fig. 4	(F)=(L)≠(V)	(V)
Fig. 5	(F)≠(L)=(V)	(F)
Fig. 7	(F)≠(L)≠(V)	(F), (L) and (V)
Fig. 10	(F)=(L)≠(V)	(V)
Fig. 12	(F)≠(L)=(V)	(F)
Fig. 13	(F)≠(L)=(V)	(F)
Fig. 14	absent from (F) and (L)	(V)
Fig. 15	absent from (F) and (L)	(V)

Table 2: Comparison of geometric diagrams and tables of numbers in manuscripts (the number mentioned is that of my edition)

Relying on the foregoing points, I propose the following stemma (fig. 1).

¹⁸ For example, “omnis numerus multiplex est unitatis” *versus* “omnis numerus multiplex unitatis est”.

¹⁹ In particular, the doubling of letters : *hiis/his*, *digittus/digitus*, *equipolet/equipolet...* I do not report here grammatical differences in person, number or tense.

²⁰ For example, *perfluat* or *profluat*.

I include the manuscript (U) as an interesting witness.

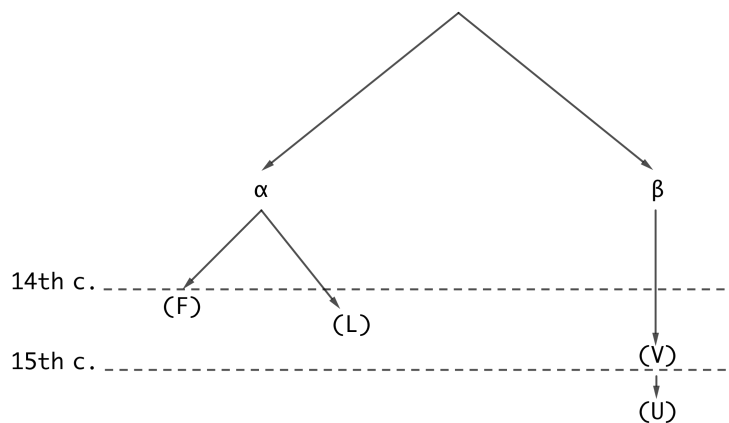


Figure 1: Viable *stemma* of the manuscripts of the *Liber restauracionis*

3. Considering the *Liber restauracionis* in a new way

3.1. Classic results in the historiography

The *Liber restauracionis* has been considered as one of the translations of al-Khwārizmī’s *Mukhtaṣar fī ḥisāb al-jabr wa-l-maqābala* [Compendious Book on Calculation by restoration and balancing], made directly from Arabic to Latin. We have already seen, concerning (V), that the *Liber restauracionis* cannot be counted among the numerous translations of Gerard of Cremona. Following (Hughes 1982; Kaunzner 1986; Hughes 1989; Franci 2003), the translation is attributed to Guglielmo of Lunis, a scholar attached to the Italy of *botteghe d’abaco* [schools of abacus]. He could also be identified with Guglielmo of Luna, a translator of Arabic philosophical works, in particular those of Ibn Rushd (1126–1198), who flourished in Toledo in the first half of the thirteenth century and in Sicily between 1258 and 1266 (Hissette 1997, 2001, 2003; Delle Donne 2007). I will not recapitulate here the various arguments in favor of this attribution, I prefer to refer to my synthesis of scholarly works in (Moyon 2017).

In the recent literature, despite the cautious recommendations of (Hissette 1997, 128–129; Hissette 2003), there seems to be a consensus to attribute the paternity of *Liber restauracionis* to Guglielmo of Lunis/Luna (and perhaps

even that of the vernacular version). Here are some examples from the most recent works.

In his critical edition and French translation of the *Mukhtaṣar*, Roshdi Rashed specifies in a footnote that it exists in three Latin translations (Rashed 2007, 86; Rashed 2009, 86). In addition to those of Gerard of Cremona and Robert of Chester, he quotes that of “Guglielmo of Luna” for which, unlike the other two, he gives no bibliographic reference.²¹

In 2011, about the introduction of al-Khwārizmī’s *al-jabr* into Europe, Nadia Ambrosetti states: “To mark the passage of this method to European culture, there are the three Latin translations made between the XIIth and XIIIth centuries in Spain, by Robert of Chester (1145), Gerard of Cremona (1170) and Guglielmo de Lunis (1250)(...)”.²²

Also, in their “History of Algebra from Antiquity to the early Twentieth Century”, Victor Katz and Karen Parshall state that, in addition to Gerard and Robert, “there is at least one other independent translator into Latin of al-Khwārizmī’s text, William de Lunis, who flourished in the first half of the thirteenth century” (Katz and Parshall 2014, 178).

In his recent synthesis on the introduction of Arabic Algebra in Europe, Ahmed Djebbar writes, with no other details than the reference to (Kaunzner 1986), that the *Mukhtaṣar* “has benefited from three Latin translations, by Gerard of Cremona (d. ca. 1187), Robert of Chester (ca. 1141) and Guglielmo of Lunis (13th century)” (Djebbar 2016, 104). Finally, it is also in these terms that Menso Folkerts and Barnabas Hughes consider these three names in their most recent *Sourcebook in the Mathematics of Medieval Europe and North Africa* (Katz et al. 2016, 103).

To my knowledge, only Jens Høyrup comes back to this attribution in a discrete note hitherto unnoticed. His conclusion is that, without new information, the *Liber restauracionis* must be considered as anonymous. Guglielmo of Lunis would be, for Høyrup, the translator of an Italian version

²¹ “There are three Latin translations: by Gerard of Cremona, by Robert of Chester and by William of Luna. On the translation by Gerard of Cremona, which is by far the best and the most literal, see (...) On the translation by Robert de Chester, see (...)” (Rashed 2009, 86, English translation from Rashed 2007, 86) Nothing is said about the aforesaid third translation.

²² *A segnare il passaggio di questo metodo alla cultura europea, stanno le tre traduzioni latine realizzate fra il XII e il XIII secolo in Spagna, 1 da Roberto di Chester (1145), Gerardo da Cremona (1170) e da Guglielmo de Lunis (1250) (...) (Ambrosetti 2011, 138)*

now lost. I reproduce here the entire note:

This redaction is often supposed to be identical with a translation made by Guglielmo de Lunis. However, all references to this translation (except a false ascription of a manuscript of the Gherardo translation) borrow from it a list of Arabic terms with vernacular explanation which is absent from the present Latin treatise. It is a safe conclusion that Guglielmo translated into Italian; that his translation is lost; and that the present redaction is to be considered anonymous. (Høystrup 2010, 13)

3.2. *My ideas*

In this part, I summarize the main results of (Moyon 2017) where I analyse the text in detail.

Editing the *Liber restauracionis*, its filiation with the *Mukhtaṣar* is not in doubt. But can we consider this text as an Arabic-Latin translation of the al-Khwārizmī's text? Indeed, I do not believe that we can because of the many divergences between the two texts. My thesis is quite different. I notice, among others (the page references are those of my edition):

- the strong differences in the geometric proofs and diagrams for the algorithms (p.20–25) and the numerical identities with square roots: $(\sqrt{200} - 10) + (20 - \sqrt{200}) = 10$ and $(20 - \sqrt{200}) - (\sqrt{200} - 10) = 30 - 2\sqrt{200}$ (p.32–34);
- the explicit reference to Euclid's *Elements*: p.20, 21, 23 and p.24 for geometrical proofs (*Book II*, see (Moyon 2007; Corry 2013, 655-659)), p.29 for the distributive property (again *Book II*, see (Corry 2016, 311-314)) and p.43 for reference to irrational numbers (*Book X*);
- the presence of new paragraphs (like the one in relation to the symbolism – see p.21) or paragraphs structured differently (as for the proportion – p.36);
- some numerical examples that are not necessarily those of the *Mukhtaṣar* (see, for example, Appendix A and Appendix B for the series of problems at the end of the text); and, more generally,
- the choice of a logical presentation that is more deductive than that of al-Khwārizmī himself. The author goes from the general rule to numerical examples, when these are given.

For me, it would therefore rather be a reworking, relatively more didactic than the *Mukhtaṣar*, which could be used to teach Algebra. Now, this reworking may have been written in Arabic, by a mathematician from the Islamic West, during the late Twelfth century or later (from the presence of algebraic symbolism and the fraction bar). In this case, the *Liber restauracionis* is *de facto* a Latin translation from an Arabic text (but not directly from al-Khwārizmī's *Algebra*) still unknown, and then Guglielmo of Lunis could be the translator.

However, the use of versification (p.17, 19, 20) to state the algorithms for solving the three composite types is, for me, a strong argument for advancing another hypothesis. Indeed, I think that we are in a case comparable to the twelfth-century mathematical treatise *Liber Mahameleth* [Book on transactions] of which Jacques Sesiano writes:

The *Liber mahameleth* thus clearly relies on Arabic sources, and this is hardly surprising for the time and place of its writing. But we may even affirm that the *Liber mahameleth* was (initially) *written in an Arabic environment*. Indeed, its reader was supposed to have, or to be able to have, access to some Arabic treatises and read them in the original. (Sesiano 2014, xviii).

That is to say that the *Liber restauracionis* would not be a book that was originally in Arabic. It would instead be an original Latin text written in the context of the Arabic algebra with the specific contributions of the Islamic West. Indeed, besides the complete absence of Arabisms in the *Liber restauracionis*, I think that it is philologically risky to claim that the versified stanzas, at least, could have been translated into Latin from an Arabic composition. Hence, the author would be a mathematician from *al-Andalus* or from the Iberian Peninsula more broadly. Whether or not he is an Arabist, he would write his text directly in Latin by knowing or directly consulting Arabic texts produced in the Islamic World and probably also the Arab-Latin translations available (at least that of Gerard of Cremona). This author is mathematically educated and particularly well informed about literature then extant, especially with regard to Euclid's *Elements* (Books II, VII and X in particular).

Finally, considering the didactic characteristics of *Liber restauracionis* that we have highlighted (particularly versification, brevity and a systematically analytical presentation), I also hypothesize that it would have been

written to spread Algebra in the environments of Latin students and scholars from Mediæval Europe, discovering the sciences of the Islamic countries. If so, and if I still had to designate a possible role for Guglielmo of Lunis, then I could only consider him as the translator of a Latin-Italian version of which (U) would be a witness.

4. Critical Edition

4.1. Critical Apparatus

In addition to the references to the three manuscripts – (F), (L) and (V) – I give the pagination of the Kaunzner’s edition (K) as, for example: /p. 50/. For convenience, I followed (Kaunzner 1986) by considering (L) as the main manuscript. I nevertheless considered, if necessary, the best readings between (F), (L) and (V) and I corrected some Kaunzner’s transcription errors. All alterations and variants between the three copies, even minor, are taken into account in my critical apparatus. As I wanted it as simple as possible, I use only few necessary abbreviations.

om. omitted (and not added otherwise by the scribe)

ad. added in a marginal or interlinear gloss

cor. corrected by the original scribe

For example,

vel] *om.* F; means: the word *vel* is omitted in (F).

quod] *ad.* LV; means: the word *quod* is added in both (L) and (V) by the scribe.

dragmas] dragm̄is *cor.* dragmas F; means: the word *dragmis* is corrected by *dragmas* in (F). Sometimes, the correction is highlighted by a sign as here.

I preserved as much as possible the signs of correction of the scribes as pointed letters, crossed out words and other underlined words (in particular in dotted line). Most of the time, these signs refer to corrections made by the scribe himself. Bold letters are used for section titles (only present in the margins of (V)) to make the text easier to read. I added also punctuation in order to enhance comprehensibility.

In my edition, all geometric letters are written in italic capital letters. In the manuscripts, they are in lowercase and framed by dots. In (L), they are also easily recognized by the lines drawn over them. For example, the square

ABGD is noted “tetragonus .abgd.” in (F) and (V), “tetragonus . $\overline{\text{abgd}}$.” in (L).

The writing of numbers is different between the three manuscripts and even within the same text. I specify in footnotes the different representations chosen by scribes: literal, Hindu-Arabic or Roman numerals (in two cases).

For the geometrical diagrams, I chose to reproduce all of them for each copy. They are often different because they were not necessarily completed in all the copies. In addition, I indicate the main differences between the three copies in footnotes, including the absence of diagrams in one or the other of the copies. I also number the figures to make reference easier.

Finally, I respected the orthography of the three scribes, including – unlike (Kaunzner 1986) – the classical mediæval diphthongs (*que* for *quae* among others). Orthographic distinctions between scribes (like doubling of letters) are included in footnotes. For the abbreviations of mediæval Latin, I consulted to the classical *Dizionario di abbreviature latine ed Italiane* (Cappelli 1912). In some cases, I keep the text of the manuscript even if the written word has no meaning for me; in that case, I inscribe (?) after this word.

4.2. Latin Text

/p. 50/ [V, fol.72r] Incipit liber, qui secundum arabes vocatur algebra et almucabala, et apud nos liber restauracionis nominatur, et fuit translatus a magistro Giurardo cremonense in toleto de arabico in latinum.²³

[L, fol.42r] [F, fol.60va] Unitas est principium numeri et non est numerus.²⁴ Numerus enim²⁵ est collectio unitatum; sicut ergo omnis numerus multiplex unitatis est,²⁶ sic eadem multiplex est ab eo denominate sue partis. Verbi gratia: ut ternarius triplex est unitatis, sic unitas tripla est²⁷ sue terciæ partis, a ternario denominate; unde necesse est, ut quemadmodum infinitas multiplicationes suscipit unitas, infinitas quoque partes eadem intellectu et ratione suscipiat. Omnis autem numerus non incongrue infra 9²⁸ terminos

²³ Incipit liber, qui secundum arabes vocatur algebra et almucabala, et apud nos liber restauracionis nominatur, et fuit translatus a magistro Giurardo cremonense in toleto de arabico in latinum] *om.* FL;

²⁴ et non est numerus] *om.* L;

²⁵ enim] *om.* F;

²⁶ unitatis est] est unitatis F;

²⁷ tripla est] est tripla V;

²⁸ 9] novem V;

artatur,²⁹ ut ex decuplatorum eorundem iteratione innumerabilis³⁰ multitudo profluat,³¹ scilicet ut habita naturali numerorum dispositione, a prima unitate usque ad 9³² terminos fiat progressus. Et hii quidem primo³³ limiti attribuuntur. Consequenter quolibet terminorum decuplato³⁴ alii 9³⁵ procreantur, qui secundum limitem custodiant, scilicet 10, 20, 30 et caetera usque in³⁶ 90. Hii quoque ad eundem modum decuplati³⁷ in tercio limite collocentur, ut 100, 200, 300 et sic usque ad³⁸ 900. Ad quorum similitudinem infinitos limites³⁹ novenis terminis dispositos generandi regula teneatur. Ad omnem compotum perutilis nichil est enim⁴⁰ computationi subserviens praeter numerum.⁴¹

Incipiuntur capitula algebre et primo de tribus minoribus.⁴²

Capitulum primum minorum.⁴³

Qualiter census equatur radicibus.⁴⁴

Numerus autem, secundum quod⁴⁵ nostre computationi necessarius est, in 3^a dividitur.⁴⁶ In⁴⁷ radicem numeri, et quadratum sive censum radicis, et numerum simplicem, nec censui nec radici comparatum. Radix autem est numerus⁴⁸, quo⁴⁹ in se ducto alter procreatur.⁵⁰ Census autem vel⁵¹ quadratus

²⁹ artatur] arteratur V;

³⁰ innumerabilis] internumerabilis L;

³¹ profluat] perfluat V;

³² 9] novem FV;

³³ primo] primi FV;

³⁴ terminorum decuplato] decuplatorum terminorum L;

³⁵ 9] novem FV;

³⁶ et caetera usque in] usque FL;

³⁷ modum decuplati] decuplati modum V;

³⁸ et sic usque ad] et caetera usque in V;

³⁹ infinitos limites] infinitos unitas limites L;

⁴⁰ nichil est enim] nichil est F; nichil enim est V;

⁴¹ praeter numerum] nisi numerus V;

⁴² Incipiuntur capitula algebre et primo de tribus minoribus] *ad.* V; *om.* FL;

⁴³ Capitulum primum minorum] Capitulum primum minorem *ad.* V; *om.* FL;

⁴⁴ Qualiter census equatur radicibus] *ad.* V; *om.* FL;

⁴⁵ quod] *ad.* V;

⁴⁶ in 3^a dividitur] in trina dividitur F; et in trina dividitur V;

⁴⁷ In] Scilicet in F;

⁴⁸ est numerus] numerus est F;

⁴⁹ quo] qui V;

⁵⁰ procreatur] producitur FV;

⁵¹ vel] *om.* F;

radicis est numerus, qui ex ducta radice in se producitur.⁵² Numerus simplex est is, qui per se producitur, nulla censum /p. 52/ vel radicem collatione⁵³ respiciens. Haec tria singula singulis collata,⁵⁴ 3⁵⁵ modos constituunt, scilicet ut census equus fit aliquot⁵⁶ radicibus, vel census numero, vel⁵⁷ radices numero.

Verbi gratia. Census radicibus, quotiens⁵⁸ dicitur: census equatur 5 radicibus, unde constat quod [L, fol.42v] radix illius est 5 et census est⁵⁹ 25, is enim suis 5 radicibus adequatur. Vel cum dicitur: tertia [F, fol.60vb] census adequatur 4⁶⁰ radicibus, ergo⁶¹ census implet 12 radices, et hic est 144. Vel sic 5⁶² census equantur⁶³ 10 radicibus, unus igitur census duabus radicibus equipollet.⁶⁴ Palam ergo quoniam radix est⁶⁵ 2,⁶⁶ census eius 4. In talibus quoque id observari⁶⁷ convenit, ut, quotiens⁶⁸ plus minusve censu alteruter reliquorum⁶⁹ comparatur, ad integritatem unius census convertatur, converso pariter reliquo sub simili cremento vel diminutione.

Capitulum secundum minorum.⁷⁰

Qualiter census equatur numero.⁷¹

Sed census⁷² numero confertur hoc modo:⁷³ census equus est 9, et est 9.

⁵² qui ex ducta radice in se producitur] qui ex dicta radice in se ducta producitur F; qui ex dicta radice in se ducta procreatur V;

⁵³ collatione] collactione V;

⁵⁴ collata] collocata F;

⁵⁵ 3] tres FV;

⁵⁶ fit aliquot] sit aliquibus V;

⁵⁷ vel] a vel V;

⁵⁸ quotiens] quociens F;

⁵⁹ est] *om.* FL;

⁶⁰ 4] quatuor V;

⁶¹ ergo] igitur V;

⁶² 5] quinque V;

⁶³ equantur] adequantur V;

⁶⁴ equipollet] equippollet L;

⁶⁵ Palam ergo quoniam radix est] Palam ergo est quoniam radix est F;

⁶⁶ 2] duo V;

⁶⁷ observari] obscurari V;

⁶⁸ quotiens] quociens F;

⁶⁹ alteruter reliquorum] alteruter reliquorum scilicet radix et numerus F;

⁷⁰ Capitulum secundum minorum] Capitulum secundum minorem *ad.* V; *om.* FL;

⁷¹ Qualiter census equatur numero] *ad.* V; *om.* FL;

⁷² Sed census] Census vero V;

⁷³ hoc modo] hoc ut F;

Vel 5 census valent 80, ergo unus census valet quintam partem de 80,⁷⁴ et hoc est 16. Vel medietas census valet 18, ergo ipse est 36.

Capitulum tertium minorum.⁷⁵

Qualiter radices equantur numero.⁷⁶

Radices vero numero equivalent⁷⁷ hoc modo: radix valet 3, ergo est 3. Vel 4⁷⁸ radices equipollent 20, una /p. 53/ igitur est 5. Vel si medietas ra[V, fol.72v]dicis valet 10, radix est 20.

Hic incipitur de tribus capitulis maioribus.⁷⁹

Ex his⁸⁰ tribus simplicibus⁸¹ tres permixti generantur, si quelibet bina tercio conferantur:⁸² census et radices numero, census et numerus radicibus, radices et numerus⁸³ censui.

Qualiter census et radices equantur numero.⁸⁴

Census et radices numero, quotiens dicitur:⁸⁵ Census et 10 radices equivalent 39 unitatibus, per quod intelligendum⁸⁶ est quidam census, cui si copulentur 10⁸⁷ radices sui, erit totum hoc 39 dragme.⁸⁸ Ad cuius scientiam haec regula⁸⁹ perducit.

⁷⁴ Vel 5 census valent 80, ergo unus census valet quintam partem de 80, et hoc est 16] Vel 5 census valent 80, ergo unus census valet quintam partem 80, hoc est 16 F; Vel 5 census valent 80, ergo unus census valet quintam partem 80, et hoc est 16 L; Vel 5 census valent 40, ergo unus census valet quintam partem de 40, hoc 8 V;

⁷⁵ Capitulum tertium minorum] Capitulum tertium minorem *ad. V; om. FL;*

⁷⁶ Qualiter radices equantur numero] *ad. V; om. FL;*

⁷⁷ equivalent] equipollent V;

⁷⁸ 4] quatuor V;

⁷⁹ Vel medietas census valet 18, ergo ipse est 36. Capitulum tertium minorum. Qualiter radices equantur numero. Radices vero numero equivalent hoc modo: radix valet 3, ergo est 3. Vel 4 radices equipollent 20, una igitur est 5. Vel si medietas radicis valet 10, radix est 20. Hic incipitur de tribus capitulis maioribus.] *om. F; ad. Super radices que numeros quoequantur et ostendit aliud allgebra (?) ad hoc signum (and followed by a circle)F;*

⁸⁰ his] hiis FL;

⁸¹ simplicibus] principibus V;

⁸² conferantur] referantur FV;

⁸³ numerus] numeri V;

⁸⁴ Qualiter census et radices equantur numero.] *om. FL;*

⁸⁵ Census et radices numero, quotiens dicitur] census et radices numero equantur quotiens dicitur F; Census et radices numero, est hac si dicat V;

⁸⁶ intelligendum] intelligendus FL;

⁸⁷ 10] decem V;

⁸⁸ dragme] dragma L;

⁸⁹ haec regula] *om. L;*

Capitulum primum.⁹⁰

Cum rebus census siquis dragmis⁹¹ dabit⁹² equum,
Res quadra medias, quadratis⁹³ adice dragmas,
Radici quorum medias res excipe demum,
Et residuum quaesiti census⁹⁴ radicem ostendet.

Verbi gratia. Radices in proposita quaestione sunt 10,⁹⁵ harum as/p. 54/sumptam medietatem, id est⁹⁶ 5, quadra, hoc est in se multiplica footnote hoc est in se multiplica] *om.* V; et erunt 25. Adice his⁹⁷ dragmas,⁹⁸ quae sunt 39, et ef[L, fol.43r]ficies 64, ex quorum accepta⁹⁹ radice, scilicet 8, dimidias radices diminue,¹⁰⁰ unde relinquuntur¹⁰¹ 3, quod est radix, census¹⁰² vero est¹⁰³ 9. Quotiens¹⁰⁴ autem plus minusve censu¹⁰⁵ in quaestione proponitur, ad unum census converte; converso pariter reliquorum utroque secundum census crementum vel diminucionem. Verbi gratia. Si proponantur 2 census et 10 radices¹⁰⁶ equivalentes dragmis 48,¹⁰⁷ per conversionem unus census et 5¹⁰⁸ radices equivalencebunt dragmis 24.¹⁰⁹

Vel si medietas census et 5¹¹⁰ radices equantur¹¹¹ 12, per conversionem [F, fol.61ra] integer census et 10 radices equabuntur¹¹² 24.

⁹⁰ Capitulum primum.] *om.* FL;

⁹¹ siquis dragmis] dragmis siquis F;

⁹² dabit] dabis V;

⁹³ quadratis] quadratum V;

⁹⁴ census] *ad.* F;

⁹⁵ 10] decem V;

⁹⁶ id est] scilicet V;

⁹⁷ his] hiis FL;

⁹⁸ dragmas] dragmīs *cor.* dragmas F;

⁹⁹ accepta] assumpta F;

¹⁰⁰ dimidias radices diminue] dimidias radicas radices diminue F; dimidias res minue V;

¹⁰¹ relinquuntur] reliquetur V;

¹⁰² census] et census V;

¹⁰³ est] *om.* FL;

¹⁰⁴ Quotiens] Quociens F;

¹⁰⁵ censu] *om.* F;

¹⁰⁶ 2 census et 10 radicem] duo census et 10 radicem F; duo census et decem radicem V;

¹⁰⁷ dragmis 48] 48 dragmis F;

¹⁰⁸ 5] quinque F;

¹⁰⁹ equivalencebunt dragmis 24] equabuntur 24 dragmis V;

¹¹⁰ 5] 5 *cor.* 2½ F (in marginalia, other hand?);

¹¹¹ equantur] equentur V;

¹¹² equabuntur] equivalencebunt V;

Qualiter census et numerus equantur radicibus.¹¹³

Census autem et numerus equipollent radicibus, quotiens¹¹⁴ dicitur: census et 21 dragme¹¹⁵ equipollent 10¹¹⁶ radicibus, per quod intelligendus¹¹⁷ est quidam census, cui si copulentur¹¹⁸ 21, erit totum hoc equale 10¹¹⁹ radicibus eiusdem census. Ad cuius noticiam hac regula perducetis.

Capitulum secundum.¹²⁰

Cum censu dragmas siquis rebus dabit equas,
Res quadra medias, quadratis abice¹²¹ dragmas, /p. 56/
Dimidiis rebus reliqui latus adde vel aufer,
Et exiens quesiti census radicem ostendet.

Verbi gratia. Radices in proposita questione sunt 10, harum medietatem quadra, hoc est 5, et efficies 25. Ex his¹²² diminue dragmas, que sunt 21, et relinquuntur¹²³ 4, cuius acceptam radicem, scilicet 2, dimidiis radicibus subtrahe vel adde, subtrahe restabunt 3,¹²⁴ quod est radix, census eius 9; vel adde et habebis 7, quod est radix, census eius 49. Experire igitur hanc questionem tum addendo tum subtrahendo, quoniam per alteram certificaberis. Hic etiam nota: si quadrate dimidie radices propositis dragmis pauci[L, fol.43v]ores¹²⁵ fuerint, questionem esse impossibilem; si vero equales, erit radix census equa¹²⁶ dimidiis radicibus. In huius modi quoque¹²⁷ quociens¹²⁸ plus minusve censu proponuntur, singula convertendo reduc in¹²⁹ unum censum.

¹¹³ Qualiter census et numerus equantur radicibus.] *om.* FL;

¹¹⁴ quotiens] quociens F;

¹¹⁵ dragme] dragma FL;

¹¹⁶ 10] decem V;

¹¹⁷ intelligendus] intelligendum V;

¹¹⁸ copulentur] copuletur FV;

¹¹⁹ 10] decem V;

¹²⁰ Capitulum secundum.] *om.* FL;

¹²¹ abice] addice L;

¹²² his] hiis FL;

¹²³ relinquuntur] relinquetur V;

¹²⁴ subtrahe vel adde subtrahe, restabunt 3] subtrahe vel adde, restabunt 3 L; subtrahe vel adde, restabunt 3 V;

¹²⁵ pauciores] pociores F;

¹²⁶ equa] eque V;

¹²⁷ In huius modi quoque] In huius quoque F; Hi quoque huius L;

¹²⁸ quociens] quotiens V;

¹²⁹ in] ad V;

Qualiter radices et numerus equantur censui.¹³⁰

Radices vero et numerus equantur censui, quotiens¹³¹ dicitur: 3 radices et 4 dragme equivalent censui,¹³² quod¹³³ hac regula comprehendes.¹³⁴ [V, fol.73r]

Capitulum tercium.¹³⁵

Si census rebus dragmisque requiritur equus,
Res quadra medias, quadratis adice¹³⁶ dragmas,
Quorum radicem mediis radicibus adde,
Et collectum quesiti census radicem indicabit.¹³⁷

Verbi gratia. Radices in proposita questione sunt 3,¹³⁸ harum assumptam medietatem quadra, et habebis 2¹³⁹ et quartam. His adice¹⁴⁰ dragmas, scilicet 4, et efficies 6¹⁴¹ et quartam, horum radicem, /p. 57/ scilicet duo et semis¹⁴², dimidiis radicibus adde, unde colligentur 4, quod est radix, census autem est¹⁴³ 16.

Predictorum 6¹⁴⁴ modorum 3¹⁴⁵ priores adeo evidentes sunt, quod probatione non egent.¹⁴⁶ Posteriores [F, fol.61rb] vero ex secundo elementorum sue necessitatis firmitatem trahunt, quorum primi – **Hic probatur qualiter census et radices equantur numero** – ¹⁴⁷ quidem, quod est: ¹⁴⁸

census et 10 radices equivalent dragmis 39. Ratio hec est: ponam tetrago-

¹³⁰ Qualiter radices et numerus equantur censui.] *om.* FL;

¹³¹ quotiens] quociens F;

¹³² 3 radices et 4 dragme equivalent censui] Tres radices et quatuor dragme equantur censui V;

¹³³ quod] quem V;

¹³⁴ comprehendes] deprehenderis V;

¹³⁵ Capitulum tercium.] *om.* FL;

¹³⁶ adice] addice L;

¹³⁷ indicabit] ostendet V;

¹³⁸ 3] tres V;

¹³⁹ 2] duo V;

¹⁴⁰ His adice] Hiis adice F; Hiis addice L;

¹⁴¹ 6] sex V;

¹⁴² duo et semis] 2 et semis F;

¹⁴³ est] *om.* FL;

¹⁴⁴ 6] sex V;

¹⁴⁵ 3] tres V;

¹⁴⁶ egent] indigent V;

¹⁴⁷ Hic probatur qualiter census et radices equantur numero] *om.* FL;

¹⁴⁸ quod est] quod qui est F; quod cum V;

num $ABGD$ censum; ¹⁴⁹ eius¹⁵⁰ radicem DG multiplicabo in 10 dragmas, que sint linea GE ¹⁵¹ directe continuata linee BG , unde proveniat superficies DE , ex quo igitur census cum 10 radicibus equatur dragmis 39. Necesse est¹⁵² superficiem quoque¹⁵³ AE 39 dragmas¹⁵⁴ implere. Divisa itaque per medium linea¹⁵⁵ GE ad punctum Z , cui directe coniuncta¹⁵⁶ est linea BG , per 6^{tam} /p. 58/ secundi elementorum¹⁵⁷ fit productum ex EB in GB , qui est superficies AE dragmarum¹⁵⁸ 39, una cum¹⁵⁹ adiuncto sibi tetragono GZ , equa¹⁶⁰ te[L, fol.44r]¹⁶¹tragono BZ . Duco igitur dimidias radices, que sunt 5, in se, unde fit tetragonus GZ ,¹⁶² cui addo multiplicationem EB in GB , que est 39 dragme, et colliguntur 64, ex quorum sumpta radice, que est BZ , diminuo dimidias radices,¹⁶³ id est GZ , et quod relinquitur est GB ,¹⁶⁴ radix scilicet census propositi. /p. 60/

Probacio qualiter census et numerus equantur radicibus.¹⁶⁵

Ratio vero secundi, quod est:¹⁶⁶ census et dragme 21¹⁶⁷ equantur 10 radicibus. Talis est: ponam censum tetragonum $ABGD$,¹⁶⁸ cuius radicem AB

¹⁴⁹ These figures are not announced in any of the three copies. Each time, they are in marginalia. In (V), here and for the two others following case, diagrams with numbers and words could be due to the same hand than the other marginal mentions.

¹⁵⁰ eius] cuius FV;

¹⁵¹ GE] \underline{EGE} F;

¹⁵² est] *om.* F;

¹⁵³ superficiem quoque] quoque superficiem V;

¹⁵⁴ dragmas] dragmis F;

¹⁵⁵ per medium linea] linea per medium F;

¹⁵⁶ coniuncta] continuata F;

¹⁵⁷ 6^{tam} secundi elementorum] $\bar{6}$ secundi elementorum L; “per sextam secundi euclidis” in marginalia, different hand (V);

¹⁵⁸ dragmarum] *om.* V;

¹⁵⁹ cum] *om.* V;

¹⁶⁰ equa] equat F;

¹⁶¹ Here, Kaunzner’s paper is false in numbering the facsimile. It is necessary to change 44r and 44v. The critical edition is correct.

¹⁶² tetragonus GZ] tetragonus 25 GZ F;

¹⁶³ diminuo dimidias radices] dimidias radices diminuo F;

¹⁶⁴ GB] BG FV;

¹⁶⁵ Probacio qualiter census et numerus equantur radicibus.] *om.* FL;

¹⁶⁶ est] *om.* L;

¹⁶⁷ et dragme 21] et 21 dragma FL;

¹⁶⁸ These figures are not announced in any of the three copies. Each time, they are in marginalia. In (V), another diagram appears truncated by the technical repair of the folio

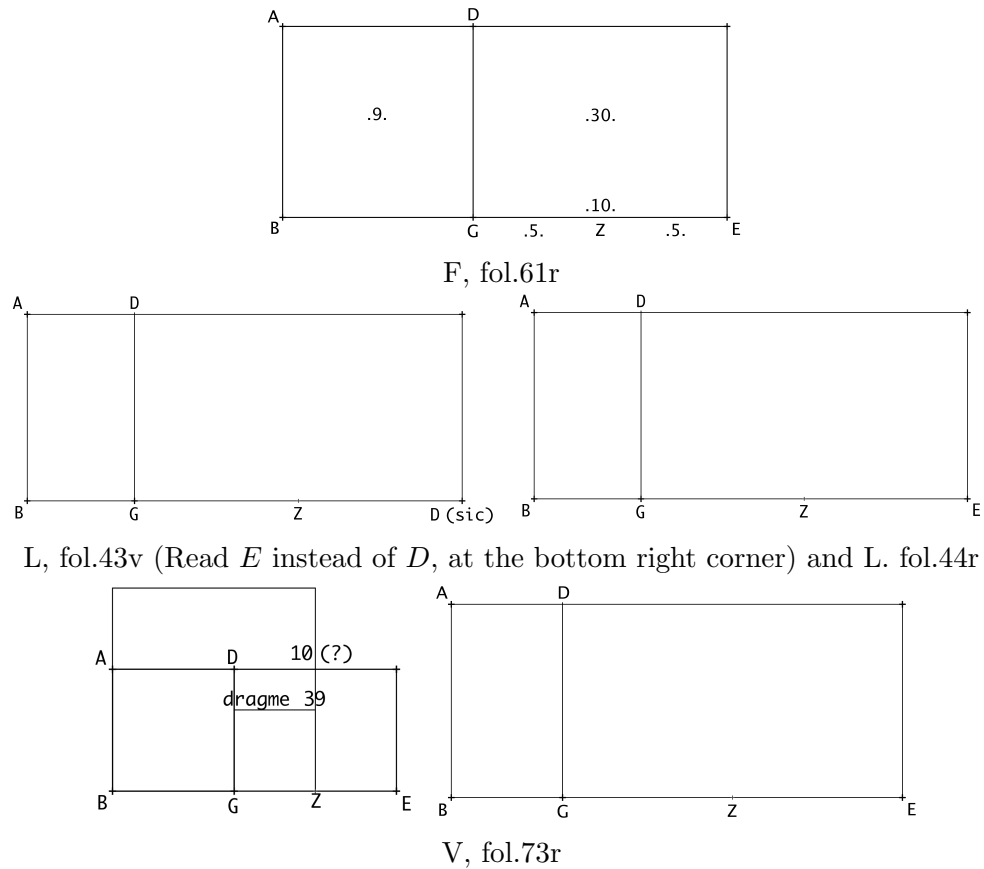


Figure 2: Geometrical diagrams for the procedure associated with $x^2 + 10x = 39$.

multiplicabo in 10^{169} dragmas, que sunt latus BE , unde proveniat superficies AE ; ex quo igitur 10 radices censui, una cum dragmis 21, coequantur. Census autem est tetragonus AG , relinquitur superficies DE /p. 61/ quantum dragme 21. Cum itaque dividitur per medium linea BE ad punctum Z , erit¹⁷⁰ hoc¹⁷¹ inter puncta $G E$ aut inter puncta $B G$.

73r.

¹⁶⁹ 10] decem V;

¹⁷⁰ erit] erit *ad. cadet* V;

¹⁷¹ hoc] ergo V;

Fit¹⁷² hoc prius inter puncta $B G$,¹⁷³ ergo per 5^{tam} secundi elementorum¹⁷⁴ tetragonus¹⁷⁵ BZ equatur¹⁷⁶ producto ex¹⁷⁷ BG in GE , una cum adiuncto sibi tetragono ZG . Duco igitur dimidias radices, que sunt BZ , in se, et fit tetragonus¹⁷⁸ 25, cui subtraho productum ex BG in GE , quod est 21, et remanentis tetragoni radicem sumo,¹⁷⁹ scilicet 4, velud GZ ¹⁸⁰ linea, qua¹⁸¹ adiuncta dimidiis radicibus, id est BZ , impleta est [F, fol.61va] BG , radix quesiti census. /p. 62/

Si autem punctum Z inter¹⁸² puncta $G E$ locabitur,¹⁸³ necesse est, ut¹⁸⁴ linea, que modo dimidiis radicibus coacervata radicem census implevit, ex eisdem sublata, radicem census in reliquo patefaciat, et hoc fuit probandum.

[V, fol.73v] **Hic probatur qualiter radices et numerus equantur censui.**¹⁸⁵

Tercii vero modi, qui est: 3¹⁸⁶ radices et 4¹⁸⁷ dragme censum equant. Ratio haec est: ponam tetragonum¹⁸⁸ $ABGD$ censum,¹⁸⁹ cuius radicem AB in 3¹⁹⁰ dragmas implico,¹⁹¹ que sint latus BE , unde proveniat super[L, fol.44v]ficies AE tribus radicibus equa,¹⁹² quod vero reliquum est tetragoni, scilicet super-

¹⁷² Fit] Sit V;

¹⁷³ Fit hoc prius inter puncta $B G$] *om.* F;

¹⁷⁴ 5^{tam} secundi elementorum] 5 secundi elementorum L; “per quintam secundi euclidis” in marginalia, different hand (V);

¹⁷⁵ tetragonus] tetragus *ad.* onus V;

¹⁷⁶ equatur] equantur V;

¹⁷⁷ equatur producto ex] equatur ex producto ex F;

¹⁷⁸ tetragonus] tetragus V;

¹⁷⁹ remanentis tetragoni radicem sumo] remanent tetragoni radicem summo F; remanentis tetragoni radicem id est sumo L;

¹⁸⁰ GZ] GE L;

¹⁸¹ qua] que V;

¹⁸² inter] ut FL; intus V;

¹⁸³ puncta $G E$ locabitur] puncta $B G$ F; puncta $G E$ loquabitur *cor.* ca V;

¹⁸⁴ ut] quod F;

¹⁸⁵ Hic probatur qualiter radices et numerus equantur censui.] *om.* FL;

¹⁸⁶ 3] tres FV;

¹⁸⁷ 4] quatuor V;

¹⁸⁸ tetragonum] tetragum *cor.* onum V;

¹⁸⁹ This diagram is introduced in the text. The diagrams in (F) and (L) are obviously wrong, $ABGD$ is clearly not a square.

¹⁹⁰ 3] tres FV;

¹⁹¹ implico] multiplico FV;

¹⁹² equa] equat F;

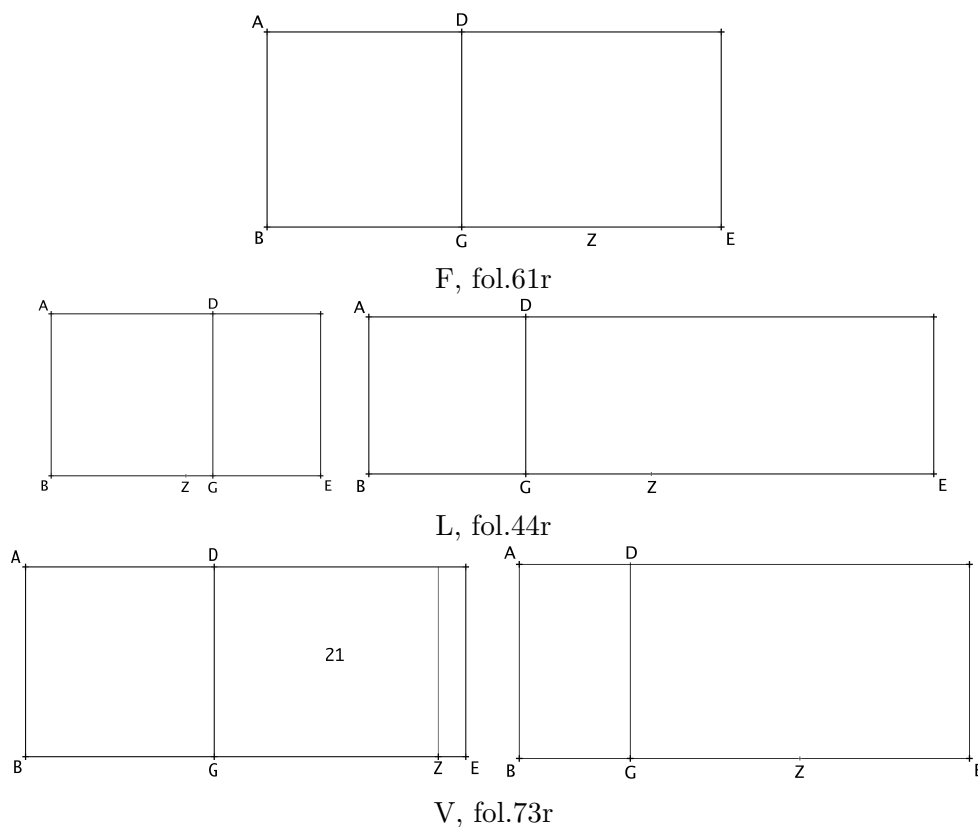


Figure 3: Geometrical diagrams for the procedure associated with $x^2 + 21 = 10x$.

ficies ED , dragmis 4 equipollet.¹⁹³ Deinde divisa¹⁹⁴ linea BE per medium¹⁹⁵ ad punctum Z , cui directe continuata est EG , fit productum¹⁹⁶ ex BG in GE , una cum tetragono ZE , equum tetragono ZG per sextam secundi euclidis.¹⁹⁷ Duco igitur ZE , id est dimidium radicem, in se, et fit 2 et quarta,¹⁹⁸

¹⁹³ equipollet] equippollet L;

¹⁹⁴ Deinde divisa] Divisa deinde FV;

¹⁹⁵ per medium] *om.* F;

¹⁹⁶ fit productum] fit per 6 secundi elementorum productum F;

¹⁹⁷ per sextam secundi euclidis] *om.* FL; “Ista probantur per sextam secundi euclidis”, “per sextam secundi euclidis” in marginalia, different hand (V);

¹⁹⁸ 2 et quarta] duo et quarta V;

cui aggrega¹⁹⁹ productum ex BG in GE , id est superficiem ED , que est 4, et colligo 6²⁰⁰ et quartam, quorum sumo radicem, scilicet ZG , que est 2²⁰¹ et semis, qua²⁰² dimidio radicem, hoc est BZ , coacervata, fit /p. 63/ propositi census radicem BG perficio, cuius figura est hec.²⁰³

Qualiter figurentur census, radices et dragme.²⁰⁴

Porro omnis comptus, qui in restauratione diminuti vel projectione superhabundantis²⁰⁵ exercetur, ad aliquod horum 6²⁰⁶ capitulorum convertibilis est, quod, ut levius fiat discenti, quedam scribendi et multiplicandi praecepta damus, quibus integre²⁰⁷ res ad invicem,²⁰⁸ necnon res, quibus diminuitur vel superhabundat numerus, aut quae diminuuntur²⁰⁹ vel superhabundant numero, multiplicentur, hoc praesumpto²¹⁰, quod ex ductu rei in rem provenit tantum census, et ex ductu rei in numerum non nisi rerum multitudo. In scribendo quidem²¹¹ haec regula teneatur: numero censuum littera c , numero radicem littera r , deorsum virgulas habentes, subterius apponantur, dragme vero sine litteris virgulas habeant, quotiens²¹² haec sine diminutione praeponuntur.²¹³

Verbi gratia: 2²¹⁴ census, /p. 64/ 3²¹⁵ radices et²¹⁶ 4 dragme sic figurentur (Fig. 5 & 6).

¹⁹⁹ aggrega] aggregabo F; agrega V;

²⁰⁰ 6] sex V;

²⁰¹ 2] duo V;

²⁰² qua] quam V;

²⁰³ perficio cuius figura est hec] proficio figura hec est. I ad tale figuram (I is a sign referring to the diagram in marginalia) F; proficio cuius figura hec est V;

²⁰⁴ Qualiter figurentur census, radices et dragme.] om. FL;

²⁰⁵ projectione superhabundantis] particione superhabundans V;

²⁰⁶ 6] sex V;

²⁰⁷ integre] integer V;

²⁰⁸ ad invicem] a diminutione L;

²⁰⁹ diminuuntur] diminuitur L;

²¹⁰ praesumpto] praesupposito V;

²¹¹ quidem] enim V;

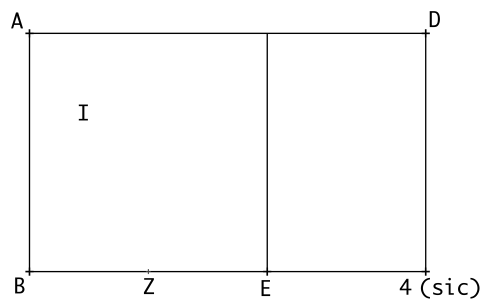
²¹² quotiens] quociens F;

²¹³ praeponuntur] proponuntur FV;

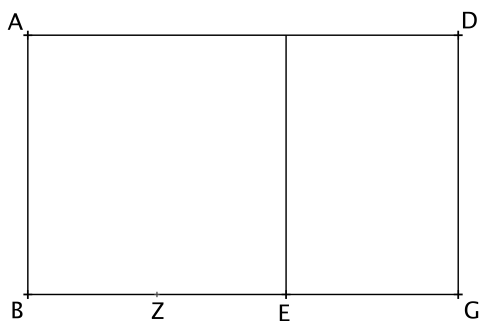
²¹⁴ 2] duo V;

²¹⁵ 3] tres V;

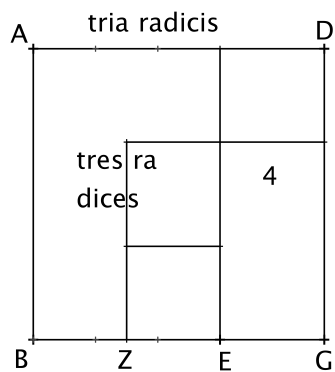
²¹⁶ et] om. FL;



F, fol.61v (Read *G* instead of 4.)



L, fol.44r



V, fol.73v

Figure 4: Geometrical diagrams for the procedure associated with $3x + 4 = x^2$.

Due terciē²¹⁷ census, 3 quarte radices,²¹⁸ 4²¹⁹ quinte unius dragme hoc

²¹⁷ terciē] tertie L;



Figure 5: From left to right: F, fol.61v (reference sign at the top left); L, fol.44v; V, fol.73v

$$\begin{array}{ccc} 2 & 3 & 4 \\ \underline{c} & \underline{r} & - \end{array}$$

Figure 6: Symbolic format for $+2x^2; +3x; +4$

modo figurentur²²⁰ (Fig. 7, 8 & 9).

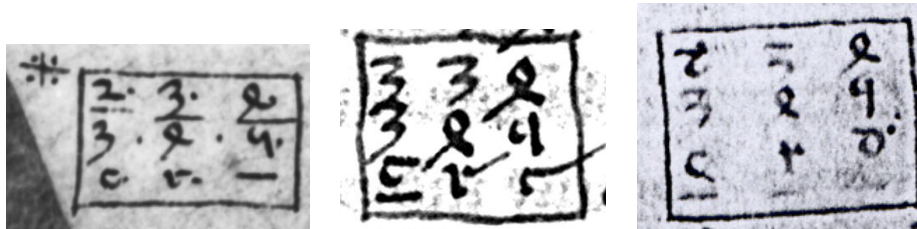


Figure 7: From left to right: F, fol.61v (reference sign at the top left); L, fol.44v; V, fol.73v

$$\begin{array}{ccc} \frac{2}{3} & \frac{3}{4} & \frac{4}{5} \\ \underline{c} & \underline{r} & - \end{array}$$

Figure 8: Symbolic format for $+\frac{2}{3}x^2; +\frac{3}{4}x; +\frac{4}{5}$

²¹⁸ 3 quarte radices] tres quarte radices F; tres quartas radices V;

²¹⁹ 4] quatuor V;

²²⁰ figurentur] om. L;

$$\begin{array}{ccc} 2 & 3 & 4 \\ 3 & 4 & 5 \\ \underline{c} & \underline{r} & - \end{array}$$

Figure 9: Symbolic format for $+\frac{2}{3}x^2; +\frac{3}{4}x; +\frac{4}{5}$

Quotiens²²¹ autem ex aliquo istorum diminutum quid proponitur,²²² aliud ei²²³ subscribatur, habens punctum loco virgule, diminutionem indicans. [L, fol.45r] Verbi gratia: 2²²⁴ census minus tribus radicibus, 2²²⁵ census minus 4 dragmis, 5²²⁶ radices minus duobus²²⁷ censibus, 5²²⁸ radices minus 4²²⁹ dragmis sic notentur²³⁰ (Fig. 10 & 11).

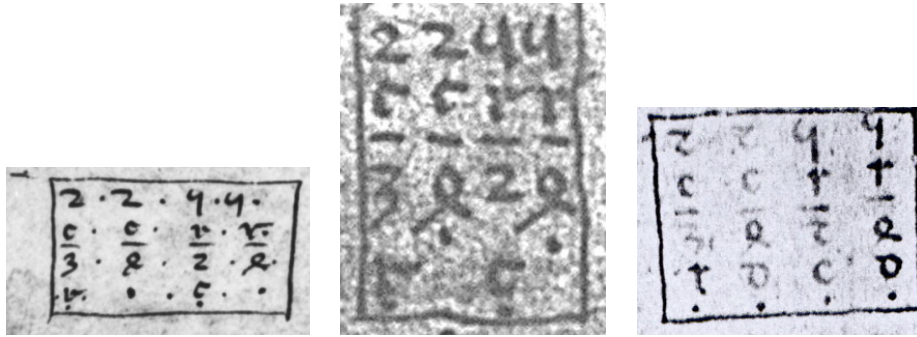


Figure 10: From left to right: F. fol.61v (reference sign at the top left); L. fol.44v; V. fol.73v

[F, fol.61vb] Deinceps in multiplicatione²³¹ rerum, quibus auctus vel diminutus proponitur numerus, aut que augentur vel diminuuntur a numero, id

²²¹ Quotiens] Quociens F;

²²² proponitur] ponitur V;

²²³ aliud ei] ad ei FL;

²²⁴ 2] duo V;

²²⁵ 2] duo FV;

²²⁶ 5] quinque V;

²²⁷ duobus] 2 F;

²²⁸ 5] quinque V;

²²⁹ 4] quatuor V;

²³⁰ notentur] notantur FV;

²³¹ in multiplicatione] multiplicatione V;

2	2	5	5
<u>c</u>	<u>c</u>	<u>r</u>	<u>r</u>
3	4	2	4
r	.	c	.

Figure 11: Symbolic format for $2x^2 - 3x$; $2x^2 - 4$; $5x - 2x^2$; $5x - 4$

ipsum, quod in multiplicatione²³²/p. 66/ digitorum et articulorum observamus, ut res sit tamquam articulus²³³ superhabundans vero vel diminutum tamquam digitus. Si enim proponantur²³⁴ digitus et articulus in digitum²³⁵ et articulum multiplicandi, quadripartimur totam multiplicationem per 4 secundi elementorum,²³⁶ scilicet ut digitus²³⁷ in digitum,²³⁸ articulus in articulum subalterne, digitus in articulum, articulus in digitum, contradictorie multiplicetur,²³⁹ ut diminutus, si diminutum [V, fol.74r] multiplicet,²⁴⁰ vel superhabundans superhabundans;²⁴¹ inde productum multiplicationi²⁴² reliquorum coacervate²⁴³ totius propositae multiplicationis summam ostendit.

Verbi gratia. Proponantur 10 et 2^{244} multiplicanda in 10 et 3. Ducimus²⁴⁵ 2^{246} in 3 et erunt²⁴⁷ 6, item 10 in 10 et erunt 100, item²⁴⁸ 2 in 10 et erunt 20,

²³² quod in multiplicatione] quod implicatione (?) L;

²³³ ut res sit tamquam articulus] et res tamquam articulus fit L;

²³⁴ proponantur] proponatur FV;

²³⁵ digitum] digittum V;

²³⁶ 4 secundi elementorum] quartum secundi elementorum F; quartum secundum elementorum V;

²³⁷ digitus] digittus V;

²³⁸ digitum] digittum V;

²³⁹ multiplicetur] multiplicantur F; Here, the Kaunzner's edition is false: "articulus in digitum, contradictorie multiplice" is forgotten, only "tur" is legible.

²⁴⁰ ut diminutus, si diminutum multiplicet] ut diminutum, si diminutus multiplicet F; *om.* L;

²⁴¹ superhabundans] *om.* L;

²⁴² multiplicationi] implecti L; see footnote 232

²⁴³ coacervate] coacervatum FV;

²⁴⁴ 10 et 2] 10 22 L;

²⁴⁵ Ducimus] Ducimur V;

²⁴⁶ 2] *om.* L;

²⁴⁷ erunt] est V;

²⁴⁸ item] *om.* LV;

item²⁴⁹ 10 in 3 et erunt 30, quorum summa 156.²⁵⁰ Proponantur quoque 10 minus duobus²⁵¹ in 10 minus tribus²⁵² ducenda. Multiplicamus 2 diminuta in 3 diminuta et erunt 6 addenda, item²⁵³ 10 in 10 et erunt 100, item²⁵⁴ duo²⁵⁵ diminuta in 10 et erunt 20 minuenda, item²⁵⁶ 10 in 3²⁵⁷ diminuta et erunt 30 minuenda,²⁵⁸ ergo reliquorum summa 56 ex multiplicatione provenit.

Qualiter census, radices et dragme inter se multiplicentur.²⁵⁹

Ad instar horum multiplicamus res, quibus diminuitur vel superhabundat²⁶⁰ numerus vel a²⁶¹ ipse numero. Verbi gratia. Proponantur²⁶² 10 dragme minus duabus²⁶³ rebus multi[L, fol.45v]plicande²⁶⁴ in 10²⁶⁵ dragmas minus tribus rebus. Multiplicamus duas²⁶⁶ res diminutas in 3²⁶⁷ res diminutas²⁶⁸ et erunt 6 census addendi, item 10 dragmas in 10 fiunt 100,²⁶⁹ item²⁷⁰ 2²⁷¹ res diminutas in 10 dragmas fiunt 20 res minuende, item²⁷² 10 in 3 res diminutas²⁷³ fiunt 30²⁷⁴ res minuende. Summa igitur huius multiplicationis est 6

²⁴⁹ item] *om.* LV;

²⁵⁰ quorum summa 156] quorum quorum summa est 156 L;

²⁵¹ duobus] 2 V;

²⁵² tribus] 3 V;

²⁵³ item] *om.* LV;

²⁵⁴ item] *om.* LV;

²⁵⁵ duo] 2 FV;

²⁵⁶ item] *om.* LV;

²⁵⁷ 3] tria F;

²⁵⁸ minuenda] diminuenda F;

²⁵⁹ Qualiter census, radices et dragme inter se multiplicentur.] *om.* FL;

²⁶⁰ superhabundat] superperhabundat L;

²⁶¹ a] *om.* FL;

²⁶² Proponantur] Ponantur V;

²⁶³ duabus] 2 F;

²⁶⁴ multiplicande] multiplicanda V;

²⁶⁵ in 10] in tribus et 10 F;

²⁶⁶ duas] 2 F;

²⁶⁷ 3] tres F;

²⁶⁸ in 3 res diminutas] *ad.* V;

²⁶⁹ item 10 dragmas in 10 fiunt 100] item ~~diminutas~~ 10 dragmas in 10 et fiunt 100 dragme F;

²⁷⁰ item] *om.* LV;

²⁷¹ 2] duas F;

²⁷² item] *om.* LV;

²⁷³ 10 in 3 res diminutas] 10 dragmas in 3 res diminutas F;

²⁷⁴ 30] 3 L;

census et 100 dragme minus 50 rebus.

Item multiplicentur 10 dragme plus duabus rebus in 10 dragmas minus tribus²⁷⁵ rebus. 2 res addite²⁷⁶ in 3 res diminutas fiunt /p. 67/ 6 census²⁷⁷ minuendi, 10 in 10 fiunt 100 dragme, 2 res addite in 10 fiunt 20 res addite,²⁷⁸ 10 in 3 res diminutas fiunt 30²⁷⁹ res minuende.²⁸⁰ Restauramus²⁸¹ ergo per 20 priores res res 30²⁸² diminutas, et sic erunt tantum 10²⁸³ res diminute. Ex hac ergo multiplicatione surgunt²⁸⁴ 100 dragme minus 6 censibus et 10 rebus. Similiter agendum est²⁸⁵ in multiplicatione numerorum cum fractionibus propositorum.

[F, fol.62ra] Proponantur enim due²⁸⁶ dragme et tercia, multiplicande in 2²⁸⁷ dragmas et unam quartam. Tercia in quartam fit 12^a,²⁸⁸ item²⁸⁹ 2 in 2²⁹⁰ fiunt 4, item²⁹¹ tercia in 2 fiunt 2^e tercie,²⁹² item²⁹³ 2 in quartam²⁹⁴ fit semis. Totum ergo fit 5 dragme et 3 duodecime.²⁹⁵

²⁷⁵ tribus] 3 F;

²⁷⁶ 2 res addite] due res F; 2 res L;

²⁷⁷ census] *om.* F;

²⁷⁸ 2 res addite in 10 fiunt 20 res addite] 2 res in 10 fiunt 20 res FL;

²⁷⁹ 30] 3 F;

²⁸⁰ minuende] diminutas V;

²⁸¹ Restauramus] Restauravimus V;

²⁸² Restauramus ergo per 20 priores res res 30] Restauramus per 20 priores res 20 F;
Restauramus ergo per 20 priores res 20 res 30 L;

²⁸³ 10] 100 L;

²⁸⁴ surgunt] insurgunt V;

²⁸⁵ est] *om.* L;

²⁸⁶ due] 2 V;

²⁸⁷ 2] duas V;

²⁸⁸ Tercia in quartam fit 12^a] Tercia in quartam ducta fit una duodecima F; Tercia in quartam fit 12 L;

²⁸⁹ Item] *om.* LV;

²⁹⁰ 2 in 2] 2 in 2 ducta F;

²⁹¹ item] *om.* LV;

²⁹² tercia in 2 fiunt 2^e tercie] una tercia in 2 fiunt 2^e tercie F; tercia in 2 fiunt due tercie V;

²⁹³ item] *om.* LV;

²⁹⁴ 2 in quartam] 2 in unam quartam F;

²⁹⁵ duodecime] 12^e V;

Item 2 dragme minus tertia in 2^{as} minus quarta ducantur.²⁹⁶ Tercia diminuta in quartam diminutam fit 12^a addenda,²⁹⁷ item²⁹⁸ 2 in 2 fiunt 4, item²⁹⁹ tertia diminuta in 2 fiunt due tercie³⁰⁰ minuende, item³⁰¹ 2 in quartam³⁰² diminutam fit semis³⁰³ minuenda.³⁰⁴ Totum ergo est 2 dragme et undecim duodecime.³⁰⁵ Et nota, quod ex re³⁰⁶ diminuta in rem provenit³⁰⁷ census diminutus. Hiis multiplicandi preceptis instructus, animum tuum in restaurationibus consequenter exercebis.³⁰⁸ Cuius facultati³⁰⁹ duo subiciamus³¹⁰ exempla [L, fol.46r] ad posteriorem tractatum perutilia.

De radicibus.³¹¹

Radice 200^{rum} ³¹² minus 10, applicata ad 20 minus radice 200^{rum} ,³¹³ id quod colligitur, equivalet 10. Verbi gratia. Radix 200^{rum} ³¹⁴ minus 10 sit /p. 69/ AB , haec applicetur³¹⁵ ad 20 minus radice 200^{rum} ,³¹⁶ quod fit³¹⁷ BC . Dico ergo quod tota AC equivalet 10. Sumamus ergo ex BC 10,³¹⁸ que sint

²⁹⁶ Item 2 dragme minus tertia in 2^{as} minus quarta ducantur] Item si 2 dragme minus una tertia in duas dragmas minus unius una quarta ducantur F; Item 2 dragme minus tertia in duo minus quartam ducatur V;

²⁹⁷ Tercia diminuta in quartam diminutam fit 12^a addenda] Tercia diminuta in quartam diminutam ducta fit una duodecima addenda F;

²⁹⁸ item] *om.* LV;

²⁹⁹ item] *om.* LV;

³⁰⁰ tertia diminuta in 2 fiunt due tercie] Item tertia diminuta in 2 fiunt 2 tercie F; tertia diminuta in 2 fiunt $\frac{2}{3}$ V;

³⁰¹ item] *om.* LV;

³⁰² quartam] unam quartam F;

³⁰³ semis] $\frac{1}{2}$ V;

³⁰⁴ minuenda] diminuenda F;

³⁰⁵ undecim duodecime] 11 duodecime F; $\frac{11}{12}$ V;

³⁰⁶ ex re] res V;

³⁰⁷ provenit] facit V;

³⁰⁸ consequenter exercebis] continenter exercebit V;

³⁰⁹ facultati] facilitatis FV;

³¹⁰ subiciamus] subiciemus FV;

³¹¹ De radicibus.] *om.* FL;

³¹² 200^{rum}] ducentorum V;

³¹³ 200^{rum}] ducentorum V;

³¹⁴ 200^{rum}] ducentorum V;

³¹⁵ haec applicetur] hec applicetur V;

³¹⁶ 200^{rum}] 200 F; ducentorum V;

³¹⁷ fit] sit V;

³¹⁸ Sumamus ergo ex BC 10] Sumemus ergo ex BC 10 F; *om.* V;

BD , per que restauretur radix 200^{rum} ,³¹⁹ ergo AD est ple[V, fol.74v]na radix 200^{rum} ,³²⁰ DC vero reliquum est 10 minus radice 200^{rum} ³²¹. Ergo tota AC est radix 200^{rum} ³²² et insuper 10 minus eadem radice, quam diminutam,³²³ loco eiusdem superhabundantis³²⁴ abicimus, et relinquitur AC 10.

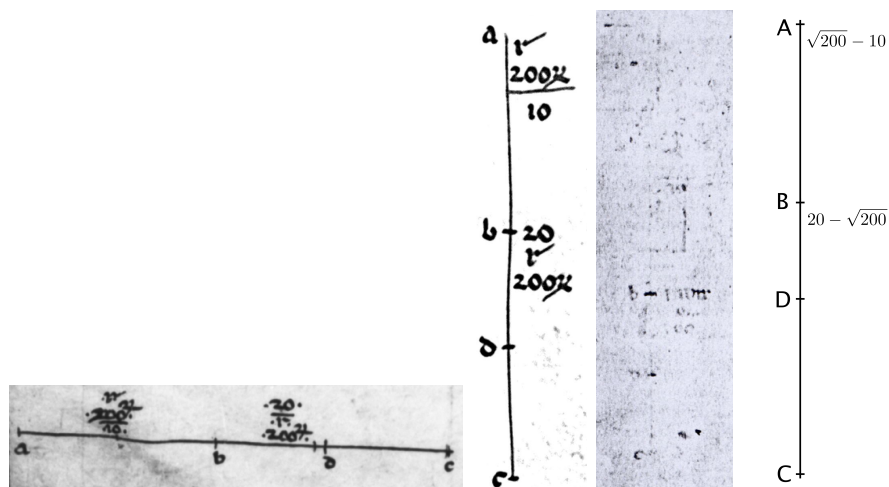


Figure 12: From left to right: F. fol.62r; L. fol.46r; V. fol.74r; modern transcription

/p. 70/ **Hic eciam tractatur de radicibus.**³²⁵

Similiter radice 200^{rum} ³²⁶ minus 10, sublata ex 20 minus³²⁷ radice 200^{rum} ,³²⁸ quod reliquum est equivalet 30 minus duabus radicibus 200^{rum} .³²⁹

Verbi gratia. Sit AB 20 minus radice 200^{rum} ,³³⁰ cui subtrahatur AC , que

³¹⁹ 200^{rum}] 200 V;

³²⁰ 200^{rum}] 200 V;

³²¹ 10 minus radice 200^{rum}] minus 10 radice 200^{rum} F; 10 minus radice de 200 V;

³²² radix 200^{rum}] radix 200^{rum} L; radix de 200 V;

³²³ diminutam] diminuta V;

³²⁴ superhabundantis] superhabundans F;

³²⁵ Hic eciam tractatur de radicibus.] *om.* FL;

³²⁶ 200^{rum}] 200 V; 200^{rum} L;

³²⁷ 20 minus] 20 minus minus F;

³²⁸ 200^{rum}] 200 V; 200^{rum} L;

³²⁹ radicibus 200^{rum}] radicibus 200 V; radicibus 200^{rum} L;

³³⁰ Sit AB 20 minus radice 200^{rum}] Sit AB 20 minus radice 200 V; Sit AB minus radice 20 200^{rum} L;

sit radix 200^{rum} minus 10.³³¹ Dico ergo³³² quod reliquum CB est 30 minus duabus³³³ radicibus 200^{rum} .³³⁴ Applicabimus igitur³³⁵ ad AB 10, que sint DA . Palam est ergo quod tota DB ³³⁶ est 30 minus radice 200^{rum} ,³³⁷ ex qua sublata DC , hoc est plena radice 200^{rum} ,³³⁸ relinquitur CB 30 minus duabus radicibus 200^{rum} .³³⁹

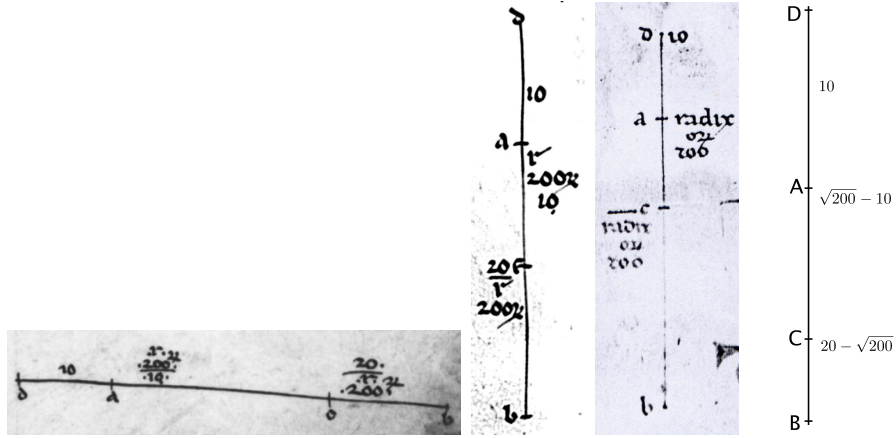


Figure 13: From left to right: F. fol.62r; L. fol.46r; V. fol.74v; modern transcription

De multiplicatione radicum.³⁴⁰

Deinceps vero³⁴¹ in accipiendo cuiuslibet radiceis quocumque³⁴² multiplice, censum eius in³⁴³ tetragonum numeri, a quo multiplex denominatur, multiplica, et provenientis summe radix propositam radicem [F, fol.62rb] se-

³³¹ radix 200^{rum} minus 10] radix 200^{rum} minus x F; radix minus 10 V; radix 200^{rum} minus 10 L;

³³² ergo] *om.* L;

³³³ duabus] 2 FV;

³³⁴ 200^{rum}] 200 V; 200^{rum} L;

³³⁵ igitur] ergo F;

³³⁶ DB] AB FV;

³³⁷ 200^{rum}] 200 V; 200^{rum} L;

³³⁸ radice 200^{rum}] radice de 200 V; radice 200^{rum} L;

³³⁹ 200^{rum}] 200 V;

³⁴⁰ De multiplicatione radicum.] *om.* FL;

³⁴¹ vero] *om.* FL;

³⁴² quocumque] quodcumque V;

³⁴³ in] *om.* V;

cundum illum numerum multiplicat. Verbi gratia. Si radicis³⁴⁴ 9 duplicem sumere volueris, 9³⁴⁵ in tetragonum binarii,³⁴⁶ a quo duplex denominatur, multiplica, et efficies 36, cuius radix, scilicet 6, propositae radicis duplex est.

Si autem radicis alicuius quamcumque³⁴⁷ submultiplicem³⁴⁸ quesieris, censum eius per tetragonum numeri, a quo submultiplex denominatur, divide, et exeuntis summae radix propositae radici³⁴⁹ secundum illum numerum submultiplex erit. Verbi gratia. [L, fol.46v] Si radicis 36 subduplum inquiris, 36 per tetragonum binarii, a quo denominatur subduplus,³⁵⁰ divide, et exibunt 9, quorum radix propositae radici subdupla³⁵¹ est. Hiis memoriter observatis, quaslibet numerorum radices, notas seu surdas,³⁵² duplicare, triplicare, dimidiare, vel terciare, facillimum est.

Quotiens³⁵³ autem radi/p. 71/cem census unius in radicem alterius ducere placuerit, censum per censum multiplica, et provenientis summae radix est quantum provenit ex multiplicatione propositarum radicum. Cum vero quemlibet duplicem³⁵⁴ radicis unius in quemlibet multiplicem radicis³⁵⁵ alterius ducere volueris, produc censum utriusque multiplicationis,³⁵⁶ unum per alterum multiplicans, unde provenientis summae radix dabit summam, que ex ductu radicis multiplicis unius in alterius multiplicatione³⁵⁷ alterius³⁵⁸ colligetur. Verbi gratia. Si duplicem radicis 9 in triplicem radicis³⁵⁹ 4 ducere placuerit, produc censum duplicis radicis³⁶⁰ 9, hoc est 36, et censum tripli-

³⁴⁴ radicis] radicem V;

³⁴⁵ 9] cum F;

³⁴⁶ binarii] binarii F;

³⁴⁷ quamcumque] quodcumque V;

³⁴⁸ submultiplicem] simplicem L;

³⁴⁹ radici] radicis V;

³⁵⁰ subduplus] subduplum F;

³⁵¹ subdupla] subduplam V;

³⁵² surdas] surdis L;

³⁵³ Quotiens] Quociens F;

³⁵⁴ quemlibet duplicem] duplicem quemlibet V;

³⁵⁵ radicis] radicem F;

³⁵⁶ multiplicationis] multiplicis FV;

³⁵⁷ multiplicatione] multiplicem V;

³⁵⁸ alterius] *om.* V;

³⁵⁹ radicis] radicem V;

³⁶⁰ radicis] *ad.* F;

cis³⁶¹ radices 4, qui³⁶² etiam³⁶³ est 36, et unum per alterum multiplica, unde efficies 1296, quorum radix³⁶⁴ est proveniens ex ductu duplilis radices 9 in triplicem radices 4.

De divisionibus radicum.³⁶⁵

In dividenda vero radice per radicem divide censum per censum, unde procedentis summe radix est, quod de divisione provenit. Sed³⁶⁶ in³⁶⁷ dividendo quolibet multiplice radices per quemlibet radices multiplicem produc censum utriusque multi[V, fol.75r]plicis, quorum uno, per alterum diviso, radix eius, quod erit,³⁶⁸ est, quod de divisione provenit.

De proporcione 4 numerorum.³⁶⁹

Inter cetera quoque manifestum³⁷⁰ est, quod quatuor³⁷¹ numerorum proporcionalium³⁷² tantum primus ex quarto quantum³⁷³ secundus ex tertio producit. Et cum duarum [L, fol.47r] radicum utraque per reliquam dividitur, que inde procedunt, /p. 72/ eadem sibi proporcione qua³⁷⁴ radicum census nectuntur,³⁷⁵ unde duo illi census coniuncti equivalent eis³⁷⁶ coniunctis, que ex ductu procedentium [F, fol.62va] de³⁷⁷ divisione in id,³⁷⁸ quod ex multiplicatione radicum provenit, generantur.³⁷⁹ Amplius in reperiendo numero, in quem multiplicata quelibet fractiones restaurent integrum, per ipsas, quotcumque³⁸⁰ fuerint, divide integrum, et serva, quod provenit. In hoc enim

³⁶¹ censum triplicis] triplicem V;

³⁶² qui] quae V;

³⁶³ etiam] *om.* L;

³⁶⁴ radix] radicem V;

³⁶⁵ De divisionibus radicum.] *om.* FL;

³⁶⁶ Sed] Set V;

³⁶⁷ in] *om.* F;

³⁶⁸ erit] exit F; exierit (?) L;

³⁶⁹ De proporcione 4 numerorum.] *om.* FL;

³⁷⁰ manifestum] maus (?) L;

³⁷¹ quatuor] iiii F; 4 V;

³⁷² proporcionalium] proporcionum L;

³⁷³ quantum] quartum L;

³⁷⁴ qua] que FV;

³⁷⁵ nectuntur] vectuntur V;

³⁷⁶ eis] caeteris V;

³⁷⁷ de] *om.* V;

³⁷⁸ id] hiis F;

³⁷⁹ generantur] generatur V;

³⁸⁰ quotcumque] quodcumque V;

multiplicate³⁸¹ fractiones integrum numerum³⁸² restituent.

Item cum primus duorum numerorum in secundum ducitur, unde prove-
niat tercius, si ducatur etiam tetragonus³⁸³ primi in tetragonum secundi,
procedet³⁸⁴ tetragonus³⁸⁵ tercii. Et cum primus duorum numerorum per se-
cundum dividitur, unde procedat tercius, si dividatur etiam tetragonus³⁸⁶
primi per tetragonum³⁸⁷ secundi, exhibit tetragonus³⁸⁸ tercii. Hiis tamquam
prestructionibus³⁸⁹ totius restaurationis³⁹⁰ facultatis³⁹¹ firmiter inspectis,³⁹²
lectoris animus ad questiones descendat.³⁹³

Formabimus³⁹⁴ igitur 6 questiones, et quomodo in 6 modos praedictos
solvantur, ostendemus.³⁹⁵

**Prima quaestio, quae imposita est in primo minorum, ubi census
equatur radicibus.**³⁹⁶

[#I] Dividimus³⁹⁷ 10 in tales partes, quarum unius tetragonus quadruplus
sit producto³⁹⁸ ex una in alteram, et quenam sint ille partes, quaestio sit.³⁹⁹
Huius vero prima⁴⁰⁰ regula talis est: ponemus lineam⁴⁰¹ AB 10, divisam ad
 C secundum praedictas partes, et esto una earum, scilicet AC , res; /p. 74/

³⁸¹ multiplicate] *om.* F;

³⁸² numerum] unum V;

³⁸³ tetragonus] tetragus V;

³⁸⁴ procedet] procedit V;

³⁸⁵ tetragonus] tetragus LV;

³⁸⁶ tetragonus] tetragus LV;

³⁸⁷ tetragonum] tetragum LV;

³⁸⁸ tetragonus] tetragus LV;

³⁸⁹ prestructionibus] premistructionibus F;

³⁹⁰ restaurationis] restauratioe FL;

³⁹¹ facultatis] facilitatis V;

³⁹² inspectis] inscriptis F;

³⁹³ descendat] descendet FV;

³⁹⁴ Formabimus] Formalibus F;

³⁹⁵ Formabimus igitur 6 questiones, et quomodo in 6 modos praedictos solvantur, osten-
demus.] *om.* V;

³⁹⁶ Prima quaestio, quae imposita est in primo minorum, ubi census equatur radicibus.]
om. FL;

³⁹⁷ Dividimus] Dividamus V;

³⁹⁸ producto] productio FL;

³⁹⁹ sit] fit V;

⁴⁰⁰ prima] *om.* FV;

⁴⁰¹ ponemus lineam] ponamus linea V;

consequens⁴⁰² est ergo, ut reliqua CB sit 10 minus re.⁴⁰³ Porro ducatur res in se et sit⁴⁰⁴ census, eadem quoque res⁴⁰⁵ ducatur in 10 minus re et sunt⁴⁰⁶ 10 res minus censu, quarum quadruplum esse censum, quem⁴⁰⁷ res ex se protulit, necesse est. Mox igitur, ut in quatuor⁴⁰⁸ multiplicatae fuerint, 10 res minus censu⁴⁰⁹ producantur⁴¹⁰ 40 res minus 4 censibus, uni censui equales. Restaurabimus ergo diminuta, idque aliter fieri nequit,⁴¹¹ [L, fol.47v] nisi 4 census diminuti⁴¹² tam radicibus quam censui accedant. Quo facto, integre 40 res 5 censibus equivalent, quibus, ad simplicem censum reductis, erit unus census 8⁴¹³ rebus equivalens.⁴¹⁴ Palam ergo, quod res una est 8⁴¹⁵, scilicet pars AC , relinquitur itaque CB 2, et sic in primum 6 minorum,⁴¹⁶ qui est: census equivalet⁴¹⁷ radicibus, quaestio nostra soluta est.

Secunda questio, imposita in secundo minorum, ubi census equatur numero.⁴¹⁸

[#II] Dividimus⁴¹⁹ itaque⁴²⁰ 10 in tales partes, quarum unius tetragonus⁴²¹ in duas dragmas et 7 nonas ductus tetragono totius equivalenceat. Quarum inven[⁴²²tionis regula haec est. Sit AB , ut consuevimus, 10, AC vero

⁴⁰² consequens] continens V;

⁴⁰³ The two following geometric representations are only in (V). In (U) – the Latin-Italian translation –, and only in (U), there is a third diagram.

⁴⁰⁴ sit] fit V;

⁴⁰⁵ res] *om.* V;

⁴⁰⁶ sunt] fiunt V;

⁴⁰⁷ quem] quam V;

⁴⁰⁸ quatuor] 4 FV;

⁴⁰⁹ 10 res minus censu] *om.* FL;

⁴¹⁰ producantur] producentur FV;

⁴¹¹ nequit] nequed V;

⁴¹² diminuti] diminute F; diminuete L;

⁴¹³ 8] scilicet F;

⁴¹⁴ 8 rebus equivalens] 8 rebus equivalentes L; equivalens 8 rebus V;

⁴¹⁵ 8] *om.* F;

⁴¹⁶ minorum] 4^{or} F; 4 L;

⁴¹⁷ equivalet] equivalens V;

⁴¹⁸ Secunda questio, imposita in secundo minorum, ubi census equatur numero.] *om.* FL;

⁴¹⁹ Dividimus] Dividamus V;

⁴²⁰ itaque] *om.* FV;

⁴²¹ tetragonus] tetragus LV;

⁴²² haec est] est haec F;

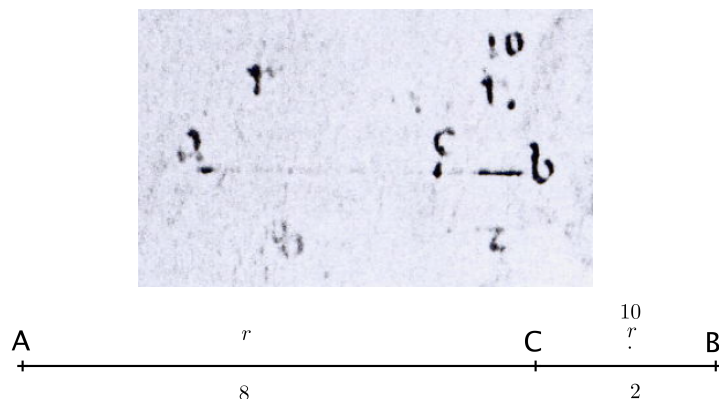


Figure 14: V. fol.75r; modern transcription

res,⁴²³ CB sit⁴²⁴ 10 excepta re. AC quoque ducatur in se et sit⁴²⁵ census. Hic ducatur in 2 dragmas⁴²⁶ et 7 nonas, consequens⁴²⁷ est ergo ut que proveniunt, scilicet⁴²⁸ 2 census cum 7 nonis, tetragono AB , hoc est 100, equivalenceant. Haec demum partiendo, ad unum censum convertantur. Consi[V, fol.75v]derato quippe, quota pars sit unus census duorum et 7 nonarum, hoc est ipsarum⁴²⁹ pars quinta⁴³⁰ et insuper eiusdem quinte partis 4⁴³¹ quinte, si tantum de utriusque collatis⁴³² eiciatur, simplex census 36 dragmis equabitur. Palam ergo quod eiusdem radix est 6, et haec est res AC . Taliter itaque quaestio est ad secundum modum, ubi census equantur⁴³³ numero,⁴³⁴ relapsa est.

Questio imposta in tercio capitulo minorum, ubi radices equan-

⁴²³ 10, AC vero] *om.* V;

⁴²⁴ sit] *om.* FL;

⁴²⁵ sit] fit V;

⁴²⁶ Hic ducatur in 2 dragmas] quod totum in duas dragmas V;

⁴²⁷ consequens] continens V;

⁴²⁸ scilicet] sit V;

⁴²⁹ ipsarum] ipsorum V;

⁴³⁰ quinta] 5^a F; $\frac{1}{5}$ V;

⁴³¹ 4] quatuor V;

⁴³² utriusque collatis] utrumque collatis F; utrumque collectis V;

⁴³³ equantur] equatur FV;

⁴³⁴ numero] *om.* V;

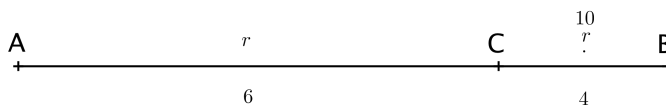
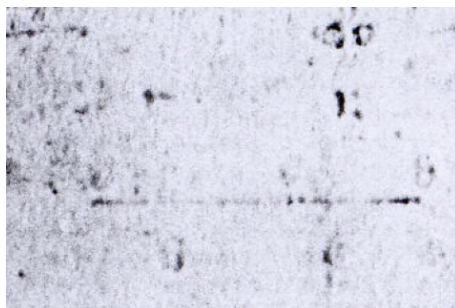


Figure 15: V. fol.75r; modern transcription

tur numero.⁴³⁵

[#III] Dividimus⁴³⁶ itaque⁴³⁷ 10 in tales partes, quarum una⁴³⁸ per alteram divisa, proveniant⁴³⁹ 4. Sit ergo, ut mos⁴⁴⁰ est, una partium res, altera 10 minus re, et quia /p. 75/ divisio 10 minus re per rem, ut constitutum est, proveniunt 4. Palam est ergo⁴⁴¹ quod ex multiplicatione rei in 4 proveniunt 10 minus re, quotiens⁴⁴² enim dividens multiplicatur in id, quod per eius divisionem ex dividendo provenit. Reddit dividendus igitur 4 res⁴⁴³ equivalent 10 minus re, et post diminuti restauracionem 5 res equivalent 10, una igitur res equatur [L, fol.48r] duobus,⁴⁴⁴ unde haec quaestio in tertium modum qui est ‘radices equivalent numero’ soluta est.

Questio imposita in primo capitulo maiorum.⁴⁴⁵

⁴³⁵ Questio imposita in tercio capitulo minorum, ubi radices equantur numero.] *om.* FL;

⁴³⁶ Dividimus] Dividamus V;

⁴³⁷ itaque] *om.* FV;

⁴³⁸ una] unam V;

⁴³⁹ proveniant] proveniat V;

⁴⁴⁰ mos] mox V;

⁴⁴¹ ergo] *om.* FL;

⁴⁴² quotiens] quociens F;

⁴⁴³ igitur 4 res] .6. 4 F;

⁴⁴⁴ duobus] duabus F;

⁴⁴⁵ Questio imposita in primo capitulo maiorum.] *om.* FL;

[#IV] Deinde quenam sit multitudo cuius tertia⁴⁴⁶ cum⁴⁴⁷ una dragma, multiplicata⁴⁴⁸ in quartam eius cum una⁴⁴⁹ dragma, 20 dragmas educat, investigamus. Sit multitudo res, et ducatur tertia rei cum dragma in quartam rei cum dragma. Unde producuntur⁴⁵⁰ 12^{a451} census et dragma et tertia ac⁴⁵² quarta rei, quas 20 dragmis equari constituimus. Hic autem restauratione opus est. Census quoque, nisi per 12 crescat, minime restauratur. Mox vero ut est⁴⁵³ eo singula per 12 creverint 12⁴⁵⁴ dragme altrinsecus⁴⁵⁵ habundantes utrobique [F, fol.63ra] eiecte fuerint, relinquitur integer⁴⁵⁶ census cum 7 radicibus 228⁴⁵⁷ dragmis equivalens. Haec quoque quaestio, ubi census una cum radicibus equivalence⁴⁵⁸ numero, per quartum modum secundum regulam supradictam solvitur, et res ibidem reperta, propositam multitudinem ostendit. Verbi gratia. Res quadra medias que sunt 3 et semis, et efficies 12 et quartam. Hiis adice⁴⁵⁹ dragmas⁴⁶⁰ et⁴⁶¹ habebis⁴⁶² 240⁴⁶³ et quartam, quorum radici,⁴⁶⁴ hoc est 15 et semis, aufer dimidias res, et relinquuntur⁴⁶⁵ 12, quae sunt res quesita.⁴⁶⁶

Questio imposita in secundo capitulo maiorum.⁴⁶⁷

[#V] Dividimus⁴⁶⁸ 10 in tales partes quarum tetragoni aggregati 58 effi-

⁴⁴⁶ tertia] tercium V;
⁴⁴⁷ cum] est LV;
⁴⁴⁸ multiplicata] multiplica L;
⁴⁴⁹ una] *om.* FL;
⁴⁵⁰ producuntur] producitur V;
⁴⁵¹ 12^a] 12 FL;
⁴⁵² ac] hac V;
⁴⁵³ est] cum FV;
⁴⁵⁴ 12] *om.* V;
⁴⁵⁵ altrinsecus] altrinsecus V;
⁴⁵⁶ integer] *om.* L;
⁴⁵⁷ 228] 223 F;
⁴⁵⁸ equivalence] equivalent V;
⁴⁵⁹ adice] addice L;
⁴⁶⁰ dragmas] dragmis *cor.* dragmas F;
⁴⁶¹ et] unde F;
⁴⁶² habebis] efficies V;
⁴⁶³ 240] The 4 could be confused with 8 L;
⁴⁶⁴ radici] radicem V;
⁴⁶⁵ relinquuntur] relinquetur FV;
⁴⁶⁶ quae sunt res quesita] nisi sunt res res quesita L;
⁴⁶⁷ Questio imposita in secundo capitulo maiorum.] *om.* FL;
⁴⁶⁸ Dividimus] Dividamus V;

ciant. Sit autem una partium res, altera 10 /p. 77/ minus re, et productum ex re in se, 1⁴⁶⁹ census, coacervatur⁴⁷⁰ producto ex 10 minus re in se, 1⁴⁷¹ censui et 100 minus 20 rebus, unde colliguntur 2 census et 100 minus 20 rebus, que data sunt equalia 58. Post restaurationem ergo 2⁴⁷² census cum 100 equivalent 58 dragmis cum 20 rebus, post eiectionem quoque dragmarum habundantium 2 census [V, fol.76r] cum 42 equivalent 20 rebus, post reductionem vero ad unum censum fit⁴⁷³ census cum 21 equalis 10 rebus.⁴⁷⁴ Palam ergo quod haec quaestio in modum, ubi census cum [L, fol.48v] numero rebus equivalet, conversus est.⁴⁷⁵ Per cuius regulam, quenam sit una pars,⁴⁷⁶ diligenter adverte,⁴⁷⁷ et 3 invenies.⁴⁷⁸

Questio imposita in tercio capitulo maiorum.⁴⁷⁹

[#VI] Consequenter illam multitudinem, cuius tertia⁴⁸⁰ in quartam⁴⁸¹ eiusdem ducta,⁴⁸² ultra ipsam 24 dragmis se porrigat, intueamur.⁴⁸³ Esto itaque multitudo res, cuius⁴⁸⁴ tertia⁴⁸⁵ in quartam ducta,⁴⁸⁶ census 12^a⁴⁸⁷ producit. Haec vero data est equalis rei et 24 dragmis. Per restaurationem ergo integer census equivalet 12 rebus cum dragmis 288, et quia sub sextum modum cadit, haec quaestio per eius regulam quantitatem⁴⁸⁸ unius rei depre-

⁴⁶⁹ 1] id est V;

⁴⁷⁰ coacervatur] coacervetur FV;

⁴⁷¹ 1] id est V;

⁴⁷² ergo 2] ergo et F; et V; 2 could be read *et* in L;

⁴⁷³ census cum 42 equivalent 20 rebus, post reductionem vero ad unum censum fit] *om.*

L;

⁴⁷⁴ rebus] radicibus V;

⁴⁷⁵ conversus est] soluitur V;

⁴⁷⁶ pars] res FV;

⁴⁷⁷ diligenter adverte] animadvertite diligenter V;

⁴⁷⁸ 3 invenies] invenies 3 V;

⁴⁷⁹ Questio imposita in tercio capitulo maiorum.] *om.* FL;

⁴⁸⁰ tertia] tertiam V;

⁴⁸¹ quartam] 4tas L;

⁴⁸² eiusdem ducta] eiusdem ductam ~~eiusdem ductam~~ V;

⁴⁸³ intueamur] intuemur V;

⁴⁸⁴ cuius] *om.* F;

⁴⁸⁵ tertia] tertiam V;

⁴⁸⁶ ducta] ductam V;

⁴⁸⁷ 12^a] 12 L;

⁴⁸⁸ regulam quantitatem] regulam quantitatem quantitatem F;

hensam per multitudinem data⁴⁸⁹ assignat.⁴⁹⁰ Horum mutatione⁴⁹¹ quaslibet quaestiones secundum restaurationem propositas in praedictos modos solubiles esse, palam est; cuius utilitas ad 10 librum elementorum⁴⁹² praecipua est [F, fol.63rb]: in inveniendis scilicet lineis alogis, id est surdis sive irrationabilibus⁴⁹³ et⁴⁹⁴ medialibus, binomiis et residuis sive reccisis,⁴⁹⁵ que per notum numerum assignari non possunt. In prac/p. 78/tica quoque geometrie et universis quaestionibus ignotorum secundum arismetica formati certissima via est.

De secundo capitulo maiore hec questio soluta est.⁴⁹⁶

[#1] Igitur⁴⁹⁷ sub forma⁴⁹⁸ praecedentium et alias quaestiones proponimus. Queritur⁴⁹⁹ quenam sint ille partes denarii, ex quarum unius multiplicatione per alteram 21⁵⁰⁰ proveniant.⁵⁰¹ Sit una illarum res, altera 10 minus re, ex quarum multiplicatione proveniunt 10 res minus censu, que data sunt equalia 21. Per restaurationem igitur diminuti fiunt 10 res censui ac 21 equales; ecce quintus modus, resolve per eum et invenies partes 3 et 7.⁵⁰²

Questio tercii capituli minorum.⁵⁰³

[#2] Item queritur quenam sint ille partes denarii quarum minoris tetragono de maioris tetragono subducto, restabit⁵⁰⁴ quantum 40. Sit una parcium⁵⁰⁵ res, cuius tetragonus est census, altera 10 minus re, et eius tetragonus⁵⁰⁶ est⁵⁰⁷

⁴⁸⁹ multitudinem data] multitudine data V;

⁴⁹⁰ assignat] assignatur V;

⁴⁹¹ mutatione] muttatione V;

⁴⁹² ad 10 librum elementorum] ad decimum libri elementorum F; ad documentum libri elementorum V;

⁴⁹³ id est surdis sive irrationabilibus] *ad. L; om. FV;*

⁴⁹⁴ et] *om. FL;*

⁴⁹⁵ sive reccisis] *om. FL;*

⁴⁹⁶ De secundo capitulo maiore hec questio soluta est.] *om. FL;*

⁴⁹⁷ Igitur] (?) F;

⁴⁹⁸ forma] formas V;

⁴⁹⁹ Queritur] quarum F;

⁵⁰⁰ 21] 2 L;

⁵⁰¹ proveniant] proveniat FV;

⁵⁰² et invenies partes 3 et 7] *om. FL;*

⁵⁰³ Questio tercii capituli minorum.] *om. FL;*

⁵⁰⁴ subducto, restabit] subductu, restabunt V;

⁵⁰⁵ parcium] *om. FL;*

⁵⁰⁶ tetragonus] tetragus L;

⁵⁰⁷ est] *om. FL;*

100 ac census⁵⁰⁸ minus 20⁵⁰⁹ rebus, ex [L, fol.49r] quibus diminuatur census, et relinquunt 100 minus 20 rebus⁵¹⁰ equales,⁵¹¹ ut supradictum est, 40. Per restaurationem quoque 40⁵¹² et 20⁵¹³ res equivalent 100, per eiectionem vero dragmarum habundantium 60 equivalent 20 rebus. Ergo 3 dragme equantur uni rei, per tertium modum.

Questio secundi capituli maiorum.⁵¹⁴

[#3] Item queritur quenam sint ille partes denarii, quarum differentia iuncta tetragonis earundem colligit⁵¹⁵ 54. Sit una partium res, altera 10 minus re, differentia 10 minus duabus rebus, ex qua et partium tetragonis coniunctis colliguntur⁵¹⁶ 110 et 2 census minus 22⁵¹⁷ rebus, que data sunt equalia 54.⁵¹⁸ Per [V, fol.76v] restaurationem /p. 80/ itaque rerum 2 census cum 110 equivalent 54 et 22 rebus, per eiectionem vero habundantis⁵¹⁹ numeri 56⁵²⁰ et 2 census 22 rebus adequantur, et per conversionem unus⁵²¹ census cum 28 equatur⁵²² 11 rebus. Resolve per quintum modum⁵²³ et res erit 4.

Questio capituli secundi maioris.⁵²⁴

[#4] Queritur etiam quenam sint ille partes denarii, ex quarum utriusque per reliquam divisionem colliguntur⁵²⁵ quantum 2⁵²⁶ dragme et sexta.⁵²⁷ Sit una partium res, altera 10 minus re, quarum tetragoni coniuncti duos⁵²⁸ cen-

⁵⁰⁸ 100 ac census] census et 100 V;

⁵⁰⁹ 20] 2 F;

⁵¹⁰ ex quibus diminuatur census, et relinquunt 100 minus 20 rebus] *om.* V;

⁵¹¹ equales] equalens V;

⁵¹² 40] 4 L;

⁵¹³ 20] 30 F;

⁵¹⁴ Questio secundi capituli maiorum.] *om.* FL;

⁵¹⁵ colligit] collige V;

⁵¹⁶ colliguntur] colligit F;

⁵¹⁷ 22] 20 FV;

⁵¹⁸ 54] 214 F; 24 *cor.* 54 (by another hand) V;

⁵¹⁹ per eiectionem vero habundantis] per eiectionem rebus vero habundantis F;

⁵²⁰ 56] 116 F;

⁵²¹ unus] unius F;

⁵²² equatur] equentur V;

⁵²³ modum] *om.* FL;

⁵²⁴ Questio capituli secundi maioris.] *om.* FL;

⁵²⁵ colliguntur] colligit F;

⁵²⁶ 2] 2^e F;

⁵²⁷ sexta] 6^a V;

⁵²⁸ duos] 2 FV;

sus et 100 minus 20 rebus efficiunt, [F, fol.63va] quibus 4,⁵²⁹ ut supradictum est, equivaleret necessario unius per alteram multiplicatio, ducta⁵³⁰ in ea, que ex⁵³¹ utriusque per reliquam divisionem proveniunt. Hoc vero datum est 2^a et sexta.⁵³² Duc ergo rem in 10 minus re et habebis 10 res minus censu, que in 2 et sextam multiplica, unde collecta 21 res⁵³³ et 2 tercias rei minus duobus censibus et sexta census confer⁵³⁴ tetragonorum coniunctioni, hoc est 2^{bus}⁵³⁵ censibus et 100 minus 20 rebus. Si ergo⁵³⁶ utraque creverint equaliter, compensatis scilicet diminutis censibus, fiunt 21 res et 2⁵³⁷ tercie rei equales 100 et 4 censibus et 6^{te}⁵³⁸ census minus 20 rebus. Et post restaurationem rerum 41 res et 2⁵³⁹ tercie rei equivalebunt 100 [L, fol.49v] et 4 censibus et⁵⁴⁰ 6^{te} quoque census,⁵⁴¹ que necesse est, ad unum censum converti. Considerandus est⁵⁴² ergo, quota pars vel partes sit unus⁵⁴³ census 4 censuum et 6^{te} census;⁵⁴⁴ est autem 5^{ta} et 5^{te} partis 5^{ta},⁵⁴⁵ et tantundem⁵⁴⁶ dragmarum et rerum ad invicem⁵⁴⁷ conferatur. Ergo unus census cum 24 equivaleret 10 rebus; resolve per 5^{tum}⁵⁴⁸ modum,⁵⁴⁹ et res erit 4.

⁵²⁹ quibus 4] *ad.* 4 quibus F; quibus V;

⁵³⁰ ut supradictum est equivaleret necessario unius per alteram multiplicatio, ducta] ut supra datum est necesse equivaleret unius per alteram multiplicata, ducta F; ut supra datum necessario equivalent unius multiplicatio per alteram, ductam V;

⁵³¹ ex] *om.* F;

⁵³² 2^a et sexta] 2 et sextam V; 2^o et sexta L;

⁵³³ 21 res] 21 rem FL; 12^a rei V;

⁵³⁴ confer] *om.* V;

⁵³⁵ 2^{bus}] duobus F; 2 V;

⁵³⁶ ergo] vero F;

⁵³⁷ 2] due V;

⁵³⁸ 6^{te}] sexte FV;

⁵³⁹ 2] due V;

⁵⁴⁰ et] *om.* L;

⁵⁴¹ et 6^{te} quoque census] et sexte census FV;

⁵⁴² est] *om.* FL;

⁵⁴³ unus] unius L;

⁵⁴⁴ 4 censuum et 6^{te} census] 4 censuum et sexte census F; quatuor censuum et sexte census V;

⁵⁴⁵ 5^{ta} et 5^{te} partis 5^{ta}] quinta et quinte partis quinta FV;

⁵⁴⁶ tantundem] tantundem V;

⁵⁴⁷ ad invicem] a diminutione L;

⁵⁴⁸ 5^{tum}] quintum FV;

⁵⁴⁹ modum] *om.* FL;

Capituli secundi maioris.⁵⁵⁰

[#5] Item queritur quenam sint ille partes denarii, quarum unius multiplicatione in 5 per reliquam divisa, si⁵⁵¹ provenientis dimidium eidem⁵⁵² multiplicationi addetur,⁵⁵³ colli/p. 81/guntur 50. Sit una partium res, et ducatur in 5, et erunt 5 res.⁵⁵⁴ Hec dividende sunt per reliquam partem, que est 10 minus re, unde provenit ignotum, cuius dimidium est 50 minus 5 rebus. Nam illud iunctum 5 rebus efficeret, ut constitutum est, 50 dragmas. At quia⁵⁵⁵ ex divisione 5 rerum per 10 minus re ignotum provenire propositum⁵⁵⁶ est, si dimidium ignoti, quod est 50 minus 5 rebus, in 10 minus re ducatur, dimidium 5 rerum, hoc est 2 et semis, produci consequens est.⁵⁵⁷ Hoc autem⁵⁵⁸ est 500 cum 5 censibus minus 100 rebus equivalet 2^{bus} ⁵⁵⁹ rebus et semis. Per conversionem ergo 100 cum uno censu minus 20 rebus equivalet⁵⁶⁰ dimidie rei, et⁵⁶¹ per restaurationem 100 cum uno censu equantur 20 rebus et semis. Resolve per quintum⁵⁶² modum⁵⁶³ et res erit 8.

Tercii capituli minorum.⁵⁶⁴

[#6] Item quaestio: 2 quantitates differunt 2,⁵⁶⁵ quarum minore⁵⁶⁶ per maiorem divisa, provenit semis. Sit quantitas minor res, maior [F, fol.63vb] sit⁵⁶⁷ res et [V, fol.77r] duo.⁵⁶⁸ Quoniam ergo ex divisa re per rem et duo

⁵⁵⁰ Capituli secundi maioris.] *om.* FL;

⁵⁵¹ si] sit V;

⁵⁵² eidem] eadem V;

⁵⁵³ addetur] addatur FV;

⁵⁵⁴ res] *om.* L;

⁵⁵⁵ At quia] atque F;

⁵⁵⁶ provenire propositum] provenire positum F; pervenire positum V;

⁵⁵⁷ dimidium 5 rerum, hoc est 2 et semis, produci consequens est.] Quorum si ex ductu alicuius in aliquo provenient aliud ergo ex ductu medietatis in id provenient medium primi, dimidium 5 rerum hoc est 2 et semis producet consequens est. F;

⁵⁵⁸ autem] *om.* V;

⁵⁵⁹ 2^{bus}] autem 2 F; duabus V;

⁵⁶⁰ equivalet] equalibus F;

⁵⁶¹ et] *om.* LV;

⁵⁶² quintum] 5 F;

⁵⁶³ modum] *om.* FL;

⁵⁶⁴ Tercii capituli minorum.] *om.* FL;

⁵⁶⁵ 2 quantitates differunt 2] due quantitates binario differunt F; Due quantitatibus binario differunt V;

⁵⁶⁶ minore] minor V;

⁵⁶⁷ sit] *om.* FL;

⁵⁶⁸ duo] 2 V;

semis⁵⁶⁹ egreditur, fit ex multiplicatione rei et⁵⁷⁰ duorum in semis, quod
equivalet rei, que deprehenditur⁵⁷¹ esse duo.⁵⁷²

[#7] Item queritur que sint ille partes denarii, quarum si⁵⁷³ differentia
multiplicationem⁵⁷⁴ unius in alteram dividat, 5 et quarta reducet.⁵⁷⁵

[#8] Item queritur quis est census, cuius 4⁵⁷⁶ radicibus in 5 radices ductis,
colligetur quantum 2 census⁵⁷⁷ et 36 dragme.

[#9] Amplius queritur quis sit census cui⁵⁷⁸ tertia sui parte et tribus
dragmis subtractis, si, quod remanet, in se ducatur, census efficitur;⁵⁷⁹ pone
illud rem, quae relinquitur.⁵⁸⁰ /p. 82/

[#10] Deinceps queritur que sint⁵⁸¹ ille partes denarii, per quarum unam
si ipse multiplicetur, tantum producit, quantum ex reliqua in se.

Formatis ad hunc modum, variis questionibus ex praedictis regulis inve-
niatur solutio.⁵⁸²

EXPLICIT⁵⁸³

⁵⁶⁹ et duo semis] 2 et semis V;

⁵⁷⁰ rei et] *om.* V;

⁵⁷¹ deprehenditur] quam comprehenditur V;

⁵⁷² duo] 2 V;

⁵⁷³ si] *om.* L;

⁵⁷⁴ multiplicationem] multiplicationis V;

⁵⁷⁵ quarta reducet] quarta deducet F; quartam deducet V;

⁵⁷⁶ 4] quatuor F;

⁵⁷⁷ 2 census] dua census F;

⁵⁷⁸ cui] cuius V;

⁵⁷⁹ census efficitur] census idem (?) efficitur F;

⁵⁸⁰ illud rem, quae relinquitur] illud rem F; et caetera V;

⁵⁸¹ sint] sunt V;

⁵⁸² solutio] soluto V;

⁵⁸³ EXPLICIT] *om.* FV;

Appendix A. The problems in the *Liber restauracionis*: Modern mathematical transcription

We have two series of problems in the *Liber restauracionis*. For easy reference, I have numbered the problems with # j ($1 \leq j \leq 6$) in Roman numerals for the first one, and # i ($1 \leq i \leq 10$) in Arabic numerals for the second one. In the first series, each type of linear and quadratic equations is represented in the same order that their algorithms of resolution were enunciated in the previous part. In the second series, the organization of the problems do not seem to meet any particular requirement.

First series:

- #I Divide 10 into two parts x and y in such a way that $x + y = 10$ and $x^2 = 4x \times y$.
- #II Divide 10 into two parts x and y in such a way that $x + y = 10$ and $(2 + \frac{7}{9})x^2 = 10^2$.
- #III Divide 10 into two parts x and y in such a way that $x + y = 10$ and $\frac{y}{x} = 4$.
- #IV Determine a number x in such a way that $(\frac{1}{3}x + 1)(\frac{1}{4}x + 1) = 20$.
- #V Divide 10 into two parts x and y in such a way that $x + y = 10$ and $x^2 + y^2 = 58$.
- #VI Determine a number x in such a way that $\frac{1}{3}x \times \frac{1}{4}x = 24 + x$.

Second series:

- #1 Divide 10 into two parts x and y in such a way that $x + y = 10$ and $x \times y = 21$.
- #2 Divide 10 into two parts x and y in such a way that $x + y = 10$ ($x < y$) and $y^2 - x^2 = 40$.
- #3 Divide 10 into two parts x and y in such a way that $x + y = 10$ and $(y - x) + (y^2 + x^2) = 54$.
- #4 Divide 10 into two parts x and y in such a way that $x + y = 10$ and $\frac{x}{y} + \frac{y}{x} = 2 + \frac{1}{6}$.
- #5 Divide 10 into two parts x and y in such a way that $x + y = 10$ and $5x + \frac{1}{2} \cdot \frac{5x}{y} = 50$.
- #6 Determine two numbers x and y in such a way that $y - x = 2$ and $\frac{x}{y} = \frac{1}{2}$.
- #7 Divide 10 into two parts x and y in such a way that $x + y = 10$ ($x < y$) and $\frac{x \times y}{y - x} = 5 + \frac{1}{4}$.

- #8 Determine a number x in such a way that $4x \times 5x = 2x^2 + 36$.
- #9 Determine a number⁵⁸⁴ x in such a way that $(x - (\frac{1}{3}x + 3))(x - (\frac{1}{3}x + 3)) = x$.
- #10 Divide 10 into two parts x and y in such a way that $x + y = 10$ and $x^2 = 10y$.

Appendix B. Comparison with the *Mukhtaṣar* of al-Khwārizmī and with the two well-known Arabic-Latin translations

All the problems of the first series are identical in the *Mukhtaṣar* of al-Khwārizmī (Rashed 2007, 144-156; Rashed 2009, 144-156), in the two well-known Arabic-Latin translations ((Hughes 1986, 217) for Gerard and (Hughes 1989, 53-56) for Robert) and in the *Liber restauracionis*. Nevertheless, we have to mention the difference for #I and #II with geometrical representations in the *Liber restauracionis*, absent from the other texts (for more details (Moyon 2017, 247-249)).

For the second series, I reproduce here the table of comparison established in (Moyon 2017, 268) with a correction for #10.

⁵⁸⁴ I give here the correct statement of the ninth problem in relation to (Moyon 2017, 267). Taking into account the use of the term *census* (*māl* in al-Khwārizmī's text) as the square of the root, one could have written the problem as follows: "Determine a square number y in such a way that $(y - (\frac{1}{3}y + 3))(y - (\frac{1}{3}y + 3)) = y$ ". The solutions are 9 and $\frac{9}{4}$, both problems are equivalent in \mathbb{Q}_+^* .

The <i>Liber restauracionis</i>	The <i>Mukhtasar</i> according to (Rashed 2007; Rashed 2009)	Robert of Chester according to (Hughes 1989)	Gerard of Cremona according to (Hughes 1986)
#1	1	1	1
#2	2	2	2
#3	3	3	3
#4	4	∅	4
#5	5	∅	5
#6	8	5	7
#7	10	∅	8
#8	18	8	9
#9	21	10	10
#10	9	∅	App.2

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