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# How language and culture affect the learning of fractions: A case study in the Kingdom of Tonga

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*In this paper, I look at how different cultural practices go hand in hand with different discourses and how the two of them together have an impact on the learning of certain formal mathematical ideas. The findings are based on fieldwork carried out in the Kingdom of Tonga in 2011, with the aim of answering the question: How do the Tongan language and Tongan cultural practices shape discourses on fractions? I examine the place of fractions in the Tongan community of discourse. Importantly, the findings provide strong evidence to support the classical idea of linguistic relativism in the form of an updated version of the Sapir-Whorf hypothesis.*

*Keywords: Linguistic relativism, fractions, the Kingdom of Tonga, communities of discourse.*

## Teaching fractions in the Kingdom of Tonga

As a teacher of mathematics and statistics at Atenisi University in the Kingdom of Tonga (a group of Islands in the South Pacific) in 1994, and again in 2010, I experienced great difficulty explaining some of the mathematical concepts involved in the domain of probability. After some investigation, I discovered that part of the problem begins with the earlier concept of fraction, the use of which is indispensable for the understanding of the canonical Western way of interpreting and measuring probability. The research developed from an observation that a group of students, who were proficient at other subjects in the mathematics curriculum, had specific difficulty understanding both fractions and probability. In this paper, I concentrate on my findings concerning fractions and only mention in passing findings relating to probability.

Preliminary observations indicated that the Tongan language does not provide Tongan students with the tools and the intuitive ideas that are so important in developing the ideas of uncertainty, probability, and fractions. Although secondary and tertiary education is supposed to be in English, my students regularly switched to Tongan when discussing what I was teaching. Tongan is the language that mediates and organises these students' lives and activities. These observations motivated the research question: How do the Tongan language and cultural practices shape discourses on fractions? More generally, the hypothesis to be tested is that the linguistic tools provided by the Tongan language differ significantly from European languages and as a result, some of the Western concepts concerning fractions do not exist in the community of native Tongan discourse. This said, as consequence of the arrival of the English language and Western forms of life, both the discourse concerning fractions and activities that require dividing whole objects into equal parts are in the process of changing in Tonga.

My research experience has led me to the conclusion that the development of discourses on fractions are directly related to some of the cultural practices concerning dividing objects into parts in specific communities. In this paper, I draw on my fieldwork on this topic in an attempt to understand and explain the discourse on fractions, which I observed in the Kingdom of Tonga. The underlying research is in process and the findings that I present are preliminary. As it will be shown, data and findings are at this stage suggestive enough to warrant further research.

## **Language, culture, and mathematical thinking**

The idea that language shapes people's views of the world (i.e., their worldviews) has a long history going back at least as far as Roger Bacon in the thirteenth century and Wilhelm von Humboldt in the nineteenth century (Gumperz & Levinson, 1993). The idea was popularised in the mid twentieth century by Benjamin Whorf and, since then, it is commonly known as the Sapir-Whorf hypothesis (Whorf, 1959). Initially there was great interest in the hypothesis and then for many years it did not receive much attention. Recently there has been a renaissance of interest (Deutscher, 2010). It is suggested that part of the reason that Whorf's ideas did not receive more support is that he over stated his case and did not base his ideas on clear definitions. He claimed that our mother tongue restricts how we think and prevents us from being able to think certain thoughts. The dominant approach today (known as neo-Whorfism or linguistic relativism) is that when we learn our mother tongue, we acquire certain habits of thought that shape our experience in significant and often surprising ways (Deutscher, 2010).

In his research, concerning the Oksampin communities in Papua New Guinea, Saxe develops the idea that not only language. However, culture and history are particularly related to how mathematical ideas are understood. He proposes a methodological approach "rooted in the idea that both culture and cognition should be understood as processes that are reciprocally related, each participating in the constitution of the other" (Saxe, 2012, p. 16). My research in Tonga originated with the idea that language affects understanding but it quickly became clear that the cultural and historical background of the community had to be taken into account.

## **Learning fractions**

An overview of research into how fractions are learnt can be found in the seminal work of Lamon (2007). In her review she states that there has been relatively little research in this field during the last twenty years and that most references go back to the 1980s or earlier. There are a number of articles by Kieren (1988) and Thompson and Saldanha (2003), which describe the steps involved, and the difficulties faced, in the development of the discourse on fractions.

For a number of reasons fractions are seen as a particularly complicated and difficult subject to teach. One of the main explanations for this is that there are numerous definitions of a fraction. Fractions can be a number (two thirds is a number between half and three quarters), an operator (give her two thirds of the cake), an adjective (two thirds majority) or a measure (two thirds of a litre). The canonical use of fractions in the western education system assumes some type of synthesis of all these definitions into one concept.

The various diverse meanings of fractions are defined by Kieren (1988), who builds an ideal network of personal rational number knowledge, which links what he refers to as the "intuitive" to the "formal" definitions of rational numbers. As summarised by Thompson and Saldanha (2003, p. 100), Kieren's basic approach is, "to break the concept of rational number into sub constructs – part-whole, quotient, ratio, number, operator and measure - and then describe rational number as an integration of those sub-constructs." On the other hand, Lamon (2007) suggests that one of the ways of understanding fractions is through the process of sharing equally (for example, sharing three biscuits between seven people). This is relevant to some of my empirical findings concerning practices of and attitudes to sharing in Tonga. Nonetheless, at this stage I have found no literature in the field on how language or cultural norms affect the learning of fractions.

## **A discursive approach**

I adopt the discursive approach in which mathematics is defined as a form of communication or discourse (Sfard, 2008). The discursive approach postulates that people are members of various

overlapping “communities of discourse”. Discourse is a specific form of communication defined by the vocabulary, the visual mediators, the communicational routines and the endorsed narratives used. A community of discourse is defined as those individuals capable of participating in any given discourse and by the endorsed narratives that they use. An individual can be a member of a number of overlapping communities of discourse. This approach supplied a powerful framework through which to understand and explain my observations in Tonga

Using this theoretical framework and for the specific topic of fractions, the aim of the research is to identify and analyse the community of discourse observed in Tonga as well as how it compares with typical Western communities of discourse. In this paper, I aim to analyse the discourse about fractions, sharing, and dividing whole objects, which I observed and experienced in Tonga.

### **Some findings concerning the use (or lack of use) of fractions in Tonga**

The seeds of this research emerged when I was trying (unsuccessfully) to help a student answer the question "What is the probability of getting two heads when two coins are tossed?" The student was an able university student. She was proficient at algebra and passed an introduction to calculus course. I recorded the following one-sided conversation:

Lecturer:           So what is half of a half?  
Student:            [Blank face]  
Lecturer:           *Ko e ha e haafe 'o e haafe?* [I translated with help the question into Tongan]  
Student:            Blank face]

As a result of this conversation I started asking colleagues, friends and acquaintances the following: “What is half of a half” [*Ko e ha e haafe 'o e haafe?*] and found that most Tongan speakers (including many well educated Tongans who speak good English) do not give the expected (Eurocentric) answer to this question. One of the most unexpected observations happened when I was discussing teaching fractions with a senior maths teacher at one of the best high schools. He described how a lack of ability with fractions causes problems when teaching other more advanced fields of mathematics. When I mentioned the question, “What is half of a half?” he answered, “I have no idea”.

Further early observations included the fact that in the fruit and vegetable market nothing is weighed and everything is sold by the "pile". Rarely is a price displayed because it is assumed that the price of a pile is three pa'anga (three Tongan dollars). When the price goes up the size of the pile goes down and when the price goes down the piles are bigger. The vegetable market is organised in a way that avoids the use of fractions either in measuring quantities or money.

These and other observations were the motivation for a systematic attempt to understand the underlying discourse on fractions in Tonga and how both the language and the cultural values of the society affect fractions use and understanding. The research started with a collection of anecdotes and progressed to include questionnaires, semi-structured interviews, lesson observations, audio and video recordings of conversations with children and adults, as well as interviews with individual professionals.

Sixty participants were interviewed from a cross section of the Tongan population. As well as a qualitative analysis of the answers, with the aim of identifying the underlying narrative, the interviews provided a quantitative basis to compare different groups in Tongan society. The questions asked in Tonga were based on an implicit assumption regarding the expected answers in a Western setting. In order to test this assumption a comparison group of fifteen Israeli participants of various ages and backgrounds were interviewed. The questions were asked in Hebrew and some questions were suitably adapted. This proved to be a rather dull procedure as with very few exceptions the comparison group gave some of the expected (Eurocentric) answers

in line with Western contexts of culture. However, it was still important to confirm or refute the anticipated assumptions concerning the expected Western responses.

### **Vocabulary for fractions**

Christian missionaries arrived in Tonga during the first half of the 19th century and were responsible for setting up an educational system based on European and Christian values. As well as introducing reading and writing, they standardised the counting system and created a Tongan vocabulary for fractions. There is no evidence of words for fractions in Tonga before the arrival of the Europeans. The missionary teachers introduced words for fractions into the language during the 19th century but they had little use outside the classroom. The expression for some fractions is cumbersome; for example eleven thirteenths is "vahe taha tolu e taha taha" [divide one-three take one-one].

There is considerable confusion caused by the order of numerator and denominator. In primary school, arithmetic is taught in Tongan and the pupils learn to express the denominator before the numerator:  $\frac{2}{3}$  is vahe tolu e ua [divide three take two]. Then in secondary school, they learn in English and the order is reversed ("two thirds"). In practice, I observed that many teachers resort to the hybrid language: ua ova tolu [two over three]. The teachers themselves seemed confused by all this and resorted to a technical definition of fraction as "the numerator divided by the denominator".

"Number" is translated into Tongan as "mata'i fika" but from my field interviews, it emerged that this phrase is understood to mean integers and does not seem to include fractions in the interpretation of Tongan people. Tongans use it in the way that we use the phrase "whole number". In an interview with an expert on Tongan language, I asked:

- Interviewer: How would you translate "the number half?"  
Expert: "koe mata'i fika haafe". But we would not say this.  
Interviewer: How would you translate "Half is greater than zero and less than one?"  
Expert: "oku lahi ange e haafe he noa pea si'isi'i ange he taha". This also does not sound right. But we would not say this, mata'i fika is a whole number, not a fraction. There is not a word in Tongan which includes both whole numbers and fractions.

If the translation of the phrase "Half is greater than zero and less than one" does not sound right to ourselves, then it is not surprising indeed that Tongan students have a specific difficulty when working with fractions. Thus, there seems to be several language issues going on in the teaching and learning of fractions.

### **Results of structured interviews and other findings**

There is only room here to give a brief written summary of some findings. I select the findings that illustrate most clearly differences between the discourse observed in Tonga and the canonical Western discourse on fractions, sharing, and dividing objects thought of as whole. I summarise these findings in the form of issues and questions relating to: i) naming and identifying fractions, ii) fractions as parts of something, iii) fractions as rational numbers, and iv) values involved in dividing objects.

That said, it is key to clarify some of the attitudes observed in the context of culture about general reciprocity. In pre-missionary Tonga there were no words for fractions and there is no evidence of objects being divided equally. Distribution of goods was based on general reciprocity so there was no need for accurate calculations as to who owed what to whom. All of the products in general use came in whole units (yams, cassava, coconuts, fish, etc). When gifts were presented they were typically counted and presented in pairs and as far as we know were never cut into parts. Tonga has undergone enormous changes in the last two centuries but a conscious attempt has been made

to preserve some of the traditional values and ways of doing things. From an early age, children are taught that giving is more important than receiving and the idea of sharing equally (so prevalent in Western culture) is not important.

#### Questions relating to naming and identifying fractions

Participants were asked to identify some simple fractions using flash cards. A large percentage (37%) of participants gave unexpected answers or did not understand the question. When asked to choose a flash card showing three quarters of a pie, 48% of the participants (29 out of 60) did not choose the expected picture. Almost all the participants in the Israeli comparison group gave the expected answers to all the relevant questions. These observations indicated that both the names and the symbols of fractions (the names of which were introduced by the missionaries) are not embedded in the Tongan discourse.

#### Questions relating to fractions as parts of something

Whereas those interviewed were fairly proficient at estimating fractions of various objects such as a fraction of the length of a stick or of a kilogram of sugar they had great problems when asked to calculate fractions of a number or fractions of another fraction. Only 38% of those questioned (23 out of 60) answered that half of a half is a quarter. When asked what is half of a half of an apple only 47% answered that it is a quarter of an apple. The most frequent other response was "half". When asked "How many minutes are there in three quarters of an hour?" only 14 respondents (23%) answered "45 minutes" and most of those took a considerable time to calculate the answer. For example, one respondent took several minutes to calculate the answer in a notebook and most others took between five and fifty seconds to answer. Again almost all the Israeli comparison group gave the expected answers to the above questions.

#### Questions relating to fractions as rational numbers

I asked a number of people to place a set of numbers (on cards) on a number line between zero and two. The aim of this question was to test the hypothesis that I was observing a specific problem when fractions are to be understood as numbers (as opposed to operators). The fractions included decimal fractions (0.5, 0.71, 1.69) and regular fractions ( $1/8$ ,  $1/2$ ,  $3/4$ ,  $11/4$ ). Of the sixteen respondents questioned, four appeared to have no idea what we wanted. To avoid embarrassing them we did not pursue the question. The remaining twelve had great difficulty placing the numbers so, for example, only one out of twelve placed " $1/8$ " in the expected position and only two out of twelve placed " $3/4$ " correctly. With one exception the Israeli comparison group put every one of the fractions in the correct position (one Israeli participant did not know where to put " $3/4$ ").

#### Questions related to values involved in dividing objects

I found strong evidence that cutting whole objects up is seen in a negative light. Respondents were asked to divide five biscuits equally between three plates. Just over 50% (31 out of 59) of the participants made a reasonable attempt to divide the biscuits equally. Other responses included placing one biscuit on each plate and placing two whole biscuits in the middle; placing two biscuits on one plate and one and a half on the other two; placing one biscuit on each plate and eating the other two; or placing one biscuit on each plate and explaining how he would break the other two biscuits "but I don't do it". The most interesting (and unexpected) observation was that, almost without exception, the respondents asked if they were allowed to break the biscuits before breaking them into parts. In comparison, Israeli respondents did not request permission before breaking the biscuits. This was the first hint that I was observing a reluctance to break whole objects into parts. In order to test this idea we devised the following question: "You have three watermelons and are going to visit two respected aunties. How would you divide the watermelons as a present? Explain" Most respondents found ways of not cutting up any of the watermelons,

typically by giving two to one aunt and one to the other. A few respondents additionally said, "One and a half each" but many of those same participants added, "This is not good in our culture". When asked to explain why they did not want to cut a watermelon in half a typical response was, "I would want to give them a melon which is complete because here in Tonga if you give something that is not complete it is selfish". The words used for selfish were Kai Po, which literally translates as eating at night and implies eating in secret by oneself.

## **The place of fractions in the Tongan community of discourse**

My findings provided strong evidence to justify the claim that a coalition between the Tongan language and daily practice make the use of fractions almost redundant in everyday discourse in Tonga. I suggest that fractions and dividing whole objects have a negative connotation in the context of culture and that, in many situations, practical alternatives have been created to avoid their use. My research supports the conjecture of a reciprocal dependence between discourses on fractions and daily activities that involve fractions:

- Discourses will only develop if they serve the needs of day-to-day activities.
- The day-to-day activities will only develop in tandem with the relevant discourse.

I have found evidence that the pre-European norms of general reciprocity and of gift giving have changed and developed over the last two hundred years but have not disappeared. Giving away excess is still seen as a value and dividing whole objects into parts still has a negative connotation.

When used, fractions function as operators rather than as objects (numbers). Thus when respondents were asked to estimate a fraction of an object the request was usually understood, but when asked to put the same fraction on a number line between zero and two this was not understood. It is thus possible to understand why so many Tongans were unable to answer the question "what is half of a half?" In their culture, the "half of" is understood as an operator that produces a part of a physical object, and it is in this sense that the expressions "half of an apple" or "half of a cake" are used in everyday discourse. In contrast, half is not understood to be a number between zero and one and therefore it is not clear how it is to be halved. It follows that half of any concrete object remains a half, even if the object itself is half of another concrete object. This explains why so many of my respondents referred to "half of a half of an apple" as "half" – it was half of the half-apple.

The research indicates that there is a need for further study concerning the effect of language and culture on how fractions, in particular, and mathematics, in general, are understood in diverse communities and contexts of culture. Mathematics cannot be taught in isolation from the linguistic and cultural practices of the students and this should be taken into account in the development of school curriculum as well as classroom practice.

All in all, this research led me to realise that when teaching mathematics, and particularly fractions and probability, it is essential to consider the linguistic rules and cultural norms of the society in which teaching and learning are placed. I learnt from my Tongan students, colleagues, and friends that there is not one all embracing approach to sharing and the need to divide objects equally. It became clear that mathematical ideas can be understood in radically different ways by different people and in different cultural contexts regardless of the mathematical definitions provided by the formal language of mathematics. At the same time, in the Tongan culture, there is a desire to learn the mathematical skills necessary to succeed in the "outside world". King George Tupou I, the founder of modern Tonga, described the necessary synthesis between respecting traditional values and adopting Western methods. In a speech to the legislative assembly in 1882 the king stressed the importance of education and added:

“If there be anything in foreign lands which will be useful to us, it is right for us to desire to get it; but it is also right if there is any Tongan custom which is useful, for us to preserve it” (Campbell, 1957, p. 69).

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