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# Using multiple representations as part of the mathematical language in classrooms: Investigating teachers' support in a video analysis

Sebastian Kuntze<sup>1</sup>, Eva Prinz<sup>2</sup>, Marita Friesen<sup>1</sup>,  
Andrea Batzel-Kremer<sup>2</sup>, Thorsten Bohl<sup>2</sup> and Marc Kleinknecht<sup>3</sup>

<sup>1</sup>Ludwigsburg University of Education, Germany; kuntze@ph-ludwigsburg.de

<sup>2</sup>University of Tuebingen, Germany, <sup>3</sup>University of Lueneburg, Germany

*When making sense of mathematical objects in classroom interaction, the use of representations is necessary. Mathematical objects can be represented by the means of language, and even if they are represented in ways that do not use words, language is mostly used for explaining the meaning and connectedness of different representations of mathematical objects. Hence, it makes sense to consider any form of dealing with representations as being part of the mathematical language used in a classroom. For the sake of research on how representations are dealt with in learning support interactions, this video study examines how teachers supported students in using representations. Results show that teachers' support in using multiple representations was rare, indicating that teachers' mathematical language in these interactions could be improved.*

*Keywords: Video analysis, mathematical language, multiple representations, learning support situations.*

## Introduction

Supporting students in building up mathematical competence is a key feature of the teachers' role in classroom interaction. This can be expected to be the case particularly in so-called learning support situations, in which the teacher has the possibility to react to students' individual questions or to interact with them individually. Such interactions do not only consist of spoken words, but sense-making may also be based on multiple other representations of mathematical objects. Since being able to represent mathematical objects in multiple ways is a core element of mathematical competence and at the same time of communicating mathematically, any form of using representations of mathematical objects or dealing with them should be regarded as an integral part of the *mathematical language* which can be observed in learning support situations. Studies which explore the teachers' support of the students with respect to the quality of the mathematical language in this sense are, however, scarce. Consequently, an analysis of video data from 30 classrooms was carried out in order to describe whether and how teachers supported their students in using multiple representations during learning support situations.

We will then first introduce the theoretical background and the research aims of this study, then describe design and methods, present results, and discuss these in a concluding section.

## Theoretical background

The way representations are dealt with is a key quality aspect of interaction processes in the mathematics classroom (e.g., Duval, 2006; Ainsworth, 2006; Kuntze, 2013). As mathematical objects can only be accessed through representations and being able to handle them as well as to change between representations is central for students' mathematical competency (Ainsworth,

2006; Lesh, Post & Behr, 1987; Duval, 2006; Dreher & Kuntze, 2015a), students should be encouraged and supported to use flexibly multiple representations. Language can play a role for such support in two ways. Firstly, mathematical objects often can be represented by means of words, for example, descriptions of situation contexts that can stand for a mathematical object (Goldin & Shteingold, 2001). Secondly, such support can also consist in explanations of how different representations, for example by symbols or in graphical representation registers (Duval, 2006), stand for a mathematical object.

Indeed, teachers' support for learners is important when dealing with multiple representations, as conversions between representation registers are complex for students and have shown to be potential obstacles for understanding (Ainsworth, 2006; Dreher & Kuntze 2015b). Thus, teacher's help for students to connect representations and to translate between them is needed.

We may conclude that representations play a key role in the mathematics-related interaction between teacher and students in the classroom, and that they are agents for mathematical sense-making, communication and reasoning, as it is the case for language. Moreover, the use of language can hardly be separated theoretically from the use of representations in class interaction. Thus, we define the mathematical language appearing in class interaction as encompassing both the use of words and any form of dealing with representations of mathematical objects.

According to relevant specific literature (e.g., Duval, 2006; Ainsworth, 2006), core aspects of the mathematical language can be considered as quality factors of classrooms:

- The mathematical language of the teacher should be rich in the sense of using multiple representations in order to avoid confusion between a mathematical object and one of its representations (Duval, 2006) and to encourage students to build up a rich concept image (Ainsworth, 2006). Students should thus be encouraged to use multiple representations.
- As conversions between representation registers can be obstacles for understanding, the teacher's mathematical language should also provide support for the learners. This implies that connections between different representation registers need to be explained and examined together with the learners, that the teacher connects to the students' representations when introducing a further representation register, and that the students are encouraged to reflect on conversions of representation registers.

These quality factors of learning support in classroom interaction merit closer examination in corresponding video studies. In a more general sense, the discussion about quality factors of learning support in the mathematics classroom has been advanced significantly from the 1990ties through large-scale international video studies (Hiebert et al., 2003; Stigler et al., 1999). In particular, the results of the TIMSS 1999 Video Study (Hiebert et al., 2003) pointed to a need for identifying specific quality factors related to mathematics education. This is important since there was not a simple observation pattern that allowed for distinguishing 'the' successful mathematics classroom from less successful teaching and classrooms. Investigating quality factors associated with the mathematical language in classrooms is hence highly relevant with this respect.

The mathematical language including interaction processes around representations can be expected to play a central role particularly in learning support situations, in which the teacher has the possibility to react to students' individual questions or to interact with them individually (Krammer, 2009; Schnebel, 2013). By learning support situations, we understand classroom situations in which there is not a whole-class dialogue, but interactions between teacher and students during seatwork phases of the students, who are working on tasks on their own, in pairs or in small groups. Students can initiate these learning support situations, for example, when asking a question. Alternatively, the teacher can initiate them, for example, when asking the student(s) or giving a hint or feedback related to the working process. Such interaction situations are a key opportunity for helping the students with conversions between representational registers

(as they are often presented or required by mathematical tasks) or to encourage them to change between representations (in order to support their ability to change flexibly between representations of mathematical objects). Hence, the quality factors for the teacher's mathematical language as introduced above apply in particular to learning support situations.

However, so far and despite the international growing relevance of these quality factors, there are to our knowledge hardly any quantitative studies about whether and how mathematics teachers encourage their students to use multiple representations and provide them with help in learning support situations.

## **Research interest and questions**

The study reported here focuses on the research need mentioned above. The key research interest consists in exploring the quality of teachers' mathematical language used in learning support situations. Accordingly, our study concentrates on the following grouped research questions:

- (a) What role does the use of multiple representations play in the teachers' interaction with students in learning support situations? Do the secondary school mathematics teachers actively promote this use? Do these teachers support the change between representational registers of their students by help focused on connecting such registers?
- (b) Which form does such help focused on connecting the different representational registers take in cases of learning support situations?

## **Design and methods**

For answering these research questions, it is advantageous to investigate mathematics lessons that concentrate on the same topic area. In this case, expectations related to usable representations can be stated in a more content-valid way and comparisons between classrooms and results of analyses are supported. For this reason, we analysed 8th-grade videotaped lessons on the topic of "increased and decreased basic value" from a learning unit on percentage calculation from classrooms of 30 different German secondary mathematics teachers. The video sample stems from a data set collected by the research group around Thorsten Bohl from the University of Tuebingen (Batzel-Kremer et al., 2013). In this video data set, teachers had been asked to introduce the topic in a first lesson and to deepen this topic with student-centred exercises in a second lesson – both lessons had been videotaped. According to the research focus on learning support situations, this analysis focuses on the second lessons. The fixed subject meant that specific representation registers as they can be labelled by "growth factor representation", "rule of proportion", "per cent bars (graphical)", or "text description" can play an important role for solving many of the tasks the students had to work on. It is of course not necessary that all of these registers are used and related to each other, but for helping the students to build up conceptual knowledge that they can use flexibly, using multiple representations can be expected to be an important element. An accordingly designed top-down coding scheme was used to code the 30 lessons – an analysis which was done by two raters independently. In all cases of disagreement, a common code could be reached in a subsequent consensus process based on the video data and the respective criteria. The coding categories focused on:

- (1) The representation registers used in the learning support situations.
- (2) The quality of the interactions related to the use of representations.
- (3) Context factors of the situations such as who is the initiator or their duration.

Codes about quality characteristics (as mentioned in (2) above) concentrated on e.g. whether the teacher connected to the students' representation register, whether the students were encouraged

to change between registers, or whether help was provided in order to help students connect representations. In line with the research questions, we will in the following concentrate on these quality characteristics. In line with research question (b), the overview approach is complemented by a deepening analysis of case examples, which uses Mayring's (2015) content analysis focusing on criteria from the theoretical background introduced above.

## Results

The descriptive results show that in 271 of 454 coded learning support situations (59.7%) it was possible to identify the representation register(s) involved in the situations from the lesson video. The results reported here concentrate on these 271 situations. In order to explore what role the use of multiple representations played in the teachers' interaction with students and whether the teachers actively promote the use of multiple representations in learning support situations (research question (a)), we first coded the number of representational registers in the interactions. Figure 1 shows the frequencies. In the majority of the situations, only one representational register was used by the teacher and the students, which implies that none of them made an attempt to introduce or to connect with a different representation register in these situations.

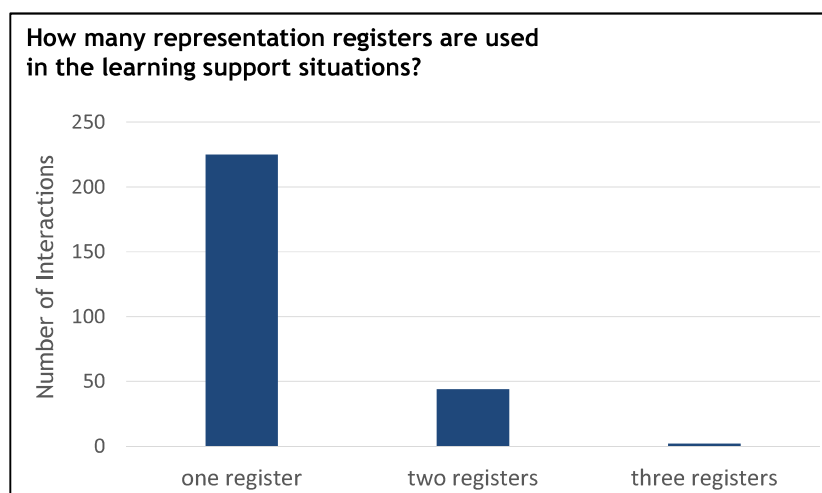


Figure 1. Frequencies of numbers of representational registers in learning support situations

In the relatively infrequent case (17%, i.e., 46 situations) in which more than one representational register was subject of the interaction, these registers were rarely connected to each other. Representational registers were coded to have remained disconnected in 38 of the 46 situations; only in eight situations, the teacher or the students connected the representational registers in the interaction (Table 1). In two of these eight situations, there was explicit help in connecting representational registers. Compared to the 271 learning support situations where representational registers were identified, explicit help in connecting registers was very rare (0.7%).

Table 1. Frequencies of codes for the learning support situations with 2 or more representational registers

<i>Code</i>	<i>Number of learning support situations</i>	<i>Relative frequency</i>	<i>Relative frequency for 271 situations with representational registers</i>
<i>Registers connected</i>	8	17,4%	3,0%
<i>Registers disconnected</i>	38	82,6%	14,0%

For exploring teachers' mathematical language in such a case in more depth, we would like to turn to the transcript in Figure 2. It shows the interaction of a teacher with a student who struggles

with the problem that 120€ has to be reduced by 20%, so that the remaining prize (which has to be calculated) is 80% of the initial prize. From the interaction it can be reconstructed that she has multiplied the 120€ by 80 so far (yielding the result 9600€). She then asks the teacher for help.

T: [...] Just draw here the two bars. A bar for the Euro amounts. Say 6 little squares for 20 €. Ok good and then?

S: [not understandable]

T: Ok.

S: This is approximately.

T: Yes good. How many per cent are the 120€?

S: 100.

T: 100%. Write it in there. And now how many per cent is it afterwards?

S: 80.

T: 80 write it in here. So. Now look at your calculation.

T: Times 80 what? 80% ok.

T: Now you have 9600€.

T: What did you change right now?

S: I put the 1 [.80] here.

T: Ok, that's to say, what was the question you were working on?

T: You have the 80% and you like to express it as a decimal number? Correct?

S: Yes.

T: Now 80% as decimal number. What did you just enter? What did you think? How many are they?

S: 1.80.

T: What would be 1.80 as per cent? What would be 1.0 in per cent?

S: 0.8.

T: Aha.

T: What would otherwise be 1.80?

S: [laughs]

T: No idea?

T: Ok these are 80% and 1.80 would be how many per cent?

S: Hum.

T: 1.0 in % are?

S: Yes 1.

S: 10. 100.

T: 100 exactly, and 1.8 are then, consequently?

S: 180.

T: 180.

T: Ok, exactly.

Figure 2. Example of transcript for one of the situations

As visible from the transcript, the teacher first encourages the student to draw an additional bar representation for the problem. He might expect that this additional representation could help the student to understand better how to proceed correctly in the calculation register. He further encourages the student to connect the given values with the bar representation. The teacher then switches to the student's calculation register and appears to encourage her to compare the two registers by asking her about the "80 what" and by drawing her attention to the 9600€. However, the situation appears to change quickly, as the student changes the 80 in the calculation into 1.80. From here, the interaction shifts to connecting the value of 80% (register of percentage values) to

a correct growth factor (register of decimal expressions for growth factors). The teacher's help to connect these registers consists in asking the student additionally to translate 1.80 and 1.0 back into the register of percentage values, a known strategy for supporting students' fluency with conversions in two directions (Duval, 2006).

Due to the length limits of this paper we are unable to elaborate on all details of our analysis of this learning support situation – yet the example shown in Figure 2 indicates how the teacher tries to connect the student's representations using other ways of representing the problem and the values contained in it. However, several issues of connecting representations and their registers might have needed further clarification or reflection in the interaction of student and teacher. The interaction takes around three minutes and compared to the multitude of issues, which emerge related to conversions of representations, it is visible that slowing down the speed of interaction and at the same time intensifying the teacher's strategies of connecting representational registers in his language might have improved the quality of the learning support in this situation.

## **Discussion and conclusions**

The results of this video study show a need of improving the quality of learning support on several levels. Some findings indicate that both opportunities for learning (such as encouraging students to use multiple representations) and for focused learning support (such as connecting representation registers and providing corresponding help) have been frequently missed in the lessons under investigation (see Figure 1, Table 1). In terms of the mathematical language used by the teachers, an enriched language use should connect students' reasoning better with multiple ways of representing mathematical objects and help them to reflect about connections between representation registers.

Beyond these main findings, considering case example situations marked by dealing with multiple representations (e.g., Figure 2) can give further qualitative insight into potential development needs of interaction in the mathematics classroom. In particular, in the case of the interaction shown in Figure 2, intensifying reflection about connections between different registers and reducing speed in interaction might be useful strategies for the improvement of the mathematical language used in learning support situations involving several representation registers.

Also on the methodological level, the representation-aware focus on the mathematical language in classrooms should be considered in follow-up research, including research on an international level. For instance, the methodology developed in the framework of this study calls for further use in intercultural video-based comparison studies that can contribute to deepened and shared knowledge about quality criteria of interaction in the mathematics classroom.

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