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Fostering a conceptual understanding of division: A language- and mathematics-integrated project in primary school

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About 20% of the 15-year-old pupils in Germany fail to develop an understanding of the four basic arithmetical operations in the course of their schooling, and of division least of all. The study presented in this paper affords an insight into the conception and evaluation of a language-sensitive intervention project involving 45 third- and fourth-graders from schools whose catchment areas have low sociographic status. Individual misconceptions of division serve as a basis for initiating a sustainable understanding amongst the children in a language-sensitive manner. The analysis shows that the development of a conceptual understanding of division depends on language structures for expressing the connection between division and multiplication and for verbalizing division concepts.

Keywords: Division, divisibility, multiplication, language-based research, primary school.

Introduction

Elementary school children are meant to acquire a confident and workable idea of the four basic arithmetical operations in their first years at school. The arithmetical operation that seems to be the most difficult to learn is demonstrably division, with children especially experiencing mathematical difficulties, and even not acquiring any concept of this arithmetical operation at all (Cawley et al., 2001; Ehlert et al., 2013; Moser Opitz, 2013; Robinson et al., 2006). While there is a relatively large number of studies delving into the operational ideas of addition, subtraction and also multiplication amongst children who are underachieving in arithmetic, division-related studies in mathematics education research tend to be rare. Some exceptions are Ehlert et al. (2013) and Robinson et al. (2006). The research findings currently at hand largely focus on two central aspects: the specific strategies children of different ages use to solve division problems, or the frequency of solutions amongst children who are perceived to have learning difficulties in mathematics in particular. And even though it is well-known that division is distinguished by specific language structures (Anghileri, 1995), owing to the two basic models of division (partitive and quotative), less is known about how to foster a conceptual understanding of division and divisors in a language- and mathematics- integrated way. The study presented in this paper focusses on these central ideas. Therefore, it is necessary to analyze the mathematical and language-based requirements for developing a division concept.

Research findings about division concepts in primary school

According to Robinson et al. (2006), typical strategies pursued by fourth- to seventh-graders to solve division problems include: factual knowledge, recourse to multiplication, partitive division, recourse to repeated addition and/or subtraction in the sense of quotative division, and derived fact strategies. In addition to this, solutions are also guessed, inappropriate strategies are pursued, or the children do not name strategies. Only the strategies of factual knowledge, recourse to multiplication and recourse to repeated addition are found on a regular basis in all grades. The strategy of repeated addition appears particularly dominant in the lower grades. Downton (2008)

resorts to a similar strategy system. She uses the additional strategies of building up as a recital of multiplication tables, as well as doubling and halving. But many research projects do not privilege or pay attention to whether the children have a conceptual understanding of division or of the inverse operation of multiplication. In fact, children can solve division tasks without any idea of for example grouping or splitting in equal parts. Especially, children who focus on strategies like knowing, recourse to multiplication or to repeated addition indeed know what to do technically and how to divide but they have no idea what is underneath and what an appropriate mental picture could look like (for representations of multiplication, e.g., see Kuhnke, 2013).

Moreover, it is well-known that correct solutions of division problems are scarcer, particularly amongst children who have difficulties in mathematics. Robinson and LeFevre (2012) show, for example, that sixth- to eighth-graders who are weak in arithmetic fail to understand the connection between multiplication and division as reverse operations. Cawley et al. (2001) furthermore show that the understanding of division amongst eighth-graders with mild disabilities tends to equal those of fifth-graders. Specifically for Germany, this discrepancy appears to be even greater. The division concept of eighth-graders with difficulties in mathematics is below the level of fifth-graders in mainstream schools (Moser Optiz, 2013). The understanding of division and the place value system is however a significant predictor for eighth-grade performance in arithmetic. Cawley et al. (2001) therefore refer to the introduction of division as a ‘cut-off’ point in mathematics teaching for many children. Without a confident understanding of division, all children lack central foundations for an understanding of divisors, and hence for elementary number theory, but also arithmetic learning contents (Feldman, 2012). Therefore, a conceptual understanding of divisibility is a central learning target for all children (Moser Optiz, 2013). A stronger networking of multiplication and division is therefore unanimously demanded to advance the understanding of division (Downton, 2008; Moser Optiz, 2013; Robinson & LeFevre, 2012).

Although a subject of little scientific attention yet, is the linguistic sensibility. In learning division, the close association between words describing a real context and the mathematical procedure for solving the related problems (Anghileri, 1995; Downton, 2008) –in the early grades– characterizes a conceptual understanding. At this stage, the interpretation of division is restricted by a somehow ‘simple’ but meaningful language. It is important that children have a conceptual understanding of dividing quantities into equal parts for example. However, many children after the third grade use these ‘simple’ words without recourse to any contextual meaning. Phrases like ‘divided by’, ‘divided into’ or ‘...time...makes’ are often used without any idea of the encoded meaning. These phrases seem to be something like a secret language in the classroom discourse: everybody uses them but only few children understand them. We have to clarify and to connect the meaning of these phrases with the conceptual idea of division. Otherwise, especially children with limited language proficiency in the language of teaching might be overtaxed by the verbal information, by differentiating between everyday words and technical terms, and by recognizing the connection between mathematical symbols and language on their own. Children with mild disabilities particularly face such situation as their language competences are still less differentiated than those of mainstream school children (Cawley et al., 2001).

What can be said in summary is that we need to know more about whether and how the children’s arithmetical skills in this regard, after the introduction of division, can be expanded or misconceptions dismantled, respectively. The support provided for this should meanwhile focus on the language- and mathematics-integrated connections between multiplication and division and on deepening the conceptual understanding of division, but also on a conceptual understanding of divisors as a preparation for future contents of mathematics learning and teaching. This is exactly the approach taken by the research project described in the next section.

Methodology: Design experiments for generating qualitative data

As a basis for this support, a design experiment was developed for the ‘factor tree’ (Figure 1) for the third and fourth grade. In a factor tree the starting number $a \in \mathbb{N}$ is progressively factorised into its natural divisors. The trivial factors one and the original number itself are left out as the factor tree would never come to an end otherwise. This task permits the connections between multiplication and division to be worked out by way of discovery: calculating from the top to the bottom shows the division, and from the bottom to the top multiplication.

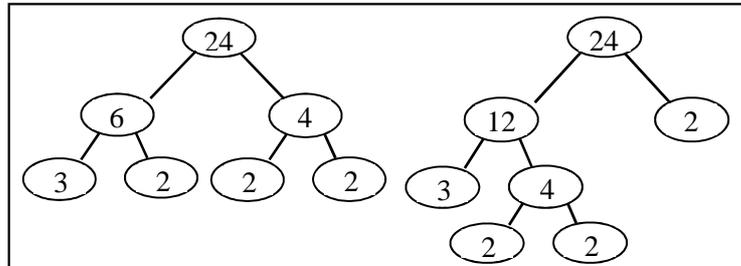


Figure 1. Two factor trees for 24

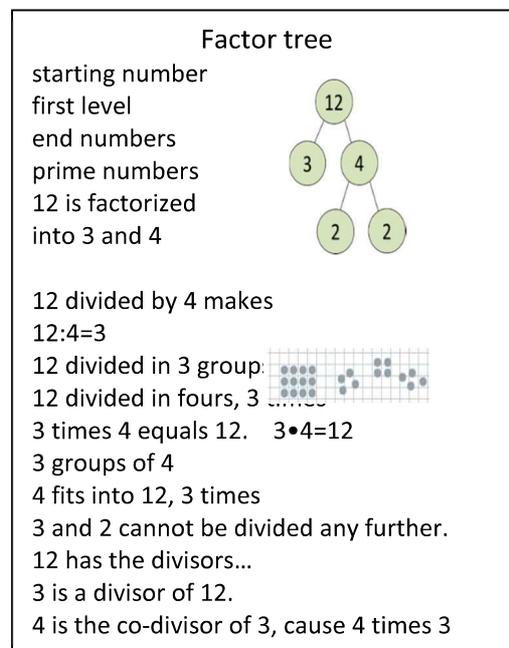


Figure 2. Example of a word list

There was support at four primary schools in major cities of the Ruhr area to 45 third- and fourth-graders in assignments for pairs or small teams (40 or 45 minutes per session). All the schools are located in districts with low socioeconomic status and each support lesson was videotaped. Four pre-service elementary teachers were trained and intensively consulted to serve as resource teachers. The language-sensitive provision of the support was performed in keeping with the scaffolding approach, as posed by Hammond and Gibbons (2005):

[T]his functional theory provided a strong framework for the deliberate and explicit focus on teaching language, teaching through language, and teaching about language. (p. 9)

This differentiates between two levels of language support for technical learning processes: macro-(designed-in) and micro-(interactional) scaffolding. Both language support levels were relied upon in the design experiment with the factor trees. On micro-level, the pre-service teachers were trained to accompany the support in a language-sensitive manner, i.e., by inviting the

children to use more precise language, by introducing technical terms and making consistent use of them but especially by helping the children expressing connections. Thus, the shared language basis should not only provide the children with access to the language of teaching and technical vocabulary, but demonstrably also support content learning (Prediger & Wessel, 2013). On macro-level, word lists that record and visualize technical terms and sentence phrases were drawn up in cooperation with the children (Figure 2). If the children wanted to verbalize their deliberations and lacked the words to do so, this word list provided them with various forms of language support. If it was necessary the children could use mathematical manipulatives to show their thinking with the help of concrete material.

The following provides insight into the individual learning pathways of three primary school children by contrasting documents and short transcripts of the videotaped episodes from the beginning of the support and from the end. The analysis in this paper could only take a local perspective asking whether the support based on the factor trees contributes the required concepts and follows the research question: How can a language- and mathematics-integrated intervention with factor trees support students' conceptual understanding of division *locally*, contrasting specific moments at the beginning and at the end of the learning process?

Empirical snapshots from the design experiments

The analysis focusses –in a local perspective– the individual learning pathways of the third graders Abbas, Hamit and Xara. Abbas' family is originally from Iraq. The family has lived in Germany for a number of years already. His performance in arithmetic is on a medium level. Hamit is from Syria. He has only been in Germany for nine months at the time of the support. The school advises him to repeat the fourth grade because his language and mathematics skills are too weak. Xara comes from a family home with little access to education. She receives additional support in mathematics. She is a very bright child, however, who loves playing an active part in discussions, as the analyses will show. In Germany, the children learn the mathematical operation of division in the second grade. Hence, these three children should be expected to be familiar with the division concept.

Snapshots from the beginning

In the first support lesson, the children initially worked out the assignment format of the factor trees for themselves. They had to analyse six completely filled factor trees. Then they had to find factor trees with self-selected numbers and describe their calculations in written form. Figure 3 shows the results of this individual work.

Abbas writes that he is calculating 'multiplication tasks' and appears to have doubled the ten. He notes this doubling in an additive manner, but still describes it as 'multiplication tasks'. He seems to connect multiplication with doubling. He moreover fails to divide the ten any further, buttressing the impression that he is solving the factor tree from the bottom up. His written text offers no valuable clues if he has understood the connection between division, multiplication, factors and divisors.

Hamit's factor trees are correct. Any further diagnosis of his mathematical skills is elusive because he has selected easily divisible numbers (21 and ten) and not made any notes, probably because of his poor German. Xara appears to develop the factor tree multiplicatively from the bottom up because she writes '2 times 6 is 12' and '9 times 2 is 18'. Due to this perspective, it makes it difficult to see that she could have divided the six and nine further. What is interesting is that she is adding the numbers twelve and 18, not multiplying them. Perhaps she does not know how two-digit numbers are multiplied. Anyhow, she only describes how she has calculated. But nothing is known about if she has an idea of what '2 times 6' really means. None of the three

children's documents provide any clues as to whether they have understood the conceptual meaning of division, divided by, multiplying or ...times... and whether they have perceived the inversion of division and multiplication in the factor tree.

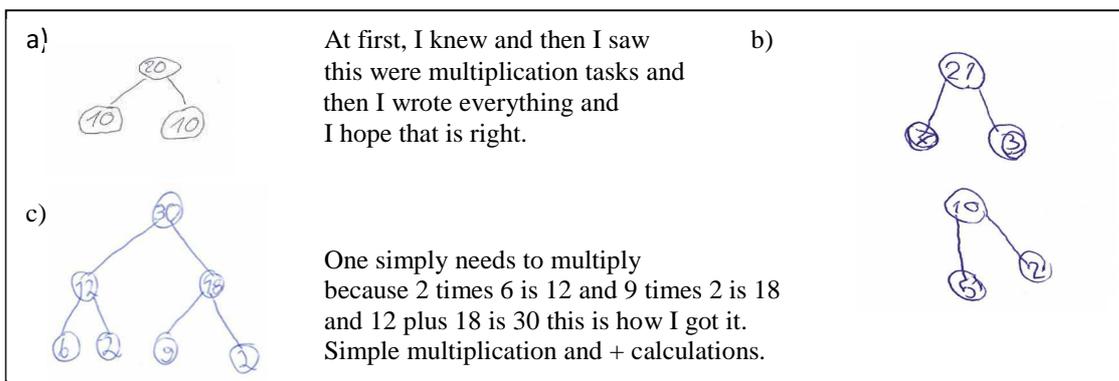


Figure 2. Written work by Abbas (a), Hamit (b) and Xara (c) in the first remedial lesson

Once the assignment format had been clarified in the first remedial lesson and the word list elaborated (see Figure 2), the game ‘Who divides last?’ was introduced in the second remedial session. In this game, the game master selects a starting number, the opponent divides it once, then the game master divides it again, etc. Whoever performs the last possible division is the winner. After a few rounds of the game, Xara suggests the use of 100 as the starting number.

- 313 Xara: With the 100 one can calculate as long as one wants. Do you want me to divide now directly?
- 314 Teacher: No, It's Hamit's turn now.
- 315 Hamit: (H notes 10 and 10. He passes the piece of paper to A)
- 316 Abbas: (A divides 10 into 5 and 2, points to the 2) Primes.
- 317 Teacher: Mhm, and what about the 5?
- 318 Abbas: Do you want me to do that as well? Oh, I know, that is also a prime just like the 2.
- 319 Xara: Can I now also divide the 10 into something else?
- 320 Teacher: Yes. You are doing a new division now.
- 321 Xara: Yippee! (...) er. OK. Let me see first what's still possible! Ah! (X takes her fingers and counts loudly) 2, 4 (...) 2, 4, 6, 8, 10. 5, um, but that is the same as just now. (...) Fiver, twoer ahhh, the 2 and the 5 are the main numbers somehow. 2 times 5 makes 10.
- 322 Teacher: Are there any other ways then?
- 323 Xara: No, with the 3 I'd be at 9 then. The 1 would work but that isn't supposed to. The 6 would be 2 too much. The 8 would be much too much in any case. No, there are no other ways, actually.

Xara appears to assume at the beginning (l. 313) that the number 100, as a particularly large number, also has more partitions and hence more divisors. From line 319 onward she starts thinking about whether the ten can possibly also be divided in any other ways than into the primes of two and five. She is taking a consistently additive approach to this, but also considers numbers such as six and eight that are completely out of the question as divisors. She seems to have a vague idea of the connection between divisors, primes, division and multiplication, but these ideas are still marked by precursor concepts of division and divisibility.

In the third support lesson, the children were invited to find all the factor trees of the numbers two to 25. The finding of all factor trees served to highlight further individual (mis-)conceptions but also problems with the specific language structures of division tasks.

- 451 Xara: Yes. I think the 9 doesn't work either.
452 Abbas: The 9? The 4 can't be divided any further and the 5 neither. Well, then the 9 is out.
453 Xara: Yes, the 9 is out.

Abbas and Xara seem to equate division with halving. This may explain why Abbas takes four and five as divisors of nine into account ($4+5=9$). Interestingly he mentions that four cannot be divided any further (l. 452) but shortly before they have noted the factor tree of four. Maybe he struggles with the expression 'divided by' and wants to express 'four and five cannot be divisors of nine'. The children seem to have individual ideas about how division works but they do not express a conceptual understanding of 'sharing equally', 'fitting in' or 'partitioning in groups' in fact and even seem to struggle with the specific language structures of division.

Snapshots from the end of the support

The finding and sorting of the factor trees and look at their commonalities combined with the *need* to express connections have turned out to be very fruitful activities for eroding misconceptions and for fostering expressing connections. The children's concept of division and divisors is expanding in this lesson in the sense of them taking all the numbers in all the factor trees into consideration as divisors of the starting number and especially in expressing the connections between divisors and division:

- 535 Abbas: Er, 6 and 4 are divisors of 24.
536 Teacher: Okay. But how can you *explain* that to me?
537 Abbas: Cause in 24 the 6 fits 4 times. 6 and 4 are divisors of 24.
538 Xara: (points to the factor tree with the first partition into 12 and 2) The same as with the 12, the 4 and the 3. 4 fits 3 times into 12.
539 Teacher: And what about the 2 and the 3, then? (points to the divisors 2 and 3 in the factor tree with the first partition into 6 and 4).
540 Abbas: 2 and 3?
541 Xara: Yes, look, 2 is a divisor cause, 24 divided into twos you get 12, 12 twos into 24. And that is the same with the 3. 3 times 8 is 24, three groups of eight into 24. I think they are divisors, too, because those further down there are also divisors of 24.

While in turn 535 Abbas considers the numbers on the first level as divisors, an enquiry by the teacher (turn 536) forces Abbas and Xara to explain the mathematical meaning of the sentence '8 and 3 are divisors of 24'. Abbas defines divisors by using a quotative strategy (turn 537): a divisor fits in an integer without remainder. He expresses a conceptual understanding of division as a '*fitting in*' concept. Xara seems to share this idea of division with Abbas (538). And in fact, it is Xara who presents another idea of divisors: an idea of partitive division. Three is a divisor of 24 because you can build three groups of eight.

16 is factorised into 4 and 4 because four times four is 16. 4 is factorised into 2 and 2 because 2 times 2 is 4.
2 and 4 are divisors of 16.
Fours and twos fit in 16.

Figure 4. Hamit's individual description

And refugee child Hamit, who has hardly actively participated in the discussions, also appears to have realized the mathematical core of division, as is evidenced by a written document at the end of the support lessons (Figure 4). He describes accurately how a factor tree needs to be calculated, using the language of teaching and technical phrases. His text reflects the connection between multiplication and division as an inversion as he argues within the factor tree from the bottom up as well as top down. And he also expresses a conceptual idea of quotative division as a 'fitting in' concept. He furthermore correctly describes *all* the numbers in the factor tree as divisors, but forgets that 16 can be factored in different ways, and hence at least the additional divisor of 8.

Discussion and conclusions

This short insight into the empirical videotaped data has shown that the deliberate support of the *connection* between multiplication and division and of expressing the relationship of multiplication and division and of divisors and factors appears to be a fruitful approach for developing a conceptual understanding of division (and for multiplication as well). It underpins the demand that "[p]lacing emphasis on the relationship between multiplication and division and the language associated with both operations before any use of symbols or formal recording needs to be a priority" (Downton, 2008, p. 177). Thus, initial misconceptions could be supplanted by first basic conceptual ideas. Especially the math underachievers Xara and Hamit impress in their individual development. With how much stability these new concepts are invested remains unanswered at this juncture. Moreover, little is known whether these children have really got the core of the relationship between multiplication and division. Much more long-term research is needed to analyse how a conceptual understanding of division and multiplication and the connection of these two operations develops.

But the phrases and technical vocabulary supported in mathematical discourses connected with the necessity of expressing mathematical understanding appears to be a central requirement for success because with the help of these terms and phrases the children have the chance to develop a language-based understanding. The phrases 'divided into', 'divided by' or '...times...makes' are often used as meaningless phrases. Using phrases like '20 divided into two groups of ten each', 'four fives in 20' or 'a divisor fits in a number without remainder' shows the thinking practice of the children – a thinking practice that is deeply connected to the 'language of schooling' (Feilke, 2012). By the way, misconceptions can be discursively discussed by the use of the technical terms and phrases in the first place. It seems to be possible to foster an understanding of division and divisors (and also of multiplication and factors) simultaneously because a potential language barrier could be deliberately avoided right from the start.

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