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## ► To cite this version:

Rachel-Ann Friesen, Marcus Schütte. Pupils' participation in collective argumentation within multi-age mathematics education at primary level. Proceedings of the IV ERME Topic Conference 'Classroom-based research on mathematics and language' (pp. 65-72), Mar 2018, Dresde, Germany. hal-01856478

**HAL Id: hal-01856478**

**<https://hal.science/hal-01856478>**

Submitted on 20 Aug 2018

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# Pupils' participation in collective argumentation within multi-age mathematics education at primary level

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*Currently in Germany, there are increasing numbers of primary schools who install multi-age education for pedagogical reasons or due to demographic changes in rural areas. Especially for mathematics education, this often causes individualised learning to take place. However, as interaction is seen as a foundational constituent of mathematical learning (Miller, 1986), this research focuses on pupils' interactions within multi-age mathematics education and has the objective of describing how pupils of different ages learn collaboratively. Because learning is seen as the increasingly autonomous participation in collective argumentation, we seek to identify how pupils participate in processes of collective negotiation of meaning and, thus, examines interaction by using the analysis of argumentation and the analysis of participation.*

*Keywords: Multi-age education, collective argumentation, participation.*

## **Multi-age education and a sociological theory of learning**

No matter which classroom one looks at, the pupils learning together are never homogeneous in their learning preconditions. Each individual is unique and therefore has differing abilities and experiences. Using age as the distinguishing factor for placing pupils in specific classes is the norm because it is seen as a possibility to make classes as homogeneous as possible. But even in learning groups which are homogeneous in age, the differences in the pupils' preconditions for learning when they enter into school can be up to five years (Lorenz, 2000, p. 22). One concept with the aim to purposefully value and meet this existing heterogeneity is merging pupils of different ages and grades in one class deliberately (Prengel, 2007, p. 69f). In English, this concept is mainly referred to as *multi-age classes*. In contrast, the terms *combination* or *multigraded classes* refer to pupils of different ages and grades being combined in classes for organisational purposes (a more detailed description of the terms can be found in Wagener, 2014). For example, in rural regions of Germany, multigraded classes have been installed in order to prevent closing small schools as a result of subsiding numbers of pupils and therefore not forcing especially younger pupils to travel long distances to school (in Saxony, multigraded classes in small primary schools are being installed since 2014). In this paper, the term multi-age classes will be used, even though the schools participating in this project have different reasons for merging classes.

When making learning groups heterogeneous in age as in multi-age classes, the diversity of learning preconditions increases and with it also the necessity for differentiation. However, mathematics is often presumed to be a subject built up rectilinear whose content can only be learned following a specific order (Lorenz, 2000, p. 19). Nührenbörger and Pust (2006) found that because of this assumption pupils are rarely given the opportunity to learn from and with others in mathematics classrooms. This often causes extremely individualised and separated learning to take place as teachers either feel they need to separate the learners back into the different age groups or they need to let each pupil learn individually in their own pace. Both have its value for certain topics but can place too great a demand on the teachers trying to accompany the learners

simultaneously in their learning process as well as waste the opportunity of collaborative learning.

However in mathematics education research, collaborative learning is not only seen as the process of pupils cooperating in order to match their individual cognitive restructuring processes, but for young learners interaction is seen as foundational constituent of learning (cf. Miller, 1986, p. 10). In this context, Miller creates a sociological theory of learning of at least two individuals – the theory of collective learning processes – in order to differentiate from psychological theories of learning which predominantly focus on isolated individuals. Hereby, he does not, on principle, question that learning is also an individual process, but rather that when learning fundamentally in the early stages of a child’s development an interactive collective process precedes or determines this process of learning something new (Jung & Schütte, 2018). In this approach, the concept of argumentation is essential. Miller differentiates argumentation, and hereinafter more specifically collective argumentation, which is based on rationality according to Habermas (1985), from “communicative action” based on something uncontroversial (p. 37f). The principal specific characteristic of collective argumentations is the aspect of rationality. Mathematical learning, therefore, takes place in the negotiation of several people about what is viewed from a mathematical perspective as rational. Krummheuer and Brandt (2001) draw on this aspect of learning in collective argumentations and create a theory of interaction of mathematical learning. For them it is of great importance for mathematical learning to occur that pupils participate in collective argumentation within classroom interaction (cf. *ibid.*, p. 20).

### **Analysing an increase of participation in various settings of learning**

The focus of multi-age mathematics education research has, therefore, been on collaborative learning because it is seen as an important element of mathematical learning processes, however, seems to lack in teaching practices. Besides overarching concepts which have been developed for collaborative learning within multi-age mathematics education (e.g., Nührenbörger & Pust, 2006), interactions between learners who are heterogeneous in age have been examined. When analysing these interactions (e.g., with regard to the construction of knowledge), the pupils show a great variety of interaction reaching from working alongside each other to the co-construction of new knowledge by either working in the “zone of proximal development” and therefore constructing new knowledge or by reflecting on pre-existing knowledge and therefore deepening it (Nührenbörger & Steinbring, 2009, p. 118). Additionally, varying qualities within the helping interaction are identified ranging from product-oriented help to process-oriented forms of help (cf. *ibid.*, p. 126). Another study by Matter (2017) shows that it is beneficial for learning when two pupils of different grades are collaborating when either both partners are subject-specifically balanced or when they are not that the social behaviour is important for learning to take place (cf. *ibid.*, p. 303). When both pupils have a very low subject-specific competence, only a low gain in learning is seen (cf. *ibid.*, p. 316).

Studies on multi-age mathematics education in primary school lay a good foundation for our project. However, they predominantly focus on dialogues between two pupils which are heterogeneous in their grade level and concentrate either on how knowledge is constructed (Nührenbörger & Steinbring, 2007) or when interactions are beneficial for learning in such settings (Gysin, 2018; Matter, 2017). In contrast, a sociological theory of learning guides our project, focusing on processes of change within the participation in the processes of negotiation of meaning and on various settings within mathematics classrooms that go beyond pair work.

### **Research question**

One of the characteristics of multi-age education is that most of the time at the beginning of a school year older pupils leave a class and younger learners join the class. This means that those

who used to be the younger learners over time become the older learners of a class. They, thus, experience a change in their role within the class. This can provide the opportunity for learners to also change their role within subject-specific negotiation of meaning and potentially participate more autonomously in collective argumentation that will lead to a broader range of possibilities for learning mathematics.

Because learning is seen as the increasing autonomous participation of pupils in collective argumentation within classroom interaction, the overall question this research seeks to address is: How do pupils participate in collective argumentation within multi-age mathematics education and is it possible to observe a development in the participation over time? In order to answer this, firstly one has to identify how this participation in collective argumentation takes place in general and if patterns of interaction can be identified. Secondly, the analyses of the interactions over a period are then compared to each other in order to answer the question whether there is a change in the degree of autonomy. As various settings (e.g., pair work, group work and whole class discussion) are seen as beneficial for multi-age education because they can all lead to bringing forth-collective argumentation, the third sub-question is raised whether there are differences in the interaction of the learners between different settings of learning within multi-age education. In this paper, only the first sub-question of how learners participate in collective argumentation with multi-age education is addressed further, as this is the basis for the other research questions.

## **Interactionist approaches of classroom research**

Methodologically, this work can be located within qualitative methods of social research which follows a reconstructive-interpretative approach and have the aim to ‘understand’ the actions of the individuals participating in class and to develop local theories (Schütte, 2011, p. 776). More specifically, this work is located within interactionist approaches of mathematics educational classroom research which have their origin in Germany in the 1980s (Krummheuer, 2011; Krummheuer & Brandt, 2001). For this study, the classes are filmed on several occasions over a period of one to two years in order to reconstruct possible age-specific changes concerning the participation in collective argumentation of the children. Through this, a potential change in the pupil’s autonomous participation in collective argumentations is to be identified and described.

The interactions of the pupils are analysed using Krummheuer and Brandt (2001), who also filmed in multi-age schools and subsequently analysed the scenes by using 1) the analysis of interaction, 2) the analysis of argumentation, and 3) the analysis of participation. Step one is the basis for the following analyses by reconstructing the processes of interactive negotiation of meaning. The subsequent analyses of argumentation and of participation are done on the basis of the summaries of the interpretations (Krummheuer, 2015, p. 53). Through the analysis of argumentation, which is based on Toulmin (1969), one can identify which children contribute to which of the four possible functional categories of an argumentation: data (undoubted statements), conclusion (inference together with the data), warrant (contribution to the legitimation of the inference) or backing (undoubtable basic convictions which refer to the permissibility of the warrant). In this research project, the analysis of argumentation is used to show potential differences in the functional categories of an argumentation when looking at the different children participating in multi-age mathematics education depending on their age or the setting they are in.

In order to identify differences in the participation of the pupils in collective argumentation the analysis of participation is used. This analysis includes the production and the recipient design and is able to show which pupils participate actively or receptively in a polyadic interaction (Krummheuer, 2011). For this paper only the production design is relevant which analyses the utterances in order to identify which individual speaker is responsible for a functional category of an argument on a syntactic and/or on a semantic level. Overall, there are four roles that pupils

can have according to this analysis: the author (who is responsible both for the content and the formulation of the utterance), the relayer (who is responsible for neither the content nor the formulation), the ghostee (who is responsible only for the content of an utterance) and the spokesman (who is responsible only for the formulation of an utterance) (Krummheuer, 2015, p. 58). By analysing these roles, the question is answered in what way there are differences in the participation of the pupils in collective argumentation regarding them taking syntactic or/and semantic responsibility within multi-age education.

Overall, this research seeks to describe how mathematical learning takes place collaboratively in multi-age education. According to the theory of interaction of mathematical learning by Krummheuer and Brandt (2001), the possibility of mathematical learning in a school context is seen within an increase of autonomy in formats of argumentation when spokesman and ghostees increasingly occur within production-designs, and when processes of argumentation with a complete “core” of an argumentation - meaning data, conclusion and warrant - are produced (p. 59f). These are seen as possible indicators of successful learning processes because e.g., spokesmen or ghostees take up previous utterances but also add new aspects to the interaction and therefore add to or deepen the thematic development of the interaction. This is seen as an advanced form of autonomy, whereas in contrast to these, being an author points to pupils already having learned something and being a relayer can be an indication for someone being at the beginning of a learning process (Brandt, 2004, p. 37f).

### **Autonomous participation of all pupils in multi-age education**

The group work analysed in this paper is from a mathematics lesson in a multigrade class including 24 children from first, second and third grade. At the beginning of the lesson the students are divided into groups of three which mostly contain one pupil of every grade level. The task given is: “Tina and her family want to decorate their Christmas tree with hand painted Christmas ball ornaments. They want every ornament to look different. To paint the ornaments they have the colours red, green and blue. With these, they can paint dots or stripes. For each ornament they want to use two colours and one pattern at the most. Which possibilities do they have when painting their ornaments?” To solve this task the pupils have material on their tables: bigger circles, stripes and dots of each colour. The group, whose interaction is analysed, includes Isabella (1st grade), Hans (2nd grade) and Elias (3rd grade). The recording starts after they read the task aloud and created two possibilities (two green circles one with three blue dots and one with two red stripes). Only a summary of the analysis of interaction is included, because the focus is here on the analysis of argumentation and of participation.

During the first part of this group work, Isabella argues that they should not only use green circles but also the two other colours because they should not only use the same colour. Elias then picks up this idea and starts suggesting to put blue dots on a red circle. Hans also follows up on Isabella’s idea and suggests to put green dots on a red circle. Elias seems to misunderstand Hans’ utterance and reiterates Isabella’s idea. Then, Hans repeats his idea to clarify it and Hans and Elias agree on creating a red circle with green dots. Hans and Elias briefly disagree on the number of dots they should use but both seem to agree implicitly that for the differentiation of the possibilities the number of dots is not relevant. After settling for three, they also disagree on which background colour they should continue with. Elias now wants to use a blue circle and Hans wants to create a second red one using the other colour left for the pattern. They then agree on using a red circle and add two blue stripes. Isabella says that she has found another possibility and points to a blue circle with one green stripe. Contrary to Elias who then wants to create another red circle with a pattern, Hans explains that they have created two blue and two red ones so far and suggests to now create blue circles. All three quickly agree that they will first use a blue circle with red dots.

We see that all three children participate in the creation of possibilities. To show how the pupils participate more specifically, the analyses of three exemplary arguments are presented below.

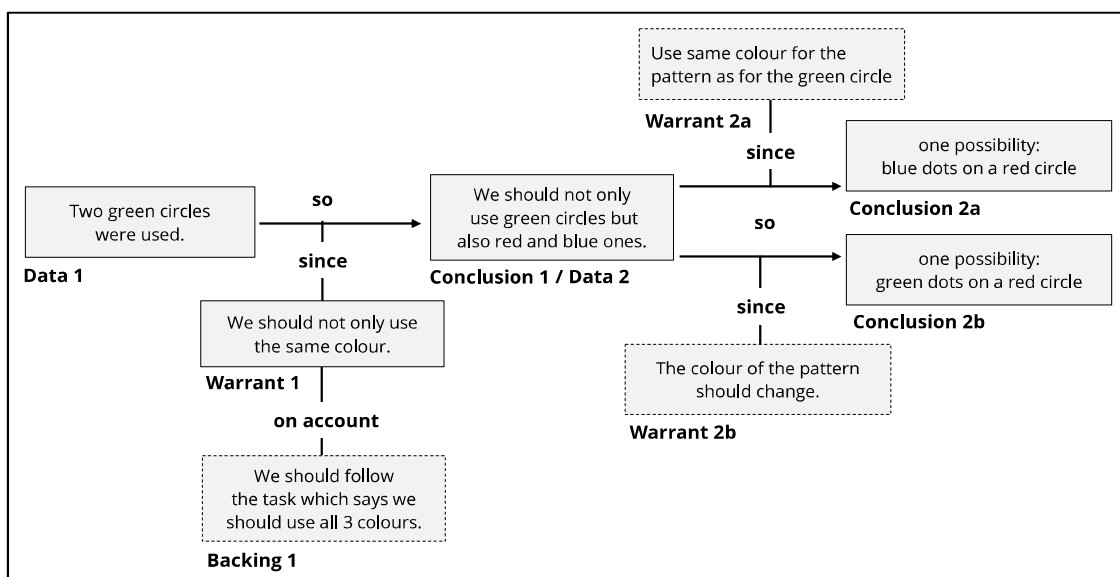


Figure 1. Analysis of argumentation for arguments 1 and 2

As shown in Table 1 (with all transcripts translated from German), the first argument is brought forth by Isabella as the author, since she has the responsibility for both the content and the formulation of warrant 1 and conclusion 1. Isabella's conclusion is then picked up by Elias as a spokesman and used as data 2 to draw the new conclusion 2a, which he is responsible for both syntactically and semantically. Hans then also picks up conclusion 1 as data and draws his own conclusion 2b as an author. Because of a misunderstanding, Elias reformulates the conclusion 1 as a spokesman by taking responsibility of the formulation of Isabella's content but goes further and is the first one to explicitly state data 1 for the first argument. He can, thus, be seen as author of data 1. Hans then acts as a relayer of his conclusion 2b by repeating it. Hans and Elias never explicitly state the warrants for their inferences, so the warrants were deducted by the analysis of interaction. The backing of the first argument was deducted from the interaction and is based on the conviction the children seem to have that the task should be followed.

Table 1. Analysis of participation for argument 1 and 2 (format based on Krummheuer, 2015, p. 60)

function of speaking person	utterance	idea (argumentative function of the utterance)
	<i>reference to a prior speaker</i>	
<b>Isabella: Author</b>	I would first not only use green but (if we) also these two colours\ . <2> <i>[points to the blue and red circles]</i> <3>	We should not only use green circles but also red and blue ones. ( <b>Warrant 1; Conclusion 1</b> )
...		
<b>Elias: spokesman</b>	on red we could <6> <i>Isabella</i>	Also use red circles ( <b>Conclusion 1 = Data 2</b> )
<b>Elias: author</b>	<i>[picks up a small blue dot]</i> <7>	One possibility: blue dots on a red circle ( <b>Conclusion 2a</b> )
...		
<b>Hans: author</b>	now green dots\ <10> <i>[takes two small green dots into his hand]</i> <11>	One possibility: green dots on a red circle ( <b>Conclusion 2b</b> )

<b>Elias: spokesman/ author</b>	no we already have green . twice\ <12> <i>[points to the first green circle and then points with two fingers of his right hand to the two green circles with patterns]</i> <13>	Two green circles were used. <b>(Data 1; Conclusion 1)</b>
	<i>Isabella</i>	
<b>Hans: relayer</b>	but green dots\ <15> <i>[puts his two green dots on the red circle in front of Elias]</i> <16>	One possibility: green dots on a red circle ( <b>Conclusion 2b</b> )
	<i>Hans</i>	

Later in the interaction another argument is brought forth which consists of these components:

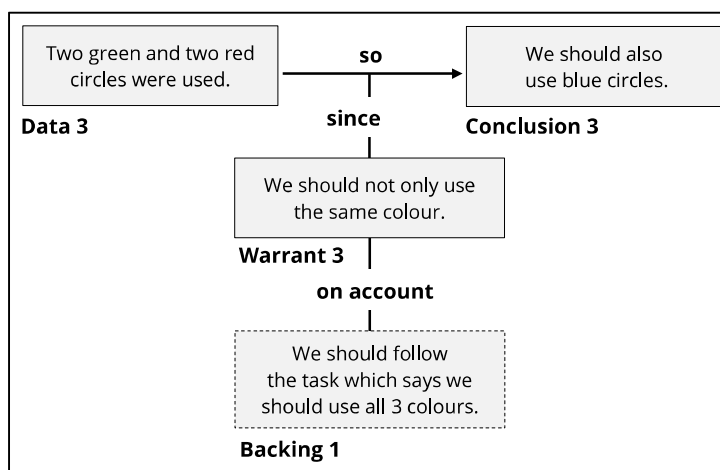


Figure 2. Analysis of argumentation for argument 3

It is similar to the first argument and the content is thus not treated as new but with reference to the prior speakers. Specifically, Hans takes up the content of Isabella's conclusion 1 and the data 1 Elias previously stated and rephrases these. Since he takes responsibility only of the formulation of these functional categories, he acts as a spokesman. Isabella then repeats conclusion 3 to use two blue ones next and becomes the relayer of Hans' and her own idea and formulation.

Table 2. Analysis of participation for argument 3

function of speaking person	utterance	idea (argumentative function of the utterance)
	<i>reference to a prior speaker</i>	
<b>Hans: Spokesman</b>	no now first blue\ <67> <i>[taps on Isabella's blue circle]</i> <68>	We should also use blue circles. <b>(Conclusion 3 ≈ C1)</b>
	<i>Isabella</i>	
<b>Hans: Spokesman</b>	we already have two red . ornaments now and two green ones\ <69> <i>[points to two red and two green circles which already have a pattern on them]</i> <70>	Two green and two red circles were used. <b>(Data 3 ≈ D1)</b>
	<i>Elias</i>	
...		
	two blue ones\ <72>	

<b>Isabella: Relayer</b>	[places another blue circle in front of herself] <73>	We should also use blue circles.
	Hans, (Isabella)	(Conclusion 3 ≈ C1)

Overall, these analyses show that within this group work in multi-age mathematics education at primary level all three pupils contribute in some way autonomously to the argumentation. Specifically, Elias is the author of data 1 and conclusion 2a and is a spokesman of the conclusion 1. Hans is the author of conclusion 2b and a spokesman of conclusion 3. Isabella is the author for the conclusion and warrant of argument 1 and a relayer of conclusion 3. In summary, possibilities of mathematical learning (according to Krummheuer and Brandt, 2001) can be found in this interaction because Hans and Elias are predominantly spokesman of utterances and by this pick up utterances of others and change them structurally. Isabella on the other hand never acts as a spokesman, however, she is the only one contributing a warrant to an argument and therefore is the only one who completes a “core” of an argument. By this she, as the youngest participant in this interaction, not only has a receptive role but also an active yet, compared to the two boys, unique role. In the future, these analyses will be compared with analyses of further group work but also of pair work and of whole class discussions. Furthermore, they will have to be compared to interactions brought forth later in time in order to identify possible changes in the participation of these pupils over time.

## Acknowledgement

‘Collaborative Learning in Multigrade Mathematics Education at Primary Level’ is part of a greater project called TUD-Sylber (Synergistic teacher education in an excellent framework) at the Technical University of Dresden, funded by the German Federal Ministry of Education and Research in the framework of the joint state and federal ‘Teacher Education Quality Offensive’.

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