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# Diversity of teachers' language in mathematics classrooms about line symmetry and potential impact on students' learning

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*In the continuity of our previous research on the impact of teaching practices on the variability of learning among students, we present a first step in order to investigate teachers' discourse more deeply. We identified crucial issues, including linguistic ones, in the conceptualization of line symmetry, related to logical aspects of the concept. This exploratory study consisting in the analysis of textbooks content and some classroom sessions suggests that these learning issues are globally hardly considered but that there are some differences in the ways teachers address them, which are likely to have differentiating impacts on students' learning.*

*Keywords: Teaching practices, language of the mathematics teacher, line symmetry, logical analysis of mathematical concepts,*

## Introduction

Our concern about language issues arose from previous research about relations between teaching practices and social inequalities in mathematics learning (Chesnais 2012, 2014, in press). Firstly, our theoretical framework for studying teacher practice and its impact on student learning is grounded in the theory of Vygotsky (1986), which indicates the importance of language as an object of learning but also as a means for conceptualization, and finally as one of the main tools of teacher activity. Secondly, as sociological research shows, language plays a crucial role in the construction of learning inequalities and their relation with the sociocultural background of students (Rochex & Crinon, 2011).

This concern about the role of language in mathematics education meets a recent preoccupation in the French community of research in didactics of mathematics (Artigue et al., 2017). The original feature of our work in the French context lies in the use of Vygotsky to investigate these questions, altogether with our interest in analyzing "ordinary" teaching practices. It also seems to largely echo some vivid preoccupations of the international – and particularly ERME's – community of research in mathematics education (Pimm 2004; Radford & Barwell 2016; Planas, Morgan & Schütte, 2018), questioning the three lines "classroom discourse", "language diversity" or "conceptualization through language" (Planas, 2016). Our originality in this landscape comes from our theorizing of the process of teaching and learning mathematics and the specific way it leads us to question classroom discourse. Firstly, our use of mathematics logic to analyze mathematical discourse seems original in the field even if it may have some common ground with some other approaches (Pimm, 2004). Secondly, our investigation of "ordinary" teachers' practices with a combined didactic and ergonomic point of view (grounded in Vygotsky, Vandebrouck, 2013), leads us to consider ordinary teachers' discourses with a "naturalistic approach" when most of the research seems to focus more on the design of experiments. Finally, our hypotheses about mathematics learning, based on a combination of Vygotskian and Piagetian theories involve a way of investigating teachers' discourse that differs from other research

influenced by cultural-historical theories (e.g., the work of Sfard 2000, 2001 or Radford, 2013). We consider that teacher telling has a role to play (Chesnais et al., submitted), which includes offering lexical means to support student activity (Pöhler & Prediger, 2015) but not solely.

We are particularly focusing here on the “logical aspects” (the arity of properties and relations) of line symmetry and reflection and the related linguistic aspects (Chesnais, 2009; Barrier et al., 2014; Chesnais et al., 2017), as we will detail below. We previously showed that these issues generate many learning and teaching difficulties in 6th grade (first grade of secondary school, 10-11 year-olds) in France (Chesnais et al., 2013; Barrier et al., 2014; Chesnais et al., 2017). The studies we are presenting here are part of two research projects: one about the transition from primary to secondary school, funded through the “chercheurs d’avenir” campaign of the Region Languedoc-Roussillon and one about the role of language in the learning of mathematics and sciences funded by the ESPE-LR (superior school of teacher training).

The questions we tackle in this paper could then be phrased as: How do 6<sup>th</sup> grade mathematics teachers deal with linguistic issues related to logical aspects of line symmetry and reflection? Are differences among teachers’ practices likely to have a differentiating impact on students’ learning? First, we present our theoretical framework and its methodological implications. Second, we expose main findings of a study of textbooks contents and some classroom sessions and we detail a demonstrative example of variability among teaching practices.

## **Theoretical framework and methodological implications**

Our theoretical framework is based on Activity theory adapted to mathematics teaching and learning in a school context (Robert & Rogalski, 2005; Vandebrouck, 2013). The main hypothesis is that learning results from students’ activity which results (mainly) from the tasks the teacher chooses for students and the way he/she implements them in the classroom. Learning is then characterized as *conceptualization*, operationalizing in a way Vergnaud’s definition of concepts, based on the theory of schemes (Vergnaud, 2009): conceptualization (as a product) of a specific piece of knowledge is characterized by its “availability” in situations in which it is relevant, with the corresponding “operator invariants”, plus its integration in the network of previous knowledge, and the use of associated “signs” (in particular linguistic ones). Managing specific linguistic signs is then consubstantial with conceptualization. Moreover, in line with Vygotskian ideas, signs (in particular linguistic ones) play a role in the *conceptualization process*. Our hypotheses on the role of the teacher in this process, resulting from combining of Vygotskian and Piagetian ideas, is that she has to support the activity of students to solve mathematical tasks, but also to ensure that the solving of mathematical tasks actively supports opportunities for conceptualization. The role of *teacher telling* seems crucial to support the dynamics between general and contextualized knowledge (Chesnais et al., submitted), in particular when tasks are not didactically very robust, which is often a reality. Hence, what we investigate in teachers’ discourse is how the linguistic elements the teacher offers to students constitute a crucial factor but also the way they contribute to construct “proximities” (Robert & Vandebrouck, 2014) between students’ real activity and the aimed knowledge.

Our methodology starts with a preliminary study of the mathematical content at stake. We chose to particularly focus on the linguistic issues related to logical aspects of line symmetry and reflection. In this specific purpose, we use logical analysis of language (Vergnaud, 2009; Durand-Guerrier, 2013). Our study of teaching practices starts with an overview of French textbooks. We consider them as “approximations” of what might be taught in classrooms and how, since textbooks in France are often written by teams including teachers (if not exclusively composed by them) and they are an important resource for teachers, even if their use is not voluntary but compulsory. Studying textbooks then allows us to access in an economical manner a wider view

on potential teaching practices – even if it necessitates confirmation by lesson analyses. Finally, we analyze four experienced 6<sup>th</sup> grade teachers’ discourse (T1, T2, T3 and T4), during sessions devoted to working on some tasks. We took videos and compared resulting opportunities for conceptualization. Overall, our analyses show that the teachers in the study potentially support learning issues related to the logical aspects of line symmetry.

## **Preliminaries about logical aspects of line symmetry**

As we first identified in Chesnais (2009) and formalized with the logical analysis of mathematical language in Barrier et al. (2014) and Chesnais et al. (2017), the concept is constituted of line symmetry as a *property* of a figure (unary predicate), but also of reflection as a *relation* – a binary one (two-place predicate) between two figures, a binary relation between a figure and a line and a ternary one (three-place predicate) involving two figures and a line. The *property* may be defined by the fact for a figure to be superimposable with itself when “flipped”. The ternary relation is defined as two figures being superimposable (one on the other) when folding along the line. The binary relation between two figures is defined by the existence of a line such that the two figures and the line satisfy the ternary relation. The binary relation between a figure and a line can be defined either as the ternary one between the two parts of the figure (situated on either side of the line) and the line, or as the case of the ternary relation where the two figures are the same one (the figure is then called *globally invariant* under line symmetry through the given line).

As Vergnaud (1998, 2009) already pointed out, the issues (and obstacles for learning) are both inseparably “linguistic and conceptual”. The conceptualization of line symmetry supposes to be able to consider, distinguish and articulate its different logical aspects in various situations (recognition of lines of symmetry in figures, construction of mirror images or completing a figure in order for it to be symmetric with respect to a given line) and to be able to use correctly the associated linguistic elements. Vergnaud (1998, p. 234) illustrated the polysemy of the French word “symétrique” with the following sentences: “1. The fortress is symmetrical. 2. Triangle A’B’C’ is symmetrical to triangle ABC in relation to line d.” The second sentence would probably more likely be phrased as “A’B’C’ is the mirror image of triangle ABC” in English, but we reproduced the *word for word* translation made by Vergnaud himself to highlight the polysemy of the word “symétrique” in French. In particular, there is no equivalent for “mirror image” or for “reflection” in French, the word “symétrique” being used in all situations –actually, the word “réflexion” is used in French to name negative isometries, but only when considering spaces of dimension greater than two. We will use *word for word* translations in the rest of the paper rather than a more regular English wording every time it seems useful for the reader’s comprehension.

This polysemy is particularly problematic because of some “expert” (with a sufficient level of conceptualization of line symmetry) linguistic forms, especially “F and G are symmetric”. This *factorized form* is ambiguous. It may either mean that the two figures share the same property (line *d* is a line of symmetry for each one of them) or that one is the image of the other under reflection through line *d*. Note that using the *expanded form* “F is symmetric to G” or adding “to each other” or “to itself/themselves” would eliminate the potential misunderstanding.

In our study, we chose to focus particularly on the issue of the conceptualization of line symmetry as a *ternary relation*: this includes distinguishing it from the property and from the binary relation, identifying the nature of the relation (as a negative isometry – materialized by folding or flipping of tracing-paper – among the isometries characterized by superimposition) and understanding the role of the third object (the line). These aspects are supposed to be tackled in France in 6<sup>th</sup> grade - not in a formal way, of course. Indeed, primary school students construct mirror images of figures using tracing-paper, folding or grids and identify lines of symmetry in elementary figures, but it is only in 6<sup>th</sup> grade that this has to be progressively formalized and unified by the

introduction of reflection as a transformation acting on points and defined by mathematical properties (perpendicularity and distance conservation). This supposes in particular to use the specific word “symétrique” with its various meanings and “par rapport à” (“with respect to”) to mention the line. However, in the learning process, some intermediate or alternative words might be used –and potentially useful.

## Empirical study

We focused on the use of the word “symétrique” and the expression “par rapport à”. The study of six textbooks (Chesnais, 2012) revealed that in all of them, the word “symétrique” is used to refer to the property and to the relation without any comment on the polysemy of the word. Forms like “A is symmetric to B” and “A and B are symmetric” seem to be distributed randomly and sentences are sometimes difficult to interpret for students. We found in a textbook in two successive exercises these sentences: “A and B are symmetric with respect to  $d$ ” and “A and B are symmetric to A' and B' with respect to  $d$ ”. In another textbook, the definition of the *ternary relation* starts with “Two figures are symmetric with respect to a line  $d$ ” and the definition of the *property*, on the next page, starts with “A figure is symmetric with respect to line  $d$ ”. Students might wonder if the meaning of the term changes when it is in the singular or in the plural. Only one textbook mentions the fact that what we called the *factorized form* is equivalent to the *expanded* one. Another finding is the frequent omission of the third element, which jeopardizes the distinction between the binary and the ternary relation.

## Comparison of teachers’ language to designate the ternary relation

The main result of our analyses of teachers’ discourse is that potentially differentiating practices are largely shared, especially the use of *factorized* forms, like in the textbooks, but also the use of more opaque forms, like “there is a symmetry” or “it is symmetric” (as showed the detailed analysis of sessions in T1 and T2’s classrooms in Chesnais et al., 2017). Note that it often echoes students’ language and that it results in ambiguities and misunderstandings about the subject of discourse (relation or property). The difference between textbooks and teachers’ discourse might be related to the fact that textbooks are written discourses. We also observed that uses differ when talking or writing, especially about the mention of the axis, as we will show in the example below. However, we also identified some singularities in one of the teachers’ practices: T3 often uses the *expanded* forms and rephrases and completes students’ answers when they are too ambiguous.

Let us detail an example of such variations to highlight their difference of potential –according to our theoretical framework– for students’ learning. T3 has taught for about ten years in a socioeconomically disadvantaged school and T4 has taught for about twenty years in a mixed one. They both use the same textbook. Their students are working on the same exercise, made of five drawings for which the question is: “are the two figures symmetric with respect to line (d)?” We focus on the third drawing.

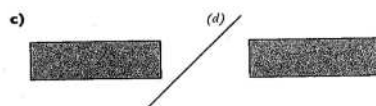


Figure 1. Third case of the exercise

The distinction between the binary and the ternary relation is at stake together with the role of the line. The two rectangles appear to be symmetric (in the binary sense) since there exists a (vertical) line with respect to which one rectangle is the mirror image of the other, but they are not symmetric *with respect to line (d)*: negation is about the *third* argument of the relation. A non-correct answer (answering “yes”) might be caused by any of the two following reasons: (1) line symmetry is considered as a binary relation between two figures, the line being obliterated: the

actual question the student answers is: “Are the two figures symmetric (to each other)?”, (2) The line is not obliterated, line symmetry being considered as a ternary relation, but the level of conceptualization of it is not sufficient. A misconception of line symmetry (Grenier, 1988) is here at stake: two figures are mirror images of each other with respect to a given line if they are globally on a horizontal direction, at the same (global) distance of the line. A right answer could also result from this misconception if the student identifies that the distance is not exactly the same (the rectangle on the left is slightly closer to the line). Answers could be based on visual perception, but could be reinforced or contradicted by the use of instruments like a ruler or tracing paper.

#### *T3’s way of pointing out the role of the line to students*

After the students have read the question, T3 asks them if the words “with respect to line d” are important. One student answers, “Yes, otherwise we cannot know if they are symmetric”. It is interesting that even if the students say yes, they do not mention the line in the answer. They did not say, for example, “Otherwise, we cannot know if they are symmetric *with respect to this line*”. After working on the task individually, numerous students answer “yes”, because they manifestly obliterated the line; some of them used tracing paper, but drew on it only the two rectangles and folded the paper to make the two rectangles match. T3 hence drives the collective discussion on the importance of the line, pointing out that it indicates where to fold the paper and that it has to be mentioned verbally using the expression “with respect to line d”, herself mentioning it systematically. T3 produces “proximities”, articulating the work on language with the solving of the task and the other dimensions – like material actions – of (real) students’ activity. At the end of the session, some students’ points of view changed. Lucien, a student with some difficulties in mathematics whose initial answer was “yes”, explained that he changed his mind and that “the two rectangles are not symmetric *with respect to line d*” but that if the line was vertical, then they would. This shows that the relation is now considered as a *ternary* one and that, simultaneously, some elements of the language of mathematics are appropriated.

#### *T4’s contradictions between action and discourse*

Right after reading the question, T4 asks the students to draw the figures on tracing paper and the line in red, but without mentioning any reason for it (drawing the line is not a necessity to solve the task). Later, after individual work, in the collective discussion this conversation arose:

T4: what can you say about figures in case c?”

Student: they are not symmetric.

T4 (*with an agreeing tone*): they are not symmetric. [*And discussion goes on case d*].

Strictly speaking, this answer is incorrect but the main issue for us is that students who answered “yes” (and there were some) did not have any chance to understand why their answer is wrong since the words used do not distinguish between the binary and the ternary relation. Here, we do not suppose that mentioning the line had been sufficient. Later in the session, writing the answer concerning the cases where the answer is “yes” produces a switch in the teacher’s discourse:

T4 (*writing on the blackboard while speaking*): the figures B, D, E are symmetric

T4 (*stopping and facing the class*): there is something very important that you have to remember and it is the figure in the middle, figure *c* that allows us to say it, actually. That is, if I had drawn the line elsewhere, in figure *c*, don’t you think it would be symmetric? [...] In other words, what is important is to say?

Student: the line.

T4: to mention the line. Then, each time, we’ll say with respect to, with respect to line *d*.

Ambiguity about case  $c$  is finally but partially clarified. An issue is how it may affect learning, since discourse seems contradictory with “action”: when solving case  $c$  minutes before, mentioning the line did not seem so important. Moreover, the use of the pronoun “it” at the end of the second T4’s intervention may prevent thinking in terms of relation between several objects.

In T3’s class, language supports action, and discourse about language is related to a situation where it is useful. Besides, the teacher’s discourse is adapted to students’ *real* activity (also because she led some space for genuine mathematical activity). In T4’s class, considerations about the importance of the line (and of the linguistic related elements) seem to be disconnected from the activity of solving the task.

Even if there is no way to assess specifically the effects of these differences on students’ mathematics learning, our theoretical background allows us to suppose that T3’s practices offer potentially better opportunities for learning (like Lucien’s answer seems to corroborate). What corroborates these assumptions is that a larger study of teaching practices of nine 6<sup>th</sup> grade teachers about line symmetry, in which we assessed the resulting learning of students by running some tests (Chesnais, 2014), attested that T3’s practices are more effective than others. Nonetheless, T4 did not participate in this larger study.

## Conclusion

This exploratory study of teaching practices based on the analyses of textbooks contents and of teachers’ discourses on a sample of four teachers suggests that language issues related to logical aspects of line symmetry are far from being systematically explicitly addressed as learning issues. We hypothesize that this might limit opportunities for conceptualization potentially offered by some tasks, at least for some students. However, we suggest that some teachers’ practices offer more opportunities than others, because they address language issues more explicitly and tend to develop students’ mathematical activity simultaneously *on and within* language.

Obviously, these assumptions need more investigations for confirmation. First, it seems necessary to investigate more teachers’ discourse in order to get a more representative sample of teaching practices and be able to refine the characterization and variability of teachers’ ways of addressing these issues. Secondly, developing methodological tools to measure the impact on students’ learning of what our theoretical framework inclines on considering as more effective teaching practices than others constitutes a crucial issue even if we consider that it would make no sense to isolate the language issue in assessing students’ learning. . One of an undoubtedly fruitful but demanding challenge remaining is also to pursue the first step that we initiated here in order to confront French didactics of mathematics research questions, frameworks and findings with international research in mathematics education on language issues in mathematics education.

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