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To cite this version:

Pierre Jehel. A multi-scale stochastic model for concrete in uniaxial cyclic loading with coupled mechanisms at meso-scale. 2nd International Conference on Multi-scale Computational Methods for Solids and Fluids, Jun 2015, Sarajevo, Bosnia and Herzegovina. pp.91-94. hal-01851319

HAL Id: hal-01851319

https://hal.science/hal-01851319v1

Submitted on 29 Jul 2018

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June 10-12, 2015
Sarajevo, Bosnia and Herzegovina

ECCOMAS Thematic Conference

2nd International Conference on Multi-scale Computational Methods for Solids and Fluids

A MULTI-SCALE STOCHASTIC MODEL FOR CONCRETE IN UNIAXIAL CYCLIC LOADING WITH COUPLED MECHANISMS AT MESO-SCALE

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Abstract — Concrete response at macro-scale results from numerous mechanisms at different scales. These latter cannot be all explicitly accounted for in numerical concrete models. In this paper, a multi-scale stochastic model for predicting concrete response in compressive uniaxial cyclic loading is presented. Heterogeneous meso-structures are randomly generated; coupled plasticity-damage model represent local constitutive response. At macro-scale, a representative element can be recovered.

Key-words — Concrete, multi-scale, stochastic vector random field, correlation, plasticity, damage.

1 Introduction

This paper briefly presents the basic ingredients of a multi-scale stochastic model developed to represent the compressive cyclic response of a concrete specimen. Such a response is characterized by the following features (see Figure 1): a backbone curve (dashed line) that is a nonlinear strain hardening phase ($0 \leq E \leq 0.0027$) followed by a strain softening phase where strength degradation is observed; unloading-reloading cycles show that stiffness decreases while loading increases, hysteresis loops are generated. This typical response is observed at macro-scale and results from numerous mechanisms of different physical and chemical nature at many different scales. In concrete, there is a continuum of scales at which heterogeneities can be observed: at macro-scale, aggregates of different characteristic sizes are surrounded by a cement paste; at lower scales, the cement paste is in particular composed of water, voids, of the products of the complete or partial hydration of the clinker particles.

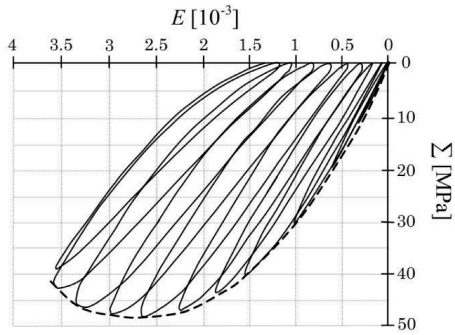

FIG. 1 – Strain-stress concrete experimental response in pseudo-static cyclic uniaxial compressive loading (adapted from [1]).

Two scales are explicitly considered in the model herein presented. At macro-scale – where the response of concrete specimen is observed as in Figure 1 – material is assumed to be homogeneous. At meso-scale, heterogeneities and uncertainties are explicitly represented. Response at meso-scale is not the same depending on which material the response corresponds to: an aggregate, the cement paste, a mix of aggregates and cement paste. At this meso-scale, the approach followed in the present work does not consist in explicitly generating a multi-phase medium with random distribution of aggregates of random geometry in a cement paste with known mechanical behavior for each phase. The path followed here consists in generating a random medium at each point of which the mechanical response

obeys a prescribed behavior that has unknown parameters.

This prescribed behavior at meso-scale is assumed to represent what could be the response of an aggregate, of the cement paste, of a void or of any combination of these later three components of concrete. This behavior is presented in the next section and the set of unknown parameters is introduced. This set of parameters is modeled as correlated random fields over an area of concrete according to the method that is presented in section 3. Response at macros-scale is recovered according to the equations in section 4. Numerical applications are presented in section 5.

2 Behavior at meso-scale

Coupled damage-plasticity behavior is assumed at meso-scale. The model is developed with a set of internal variables that are the history of the material. The basic ingredients are as follows:

$$\epsilon = \epsilon^d + \epsilon^p \tag{1}$$

$$\psi(\epsilon, D, \epsilon^p) = \sigma(\epsilon - \epsilon^p) - \frac{1}{2}\sigma D \sigma \tag{2}$$

$$\phi(\sigma) = |\sigma| - \sigma_y \leq 0 \tag{3}$$

Potential ψ and criterion function ϕ are expressed with the stress at meso-scale σ, the damage and plastic deformations ϵ^d and ϵ^p, the damage compliance D and the yield stress σ_y. The internal variables are D and ϵ^p.

Then, we introduce the following damage-plasticity coupling relations:

$$\dot{\gamma}^d + \dot{\gamma}^p = \dot{\gamma} \quad \text{with} \quad r\dot{\gamma} = \dot{\gamma}^d \ , \ (1-r)\dot{\gamma} = \dot{\gamma}^p \quad \text{and} \quad 0 \leq r \leq 1 \tag{4}$$

and the following equations can be derived within the framework of thermodynamics with internal variables (see e.g. [2] for the numerical aspects regarding this framework):

$$\epsilon^d = D\sigma \quad ; \quad \dot{\gamma} \geq 0 \ ; \ \phi \leq 0 \ ; \ \dot{\gamma}\phi = 0 \quad ; \quad \dot{D} = r\dot{\epsilon}/\sigma \quad \text{and} \quad \dot{\epsilon}^p = (1-r)\dot{\epsilon} \tag{5}$$

Initially, previous to any damage, we set $D = 1/C^e$ where C^e is the elastic tensor.

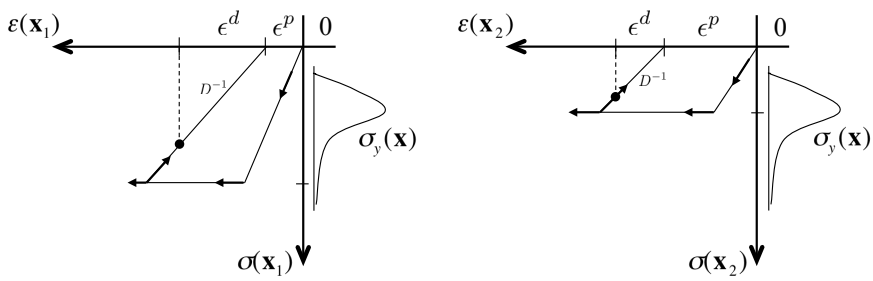

FIG. 2 – 1D behavior at meso-scale at two distinct positions $\mathbf{x}_1$ and $\mathbf{x}_2$ over a concrete area.

3 Model of the parameters heterogeneity and uncertainty

In a 1D setting, the model presented above is developed using 3 parameters: elastic modulus C^e, yield stress σ_y and damage-plasticity ratio r. Thereafter, we focus on a square material area $\mathcal{R} = d \times d$ and the spatial heterogeneity of these parameters over $\mathcal{R}$, along with their uncertain nature, has to be represented. Some correlation between the parameters is also expected. Suppose indeed that the material point of interest at meso-scale corresponds to an aggregate, then the elastic modulus is larger than at any other point, the yield stress is much larger too and there is no plasticity. Consequently, the 3 parameters C^e, σ_y and r should be correlated accordingly.

2-dimensional 3-variate homogeneous nonGaussian random vector fields are generated to model

these specific features at meso-scale. The method used is based on the so-called Spectral Representation Method with the following assumptions for material properties as introduced in [3]: (i) material properties possess quadrant symmetry, (ii) the cross-spectral density can be written as:

$$\mathbf{S}(\mathbf{k}) = S(\mathbf{k}) \begin{pmatrix} 1 & \rho_{12} & \rho_{13} \\ \rho_{12} & 1 & \rho_{23} \\ \rho_{13} & \rho_{23} & 1 \end{pmatrix} = S(\mathbf{k})\,\mathbf{s} \tag{6}$$

Where the correlation coefficients satisfy $|\rho_{12}| < 1$, $|\rho_{13}| < 1$ and $|\rho_{23}| < 1$.

Then, considering three independent 2D homogeneous Gaussian random fields $g_s(\mathbf{x})$, $s = 1, 2, 3$, the homogeneous Gaussian vector field is calculated as:

$$f_r(\mathbf{x}) = \sum_{s=1}^{r} h_{rs} g_s(\mathbf{x}) \tag{7}$$

where matrix $\mathbf{h}$ is computed from the Cholesky decomposition $\mathbf{s} = \mathbf{h}\mathbf{h}^T$. The Gaussian fields can then be translated to nonGaussian fields.

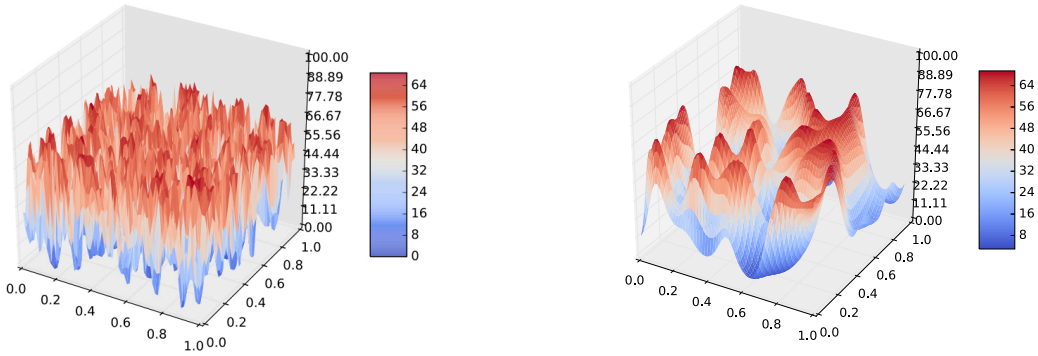

FIG. 3 – Yield stress field [MPa] generated over a unit area $\mathcal{R} = 1 \times 1$ with two different correlation length: $b = 0.04$ [left] and $b = 0.17$ [right].

4 Response at macro-scale

Macro-scale response is calculated assuming:

$$E(\mathbf{X}) = \epsilon(\mathbf{x}),\ \forall \mathbf{x} \in \mathcal{R} \qquad \text{and} \qquad \Sigma(\mathbf{X}) = \langle \sigma \rangle(\mathbf{x}) = \frac{1}{|\mathcal{R}|} \int_{\mathcal{R}} \sigma(\mathbf{x}; \mathbf{X}) d\mathbf{x} \tag{8}$$

Then, introducing the tangent moduli at both scales: $\Delta\Sigma = \mathcal{D}^{eff} \Delta E$ and $\Delta\sigma = \mathcal{D}\Delta\epsilon$, it comes:

$$\mathcal{D}^{eff}(\mathbf{X}) = \frac{1}{|\mathcal{R}|} \int_{\mathcal{R}} \mathcal{D}(\mathbf{x}; \mathbf{X}) d\mathbf{x} \tag{9}$$

Both the stress and the tangent modulus at macro-scale are possibly random quantities because they are computed from a random meso-scale generated over material area $\mathcal{R}$.

5 Numerical application: Concrete uniaxial compressive cyclic response

The model developed in the 3 preceding sections is implemented in a computer program that can solve nonlinear problems. Material area $\mathcal{R} = 1 \times 1$ is meshed by 64×64 identical squares of size $\Delta x = 1/64 = 0.016$; behavior law as presented in section 2 is assigned to the centroid of each of these squares. The random vector fields are generated with identical properties in both directions over $\mathcal{R}$; all correlation coefficients are taken as equal to $\rho = 0.9$; spectral density function $S(k)$ is assumed to have Gaussian shape; log-normal distribution with mean $\mu_{C^e} = 30$ GPa and standard deviation $\nu_{C^e} = 15$ GPa is assumed for elastic modulus C^e; uniform distribution is assumed for both the yield stress σ_y and the damage-plasticity ratio r with $\sigma_y \in [0, 70]$ MPa and $r \in [0, 0.6]$. The response of the material area $\mathcal{R}$ in compressive loading obtained with this model is illustrated in Figure 4 for different

values of the correlation length b.

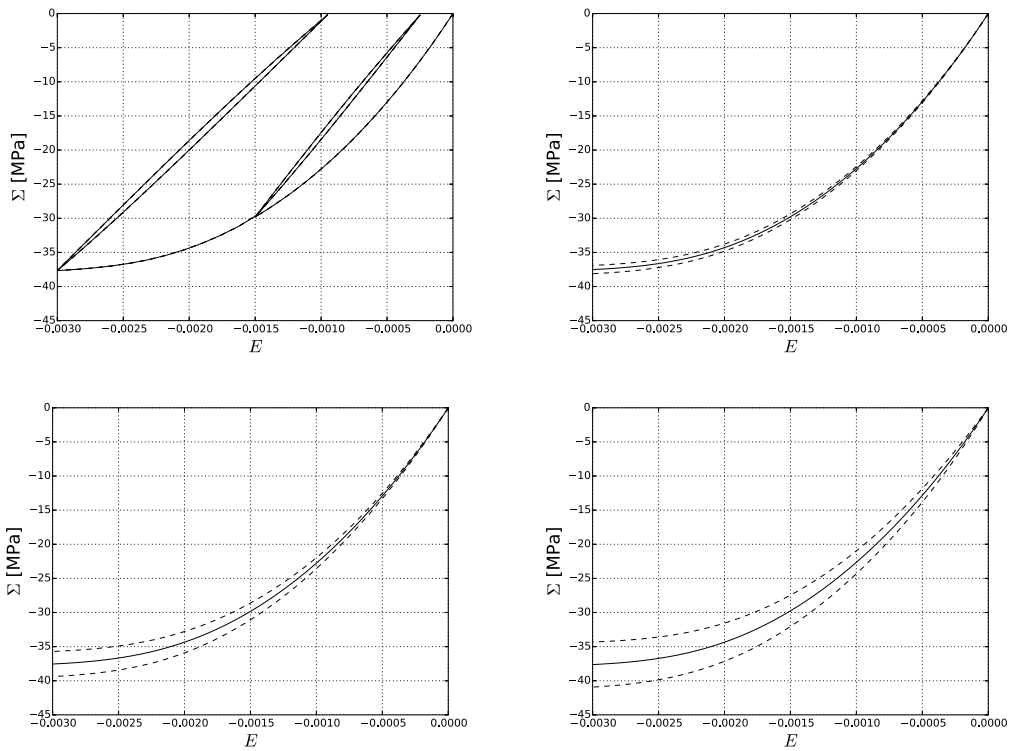

FIG. 4 – [Top left] $b = 0.04$, 1 realization of the meso-structure, uniaxial cyclic compressive loading. Mean response along with mean +/- standard deviation (dashed lines) for 200 realizations of the random meso-structure with $b = 0.04$ [top right], $b = 0.09$ [bottom left] and $b = 0.17$ [bottom right].

6 Conclusions and ongoing work

It is shown in Figure 4 that, for $b = 0.04$, a representative material area $\mathcal{R}$ can be recovered in the sense that the response at macro-scale is almost independent of the realization of the meso-structure. Besides, for $b = 0.04$ again, Figure 4 shows that the model is capable of representing salient features of concrete response in compressive uniaxial cyclic loading. Compared to the work presented in [4], the model presented here can represent loss of stiffness. This comes from the fact that coupled damage and plasticity mechanisms have been introduced at meso-scale. Other methods to couple plasticity and damage at meso-scale, parametric analysis that further explores the potential of the model, investigation of the effects of the nonlinear translation of the pseudo spectral density function are among the points to be further investigated. Also, there is a need for experimental data from material scientists for identification purposes; the model presented here can show which data would be useful.

Acknowledgement

This research is supported by a Marie Curie International Outgoing Fellowship within the 7th European Community Framework Programme (proposal No. 275928).

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