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Comments on the computation of multiple Lyapunov-like functions for switched hybrid systems

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1 Introduction

Switched hybrid systems representing a wide range of physical and cyber-physical systems, the analysis of hybrid stability is of particular interest yet the more difficult. The use of multiple Lyapunov functions here is well known [1]. In the work of [Zheng *et al.* 2], the authors aim to iteratively compute multiple Lyapunov-like functions to estimate the domain of attraction. Notwithstanding, the approach bears formal and conceptual shortcomings, which we shall present in the following. In consequence, as we argue, the proposed algorithm might not yield a correct estimate.

Notation. The state-space in mode i be \mathcal{D}_i ; the Lyapunov-function candidates $V_i : \mathbb{R}^n \rightarrow \mathbb{R}$; and the estimated domain of attraction \mathcal{D} . The set of sum of squares polynomials is denoted as $\Sigma[\mathbf{x}]$ and the zero-polynomial as $0(\mathbf{x}) \equiv 0$.

2 Corrections

2.1 Increase of the estimate

In order to properly enlarge the estimate from iteration k to $k + 1$, the authors require [2, Eq. 3]

$$\mathcal{D}_k \subsetneq \mathcal{D}_{k+1}, \quad (1)$$

where $\mathcal{D}_k = \bigcup_i (\mathcal{D}_{k,i} \cap \mathcal{D}_i)$, and therefore check [2, cf. Eq. 8]

$$\mathcal{D}_{k,i} \subsetneq \mathcal{D}_{k+1,i} \quad (2)$$

for all i . However, (2) does not imply (1) in general, which can be easily seen:

Counterexample. Let $\mathcal{D}_{k+1,i} = \mathcal{D}_{k,i} \cup \{\mathbf{x}_i\}$ for all i with $\mathbf{x}_i \notin \mathcal{D}_{k,i}$ and suppose $\mathbf{x}_i \notin \mathcal{D}_i$; then,

$$\bigcup_i (\mathcal{D}_{k+1,i} \cap \mathcal{D}_i) = \bigcup_i (\mathcal{D}_{k,i} \cap \mathcal{D}_i)$$

although $\mathcal{D}_{k,i} \subsetneq \mathcal{D}_{k+1,i}$. \square

Even we assumed $\mathbf{x}_i \in \mathcal{D}_i - \mathcal{D}_{k,i}$, this would not guarantee $\mathbf{x}_i \notin \mathcal{D}_k$ as the state-spaces \mathcal{D}_i are not pairwise disjunct, thus disproving the implication. Whether or not the implication might still hold for the assumptions taken, has not been proven here. The iteration might therefore still yield a larger estimate. The restriction to $\mathcal{D}_{k+1,i} = \{\mathbf{x} | V_{k+1,i}(\mathbf{x}) \leq 1\}$ as partial estimate further unnecessarily neglects possibly stable level sets of $V_{k+1,i}(\cdot)$ larger than 1. It is finally questionable whether the iteration will actually approximate the “true” domain of attraction, since a set \mathcal{D}' which is by an arbitrary measure larger than \mathcal{D}_k does not necessarily imply $\mathcal{D}_k \subset \mathcal{D}'$.

2.2 Continuity of the Lyapunov-functions

Multiple Lyapunov-function candidates must be continuous along the switching surfaces, here $SS_{i,j} \subset S_{i,j} = \bigcup_r \{\mathbf{x} | \mathbf{h}_{i,j,r}\mathbf{x} = 0\}$ for $i \neq j$. The constraint [2, Eq. 6]

$$S_{i,j} \subseteq \{\mathbf{x} | V_i(\mathbf{x}) \geq V_j(\mathbf{x})\} \quad (3)$$

is then supposed to be encoded as [2, Eq. 11]

$$-s_{i,j}(\mathbf{x}) \prod_r \mathbf{h}_{i,j,r}\mathbf{x} - (V_j(\mathbf{x}) - V_i(\mathbf{x})) \in \Sigma[\mathbf{x}] \quad (4)$$

and $s_{i,j}(\cdot) \in \Sigma[\mathbf{x}]$ for all $i \neq j$. Yet, (4) actually implies [3, Lemma 2]

$$\left\{\mathbf{x} \left| \prod_r \mathbf{h}_{i,j,r}\mathbf{x} \geq 0 \right.\right\} \subseteq \{\mathbf{x} | V_i(\mathbf{x}) - V_j(\mathbf{x}) \geq 0\}, \quad (5)$$

which is a much stronger constraint than (3).

2.3 Continuity along the boundary

Finally, the authors require [2, Eq. 7]

$$SS_{i,j} \cap \{\mathbf{x} | V_i(\mathbf{x}) = 1\} \subseteq \{\mathbf{x} | V_j(\mathbf{x}) = 1\}, \quad (6)$$

which they encode as [2, Eq. 12]

$$-s_{1,i,j}(\mathbf{x}) \prod_r \mathbf{h}_{i,j,r}\mathbf{x} - s_{2,i,j}(\mathbf{x})(1 - V_i(\mathbf{x})) \pm s_{3,i,j}(\mathbf{x})(1 - V_j(\mathbf{x})) \in \Sigma[\mathbf{x}] \quad (7)$$

and $s_{1,i,j}(\cdot), s_{2,i,j}(\cdot), s_{3,i,j}(\cdot) \in \Sigma[\mathbf{x}]$. Eq. (7), we are afraid to say, does not imply (6) at all. We recall the primal lemma [3, Lemma 2]:

Lemma 1. *Let g_0, g_1, \dots, g_m be polynomials in \mathbf{x} ; if there are $s_1, \dots, s_m \in \Sigma[\mathbf{x}]$ such that $g_0(\mathbf{x}) - \sum_{i=1}^m s_i(\mathbf{x})g_i(\mathbf{x}) \in \Sigma[\mathbf{x}]$, then $\{\mathbf{x} | g_1(\mathbf{x}), \dots, g_m(\mathbf{x}) \geq 0\} \subseteq \{\mathbf{x} | g_0(\mathbf{x}) \geq 0\}$.*

In the case of (7), we have $g_0 = 0(\mathbf{x})$ and, subsequently, $\{\mathbf{x} | g_0(\mathbf{x}) \geq 0\} = \mathbb{R}^n$. Indeed, $s_{1,i,j} = s_{2,i,j} = s_{3,i,j} = 0(\mathbf{x})$ satisfy (7) for any $\mathbf{h}_{i,j,r}$, $V_i(\cdot)$, and $V_j(\cdot)$. That is, the constraint in (7) is a tautology.

3 Conclusions

The estimation of the domain of attraction of a switched hybrid system does not work as the authors [2] suppose. With (1) not proven, an increase of \mathcal{D}_{k+1} is possibly not enforced. The effectively imposed constraint of (5) is much stricter than suggested. In any case, as (7) is void, the output is not necessarily a domain of attraction in the sense of the author's first lemma.

References

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