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Frequency-domain data-driven control design in the Loewner framework

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Abstract: In this article, a direct data-driven design method, based on frequency-domain data, is proposed. The identification of the plant is skipped and the controller is designed directly from the measurements. The identification task is reported on a fixed-order controller using for the first time the Loewner approach, known for model approximation and reduction. The method is validated on two numerical examples including the control of an industrial hydroelectric generation plant, modelled by irrational equations.

Keywords: Data-driven control, Model reference control, Loewner interpolation

1. INTRODUCTION

In many control engineering applications, no mathematical description of the plant is available or easily accessible. Given some input-output data collected on a system, one can either identify a model of the plant and then, design a controller using any kind of model based technique (indirect methods), or directly use the experimental data to design a controller (direct methods).

In this paper, we are interested in the latter case. Indeed, direct methods are particularly interesting when a model of the system would be too time-consuming, too complex or too costly to obtain. Another application is for industrial processes which tend to exhibit various parametric variations. Modelling and identifying these behaviors and taking them into account in the controller design is a difficult task which is avoided using a direct data-based method. Moreover, it is worth noting that a model is always a compromise between complexity and reliability, and a major difficulty is to determine which parts of the dynamics have to be considered. Furthermore, the identification can result in a complex structure for the plant, and consequently for the controller, and a reduction step (for the controller) might be needed, which is a complex task (see Antoulas et al. (2015)). Of course, authors are conscious that, with direct methods, the identification problem is shifted to the control one, and the problem is then to know which parts of the dynamics are to be controlled. However, since the selection of the controller is done directly from the experimental data, direct data-driven methods seem less conservative, and not sensitive to modelling errors. Moreover, they are less time-consuming since the modelling and/or identification steps are skipped and the resulting control law is tailored to the actual system. However, authors are conscious that an advantage of indirect methods over direct ones is that the specifications can be more advanced and that the model can be

used for other purpose (stability and robustness analysis, simulation, etc...).

Numerous direct methods, also referred to as "model free" control, have been proposed to improve the results and try to achieve the best possible performance without using any plant model. One of the first direct methods relies the unfalsified control concept, see Safonov and Tsao (1994). Another one is the Iterative Feedback Tuning (IFT, Hjalmarsson (2002)) which finds the controller parameters thanks to an adaptative and iterative control algorithm based on explicit criterion minimization. Note that Kammer introduced in Kammer et al. (2000) a frequency-domain variant of the IFT. Among all these techniques, the Virtual Reference Feedback Tuning (VRFT) is a direct method that has been introduced in Campi et al. (2002) in its original version. It consists in minimizing a performance criterion, which is a convex function for an a priori fixed poles selection. It finds the controller (in a class a priori selected) so that the difference between the closed-loop and an objective transfer is minimized. See also Formentin et al. (2013) for an application of the VRFT and Formentin et al. (2014) for a comparison with indirect methods. In an applicative set-up, direct methods such as the VRFT are really useful when a quick and low-cost controller has to be synthesized. One of the main problem is that selecting a class of controllers, as it is required in the methods mentioned above, can be particularly difficult.

In this article, a one shot direct method using frequency-domain data is proposed. In this approach, the user does not have to choose a structure for the controller, which makes it really easy to use. It can be applied to MIMO plants and it is suitable for large scale problems. As the VRFT, it is a model reference-based control technique: the specifications are expressed as a reference transfer function which represents the desired behavior in closed-loop. The problem considered (see Figure 1) is to design a controller for an unknown plant P on the basis of n_u input- n_y output,

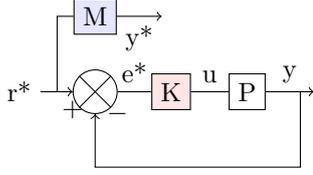


Fig. 1. Problem formulation: M is the reference transfer, P is the plant and K the controller to be identified

respectively denoted u and y , measurement data without identifying a dynamical model of the plant.

The goal is to identify a controller \hat{K} that gives a closed-loop as close as possible to the reference transfer M if inserted in the closed-loop. The measurements can be given as a data-set $\{u(t_{i_t}), y(t_{i_t})\}$, $i_t = 1 \dots N_t$, where $u(t_{i_t}) \in \mathbb{R}^{n_u}$ and $y(t_{i_t}) \in \mathbb{R}^{n_y}$, or directly as samples of the frequency response of the plant $\{\omega_{i_f}, \Phi_{i_f}\}$, $i_f = 1 \dots N_f$, where $\Phi_{i_f} \in \mathbb{C}^{n_y \times n_u}$ and $\omega_{i_f} \in \mathbb{R}_+$.

The closed-loop objective transfer M and the open-loop experimental data of the plant are used to get the frequency response of the “ideal” controller denoted $K(\omega_{i_f})$ for a limited frozen set of frequency values. This controller is “ideal” in sense that it would give exactly the objective transfer function M if inserted in the closed-loop. Then, the “ideal” controller, known over a limited data samples, will be approximated by \hat{K} , a linear time-invariant system defined by the following generalized state-space form:

$$\hat{K} : \begin{cases} E\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{e}(t) \\ \mathbf{u}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{e}(t) \end{cases}, \quad (1)$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ the state vector, $\mathbf{e}(t) = \mathbf{r}^*(t) - \mathbf{y}(t) \in \mathbb{R}^{n_y}$ the input vector of the controller, which is the tracking error, $\mathbf{u}(t) \in \mathbb{R}^{n_u}$ the output vector (i.e. the control signal) and $E, \mathbf{A} \in \mathbb{R}^{n \times n}$, $\mathbf{B} \in \mathbb{R}^{n \times n_y}$, $\mathbf{C} \in \mathbb{R}^{n_u \times n}$ and $\mathbf{D} \in \mathbb{R}^{n_u \times n_y}$. Its associated transfer function is $\hat{K}(s) = \mathbf{C}(sE - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}$.

The direct data-driven control design technique proposed in this paper relies on two steps: (i) the obtention of an “ideal” frequency response of the controller $K(\omega_{i_f})$ and (ii) its identification \hat{K} through the Loewner framework in the frequency domain. As said earlier, this method, called **L-DDC** (Loewner Data-Driven Control), does not require any a priori selection of a class of controllers, the user does not have to determine a priori the poles which is a long and difficult task that do not guarantee the selected poles optimality. Moreover, it is applicable to MIMO systems, a single set of input-output measurements is needed and, as shown in the rest of the paper, is easy to apply.

Let us denote ι the complex variable and \dagger the Moore-Penrose inverse. $\bar{s} \in \mathbb{C}$ denotes the complex conjugate of $s \in \mathbb{C}$. The (A, E) pencil denotes the generalized eigenvalues of $A - \lambda E$. $\mathcal{F}(\cdot)$ denotes the Fourier operator.

This article is organized in five sections. Section 2 introduces the Loewner framework which is the frequency-based interpolation technique used in this work and which plays a pivotal role in the proposed approach. Then, the proposed method to design a controller on the basis of frequency-domain data is exposed in Section 3. Finally, two applications are considered in Section 4. The first one is an academical example concerning a flexible transmis-

sion. The second one consists in designing a controller for an open-channel hydraulic system described by an irrational transfer function. Conclusions and outlooks are finally given in Section 5.

2. PRELIMINARY RESULTS: LOEWNER-BASED IDENTIFICATION

The Loewner approach, exposed in Mayo and Antoulas (2007), is an interpolatory method initially used for model approximation and reduction. In its original version, it constructs a descriptor model in state-space form directly from the frequency-domain data so that the model performs a barycentric Lagrange interpolation (see Ionita (2013) for further details).

This model can be reduced to balance between its complexity and its accuracy. In order to construct such a realization, the following inputs are required: (i) left interpolation point $(\mu_j)_{j=1 \dots q} \in \mathbb{C}$ and left tangential directions $(\mathbf{l}_j)_{j=1 \dots q} \in \mathbb{C}^{n_y}$, and (ii) right interpolation points $(\lambda_i)_{i=1 \dots k}$ and right tangential directions $(\mathbf{r}_i)_{i=1 \dots k} \in \mathbb{C}^{n_u}$. The following vectors are then defined from the input data: $\mathbf{v}_j^T = \mathbf{l}_j^T K(\mu_j) \forall j = 1 \dots q$ $\mathbf{w}_i = K(\lambda_i) \mathbf{r}_i \forall i = 1 \dots k$.

The interpolation points correspond to the data of the model to be identified, which are, in our approach, the samples of the frequency response of the “ideal” controller $\{\omega_{i_f}, K(\omega_{i_f})\}$, $i_f = 1 \dots N_f$. The computation of the samples $K(\omega_{i_f})$ and the separation of the data between left and right interpolation points, respectively $(\mu_j)_{j=1 \dots q}$ and $(\lambda_i)_{i=1 \dots k}$, are explained in Section 3. Note that in case of a SISO system, the tangential directions $(\mathbf{l}_j)_{j=1 \dots q}$ and $(\mathbf{r}_i)_{i=1 \dots k}$ are useless and can be fixed to 1. In the Loewner approach, one seek for a model \hat{K} that interpolates the data as follows: $\begin{cases} \mathbf{l}_j^T \hat{K}(\mu_j) = \mathbf{l}_j^T K(\mu_j) = \mathbf{v}_j^T \forall j = 1 \dots q \\ \hat{K}(\lambda_i) \mathbf{r}_i = K(\lambda_i) \mathbf{r}_i = \mathbf{w}_i \forall i = 1 \dots k \end{cases}$, where K represents the ideal controller (Section 3 explains how to obtain samples of its frequency response for a given set of frequency values).

Based on the $(\mu_j, \mathbf{l}_j^T, \mathbf{v}_j^T)$ and $(\lambda_i, \mathbf{r}_i, \mathbf{w}_i)$ data, one can construct the Loewner and shifted Loewner matrices \mathbb{L} and \mathbb{L}_σ as follows for all $j = 1 \dots q$ and $i = 1 \dots k$:

$$[\mathbb{L}]_{j,i} = \frac{\mathbf{v}_j^T \mathbf{r}_i - \mathbf{l}_j^T \mathbf{w}_i}{\mu_j - \lambda_i}, \quad [\mathbb{L}_\sigma]_{j,i} = \frac{\mu_j \mathbf{v}_j^T \mathbf{r}_i - \lambda_i \mathbf{l}_j^T \mathbf{w}_i}{\mu_i - \lambda_j}. \quad (2)$$

As explained in Mayo and Antoulas (2007), one of the main advantages of the Loewner framework is that the minimal Mc Millan order of the interpolating model \hat{K} can be obtained by evaluating $r = \text{rank}[\mathbb{L}, \mathbb{L}_\sigma]$. The matrices of the model \hat{K} are then computed as follows :

$$E = -Y^* \mathbb{L} X, \quad A = -Y^* \mathbb{L}_\sigma X, \quad B = Y^* V, \quad C = W X, \quad (3)$$

where $X \in \mathbb{C}^{k \times r}$ and $Y \in \mathbb{C}^{q \times r}$ are two matrices determined by the singular value decomposition of the Loewner pencil $(\mathbb{L}_\sigma, \mathbb{L})$ as follows:

$$[\mathbb{L}, \mathbb{L}_\sigma] = Y \Sigma_l \tilde{X}^*, \quad \begin{bmatrix} \mathbb{L} \\ \mathbb{L}_\sigma \end{bmatrix} = \tilde{Y} \Sigma_r X^*, \quad (4)$$

where $\Sigma_l, \Sigma_r \in \mathbb{R}^{r \times r}$.

Computing $\nu = \text{rank}(\mathbb{L})$ allows to know whether the model is strictly proper rational or not. If not, a D-term

or a polynomial term is added. More details are available in Antoulas et al. (2015).

In addition to determining the smallest exact interpolating model, the Loewner framework allows to control the complexity of the identified model: by keeping the n largest singular values of the decomposition of the Loewner pencil only (4), the obtained realization is a n -th order one. In our case, n is the objective order for the controller ($n \leq r$).

Usually, the Loewner framework presented here is used for model order reduction, see Dalmas et al. (2016) for an industrial application. In the context of the proposed data-driven controller design, the Loewner framework will be used for the first time to identify directly a controller of a limited complexity. The proposed procedure is detailed in the next section.

3. MAIN CONTRIBUTIONS: LOEWNER BASED DATA-DRIVEN CONTROL DESIGN

As introduced above, we propose a model reference control technique using frequency-domain data. The proposed procedure is indicated in the following **L-DDC** algorithm (Algorithm 1).

The idea is to use the interconnection of Figure 1 and to exploit experimental data $\{\omega_{i_f}, \Phi_{i_f}\}$, $i_f = 1 \dots N_f$ to determine the frequency response of the “ideal” controller $K(\omega_{i_f})$. The “ideal” controller is the one which would allow to get exactly the desired closed-loop transfer materialized by the function M in Figure 1. Then, we propose to identify the controller using the Loewner framework which principles are recalled in Section 2.

Our approach does not require to select a class of controllers *a priori*, contrary to the VRFT which allows to tune controllers with a prescribed structure. This is actually, from a practical viewpoint, one of the main strengths of our approach and is due to the interpolatory property of the Loewner framework.

As in the classic VRFT method, the performance objective is fixed by a transfer function M which represents the dynamic one wants to achieve in closed-loop.

If a time data-set $\{u(t_{i_t}), y(t_{i_t})\}$, $i_t = 1 \dots N_t$, of input-output measurements of the plant is available, a Fourier transform of the input and output signals is done and samples of the plant frequency response are estimated: $\Phi_{i_f} = \frac{Y(\omega_{i_f})}{U(\omega_{i_f})} = \frac{\mathcal{F}(y(t_{i_t}))}{\mathcal{F}(u(t_{i_t}))}$, $\forall i_f = 1 \dots N_f$. Obviously, the signal u should sufficiently excite the system to get rich-enough information in terms of frequencies.

With reference to Algorithm 1, first step (1) aims at obtaining the frequency response of the “ideal” controller $K(\omega_{i_f})$ at given frequencies $(\omega_{i_f})_{i_f=1 \dots N_f}$. Then, steps (2) to (6) consist in identifying the controller from the “ideal” frequency response computed above. This is achieved through the Loewner framework. The data are equally separated between left and right interpolation points. In Ionita (2013), the author recommends to alternate between left and right to avoid rank loss in the Cauchy-like Loewner matrix \mathbb{L} . Let us consider the following shift selection: $\omega = \begin{bmatrix} \mu_1 & \lambda_1 & \dots & \mu_{\frac{N_f}{2}} & \lambda_{\frac{N_f}{2}} \end{bmatrix}$. Finally, to

deal with real arithmetic only, the complex conjugate values are considered, ensuring \mathbb{L} realness (see Ionita (2013)), so the left and right interpolation points are given by:

$$\mu = \left[\mu_1 \ \bar{\mu}_1 \ \dots \ \mu_{\frac{N_f}{2}} \ \bar{\mu}_{\frac{N_f}{2}} \right], \quad \lambda = \left[\lambda_1 \ \bar{\lambda}_1 \ \dots \ \lambda_{\frac{N_f}{2}} \ \bar{\lambda}_{\frac{N_f}{2}} \right],$$

where $q = k = N_f$. Note that $K(-\omega) = \overline{K(\omega)}$ (in order to compute the vectors $\mathbf{v}_j^T \forall j = 1 \dots q = N_f$ and $\mathbf{w}_i \forall i = 1 \dots k = N_f$).

Concerning the tangential directions, if the system is SISO, the tangential directions are fixed to 1. In a multivariable case, the choice of the tangential directions is still an open problem and it could be a way to improve our method in the future. If not choosing them randomly, an idea is to spread the directions on the unit sphere. See Ionita (2013) for further details on these problems.

Data:

- Samples of the frequency response of the plant $\{\omega_{i_f}, \Phi_{i_f}\}$, $i_f = 1 \dots N_f$. Note that $q = k = N_f$.
- Objective order n for the controller
- Reference transfer function M

Solution:

- (1) Compute the samples of the frequency response of the “ideal” controller as follows: $\forall i_f = 1 \dots N_f$,

$$K(\omega_{i_f}) = (\Phi_{i_f} - \Phi_{i_f} M(\omega_{i_f}))^\dagger M(\omega_{i_f}).$$

- (2) Divide the samples between left and right interpolation points, respectively $(\mu_j)_{j=1 \dots q}$ and $(\lambda_i)_{i=1 \dots k}$. Choose the left and right tangential directions, respectively $(\mathbf{l}_j)_{j=1 \dots q} \in \mathbb{C}^{n_y}$ and $(\mathbf{r}_i)_{i=1 \dots k} \in \mathbb{C}^{n_u}$. See the comments above for the choice of the repartition and the directions.
- (3) Compute the Loewner pencil $(\mathbb{L}, \mathbb{L}_\sigma)$ following the definitions of (2).
- (4) Compute the dimension of the minimal representation $r = \text{rank}[\mathbb{L}, \mathbb{L}_\sigma]$.
- (5) Perform the Singular Value Decomposition of the Loewner pencil as in (4). If $n \leq r$, only the n largest singular values are kept. Otherwise, the controller will be of order r .
- (6) Compute the descriptor controller model as indicated by (3) to obtain the matrices $E \in \mathbb{R}^{n \times n}$, $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times n_y}$, $C \in \mathbb{R}^{n_u \times n}$ and $D \in \mathbb{R}^{n_u \times n_y}$. The controller \hat{K} is then given as in (1).

Algorithm 1. L-DDC algorithm

Remark 1. An optional filtering of the frequency-domain data ω_{i_f} , $i_f = 1 \dots N_f$, can be applied before or after computing the ideal frequency response at step (1). We have not proposed any filtering option yet, this would be an interesting outlook for further research.

Remark 2. Note that the complexity of the **L-DDC** algorithm resides in the Singular Value Decomposition computed at step (5) of the algorithm. Consequently, it is well tailored to large scale problems (i.e. with a large number of data), and there is no limitations to MIMO cases, see Demourant and Poussot-Vassal (2016) for a multivariable example.

4. NUMERICAL APPLICATIONS

In order to illustrate the main features of the **L-DDC** algorithm, simulation examples are presented. The first one is a simple academical example while the second one

is an industrial simulation one (it consists in an application to an open channel flow described by an irrational transfer function).

4.1 Flexible transmission system

Here, the considered plant is the well-known flexible transmission system. This example has been used in Campi et al. (2002) and in Hjalmarsson et al. (1995) as a benchmark for the presentation of the VRFT using time-domain data. The system consists of three horizontal pulleys connected by two elastic belts (see Campi et al. (2002)).

The system input is the angular position of the first pulley and its output is the angular position of the third pulley. The control objective is to pilot the angular position of the third pulley. The behavior of the plant is described by the following discrete-time linear transfer function (using a sampling time $T_s = 0.05s$): $P(z) = z^{-3} \frac{B(z)}{A(z)}$, where $B(z) = 0.28261 + 0.50666z^{-1}$ and $A(z) = 1 - 1.41833z^{-1} + 1.58939z^{-2} - 1.31608z^{-3} + 0.88642z^{-4}$. The control objective is defined, as in Campi et al. (2002), by the following closed-loop reference transfer function: $M(z) = \frac{z^{-3}(1-\alpha)^2}{(1-\alpha z^{-1})^2}$, with $\alpha = e^{-T_s \bar{\omega}}$ and $\bar{\omega} = 10$ the desired bandwidth.

For the VRFT technique, the selected class of controllers proposed in Campi et al. (2002) is defined as $K_{VRFT}(z, \vartheta) = \frac{\vartheta_0 + \vartheta_1 z^{-1} + \vartheta_2 z^{-2} + \vartheta_3 z^{-3} + \vartheta_4 z^{-4} + \vartheta_5 z^{-5}}{1 - z^{-1}}$, where $\vartheta_{i,1} = 1 \dots 5$ are the decision variables. In the noise free simulation case, the parameters found by the VRFT are (see Campi et al. (2002)) $\vartheta^2 = [0.32905, -0.59771, 0.70728, -0.64010, 0.46499, -0.11763]^T$.

The input signal used for the simulation was a linear swept-frequency signal from $\underline{f} = 1\text{Hz}$ to $\bar{f} = 500\text{Hz}$ ($N_f = 750$). Now, to stick to the **L-DDC** approach, a Fourier transform of the input-output signals allows us to compute samples of the frequency response of the plant: $\Phi_{i_f} = \frac{Y(\omega_{i_f})}{U(\omega_{i_f})}$, $\forall i_f = 1 \dots N_f$. Then, following the **L-DDC** algorithm, the frequency response of the ideal controller is computed and data are shown as blue dots in Figure 2: $\forall i_f = 1 \dots N_f$, $K(\omega_{i_f}) = \frac{M(\omega_{i_f})}{\Phi_{i_f} - \Phi_{i_f}(\omega_{i_f})}$.

The model of the 5-th order controller obtained by the **L-DDC** algorithm is visible in Figure 2. First, note that the frequency response of the ideal controller is really similar to the VRFT controller one, which allows to get quite exactly the reference transfer in closed-loop.

The **L-DDC** controller, obtained by applying the algorithm of Section 3, fits the data of the ideal controller with a 5-th order (same order as the VRFT controller). Note that the minimal order giving an exact interpolation of the data is $r = 72$. However, the singular values of the Loewner matrix \mathbb{L} , given in Figure 3, show that the five first singular value are enough to describe the controller. The Bode magnitude plots of the plant P and the reference model M are shown in Figure 4 (solid red and solid blue respectively). The closed-loops obtained by the two different controllers are also visible in Figure 4. The VRFT controller allows to get a similar closed-loop to the reference transfer. However the **L-DDC** controller gives

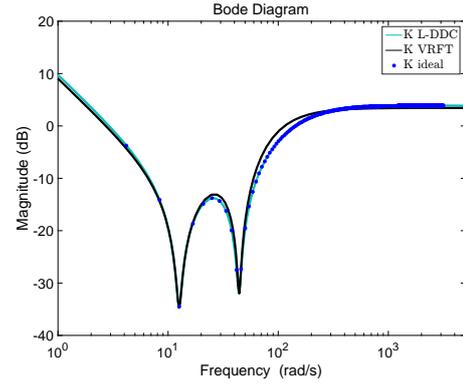


Fig. 2. Identification of the controller by the L-DDC algorithm (solid clear blue) and VRFT controller (solid black). The blue dots corresponds to the frequency response of the ideal controller computed from the data and the objective transfer function M .

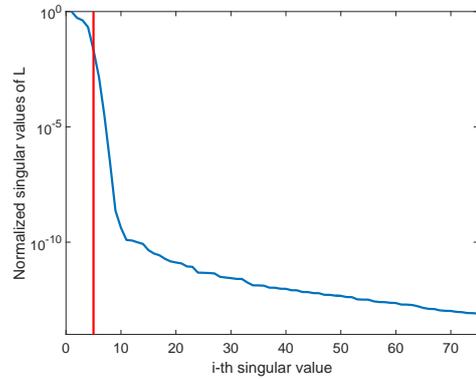


Fig. 3. Decrease of the singular values of the Loewner matrix \mathbb{L} in blue and limit at the 5-th singular value in red

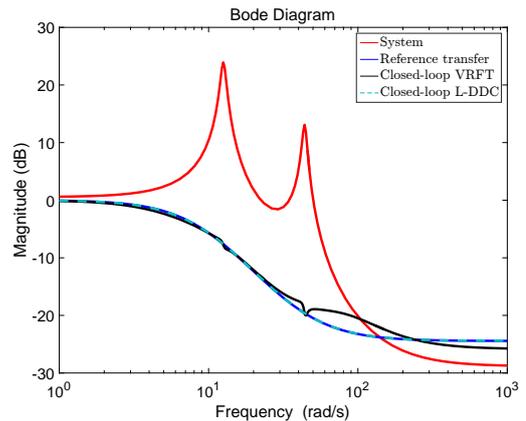


Fig. 4. Closed-loops obtained with the VRFT (solid black) and Loewner (dashed clear blue) controllers, compared with the initial plant (solid red) and the reference transfer (solid blue).

a closed-loop that almost perfectly match the objective function M , with the exact same complexity.

The fact that the **L-DDC** controller allows a better fit of the reference transfer can be explained by the fact that the user does not have to select a structure a priori, it is directly identified from the “ideal” frequency response. The absence of a user-defined structure adds degrees of freedom to the problem that are used toward the realization of the objective transfer function M . Not choosing a structure for the controller is a strength of our method and makes it really appealing in practice.

4.2 Application to an open-channel flow for hydroelectricity (MISO case)

Now let us move to an industrial problem provided by the French power producer EDF (Electricité de France). EDF uses water resources to generate green energy with run-of-the-river power plants. They rely on open-channel hydraulic systems that are non-linear and which dynamic depends on the operating point. Here, for simplicity we will consider one single operating point only.

Their physical model requires partial differential equations (namely Saint-Venant equations). In Dalmas et al. (2016), a new irrational transfer function is proposed for open channels to represent the level-to-flow variations for any operating point. It is the solution of Saint-Venant equations under many assumptions. The system has two inputs, the entering and the outgoing flows q_e and q_s , and one output, the water depth h . The transfer is given by:

$$\begin{aligned} h(x, s, Q_0) &= G_e(x, s, Q_0)q_e(s) + G_s(x, s, Q_0)q_s(s) \\ &= P(x, s, Q_0) \begin{bmatrix} q_e \\ q_s \end{bmatrix}, \end{aligned}$$

where

$$\begin{aligned} G_e(x, s, Q_0) &= \frac{\lambda_1(s)e^{\lambda_2(s)L+\lambda_1(s)x} - \lambda_2(s)e^{\lambda_1(s)L+\lambda_2(s)x}}{B_0s(e^{\lambda_1(s)L} - e^{\lambda_2(s)L})} \\ G_s(x, s, Q_0) &= \frac{\lambda_1(s)e^{\lambda_1(s)x} - \lambda_2(s)e^{\lambda_2(s)x}}{B_0s(e^{\lambda_1(s)L} - e^{\lambda_2(s)L})} \end{aligned}$$

where x is the position of the measurement point on the channel, Q_0 the nominal flow, L the length of the open channel. B_0 , $\lambda_1(s)$ and $\lambda_2(s)$ depend on the canal configuration and the nominal flow (see Dalmas et al. (2016)). The system, which dynamic is visible in Figure 5, is extremely slow, has a delay behavior and a pole in limit of stability. Moreover, it has an infinite number of poles since the transfer function is irrational.

The system have been approximated in Dalmas et al. (2016) by a 21-st order rational transfer function depending on the nominal flow with input time delays: $\tau_e \simeq 500s$ and $\tau_s \simeq 1500s$, on q_e and q_s respectively. This approximation is shown in Figure 5 (dashed red).

The objective is to stabilize the system and to obtain a faster dynamic. The reference closed-loop M is chosen to be a second order continuous transfer function: $M(s) = \frac{1}{1 + \frac{2\xi}{\omega_0}s + \frac{s^2}{\omega_0^2}}$, with $\omega_0 = 10^{-5}rad.s^{-1}$ and $\xi = 1$. In this example, the frequency approach is interesting since the system is represented by an irrational transfer function. Therefore, one cannot have a time-domain simulation. However we still can estimate samples of the frequency response of the system $\{\omega_{i_f}, \Phi_{i_f}\}$, $i_f = 1 \dots N_f$, from which the ideal controller’s frequency response can be deduced.

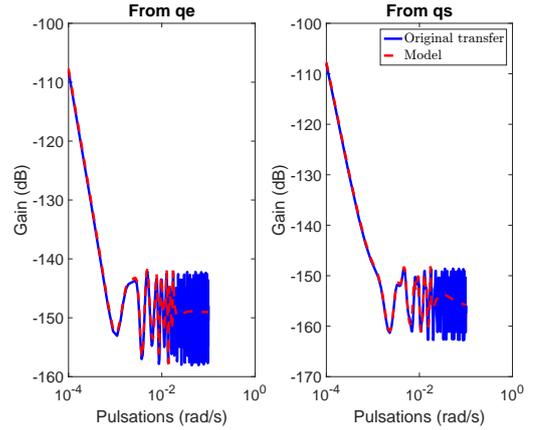


Fig. 5. System dynamic for nominal flow $Q_0 = 1400 m^3.s^{-1}$: original plant in solid blue and approximation obtained by Dalmas et al. (2016) in dashed red.

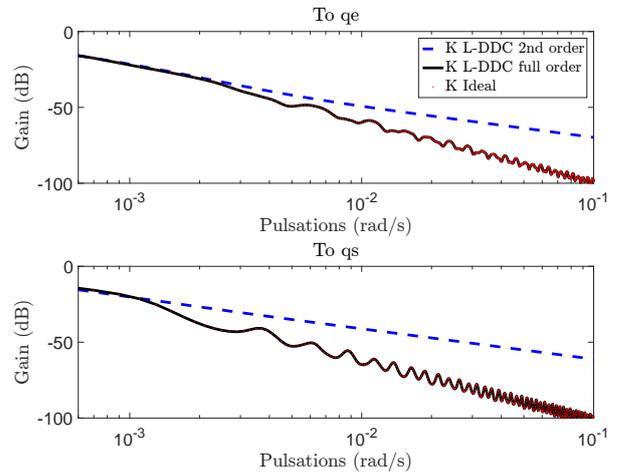


Fig. 6. **L-DDC** controller synthesized of order 2 (dashed blue), full order $r = 215$ (solid black) compared with the ideal frequency response (red dots).

The “ideal” controller $K(\omega_{i_f})$, which exactly provides the desired closed-loop behavior dictated by M when placed in the closed-loop, is obtained as follows (step (1) of the **L-DDC** algorithm):

$$\forall i_f = 1 \dots N_f, K(\omega_{i_f}) = (\Phi_{i_f} - M(\omega_{i_f})\Phi_{i_f})^\dagger M(\omega_{i_f})$$

The samples of the frequency response $\Phi_{i_f} = P(\omega_{i_f})$, $i_f = 1 \dots N_f = 500$ are extracted from the irrational transfer function P . Note that the Moore-Penrose inverse \dagger is used because it is no longer a SISO case. Therefore, the pseudo-inverse should be employed.

The result of the identification step is given in Figure 6. The smallest order giving an exact interpolating model for the considered data is $r = rank[\mathbb{L}, \mathbb{L}_\sigma] = 215$. Even if a controller with an higher order would be more accurate in terms of fitting the ideal frequency response, the second order controller \hat{K} , $n = 2$, shown in Figure 6, which is an approximation of the ideal controller, should be sufficient to obtain good performances.

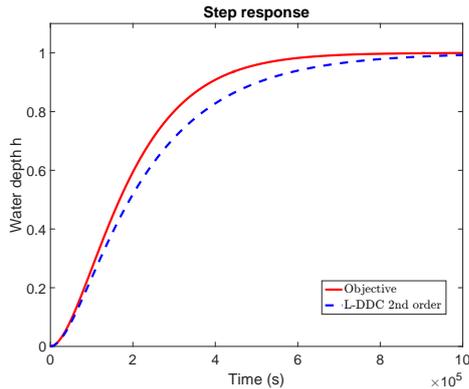


Fig. 7. Step response of the closed loop using the 2nd order **L-DDC** controller (dashed blue) and of the objective transfer (solid red).

Since we had no access to the EDF simulator, the 21-st order rational transfer function of Dalmas et al. (2016) is used to simulate the closed-loop with the 2-nd order controller obtained by the **L-DDC**. The results are shown in Figure 7. The closed-loop dynamic is similar to the objective behaviour. Note that the command signal (not shown here for space limitations) is reasonable, the maximum flow variation is around $3m^3.s^{-1}$, which is in the acceptable range for this application with a controller of order 2. In this case, the **L-DDC** method is appealing since it does not require to simulate the complex system described by an irrational transfer function to obtain time-domain data. Only samples of the frequency response of the plant are needed, which can be estimated directly from the irrational transfer function. Moreover, one should notice that controlling such an infinite order model is also quite challenging even for model-based methods. A perspective is to try this controller on the EDF simulator instead of using the approximate model to validate the performances.

5. CONCLUSION AND DISCUSSION

In this paper, the **L-DDC** (Loewner Data-Driven Control), which is a direct data-driven method involving frequency-domain data, has been proposed. It is a model reference technique which specifications are imposed easily through a reference transfer function representing the desired closed-loop behavior. Despite its simplicity, the **L-DDC** approach seems to provide good performances and, thanks to the Loewner framework, a set of interesting properties.

First, the frequency response of the “ideal” controller is computed thanks to frequency-domain data from the plant and the reference transfer. This controller is called “ideal” because it would give exactly the objective if inserted in the closed-loop. Then, the Loewner framework is used to identify the controller on the basis of this “ideal” frequency response. The Loewner framework provides a nice bundle of properties we pretend to exploit in the future of this work (such as the rank, the interpolary features or the potential structuration of the controller for example).

The main advantage of the **L-DDC** method relies on its simplicity, the user does not have to choose a structure for

the controller, only the order, which becomes a tunable parameter allowing to find a compromise between complexity and reliability. This technique is appealing for engineers for applications when a controller should be synthesized quickly and for which it would be too costly or too complex to identify a model. It is a one shot method and the obtained controller is tailored to the actual system. It can be applied to MIMO plants and it is suitable for large scale problems since the computational cost is low. The use of more properties of the Loewner framework is promising. However, the **L-DDC** technique, as other direct methods, does not allow the use of model-based analysis of stability and robustness and further research will address this point.

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