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► **To cite this version:**

Ghislaine Gueudet, Pierre-Vincent Quéré. “Making connections” in the mathematics courses for engineers: the example of online resources for trigonometry. INDRUM 2018, INDRUM Network, University of Agder, Apr 2018, Kristiansand, Norway. hal-01849973

HAL Id: hal-01849973

<https://hal.science/hal-01849973v2>

Submitted on 10 Aug 2018

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“Making connections” in the mathematics courses for engineers: the example of online resources for trigonometry

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Abstract: This paper concerns the teaching of mathematics for future engineers, focusing on the theme of trigonometry. We claim that the use of trigonometry in engineering courses requires different kinds of connections: connecting different domains, different concepts, frames and registers. We use here the concept of connectivity, developed in the frame of e-textbooks analysis, to analyse online courses for future engineers in France. We evidence that these courses propose some connections; but their connectivity is not developed enough to meet the requirements of engineering courses.

Keywords: *Connectivity, Teaching and learning of specific topics in university mathematics, The role of digital and other resources in university mathematics education, Teaching and learning of mathematics in other fields, Mathematics for engineers.*

INTRODUCTION

How can a teaching of mathematics answer the needs of engineering courses, i.e. can provide the mathematics needed to understand the course and solve the problems proposed? Which should be the features of such a teaching, and do existing courses present such features? This is the general theme of the work presented here. Previous works addressing this theme observe a gap between the mathematics taught in “mathematics courses” and the use of them to solve problems in engineering courses (e.g. Redish 2005; Biehler, Kortemeyer & Schaper 2015). Winsløw, Gueudet, Hochmut and Nardi (2018) note that several works presented at CERME conferences identify “a lack of connectedness of curricula integrating mathematics and other disciplines”. Interviewing French engineers about their mathematical needs in the workplace (Quéré 2017), and how the mathematics courses they followed as students answered or not these needs, we noticed that several of them declared that the mathematics courses “did not make enough connections”. These connections, according to the engineers, can be of different kinds: links between mathematics and the real world, between different mathematical contents, between different representations etc. We consider that this is an important issue for understanding mathematics applied to engineering and their teaching. Which kinds of connections should propose a teaching of mathematics for engineering?

Moreover, in the frame of another starting study, we are interested by the possible design of innovative curriculum resources for the teaching of mathematics for

STEMSS (Science, Technology, Engineering, Medicine and Social Sciences) courses. Before starting the design of such resources, we have investigated existing resources, trying in particular to observe whether they tried to build the kind of connections evoked by the engineers.

This investigation led us to choose a focus on trigonometry. Indeed, trigonometry appeared as extensively used in different kinds of engineering courses, and offering many possibilities of connections of all the kinds evoked above. In what follows, we firstly introduce our theoretical tools and research questions; then we present related works on the teaching of trigonometry. To exemplify the mathematical needs, we analyse the use of trigonometry in an electricity course for first year students; then we consider the trigonometry content in two online mathematics courses for engineering students.

THEORETICAL PERSPECTIVE AND RESEARCH QUESTIONS

The overarching perspective guiding our work is an institutional perspective (Chevallard, 2006). We consider that the mathematics taught are shaped by the institution where they are taught. Engineering studies at university or in engineers' schools constitute an institution, different from mathematics courses for maths majors. Within engineering studies, engineering courses also constitute an institution different from mathematics courses. Our aim here is to compare how these two institutions shape the mathematics contents, more precisely the trigonometry contents.

Our central focus is on “making connections”. Students' understanding in terms of mathematical reasoning and problem-solving has been linked by several authors to “making connections” and “connectivity” (e.g. Hiebert & Carpenter 1992). Drawing on these works, we have chosen to look for:

“connections in, between, and across individuals' cognitive/learning tasks and activities, and how e-textbooks may support those (micro level); as well as for ‘connected learning’ between and across groups of individuals, teachers or students (macro level).” (Gueudet, Pepin, Sabra, Restrepo & Trouche to appear)

We have therefore proposed a concept of “connectivity” to analyse e-textbooks (encompassing various kinds of digital curriculum), with the intention to evaluate their potential for the building of connections for the students.

Thus, connectivity has two components: “macro-level connectivity”, which considers the e-textbook as a whole; and “micro-level connectivity”, where the focus is on a particular mathematical topic (trigonometry here). In this paper we only use “micro-level connectivity”; more precisely, we observe in curriculum resources available online the presence of:

- Connections between different topic areas or frames;

- Connections between different semiotic representations (e.g. text, figures, static and dynamic); [...]
- Connections between different concepts [...]” (Gueudet et al. to appear).

Here we want to compare the connections concerning trigonometry when it is used in engineering courses and the connections concerning trigonometry in mathematics courses for engineers, more precisely the connectivity of online courses for engineers. Our purpose is not to discuss whether trigonometry should be introduced in mathematics courses or engineering courses, but to compare how it is introduced/used in these institutions. Hence the research questions we study here can be formulated as:

- Which connections concerning trigonometry appear in non-mathematical engineering courses?
- Which connectivity, concerning trigonometry, can be observed in online resources for mathematics courses for engineers, and how does this compare with the connections in non-mathematical courses?

In terms of methods, we have searched for curriculum resources and online courses on three major websites used in France: Unisciel¹, meaning “online science university”, gathering many online courses; IUTenligne², a website for technicians institutes within universities; and FUN³, meaning France Digital University, the national platform offering MOOCs. We have selected all the resources corresponding to mathematical courses for engineers or technicians on trigonometry, and have looked at the same time for resources on non-mathematical subjects using trigonometry. We have eventually chosen the theme of electrical engineering, because we have identified in it specific mathematical needs in trigonometry. Before analysing these resources, we now consider some works about trigonometry in mathematics education, and how they enlighten the connections issue.

CONTEXT AND RELATED WORKS

In France, trigonometry is firstly introduced in grade 8 or grade 9 through the definitions of \cos , \sin and \tan as quotients of lengths in a right-angled triangle. The angles are measured in degrees. In grade 10, the unit circle and the radian are introduced, together with a new frame for \cos and \sin , which are now the coordinates of a point on the unit circle. In grade 11, \cos and \sin are studied as functions. Hence connections between frames and registers are extensively present in this teaching, and these connections can raise difficulties for the students (Berté *et al.* 2004). For

¹ <http://ue1.unisciel.fr>

² <http://www.iutenligne.net/>

³ <https://www.fun-mooc.fr/>

example, grade 9 students already use for solving geometry exercises the “ \cos^{-1} ” key on their calculator; nevertheless, this key refers to a reciprocal trigonometric function, which is only presented at university, and moreover belongs to the functional frame (Bueno-Ravel & Gueudet 2010).

The international research on the teaching and learning of trigonometry acknowledge the existence of all these different registers and representations and investigate their consequences. Kendal and Stacey (1997) compare two teachings in grade 10 in Australia, using respectively ratios and the unit circle to introduce sin, cos and tan; they conclude that the ratio method appears as a better choice. Also at the university level, trigonometry remains a difficult subject for the students (Weber 2005). Mesa and Goldstein (2017), studying the presentation of trigonometry in college textbooks, have evidenced that these textbooks propose different conceptions of angles, trigonometric and inverse trigonometric functions; depending on these conceptions, some problems can be delicate to tackle for the students. The textbooks do not try to link different conceptions, and do not highlight which one is more relevant for a given problem.

Trigonometry clearly requires many connections between frames and registers. It is moreover linked with many different mathematical subjects (geometry, functions, but also complex numbers); and is extensively used in physics. Several researchers have also studied trigonometry within physics courses. Chiu (2016) studied the impact of a new curriculum in Taiwan, where contents of physics requiring trigonometry are taught before the corresponding mathematics courses. She observes that, while curriculum designers are positive on the possible consequences of a teaching of trigonometry by the physics and then by the mathematics teachers, the students and the teachers are mostly negative about this experience. A teaching of trigonometry in mathematics courses seems necessary before using it in physics.

In his comparative study between France and Vietnam, Nguyen Thi (2013) shows that in both countries, trigonometry is present in physics courses with mathematical models for periodical phenomena, under two forms: uniform circular motion (represented by a point moving on a circle, in a graphic or algebraic register) and harmonic oscillation (represented by functions in a graphic, algebraic or vectorial register). Nevertheless only a few exercises propose modelling activities (in physics as well as in mathematics); and the two models are almost never connected.

TRIGONOMETRY IN ELECTRICAL ENGINEERING COURSES

In this section we draw on the content of several courses for future engineers or technicians, available on French websites (e.g. Piou 2014). In electrical engineering, one of the major subjects is the study of the “alternating sinusoidal regime”. In circuits with such a regime, the different signals (current and voltage in particular) are of the form: $s(t) = A\sqrt{2}\sin(\omega t + \phi)$.

In a circuit where reactive loads are present (like capacitors or inductors) energy storage in the loads results in a phase difference between the voltage (u) and the current (i) waveforms. This difference is firstly introduced in the context of functions, with formulas like: $u(t)=U\sqrt{2}\sin(\omega t)$; $i(t)=I\sqrt{2}\sin(\omega t-\varphi)$, and associated with a graphical representation as two curves on the time axis (Figure 1, left part).

A connection is immediately established with a geometrical representation of these signals, through “Fresnel vectors”. A signal defined by $s(t) = A\sqrt{2}\sin(\omega t + \phi)$ (where A is positive) can be represented by a vector of length A , and a direction forming an angle ϕ with the horizontal direction. Hence $u(t)$ can be represented as a horizontal vector, and $i(t)$ as a vector forming an angle $(-\varphi)$ with it (figure 1, right part). Some courses also propose an interpretation in terms of complex numbers; for the sake of brevity, we do not present it here.

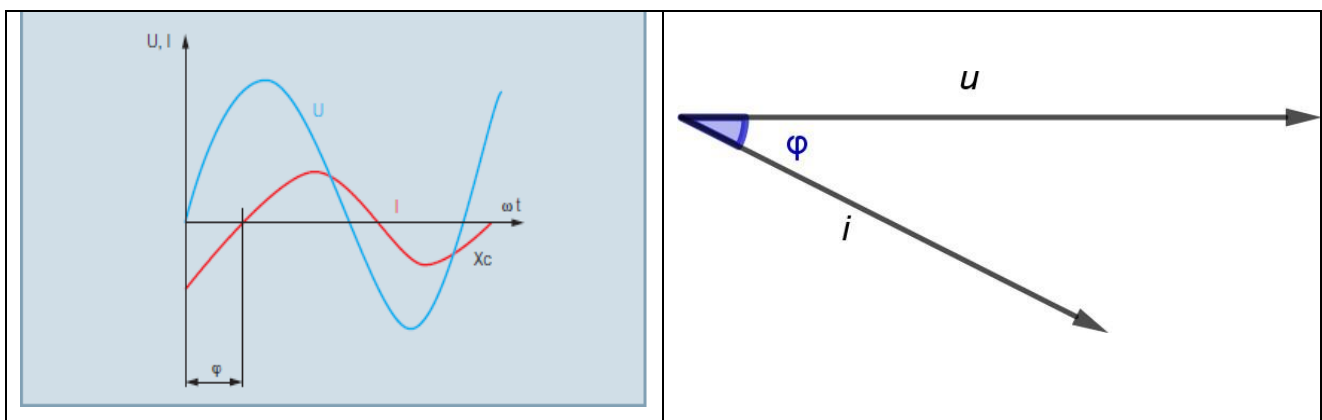


Figure 1. Signals in the alternating sinusoidal regime and phase difference. On the left: functional frame; on the right, Fresnel vectors in the vectorial frame.

We argue that the students in this case have to master connections between these two representations of signals: as two curves with a gap of φ on the horizontal axis; or as two vectors forming an angle φ . Some courses propose animated pictures or exercises to work explicitly on this connection (see figure 2 for an example extracted from a teacher’s website, <http://fisik.free.fr/>.)

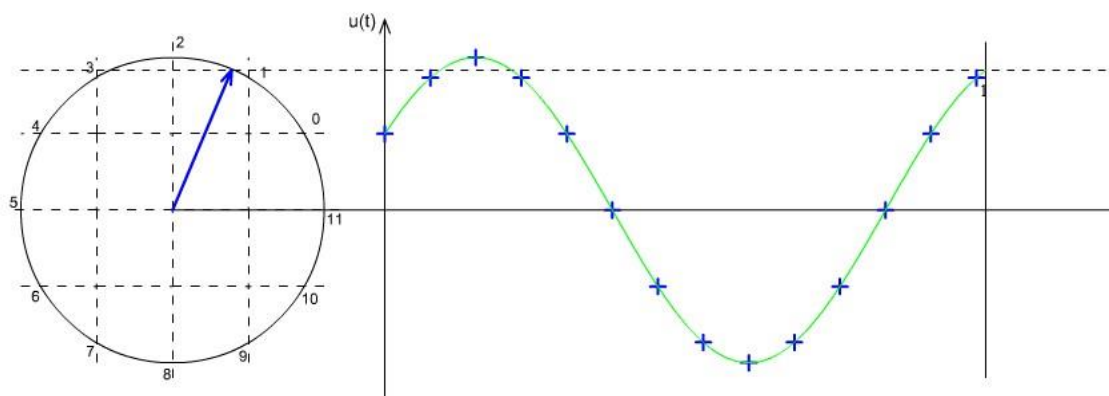


Figure 2. Connection between a rotating vector and the corresponding curve, extract of an animated picture. The blue points on the right appear when the vector rotates.

In terms of micro-level connectivity, electrical engineering courses naturally connect trigonometry with electrical engineering; they also connect different concepts: in the example we considered here, functions and vectors, and in other courses also complex numbers. Since vectors are represented as arrows, and functions represented by their graphs, these two kinds of representations are also connected in the text of the course.

TRIGONOMETRY IN MATHEMATICS ONLINE COURSES FOR ENGINEERS

A MOOC presenting the basics of mathematics for future engineers

The MOOC “Basics of mathematics”, available on the FUN platform (freely available after inscription) presents its objective on its first page: “This MOOC aims at revising the basic notions of mathematics, needed to start engineering studies”. The MOOC lasts 12 weeks, corresponding to 12 chapters. Chapter 2 is entitled “trigonometry”. It comprises 7 course videos (from 5 to 12 minutes), interactive multiple-choice questions, and a final assessment. Most of the notions presented in chapter 2 of the MOOC are taught in France at secondary school; nevertheless the reciprocal functions (like \sin^{-1}) are only presented during the first year of university.

The course proposes almost no connection with engineering activities. The first video says that “Trigonometry appears in many domains, like drawing plans, navigating or mechanics”; the last video mentions the task of “triangulation”, without definition. Three exercises are associated with each video. Most of these exercises are situated in the geometrical frame; some of them are in the frame of trigonometric equations. There is only one exercise in the whole chapter with a concrete context: computing the length of a cable joining the top of a pole to the ground, over a mountain with slope of 15%. The final evaluation is composed of two problems; the first one has a concrete context, measuring lengths, and the second one concerns trigonometric equations.

The definitions of \cos , \sin , \tan are introduced in the geometrical frame in the right triangle (video 1). In video 2, a software (Maple reader) is used to establish a connection between \cos , \sin and \tan in the unit circle and their graphs as functions (figure 3). The animated picture supports the discourse of the teacher explaining this link. This picture associates in fact the unit circle with the graphs of the functions, but also with the triangle. In fact the radius of the circle can be changed; it is not only the unit circle, but any circle; and the words “Adjacent” and “Hypotenuse” on the left of the screen link the circle and the other geometrical view as a triangle.

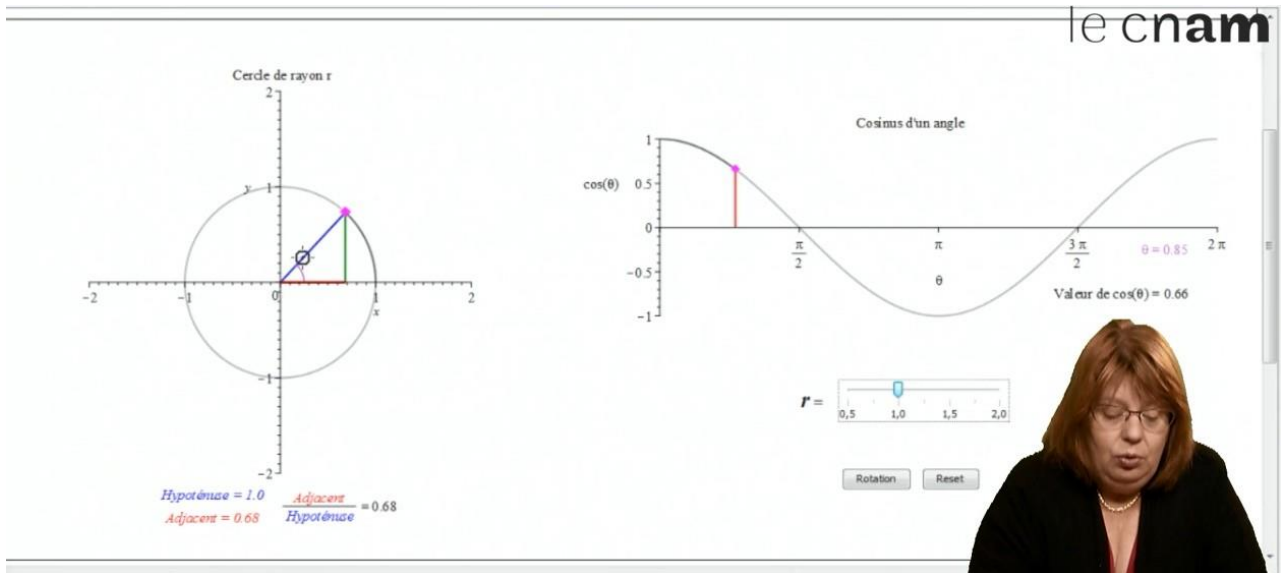


Figure 3. A dynamic representation making connections in the Mooc, case of cos.

This dynamic image is clearly used to connect the geometrical frames (triangle and unit circle) and the analytical frame; nevertheless, it might be very difficult to understand for participants who do not remember their school courses.

As a summary, we retain that micro-level connectivity in this MOOC comprises some connections between concepts and representations, including dynamic representations; but almost no connections with other domains or engineering contexts. Moreover the connections with dynamic representations can remain unclear for students because of a lack of explanations.

“Mathematical tools for physics”, an online course

“Mathematical tools for physics”⁴ is an online course addressed to first year students in physics, freely accessible (without inscription). It belongs to a complete first year course, which is always organised in three sections: “learning” (course) “practice” (exercises) and “self-assessment”. It comprises 11 chapters; chapter 6 is entitled “Circular trigonometry – Hyperbolic trigonometry”. This chapter comprises 8 subsections; here we are only interested in the 4 subsections concerning circular trigonometry.

After recalling the definition of an angle, the first subsection defines sin, cos and tan in the frame of the unit circle. Nevertheless, an animated picture proposes a link with the frame of the right-angled triangle. A subsection about formulas associates a functional frame: specific values of sin, cos and tan and register of the unit circle. Then two sections are dedicated to the properties of the “direct circular functions” and of the “reciprocal circular functions”, and only mention the functional frame. This course makes no link with physics or any real-life context.

⁴ http://uel.unisciel.fr/physique/outils_nancy/outils_nancy/co/outils_nancy.html

There is only one problem, divided in three questions, in the “practice” section. Interestingly, it is a problem of physics: “the Compton effect” (scattering of a photon by an electron). To solve this problem, the students must master contents of physics: the law of conservation of energy and quantity of movement (and the associated formula). The initial model is in the frame of vectors; these vectors are projected on the two axes, and the students have then to use trigonometric formula, and finally to work in the frame of functions. The difference between the wavelengths before and after the scattering is indeed of the form $\lambda(1-\cos\theta)$, where θ is the angle characterizing the direction of the photon after the scattering; the students must observe that this function is increasing over $[0, \pi]$: a larger angle corresponds to a larger change in the wavelength.

The “assessment” section comprises five parts: 2 on circular trigonometry, 2 on hyperbolic trigonometry and one entitled “composition of vibratory motions”. There is here again a connection between trigonometry and physics. Nevertheless its mathematical part remains entirely in the functions’ frame.

Finally, concerning the micro-level connectivity of this online course, we retain that it proposes some connections with physics in the exercise and assessment part (but no such connection in the course part). The connections between concepts and between registers are restricted to the case of a single problem.

CONCLUSION

Trigonometry is a domain of mathematics where many different concepts (angles, vectors in geometry; functions) and semiotic registers (triangles, arrows, circle in geometrical register; curves in a graphical register; equations etc.) can be connected. It is recognised as a difficult domain for students. Nevertheless, within mathematics some exercises are limited to a single register: the study of trigonometrical functions for example does not always require thinking in terms of angles. Using trigonometry in engineering courses, on the opposite, always requires such connections. The students must be able to associate an expression like $s(t) = A\sqrt{2}\sin(\omega t + \phi)$ both with a function and its graph; and a vector represented by an arrow. Engineering courses have a high degree of micro-level connectivity, for trigonometry. Our analyses of two online courses of trigonometry for engineers in France lead us to observe that they have a reduced level of micro-level connectivity: limited to connections between concepts and semiotic registers for the first one, while the second one on the opposite offers more connections between trigonometry and physics, but reduced connections between concepts and semiotic registers.

Our exploratory work leads to formulate recommendations for mathematics courses for future engineers, concerning trigonometry (and possibly other topics). Our study of engineering courses confirms that developing the ability of these students to make connections is an important aim. Connections between frames, between registers, but also connections with engineering are possible, as evidenced by Nguyen Thi (2013).

Online resources could propose such connections, and moreover could draw on the possibilities offered by dynamic representations and various kinds of software. Contributing to the development of such resources is an important aim for mathematics education research, to address the need for students to make a relevant use of the mathematics they learn at university in and for their engineering courses.

Acknowledgments

The authors warmly thank Alain Jameau and Damien Grenier for their contribution on electrical engineering.

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