Discussing Mathematical Learning and Mathematical Praxeologies from a Subject Scientific Perspective

Reinhard Hochmuth

Discussing Mathematical Learning and Mathematical Praxeologies from a Subject Scientific Perspective

Reinhard Hochmuth

Leibniz University Hanover, Germany, hochmuth@idmp.uni-hannover.de

This programmatic contribution discusses the link between concepts from Anthropological Theory of Didactics (ATD) and the "subject-scientific point of view" according to Holzkamp (1985, 1993). The main common concern of ATD and the subject-scientific approach is to conceptualize and analyse "objects" like "institutionalized mathematical knowledge" and "university" not as conditions that cause reactions but essentially as meanings in the sense of generalized societal reified action possibilities. The link of both approaches is illustrated by the issue of "real numbers" in the transition from school to university: Hypotheses are derived for further actual-empirical research, which intrinsically incorporate content- and subject related perspectives as well as societal and school-related findings.

Keywords: Curricular and institutional issues concerning the teaching of mathematics at university level, transition to and across university mathematics, subject scientific approach, mathematical praxeologies, real numbers.

INTRODUCTION

This paper contributes to an ongoing major research project that describes and analyses form and content of advanced mathematics and its teaching and learning from a subject scientific point of view. This approach is grounded in "Critical Psychology", framed by Holzkamp (1985) (see Tolman (1991) and Schraube & Højholt (2015) for English written introductions). Recently this theory becomes internationally more known in the mathematics education community due to Roth & Radford (2011), who assessed "German Critical Psychology" as a further development of the culture-historical activity approaches by Leontjev (1978) and Vygotsky (1978). It's beyond the scope of this paper to describe and analyse in which respects "German Critical Psychology" differs and goes beyond culture-historical activity theory. Instead the paper intends to point out its compatibility and (partial) complementarity with ATD as well as to illustrate its potential relevance for further research concerning university mathematics education.

Main features of "Critical Psychology" and its subject-scientific point of view are well elaborated psychological categories (roughly: basic notions, see for details (Holzkamp, 1985, pp. 28)) for describing and analysing cognitive and emotional-motivational dimensions of subject [individual] related experiences, in particular thoughts, actions and learning, in a way that major societal aspects are inherently be incorporated. It aims (besides others) to provide individuals with analytic tools for their self-reflection of problematic experiences and situations to reveal their inherent dependencies and circumstances, thus allowing individuals to achieve a more reflective learning. Within this framework, there is so far a lack of research that

relates to mathematical learning in general and in university in particular. "Critical Psychology" provides points of contact for incorporating research results concerning the societal and historical genesis of knowledge and reference structures as well as institutionally framed (e.g. school, university, study courses) external and internal transposition processes (Chevallard, 1991).

ATD's praxeological analyses could principally inform any psychological or socio-logical theory considering teaching and learning. Already in Castela (2015) a link between ATD and cultural historical activity theory is discussed, in particular between Roth's concept of crossing boundaries between different socio-cultural contexts and the issue of inter-institutional transitions. This basic idea is in the following taken up in a broader sense.

In view of the ongoing major research project, this contribution has the status of an intermediate step presenting programmatic ideas about linking ATD with the subject scientific approach. This combination might in particular be fruitful for research connecting detailed analyses of mathematical practices with a complex vision of learners and teachers in a way that (with respect to both sides) their intrinsic societal mediatedness is systematically incorporated. Though this rather pretentious goal can easily be formulated (at least by using abstract notions, which would require a lot of pages to be embedded in a coherent theory and to be explained in detail (Holzkamp, 1985, 1993)), the following lines also indicate that there is still a way to go combining both approaches in actual-empirical research (Holzkamp, 1985, pp. 509).

The structure of the paper is organized as follows: In the first two sections we intro-duce some notions from ATD and the subject scientific approach. Then we discuss the link of both approaches. Finally we illustrate the link and some of its aspects and opportunities considering the issue of "real numbers" in the transition from school to university: After an ATD-orientated overview about the nowadays typical treatment of real numbers in German secondary schools and considering various options of extending this discourse in the transition from school to university , we discuss subject scientific related aspects taking into account societal and school-related findings and how they might contribute to validate mathematical and didactical practices. This theoretical analysis exemplarily demonstrates how the link intrinsically integrates content- and subject-related perspectives and leads to hypotheses for actual-empirical research projects.

SOME NOTIONS FROM ATD

ATD (Chevallard, 1992; Winslow, Barquero, Vleeschouwer & Hardy 2014) aims at a precise description of knowledge and its epistemic constitution. Its concepts make possible to explicate institutional specificities of knowledge and related practices. An underlying conviction of this approach is that cognitive-oriented accesses tend to misinterpret contextual or institutional aspects of practices as personal dispositions. A basic concept of ATD are praxeologies represented in so called "4T-models (T,τ,θ,Θ)" consisting of a practical and a theoretical or logos block. The practical

block (know-how, "doing math") includes the type of task (T) and the relevant solving techniques (τ). The logos block (knowledge block, discourse necessary for interpreting and justifying the practical block, "spoken surround") covers the technology (θ) explaining and justifying the used technique and the theory (Θ) justifying the underlying technology. Praxeologies give descriptions of mathematics by reference models that are activity oriented (techniques, technologies). The interconnectedness of knowledge is modelled in ATD by means of local and regional mathematical organizations that allow contrasting and integrating practical and epistemological aspects in relation to different institutional contexts. Therefore ATD is in particular helpful for analysing institutional realizations of mathematical knowledge within different learning contexts, e.g. the use of mathematics in signal theory (Hochmuth & Schreiber, 2015).

More than 15 years ago Chevallard has introduced the additional notion of "scale of levels of codeterminations" that in the meanwhile has become rather important in ATD analyses (Bosch & Gascón, 2006). The hierarchical sequence of levels covers civilisation, society, school, pedagogy, discipline, domain, sector, theme and subject (in the sense of topic). Each level provides some kind of framework, within among others actions on lower levels are possible, supported or hindered and praxeologies are in a certain sense embedded. In Barbé et al. (2005) is shown, for example, how general didactic restrictions for teaching mathematical topics in school can affect teachers' practices and their established praxeologies, in particular the shaping of the practical and the logos block and the relations between them. To mention one further example, Job & Schneider (2014) argue that smoothing the transition-gap from calculus to analysis shows at least the tendency to blur the distinction between the different discourses in school and university, which tends to reinforce an empirical positivist attitude by students as an epistemological obstacle to learning (ditto; p. 641). Generalizing their arguments, one might say that there are issues relating to general world views (society) that affect institutionally settled praxeologies.

Moreover, Chevallard (1991) introduced the notion $R_I(x, o)$ indicating the relation of a position x (a typical position of an individual) within an institution I to a praxeology o. The "scale of levels of codeterminations" underlines that the institution and, with the institution, the position x and the praxeology o have to be considered as being societal situated, i.e., that in their analyses specific emphasize also has to be put to societal assignments that are related to societal mediation processes. The subject scientific perspective, which is introduced and discussed in the next both sections, allows to further specify positions x keeping the significance of societal and institutional mediatedness (in a materialistic sense, see for example (Arndt, 2013)).

SOME NOTIONS CONCERNING THE SUBJECT SCIENTIFIC APPROACH ("CRITICAL PSYCHOLOGY")

"Critical Psychology" claims to present a scientific discussable and criticisable elaboration of basic psychological concepts (categories). The starting point is a historical-

empirical investigation of general historical-specific characteristics of relations between societal and individual reproduction as well as its dialectic mediatedness. One of the central subject related categories is "action potence", which is the potence to ensure the disposal about the subject's individual living conditions together with others (Holzkamp, 1985, pp. 239).

Within the context of this paper there are three important points to notice: First, the actual historical-specific form of subjectivity is characterized by the "possibility relation" regarding the societal reality, which includes in particular the basic experience of intentionality. Second and connected to the first, the specific modality of subjective experiences comprises the discourse form "reasoning discourse": "I" speak about my "own" actions in terms of subjective reasonable (not necessarily "rational") activities and of premises in the light of "my" life interests. A third crucial point is that the "human's relationship to the environment is almost always mediated. […] Categories of psychology like learning, emotion, motivation and cognition cannot fail to be significantly altered by the fact of our existence's social mediatedness. The most important mediation category is meaning." (Tolman, 1991, pp. 14-15)

These three interrelated issues are combined in the assertion that conditions are given to "me" in terms of meanings in the sense of generalized societal action possibilities and that reality aspects, which are relevant for "me", denoting again the generalized subject standpoint, become premises for "me" in the light of "my" life interests. Therefore, subject scientific considerations are essentially given by meaning-premises-reasons-relations, which a priori situate experience and activities of the (individual) subject "within the world" Accordingly, Holzkamp (1985, pp. 342) figured out the level of subjective action reasoning as the main subject specific level: It represents the level with respect to which individual experiences and activities (e.g. learning) has to be reconstructed and analysed.

Via the specific notion of meaning, human activities, like teaching and learning, are intrinsically thought as societal mediated. This implies that any analysis of subjective actions requires the reconstruction of subjectively relevant conditions in the sense of generalized action possibilities and the consideration of their societal mediatedness. Since meanings appear (via objective-subjective premises) as the medium within which subjects' reasoning discourses are grounded, their study is a prerequisite for describing and analysing related cognitive, motivational and emotional processes as aspects of subjects' activities. But, although meanings in the indicated sense are rather relevant for acting and thinking, they do not determine them. Instead, they represent action possibilities that might become premises in the light of subjectively perceived "life interests".

THE LINK BETWEEN ATD AND THE SUBJECT SCIENTIFIC APPROACH

The position x within an institution can be (re-)interpreted as the "position" and/or "situation of life" from the subject point of view, which includes intentionality, the modus of reasoning discourse and societal mediatedness. As an "element" (a

"position x") of an institution a subject is typically not confronted with the whole world but only with a local "situated" section represented by those meanings typically produced and reproduced within the institutional context. Hence, institutional contexts provide specific frameworks for premise-reasoning-patterns. In this sense ATD's praxeological analyses contribute to concretisations of meaning-premise-reasoning-patterns that are typical within an institution at the position x. In particular the concept of praxeologies allows capturing substantial aspects of mathematical practices in such a way, that they can be injected as facets of action related meanings, i.e., they can be (re-)interpreted as generalized societal action possibilities, which were potentially reflected in subject related reasoning schemes as premises and/or reasons. In this sense praxeological analyses can be seen as one non-trivial first step within subject scientific research projects: They might inform microanalyses of task solution processes by exploring institutional established practices. They are relevant for describing and analysing related activities, since they appear as institutionalized medium, within subjective action reasoning grounds. With respect to premises-reasoning-patterns the technological dimension of praxeologies, i.e. the justification and validation of techniques, is of specific importance. But, see above, praxeologies do not determine subjects' activities, since there is an unconscious-conscious step by subjects of selecting, neglecting or highlighting facets of praxeologies in view of their evaluation of "life interests" and how they are perceived by them at "position x" in the "institution I" in view of all prospects addressed by "the scale of level of codeterminations". Thus, the latter is rather relevant for both, the analysis of meanings (essentially by ATD) and the analyses of premises-reasons-relations (essentially by the subject scientific approach). In fact, both strands can't be seen as totally separated but as dialectically interrelated, since institutionalized practices live through subjects' [individual] activities.

THE ISSUE OF "REAL NUMBERS"

In this section we give first a short overview of the nowadays typical treatment of real numbers in German secondary schools in grade eight or nine. Because of the space limitation a detailed praxeological analysis can't be presented.

The Treatment of Real Numbers in German Schools

The treatment of real numbers in German schools presumes that rational numbers are known and can be represented by ratios, decimal fractions and points on the number line. Moreover it is presumed that students are able to switch between those representations. In particular basic calculations should be understood and can be executed with respect to the different representations. The typical starting point for the introduction of real numbers is the observation or proof (sometimes!) that there are quadratic equations like $x^2 = 2$ without rational solutions. Next it is observed (but typically not proven) that one can find approximations by proper decimal fractions that fulfil those equations up to an arbitrary chosen error. On the other hand it is (geometrically) argued that there is a magnitude x, the length of the square

diagonal, satisfying $x^2 = 2$ that corresponds to a certain point on the number line. The intuitive conviction about the existence of those points on the number line supports the idea, that (somehow converging) infinite sequences of approximating proper decimal fractions give a (unique!) final finite result, a number, that can be represented by a non-terminating decimal fraction. This type of discourse justifying the existence of infinite-finite objects (i.e., infinite processes giving in a certain sense a finite result) has in particular been considered in Lakoff & Nūñes (2002) as basic for the whole analysis and denoted as "basic metaphor of infinity". The new objects of non-terminating and non-periodic decimal fractions are called irrational numbers and build, together with the already known rational numbers, the set of real numbers. Moreover, the calculation rules that are known for rational numbers are assumed to be also true for all real numbers. Whereas in former years one can at least find Descartes' geometrical arguments for explaining multiplication and division for general real numbers such arguments are nowadays missing. Corresponding to the sketched treatment of real numbers there are nearly no tasks that are related to structural aspects of real numbers or that enforce to reflect arguments of the discourse concerning limits or the existence of points or numbers respectively. Instead the tasks focus on various isolated techniques that are locally established, for example approximation techniques like interval bisections and the Heron algorithm or the use of calculation rules.

In terms of the 4T-model the established mathematical praxeologies can be characterized as essentially punctual (or at most local) with isolated types of tasks and corresponding isolated techniques, where the tasks can be solved without referring to superordinated technological aspects, i.e., there are praxeologies $(T_i, \tau_i, \theta_i^{weak},)$ with technologies θ_i^{weak} having in particular weak connections to T_j, τ_j for $j \neq i$.The technological and theoretical discourse remains (so far it is represented at all) mostly implicit and essentially incomplete. These observations blend with those presented by González's et al. (2013) institutional analysis and with results from a qualitative study by Bauer, Rolka & Törner (2005). For corresponding results considering prospective secondary mathematics teachers we refer to (Sirotic & Zazkis, 2007) and for an actual rather detailed study, which problematizes in particular the use of the number line and investigates the knowledge of fresh French university students see (Durand-Guerrier, 2016).

Real Numbers in University Education and Potential Foci for Transitions

First of all it is interesting to notice that mostly the treatment of real numbers in university is either done axiomatically (courses for math majors) or (more or less) skipped (courses for engineers and natural scientists), but does generally not intend to connect or complete school praxeologies, i.e. for example: showing the one-to-one correspondence of number line and decimal fraction views (Kirsch, 1966); discussing geometrically the completeness of the number line (Artmann, 1983); showing how to add and multiply non-terminating and non-periodic decimal fractions.

For improving this situation one might think of transition measures that are adjusted to study courses for, e.g., math majors, prospective grammar school teachers or engineering students. A general scheme suitable for describing and analysing desirable transitions is given as follows (Hochmuth, 2018):

$$R_S(s,o) \rightarrow R_U(\sigma,\omega[X(o)])\ with\ X \in \wp\{\tau,\theta,\Theta\},$$

where o represents a praxeology and *s* a student within the institution *S* (school), ω a praxeology and σ a student within the institution *U* (university) in relation to one or several blocks of the praxeology $o = [T,\tau,\theta,\Theta]$ and $\wp$ is the power set symbol. The scheme works as a heuristic tool and allows to express that techniques, technologies or theories of o might be differently relevant for the relation of S within U to a (perhaps new) praxeology ω (see (Biehler & Hochmuth, 2017) for a slightly more restricted scheme).

Applying the scheme we illustrate next various transition foci: At first the focus might be on techniques τ_i and technologies θ_i^{weak} of o such that related skills are improved, but tasks, techniques and technologies are only slightly extended, for example: ordering of square roots and decimal fractions; applying calculation rules for simplifying terms. This might be important for all above mentioned study courses. Secondly the focus might be on technologies θ_i^{weak} and their further development (possibly also their theoretical embedding), for example: knowing, that square roots like $\sqrt{2}$ or $\sqrt{3}$ can only be approximated by finite decimal fractions; justifying real exponentials and powers (Winsløw & Grønbæk, 2014). This might in particular be important for prospective grammar school teachers. Thirdly the focus might be on techniques τ_i and the replacement of technologies θ_i^{weak} by technologies θ_i that are strongly and systematically embedded in real analysis, for example: constructing the set of real numbers by Dedekind cuts or Cauchy sequences; starting with axioms for $\mathbb{R}$ and identifying natural and rational numbers within this new set of objects. This might in particular be important for math majors. Within the ATD-framework each case could be analysed, explored and specified in greater details and in view of the scale of level of codeterminations. We skip further details in this paper.

Links to the Subject Scientific Approach

From the subject scientific point of view the analysed praxeologies represent meanings in the sense of institutionalized action possibilities. Students' reasoning and activities ground in those praxeologies but select, neglect or highlight them in view of an evaluation of their "life interests" and how they are perceived in view of the institutions *S* and *U*, corresponding to positions s and σ, and in particular aspects related to levels of codetermination. It is well-known that in the transition a lot of issues play a role, see e.g. Gueudet (2008). In the following we will discuss two different but complementing issues that are specifically linked to "Critical Psychology".

The first issue refers to the level "society" and in particular to Dowling's sociological analysis of myths, exemplarily the myth of "reference" (Dowling, 2002). Within our frameworks Dowling's myths can be (re-)interpreted as technological aspects of mathematical praxeologies in school that are related to the societal significance of educational processes. With respect to the latter Dowling differentiates between assignments concerning general societal aspects and those concerning their historic specificity. Considering the historic specificity the exchange-value aspect comes into play, which somehow undermines the use-value aspect and establishes a problematic mixture of both. This fits to the observation that actual school introductions of real numbers typically refer to "real world" problems like doubling the area of a quadratic piece of chocolate. Such references dominate the justification of introducing real numbers although treating the "real world" problems does not require "exact" solutions and, moreover, algebraic extensions would be sufficient to resolve this issue. Following Dowling, the myth of "reference" is not only a didactical issue that relates to illustrative introductions, but possibly leads to problematic technological and theoretical ideas, which do not disappear by establishing new, for example, axiomatic praxeologies, instead they possibly survive and constitute a strong epistemological (and motivational) obstacle (similar to the epistemological obstacle considered in (Job et al., 2014)) for students' learning of university mathematics, in particular for future grammar school teachers. This could in particular happen, since Dowling's myths might dominate students' general view of their "situation of life" and therefore their accentuating of meanings.

The second issue relates to the organization of learning in school and to the "school-and-exam system" mentioned for example in Chevallard (2013). Partly because of this issue Holzkamp (1993) introduced the notion of defensive learning, a learning which primarily intends to prevent negative consequences. An important aspect of this notion is the opposition between ostensive and conceptual thinking that represents, according to (Holzkamp, 1985), the historic-specific societal concretization of the cognitive aspect of human activities: Ostensive thinking is essentially characterized by taking things as they appear to be and, in terms of the 4T-model, by strongly focusing on locally situated technical and technological issues, which blend with the above described praxeological organisation of "real numbers" in school.

Again, corresponding "ostensive" students' views on their "situation of life" and related meanings-premises-reasoning-patterns might let transition measures' intentions fail and in particular the incorporation of technological school-blocks within technological university-blocks, which results in new isolated praxelogies with new but still weak technologies. It is an empirical open question how this tendency is amplified by actual initiatives aiming to reduce transition problems by establishing a "university-and-exam system".

FINAL REMARK

The hypotheses derived in the last section illustrate the necessity that an analysis of measures supporting students in the transition from school to university have

systematically to take into account both approaches, the praxeological and the subject scientific as well as, with respect to both, the scale of level of codeterminations. The established link between ATD and the subject scientific approach facilitates theoretical and actual-empirical studies factoring in systematically aspects, which are intrinsically connected to the institutional and societal level and have impact both on institutionalized praxelogies and subjects' meaning-premises-reasoning-patterns.

REFERENCES

Arndt, A. (2013): *Unmittelbarkeit.* Owl of Minerva Press.

Artmann, B. (1983). *Der Zahlbegriff.* Göttingen: Vandenhock & Ruprecht.

Barbé, J., Bosch, M., Espinoza, L. & Gascón J. (2005). Didactic Restrictions on the Teacher's Practice: The Case of Limits of Functions in Spanish High Schools. *Educational Studies in Mathematics 59* (1-3), 235-268.

Bauer, L., Rolka, K., & Törner, G. (2005). Mentale Repräsentationen von Irrationalzahlen – eine Analyse von Schülerinnenaufsätzen. *JMD, 26(1),* 3-27.

Biehler, R., & Hochmuth, R. (2017). Relating different mathematical praxeologies as a challenge for designing mathematical content for bridging courses. In: *Didactics of Mathematics in Higher Education as a Scientific Discipline. khdm-Report 17-05* (pp. 14-20). Kassel: Universität Kassel.

Bosch, M. & Gascón, J. (2006). Twenty-Five Years of the Didactic Transposition. *ICMI Bulletin No. 58,* 51-65.

Castela, C. (2015). When Praxeologies Move from an Institution to Another One: The Transpositive Effects. In: *23rd annual meeting of the Southern African association for research in mathematics, science and technology*, SAARMSTE, 6-19.

Chevallard, Y. (1991). *La transposition didactique. Du savoir savant au savoir enseigné, 2nd edition*. Grenoble: La Pensée Sauvage Éditions.

Chevallard, Y. (1992). Fundamental Concepts in Didactics: Perspectives Provided by an Anthropological Approach. *Recherches en didactique des mathématiques, Selected Papers. La Pensée Sauvage, Grenoble,* 131–167.

Chevallard, Y. (2013). Teaching mathematics in tomorrow's society: A case for an oncoming counter paradigm. Regular lecture at ICME-12 (Seoul, 8–15 July 2012).

Dowling, P. (2002). *The sociology of mathematics education: Mathematical myth/pedagogical texts.* London: Routledge.

Durand-Guerrier, V. (2016). Conceptualization of the Continuum, an Educational Challenge for Undergraduate Students. *International Journal of Research in Undergraduate Mathematics Education*, 2(3), 338-361.

Gueudet, G. (2008). Investigating the Secondary–Tertiary Transition. *Educational Studies in Mathematics, 67*(3), 237-254.

González-Martin, A.S., Giraldo, V. & Souto, A.M. (2013). The introduction of real numbers in secondary education – an institutional analysis of textbooks. *Research in Mathematics Education 13(3),* 230-248.

Hochmuth, R. (2018). *A General Scheme for a Heterogeneous Manifold of Transitions*. Poster, accepted for citad6 (6th International Congress of Anthropological Theory of Didactics), Grenoble (22nd -26th Jan. 2018).

Hochmuth, R., & Schreiber, S. (2015). Überlegungen zur Konzeptualisierung mathematischer Kompetenzen im fortgeschrittenen Ingenieurwissenschaftsstudium am Beispiel der Signaltheorie. In: *Lehren und Lernen von Mathematik in der Studieneingangsphase*. Wiesbaden: Springer, 549-566.

Holzkamp, K. (1985). *Grundlegung der Psychologie*. Frankfurt/M.: Campus.

Holzkamp, K. (1993). *Lernen: Subjektwissenschaftliche Grundlegung*. Frankfurt/M.: Campus.

Job, P., & Schneider, M. (2014*).* Empirical positivism, an epistemological obstacle in the learning of calculus. *ZDM, 46(4),* 635–646.

Kirsch, A. (1966). Zur Behandlung der reellen Zahlen im Oberstufenunterricht. *Der Mathematikunterricht im Gymnasium*, 215-227.

Lakoff, G., & Núñez, R. E. (2000). *Where Mathematics comes from: How the embodied mind brings mathematics into being*. Basic books.

Leontjev, A.N. (1978). *Activitiy, consciousness and personality (M.J. Hall Trans.).* Englewood Cliffs, NJ: Prentice Hall.

Roth, W.-M., & Radford, L. (2011). *A cultural-historical perspective on mathematics teaching and learning.* Rotterdam: Sense.

Schraube, E., & Højholt, C. (Eds.). (2015). *Psychology and the conduct of everyday life*. Routledge.

Sirotic, N., & Zazkis, A. (2007). Irrational numbers: The gap between formal and intuitive knowledge. *Educational Studies in Mathematics, 65(1),* 49-76.

Tolman, C. W. (1991). Critical Psychology: An Overview. In C. W. Tolman & W. Maiers (Eds.), *Critical Psychology: Contributions to an historical science of the subject* (pp. 1-22). Cambridge etc.: Cambridge Univ. Press.

Vygotsky, L.S. (1978). *Mind in Society: The development of Higher Psychological Processes.* Cambridge, M.A.: Harvard University Press.

Winsløw, C., Barquero, B., Vleeschouwer, M. de, & Hardy, N. (2014). An institutional approach to university mathematics education: from dual vector spaces to questioning the world. *Research in Mathematics Education, 16*(2), 95–111.

Winsløw, C., & Grønbæk, N. (2014). Klein's double discontinuity revisited: contemporary challenges for universities preparing teachers to teach calculus. *Recherches en didactique des mathématiques*, *34*(1), 59-86.