



HAL
open science

University Teachers-Researchers' Practices: the Case of Teaching Discrete Mathematics

Theresia Tabchi

► **To cite this version:**

Theresia Tabchi. University Teachers-Researchers' Practices: the Case of Teaching Discrete Mathematics. INDRUM 2018, INDRUM Network, University of Agder, Apr 2018, Kristiansand, Norway. hal-01849936

HAL Id: hal-01849936

<https://hal.science/hal-01849936>

Submitted on 26 Jul 2018

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

University Teachers-Researchers' Practices: the Case of Teaching Discrete Mathematics

Theresia Tabchi

University of Reims Champagne-Ardenne, CEREP EA 4692, Reims, France,
theresia.tabchi@etudiant.univ-reims.fr

Our work is part of a thesis which deals with the teaching practices of university mathematics professors. Practices of university professors concur specificities: an example is the articulation between teaching and research. We seek to characterize the research impact in the teaching practices at university. We choose discrete mathematics as an object of study. It is an area whose didactic transposition is not fully achieved while the links with other mathematical fields remain undefined. These elements make the choice of resources to be mobilized and conceived quite complex. Our exploratory study draws on interviews with university professors aiming to characterize the interaction with resources; which can help us clarify the research activities impact on teaching practices in the case of discrete mathematics.

Keywords: Teachers' practices at university level, discrete mathematics, undergraduate education, impact of research, resources.

INTRODUCTION

Our work is part of an ongoing thesis which deals with the practices of university mathematics professors. The study belongs to the growing body of research on university professors' practices (Biza, Giraldo, Hochmuth, Khakbaz, & Rasmussen, 2016). At this level of our study, we designate by "University professors" the university mathematics teachers who occupy teaching/research positions.

While we are interested in teaching practices at university, we try to characterize the different factors that impact them (institutional, didactic and epistemological), in particular, the impact of the research activity on the teaching practices. Throughout the text, we will be relying on the following definitions:

- "Teaching practices" and "university teachers' practices" to refer to the "teaching" aspect of the university professors' work;
- "Research activity" to refer to the research aspect of the profession.

Our study focuses on the place of discrete mathematics in undergraduate level. Discrete mathematics holds an epistemological importance, and it is a branch whose topics are not stabilized in mathematics curricula (Heinze, Anderson, & Reiss, 2004; Hart & Sandefur, in press). Hence, it is particularly interesting to study university mathematics professors' practices in the field of discrete mathematics, a choice that we will justify later in the document.

In a study of ICME [1], Biza et al. (2016) conducted a survey of existing research on mathematics education at university. Drawing on journal publications and conference proceedings on university mathematics education published since 2014, Biza et al.

identified the current trends and the most recent advances in the field. They classified the existing work according to the topic of research. Among these, the influence of teachers' research activity on their teaching approaches in particular contexts: the use of examples in mathematics tutorials and the use of graphic representations. The study highlights the emerging interest in taking into consideration resources in teaching at university, in particular, interactions with resources for mathematics education and their impact on teachers' professional development. The study points as well the need for further research on practices of university mathematics teachers and the possible impact of their research activity on their teaching practices.

The work that we present in this text is a contribution to the growing body of research in mathematics education on the practices of mathematics teachers at university level. We will present first the context of the study and the research questions. Then, we will describe the first steps in the construction of our theoretical framework and the collection of data. Finally, we will present the methodology we used in our study as well as some extracts, along with some preliminary results and perspectives.

CONTEXT AND RESEARCH QUESTIONS

The study of the practices of university professors is part of a field of research currently in development, but still largely unexplored. There are similarities in many aspects of the practices of secondary and university teachers: the preparation of the courses and tutorials, the design of instruction, the conception of evaluations, the classroom management, and the interactions with students. However, the practices of university professors conquer specificities, from an institutional point of view and given their training and their academic background.

University professors have some freedom in the design of their courses and the choice of contents to be taught, and in the selection and development of resources. Moreover, their research activity can impact (or not) different aspects of their teaching practices (Biza et al, 2016; Mali, 2015) such as the instructional approach, the form of interaction with students, and the choice of new knowledge to be taught.

University professors, in their teaching practices, maintain a constant interaction with various resources (Gueudet, 2017) derived from their teaching practices as well as their research activity (old resources, software, online resources, videos and podcasts, manuals, research articles, etc.). The interaction with the resources can take place at different moments of their teaching practices: in the design of the sessions, the choice of contents to be taught, the teaching in the classroom, and the evaluation of learning. In the working environment of a university professor, the same resources can be used in his teaching practices and his research activity (numerical computation software for example). Other interactions between teaching and research can take place, but in a less tangible way. The study of the university professors' interactions with the resources could be a first step in clarifying the nature of the relationship between their teaching practices and their activity of research: in the choice of contents to be

taught, in the teaching mode of different mathematical contents, and in the learning they wish to develop with students (reasoning, application of properties, etc.).

In mathematics education, teaching practices are not considered independently of the contents taught. The relationship between university teachers' practices and their research activity may depend on several parameters, including the mathematical branches addressed in teaching and research; discrete mathematics, in particular, offer an interesting experience. Discrete mathematics have a real epistemological importance as they unfold in different mathematical domains; the objects are easy to access and can contribute to the understanding of other branches of mathematics (Grenier & Payan, 1998, Maurer, 1997). They provide an introduction to modeling, optimization, operational research, and experimental mathematics (Grenier & Payan, 1998, Maurer, 1997). They promote the learning of proof and the development of heuristic processes in students (DeBellis & Rosenstein, 2004, Goldin, 2004).

Discrete mathematics is a field in expansion, with significant achievements in society. Attempts to integrate discrete objects in the curricula appear in several countries at the international level. In France, the place of discrete mathematics in the curriculum is still not sufficiently stabilized and takes various forms depending on the educational context (Ouvrier-Buffer, 2014); which makes this branch very interesting to study the relationship between research activity and teaching practices of university professors. Hence the research questions can be formulated as follows:

- How do university professors interact with the resources in / for the teaching of discrete mathematics?
- How can we characterize the relationship between the teaching practices of discrete mathematics at the tertiary level and research activities in the same field?

Trying to answer these questions requires specific theoretical and methodological developments.

TOWARDS THE CONSTRUCTION OF A THEORETICAL FRAMEWORK

We conducted a first theoretical choice: *the documental approach* (Gueudet & Trouche, 2009). We will justify the choice in what follows. We note that the choice of the documental approach constitutes a tool for a first exploration of the field.

Gueudet (2017) based her work on the documental approach (Gueudet & Trouche, 2009) to analyze the interactions of university professors with *resources* derived from their teaching practices. In the documental approach, there is a distinction between *resources* and *documents*. University professors select, mobilize and use various resources (contents to be taught in class, written evaluations, old resources, etc.). This interaction with the resources generates a document, which is the association of resources and a *scheme of use* of these resources. A scheme is used here as it was defined by Vergnaud (2009) as the invariant organization of conduct

for a set of situations having the same aim. According to Vergnaud (1998), a scheme is a dynamic that has four interacting components:

- An *aim* that can be easily identified and that indicates intentionality in the organization of the activity;
- *Rules of actions* which are the ways of acting generated by the scheme in order to achieve a specific aim;
- *Operational invariants* that influence the rules of action. They can be *theorems-in-action* (propositions considered as true by the subject, but may be true or false) or *concepts-in-action* (considered as relevant in a given situation);
- Possibilities of *inferences*, that are the adaptations that the subject can bring to his activity in order to respond to the specificities of a situation corresponding to an aim.

A scheme developed by a subject is associated with a class of situations (Vergnaud, 2009). A class of situations includes all situations having the same aim. In her work, Gueudet (2017) considered classes of situations for specific aims (for example, preparing an assessment in linear algebra) and thus corresponding to a single document, and larger classes of situations that are independent of the mathematical content (for example, preparing an assessment). She made these choices in order to observe the organization of the resources of university mathematics professors globally, and to analyze the operational invariants and the rules of actions associated to mathematical contents in more restricted classes of situations. Although Gueudet (2017) recognizes the impact that university professors' research activity can have on their teaching practices, she did not focus on this aspect in her study.

We rely on the methodology developed by Gueudet (2017). We seek to analyze the choices, of contents and resources, of university professors in their teaching of discrete mathematics. Furthermore, we are interested in the impact of the research activity on the teaching practices. Therefore, we will adapt and develop Gueudet's methodology (2017) in a way to be able to consider the particular field of discrete mathematics and to take into consideration the research activity's impact on teaching.

ELABORATION OF INTERVIEW GUIDELINES AND DATA COLLECTION

The methodology of our exploratory study is based on interviews. We consider university professors whose field of research is discrete mathematics in Lebanon and in France. The interviews, of approximately 90 minutes, took place in workplaces of the Lebanese professors in order to have access to their resources. For French professors, the interviews were conducted via Skype.

We have developed the interview guidelines in order to characterize the interactions of university professors with various resources, resulting from their teaching practice and activity of research. We asked questions such as "What are the main resources you use for your teaching? How do you select the contents of your courses?" these

questions intended to give a panoramic view of the resources selected, mobilized and used in the different classes of situations (preparation of lectures and tutorials, evaluation); We believe that the type and nature of the resources (resources resulting from research or teaching) can inform us about the relations between the research activity and the teaching practices of university professors; furthermore, the choice of contents (lectures, tutorials and evaluations) can give us insight about the different rules of action and the operational invariants corresponding to the different classes of situations. Another question was “What are the conditions and constraints that guide your different choices?” Learning about the institutional conditions and constraints (related to the educational institution or to the research institution) can contribute in characterizing the research activity’s impact on teaching practices. Other questions were about the links with other mathematical fields, the collective work, the experimentation in classrooms, etc.

We conducted pilot interviews with two discrete mathematics university professors, one in Lebanon and the other in France, to test the interview guidelines. We recorded and transcribed the interviews. To keep the anonymity of the professors interviewed, we will call them Michel (Lebanon) and Bertrand (France) throughout the document.

THE METHODOLOGY OF ANALYSIS

Our methodology of analysis consists of two steps which we will be presenting next.

The first step is analyzing the practices of university professors in terms of their interactions with resources. For that, we detected the aims, mentioned by the university professors in the interviews, related to their teaching practices. In other words, we identified possible classes of situations based on statements, such as “preparing a lecture” or “preparing an assessment”. For each aim, we identified the associated resources (manuals, books, scientific articles, computers, online resources etc.) that were explicitly mentioned in the interview. For the rules of action, we relied on the university professors’ declarations; they described the ways they behaved in order to achieve an aim, and the adaptations they brought to their actions in accordance with the peculiarities of each situation. These are the conditions and actions that can be expressed in statements of the form “if (condition) ... then (rules of action) ...” or “for (aim) ... I (action) ...”. The regular ways of acting in specific situations reveal the presence of operational invariants that university professors do not always express in their discourse. To detect them, we identified in the interviews, the statements justifying the rules of actions, the propositions held to be true by the university professors (theorems-in-action) and the logical reasoning underlying the choice of actions to be conducted according to the specificities of each situation.

We constructed a table for the documents of each university professor; horizontally, in each table, we come across a document associated with a given class of situations

(the resources used and the scheme of use). Vertically, we find the sets of the resources used, the rules of action and the operational invariants.

The second step consists in studying the impact of the research activity on the teaching practices in classes of situations related to teaching. For each class of situations, first we choose to detect, in the set of resources used, the presence / absence of an impact of the research activity (for example, the choice of contents to be taught in class might be influenced by the research activity of the university professors or not). Then, we will define a typology of operational invariants; it would allow us to classify the reasons behind some rules of actions of the university professors and behind the “transfer” (use, selection and modification) of some resources from their research activity to their teaching: beliefs developed by the activity of research, gestures acquired with professional experience or institutional constraints and conditions. We will eventually search for additional tools for a deeper study of the impact of research activity on teaching practices, following the results of the pilot interviews.

ANALYSIS OF THE FIRST PILOT INTERVIEW

Our first interview was with Michel (Lebanon), whose research domain is graph theory. Michel teaches graph theory for students in masters’ degree and discrete mathematics for students majoring in mathematics in year 2. We detected, in Michel's declarations, the resources produced and / or used in his teaching practice.

Aims	Resources	Rules of actions	Operational invariants
Prepare a course of discrete mathematics in second year	well-known books in the world Scientific publications Typical texts	He tries to convey the basic ideas in discrete mathematics. He uses many examples. He chooses contents that allow students to discover ideas.	The contents must be aligned with the official instructions of the major. Definitions in discrete mathematics are simple, there is a need for applications to initiate the work. Understanding the ideas is very important in discrete mathematics.
Implementing a discrete mathematics course in class	well-known books in the world Typical texts	He engages students in writing. He encourages students to read typical texts. Before writing the proof of a theorem,	It is important that students practice writing. It takes examples and applications to familiarize students with discrete mathematics. It is very important to illustrate

		he illustrates the ideas.	the ideas in discrete mathematics.
--	--	---------------------------	------------------------------------

Table 1- Extract of Michel's documents table from his teaching practices (aims, resources, rules of action and operational invariants)

The operational invariants that justify the actions of Michel can be classified as follows. From a didactical point of view, the importance of understanding and writing ideas, as well as illustration; and the importance of examples and applications in familiarizing students with the field of discrete mathematics. From an epistemological point of view, the characteristics of proofs in discrete mathematics such as the accessibility of discrete objects and the simplicity of definitions. From an institutional point of view, the alignment of contents to ensure a smooth progressivity.

In developing the contents of his courses, Michel uses examples and “technical proofs”. He believes that students need time to get used to reasoning in discrete mathematics. He tries to convey the basic ideas in the theory of graphs such as the definitions and characteristics of objects; a condition that guides his choice of contents is the compliance with the instructions of the major.

In teaching, Michel stresses the importance of proof and illustration, and engages his students in reading and writing proofs. He teaches discrete mathematics in a theoretical way without real life applications; there is a lack of experimentation in his courses. He recognizes the importance of computers for research (for expanding the empirical space), but he does not use them for teaching in his classes.

The interview with Michel was set up to test the guidelines. We have identified areas for improvement and adapted the guideline before the second interview.

ANALYSIS OF THE SECOND PILOT INTERVIEW

The second interview was with Bertrand. Bertrand (France) is a researcher in graph theory and a research director at the National Center of Scientific Research. He teaches by choice: an optional course of combinatory games and mathematical reasoning to students in first and second year and a course of graph theory for master's degree students.

We are particularly interested in his teaching practices at university. Therefore, we will identify the different classes of situations related to his teaching practices. We present in Table 2, an extract from the table of documents of Bertrand.

Aims	Resources	Rules of actions	Operational invariants
Develop a course of combinatory games and	Research problems such as hunting the	He selects problems arising from research situations.	It is important that students experience research problems. Problems in discrete mathematics are easily

mathematical reasoning for students in first and second year of university	beast and problems issued from situations proposed in workshops.	He makes choices that can put emphasis in his teaching on proofs.	accessible. Discrete mathematics makes it possible to teach mathematical knowledge and knowledge related to the proof.
Implement a discrete mathematics course in class	Research problems such as hunting the beast and problems issued from situations proposed in workshops.	He focuses more on proofs than on mathematical objects. He puts the students in the position of researchers. He uses material objects for experimentation. He does not use computers in teaching.	Doing mathematics means working on proofs. It is important that students experience research problems in mathematics. In discrete mathematics, it is important to experiment with material objects. A computer can be interesting for research purposes, but does not contribute in the teaching of discrete mathematics.

Table 2 - Extract of Bertrand's documents table from his teaching practices (aims, resources, rules of action and operational invariants)

The operational invariants that justify Bertrand's actions can be classified as follows. From a didactical point of view, the role of discrete mathematics in developing reasoning skills, the importance of experiencing research moments for the students. From an epistemological point of view, the importance of proofs, the characteristics of problems in discrete mathematics (fun and accessible). From an institutional point of view, the freedom of a researcher in the choice of contents of his courses.

To develop a course in discrete mathematics, Bertrand selects research problems and problems arising from situations experienced in workshops. The course of discrete mathematics to students in first and second year of university is optional and students are not all in scientific majors. Bertrand has some freedom in the choice of contents. He is convinced that the course should develop students' reasoning and knowledge about proofs, as well as knowledge related to discrete mathematics; these convictions guide his choice of contents. He focuses in his teaching on the activity of proof.

Bertrand's classroom management is based on putting students in situations of research. He does not provide the answers, he believes that they must experience research situations like real scientists. He engages students in experimentations with games such as hunting the beast. He thinks that computers are not useful in class.

RESULTS AND DISCUSSION

First results at the interview grid and difficulties encountered in the analysis

In the analyses, we realized the impact of the level of education (Bachelor or Master) on the interactions of university professors with the resources in their teaching practices and on the relationship between teaching practice and research activity.

We relied on the methodology developed by Gueudet (2017) to study the interactions of university professors with resources derived from their teaching practices. It seemed to us that the institutional context has a large impact on teaching practices in university (distinction between Bachelor/Master, progressivity in certain fields, training of non-scientists, etc.). Taking into account the institutional context implies new theoretical and methodological choices.

First results at the research questions

Our first research question is “How do university professors interact with the resources in / for the teaching of discrete mathematics?” There is a big difference between Michel and Bertrand. Michel’s course of discrete mathematics in the second year of university aims to prepare the students for a Master’s degree in graph theory; a constraint on the choice of contents is the compliance with the instructions of the major. On the other hand, Bertrand's course is optional, which allows him a larger degree of freedom in the selection of contents. Bertrand uses, in teaching, resources derived from his research activity, which may be due to his involvement in research.

Both professors insist on the importance of reasoning and proofs, therefore they select contents that contribute to develop students’ ability to write proofs. Bertrand uses games and material objects for experimentation while Michel’s approach focuses more on theories. Both professors state that the use of software cannot contribute to reach the teaching objectives in class, although it can be useful for research purposes.

The second research question is “How can we characterize the relationship between the teaching practices of discrete mathematics at the tertiary level and research activities in the same field?” Bertrand holds a position of research. He adopts a researcher’s attitude in class; he engages students in research and adapts the plan of his course according to their results. On the other hand, Michel follows a stiffer plan in his teaching. Quoting Bertrand “A teacher has to have the knowledge and pass it on, a researcher does not know everything, seeking answers is part of the research”.

The analysis of the pilot interviews shows that the choice to consider the interaction with the resources is revealing for the study of the relations between teaching practices and research activity. The operational invariants identified are based on experience in teaching as well as experience in research. It seems interesting to define, according to the interviews, a typology of operational invariants (for classes of teaching situations). We plan to complete the data collection with a questionnaire that will be disseminated in universities in Lebanon and France for an analysis of the

institutional context of discrete mathematics education. The results of this questionnaire will give us a map that will allow us to better study the case.

[1] International Congress on Mathematical education

REFERENCES

- Biza, I., Giraldo, V., Hochmuth, R., Khakbaz, A., & Rasmussen, C. (2016). Research on Teaching and Learning Mathematics. *ICME-13 Tropical surveys*. Hamburg: springer.
- DeBellis, V. A., & Rosenstein, J. G. (2004). Discrete Mathematics in Primary and Secondary Schools in the United States. *ZDM*, 36(2), 46-55.
- Goldin, G. A. (2004). Problem Solving Heuristics, Affect, and Discrete Mathematics. *ZDM*, 36(2), 56-60.
- Grenier, C., & Payan, D. (1998). Spécificités de la preuve et de la modélisation en mathématiques discrètes. *Recherche en Didactique des Mathématiques*, 18/1, 59-100.
- Gueudet, G. (2017). University Teachers' Resources Systems and Documents. *International Journal of Research in Undergraduate Mathematics Education*, 40753, 1 – 27.
- Gueudet, G., & Trouche, L. (2009). Towards new documentation systems for mathematics teachers? *Educational Studies in Mathematics*, 71(3), 199-218.
- Hart, E. W., & Sandefur, J. (in press, Eds). Teaching and Learning Discrete Mathematics in the School Curriculum Worldwide. *an ICME-13 Monograph* . Springer.
- Heinze, A., Anderson, I., & Reiss, K. (2004). Discrete mathematics and Proof in the High School . *Zentralblatt für Didaktik der Mathematik*, 36(2), 44-84.
- Mali, A. (2015). Characterising university mathematics teaching. In K. Krainer & N. Vondrová (Eds.), *Proceedings of CERME9* (pp. 2187–2193). Prague, Czech Republic: Charles University in Prague, Faculty of Education and ERME.
- Maurer, S. B. (1997). What is Discrete Mathematics? The many answers. In *Discrete Mathematics in the Schools* (pp. 121-132). American Mathematical Society.
- Ouvrier-Bufferet, C. (2014). Discrete mathematics teaching and learning. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 181-186). Dordrecht: Springer.
- Vergnaud, G. (1998). Towards a cognitive theory of practice . In K. Jeremy, & S. Anna (Eds.), *Mathematics education as a research domain: A Search for identity* (pp. 227-241). Dordrecht: Kluwer academic publisher.
- Vergnaud, G. (2009). The theory of conceptual fields. *Human development*, 52, 83-94.