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# The great table of Description Logics and formal ontology notations

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Description Logics (DL) are the logics used to formalize ontology [1]. Many notations are used to express DL, *e.g.* in equations in scientific papers, in editor software like Protégé, or in programming languages. Moreover, the semantics of DL is usually expressed in first-order logics or as set formula. These notations are difficult to understand and to translate from one to another.

This is why I propose here a big table (next page) that compares 5 notations related to DL and formal ontologies:

1. DL syntax (as commonly used in equations and scientific papers)
2. Protégé editor (expression editor syntax)
3. Owlready2 (a package for ontology-oriented programming in Python [2, 3])
4. Semantics in first-order logic
5. Semantics in set formula

This table is an augmented and improved version of the one I presented in [2] and in my habilitation thesis [4].

## Background on DL semantics

DL have a model-theoretic semantics, which is defined in terms of interpretations. For a given ontology  $\mathcal{O}$ , an interpretation  $\mathcal{I} = (\Delta, f)$  is a tuple where the domain  $\Delta = \{\dots\}$  is a non-empty set of objects and the interpretation function  $f$  is a function that associates each individual  $i$ , class  $A$ , role  $R$ , composed expression (defined with semantic connectors) and axiom with its interpretation over  $\Delta$ , as follows:

$$\begin{aligned} f(i \in \mathbb{I}) &\in \Delta \\ f(A \in \mathbb{C}) &\subseteq \Delta \\ f(R \in \mathbb{R}) &\subseteq (\Delta \times \Delta) \end{aligned}$$

Note:  $f$  and  $\Delta$  are sometimes written  $\cdot^I$  and  $\Delta^I$ ; in this case  $x^I = f(x)$ .

## References

- [1] F Baader, D Calvanese, D L McGuinness, D Nardi, and P L Patel-Schneider. *The description logic handbook: theory, implementation and applications*. Cambridge University Press, 2007.
- [2] Lamy JB. Owlready: Ontology-oriented programming in Python with automatic classification and high level constructs for biomedical ontologies. *Artif Intell Med*, 80:11–28, 2017.
- [3] Lamy JB. Ontology-Oriented Programming for Biomedical Informatics. *Stud Health Technol Inform*, 221:64–68, 2016.
- [4] Lamy JB. *Représentation, iconisation et visualisation des connaissances : Principes et applications à l'aide à la décision médicale*. PhD thesis, Université de Rouen-Normandie, 2017.

Available at:

- [http://www.lesfleursdunormal.fr/\\_downloads/article\\_owlready\\_aim\\_2017.pdf](http://www.lesfleursdunormal.fr/_downloads/article_owlready_aim_2017.pdf)
- [http://www.lesfleursdunormal.fr/static/\\_downloads/hdr.pdf](http://www.lesfleursdunormal.fr/static/_downloads/hdr.pdf)

	DL syntax	Protégé	Python + Owlready2	First-order logic	Semantics in set formula
Top	$\top$	Thing	Thing	$\top$ , such as $\forall x, \top(x) = true$	$\Delta$
Bottom	$\perp$	Nothing	Nothing	$\perp$ , such as $\forall x, \perp(x) = false$	$\emptyset$
Subsumption	$A \sqsubseteq B$	A subclass of B	class A(B): ... A.is_a_append(B) issubclass(A, B)	$\forall x, A(x) \rightarrow B(x)$	$f(A) \subseteq f(B)$
Equivalence	$R \sqsubseteq S$	R subproperty of S	(same as above)	$\forall x \forall y, R(x, y) \rightarrow S(x, y)$	$f(R) \subseteq f(S)$
	$A \equiv B$	A equivalent to B	A.equivalent_to_append(B) (as.) B in A.equivalent_to	$\forall x, A(x) \leftrightarrow B(x)$	$f(A) = f(B)$
Instantiation	$A(i)$	i type A	i = A() i.is_instance_of_append(A)	$A(i)$	$f(i) \in f(A)$
Relations	$R(i, j)$	i object property assertion j	i.R = j (R is functional)	$R(i, j)$	$(f(i), f(j)) \in f(R)$
		i data property assertion j	i.R.append(j) (otherwise)		
Complement	$\neg A$	not A	Not(A)	$\neg A(x)$	$\Delta \setminus f(A)$
Intersection	$A \sqcap B$	A and B	A & B (or) And([A, B, ...])	$A(x) \wedge B(x)$	$f(A) \cap f(B)$
Union	$A \sqcup B$	A or B	A   B (or) Or([A, B, ...])	$A(x) \vee B(x)$	$f(A) \cup f(B)$
Extension	$i, j, \dots$	$\{i, j, \dots\}$	OneOf([i, j, ...])	$x \in \{i, j, \dots\}$	$\{f(i), f(j), \dots\}$
Inverse	$R^-$	inverse of R	Inverse(R) (construct) S.inverse = R	$\forall i \forall j, S(i, j) = R(j, i)$	$\{(a, b) \mid (b, a) \in f(R)\}$
Transitive closure	$R^+$	-	-		$\cup_{i \geq 1} (f(R))^i$
Composition	$R \circ S$	R o S	PropertyChain([R, S])		$\{(a, c) \in \Delta \times \Delta \mid \exists b, (a, b) \in f(R) \wedge (b, c) \in f(S)\}$
Existential quantifier	$\exists R.B$	R some B	R.some(B)	$\exists y, R(x, y) \wedge B(y)$	$\{a \in \Delta \mid \exists b, (a, b) \in f(R) \wedge b \in f(B)\}$
Universal quantifier	$\forall R.B$	R only B	R.only(B)	$\forall y, R(x, y) \rightarrow B(y)$	$\{a \in \Delta \mid \forall b, (a, b) \in f(R) \rightarrow b \in f(B)\}$
Number restrictions	$= 2R.B$	R exactly 2 B	R.exactly(2, B)	$ \{y \mid R(x, y) \wedge B(y)\}  = 2$	$\{a \in \Delta \mid  \{b \mid (a, b) \in f(R) \wedge b \in f(B)\}  = 2\}$
	$\leq 2R.B$	R max 2 B	R.max(2, B)	$ \{y \mid R(x, y) \wedge B(y)\}  \leq 2$	$\{a \in \Delta \mid  \{b \mid (a, b) \in f(R) \wedge b \in f(B)\}  \leq 2\}$
	$\geq 2R.B$	R min 2 B	R.min(2, B)	$ \{y \mid R(x, y) \wedge B(y)\}  \geq 2$	$\{a \in \Delta \mid  \{b \mid (a, b) \in f(R) \wedge b \in f(B)\}  \geq 2\}$
Role filler	$\exists R.\{j\}$	R value j	R.value(j)	$R(x, j)$	$\{a \in \Delta \mid (a, f(j)) \in f(R)\}$
Disjoint	$A \sqcap B \sqsubseteq \perp$	A disjoint with B	AllDisjoint([A, B])	$\forall x, \neg(A(x) \wedge B(x))$	$f(A) \cap f(B) = \emptyset$
Property domain	$\exists R.T \sqsubseteq A$	R domain A	R.domain = [A]	$\forall x, (\exists y, R(x, y)) \rightarrow A(x)$	$f(R) \subseteq \{(a, b) \mid a \in f(A)\}$
Property range	$\top \sqsubseteq \forall R.B$	R range B	R.range = [B]	$\forall x \forall y, R(x, y) \rightarrow B(y)$	$f(R) \subseteq \{(a, b) \mid b \in f(B)\}$
Role filler as class property	$A \sqsubseteq \exists R.\{j\}$ $\wedge (\exists R^-.A)(j)$	-	A.R = j (R is functional) A.R.append(j) (otherwise)	-	-
Local closed world	-	-	close_world(A)	-	-