



HAL
open science

The great table of Description Logics and formal ontology notations Background on DL semantics

Jean-Baptiste Lamy

► **To cite this version:**

Jean-Baptiste Lamy. The great table of Description Logics and formal ontology notations Background on DL semantics. [Technical Report] LIMICS. 2018. hal-01849822

HAL Id: hal-01849822

<https://hal.science/hal-01849822v1>

Submitted on 26 Jul 2018

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

The great table of Description Logics and formal ontology notations

Jean-Baptiste Lamy / jean-baptiste.lamy @ univ-paris13.fr

June 22, 2018

Description Logics (DL) are the logics used to formalize ontology [1]. Many notations are used to express DL, *e.g.* in equations in scientific papers, in editor software like Protégé, or in programming languages. Moreover, the semantics of DL is usually expressed in first-order logics or as set formula. These notations are difficult to understand and to translate from one to another.

This is why I propose here a big table (next page) that compares 5 notations related to DL and formal ontologies:

1. DL syntax (as commonly used in equations and scientific papers)
2. Protégé editor (expression editor syntax)
3. Owlready2 (a package for ontology-oriented programming in Python [2, 3])
4. Semantics in first-order logic
5. Semantics in set formula

This table is an augmented and improved version of the one I presented in [2] and in my habilitation thesis [4].

Background on DL semantics

DL have a model-theoretic semantics, which is defined in terms of interpretations. For a given ontology \mathcal{O} , an interpretation $\mathcal{I} = (\Delta, f)$ is a tuple where the domain $\Delta = \{\dots\}$ is a non-empty set of objects and the interpretation function f is a function that associates each individual i , class A , role R , composed expression (defined with semantic connectors) and axiom with its interpretation over Δ , as follows:

$$\begin{aligned} f(i \in \mathbb{I}) &\in \Delta \\ f(A \in \mathbb{C}) &\subseteq \Delta \\ f(R \in \mathbb{R}) &\subseteq (\Delta \times \Delta) \end{aligned}$$

Note: f and Δ are sometimes written \cdot^I and Δ^I ; in this case $x^I = f(x)$.

References

- [1] F Baader, D Calvanese, D L McGuinness, D Nardi, and P L Patel-Schneider. *The description logic handbook: theory, implementation and applications*. Cambridge University Press, 2007.
- [2] Lamy JB. Owlready: Ontology-oriented programming in Python with automatic classification and high level constructs for biomedical ontologies. *Artif Intell Med*, 80:11–28, 2017.
- [3] Lamy JB. Ontology-Oriented Programming for Biomedical Informatics. *Stud Health Technol Inform*, 221:64–68, 2016.
- [4] Lamy JB. *Représentation, iconisation et visualisation des connaissances : Principes et applications à l'aide à la décision médicale*. PhD thesis, Université de Rouen-Normandie, 2017.

Available at:

- http://www.lesfleursdunormal.fr/_downloads/article_owlready_aim_2017.pdf
- http://www.lesfleursdunormal.fr/static/_downloads/hdr.pdf

	DL syntax	Protégé	Python + Owlready2	First-order logic	Semantics in set formula
Const.	\top	Thing	Thing	\top , such as $\forall x, \top(x) = true$	Δ
	\perp	Nothing	Nothing	\perp , such as $\forall x, \perp(x) = false$	\emptyset
Axioms	$A \sqsubseteq B$	A subclass of B	class A(B): ... A.is_a_append(B) issubclass(A, B)	$\forall x, A(x) \rightarrow B(x)$	$f(A) \subseteq f(B)$
	$R \sqsubseteq S$	R subproperty of S	(same as above)	$\forall x \forall y, R(x, y) \rightarrow S(x, y)$	$f(R) \subseteq f(S)$
	$A \equiv B$	A equivalent to B	A.equivalent_to.append(B) (as.) B in A.equivalent_to (test)	$\forall x, A(x) \leftrightarrow B(x)$	$f(A) = f(B)$
	$A(i)$	i type A	i = A() (assertion) i.is_instance_of.append(A) (test)	$A(i)$	$f(i) \in f(A)$
	$R(i, j)$	i object property assertion j i data property assertion j	i.R = j (R is functional) i.R.append(j) (otherwise)	$R(i, j)$	$(f(i), f(j)) \in f(R)$
	$\neg A$	not A	Not(A)	$\neg A(x)$	$\Delta \setminus f(A)$
	$A \sqcap B$	A and B	A & B (or) And([A, B, ...])	$A(x) \wedge B(x)$	$f(A) \cap f(B)$
	$A \sqcup B$	A or B	A B (or) Or([A, B, ...])	$A(x) \vee B(x)$	$f(A) \cup f(B)$
	i, j, \dots	$\{i, j, \dots\}$	OneOf([i, j, ...])	$x \in \{i, j, \dots\}$	$\{f(i), f(j), \dots\}$
	R^-	inverse of R	Inverse(R) (construct) S.inverse = R (assertion)	$\forall i \forall j, S(i, j) = R(j, i)$	$\{(a, b) \mid (b, a) \in f(R)\}$
	R^+	-	-	-	$\cup_{i \geq 1} (f(R))^i$
	$R \circ S$	R o S	PropertyChain([R, S])	-	$\{(a, c) \in \Delta \times \Delta \mid \exists b, (a, b) \in f(R) \wedge (b, c) \in f(S)\}$
	$\exists R.B$	R some B	R.some(B)	$\exists y, R(x, y) \wedge B(y)$	$\{a \in \Delta \mid \exists b, (a, b) \in f(R) \wedge b \in f(B)\}$
	$\forall R.B$	R only B	R.only(B)	$\forall y, R(x, y) \rightarrow B(y)$	$\{a \in \Delta \mid \forall b, (a, b) \in f(R) \rightarrow b \in f(B)\}$
	$= 2R.B$	R exactly 2 B	R.exactly(2, B)	$ \{y \mid R(x, y) \wedge B(y)\} = 2$	$\{a \in \Delta \mid \{b \mid (a, b) \in f(R) \wedge b \in f(B)\} = 2\}$
	$\leq 2R.B$	R max 2 B	R.max(2, B)	$ \{y \mid R(x, y) \wedge B(y)\} \leq 2$	$\{a \in \Delta \mid \{b \mid (a, b) \in f(R) \wedge b \in f(B)\} \leq 2\}$
	$\geq 2R.B$	R min 2 B	R.min(2, B)	$ \{y \mid R(x, y) \wedge B(y)\} \geq 2$	$\{a \in \Delta \mid \{b \mid (a, b) \in f(R) \wedge b \in f(B)\} \geq 2\}$
	$\exists R.\{j\}$	R value j	R.value(j)	$R(x, j)$	$\{a \in \Delta \mid (a, f(j)) \in f(R)\}$
	$A \sqcap B \sqsubseteq \perp$	A disjoint with B	AllDisjoint([A, B])	$\forall x, \neg(A(x) \wedge B(x))$	$f(A) \cap f(B) = \emptyset$
	$\exists R.T \sqsubseteq A$	R domain A	R.domain = [A]	$\forall x, (\exists y, R(x, y)) \rightarrow A(x)$	$f(R) \subseteq \{(a, b) \mid a \in f(A)\}$
	$\top \sqsubseteq \forall R.B$	R range B	R.range = [B]	$\forall x \forall y, R(x, y) \rightarrow B(y)$	$f(R) \subseteq \{(a, b) \mid b \in f(B)\}$
	$A \sqsubseteq \exists R.\{j\}$	-	A.R = j (R is functional)	-	-
	class property $\wedge (\exists R^-.A)(j)$	-	A.R.append(j) (otherwise)	-	-
	Local closed world	-	close_world(A)	-	-