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# New criteria for iterative decoding

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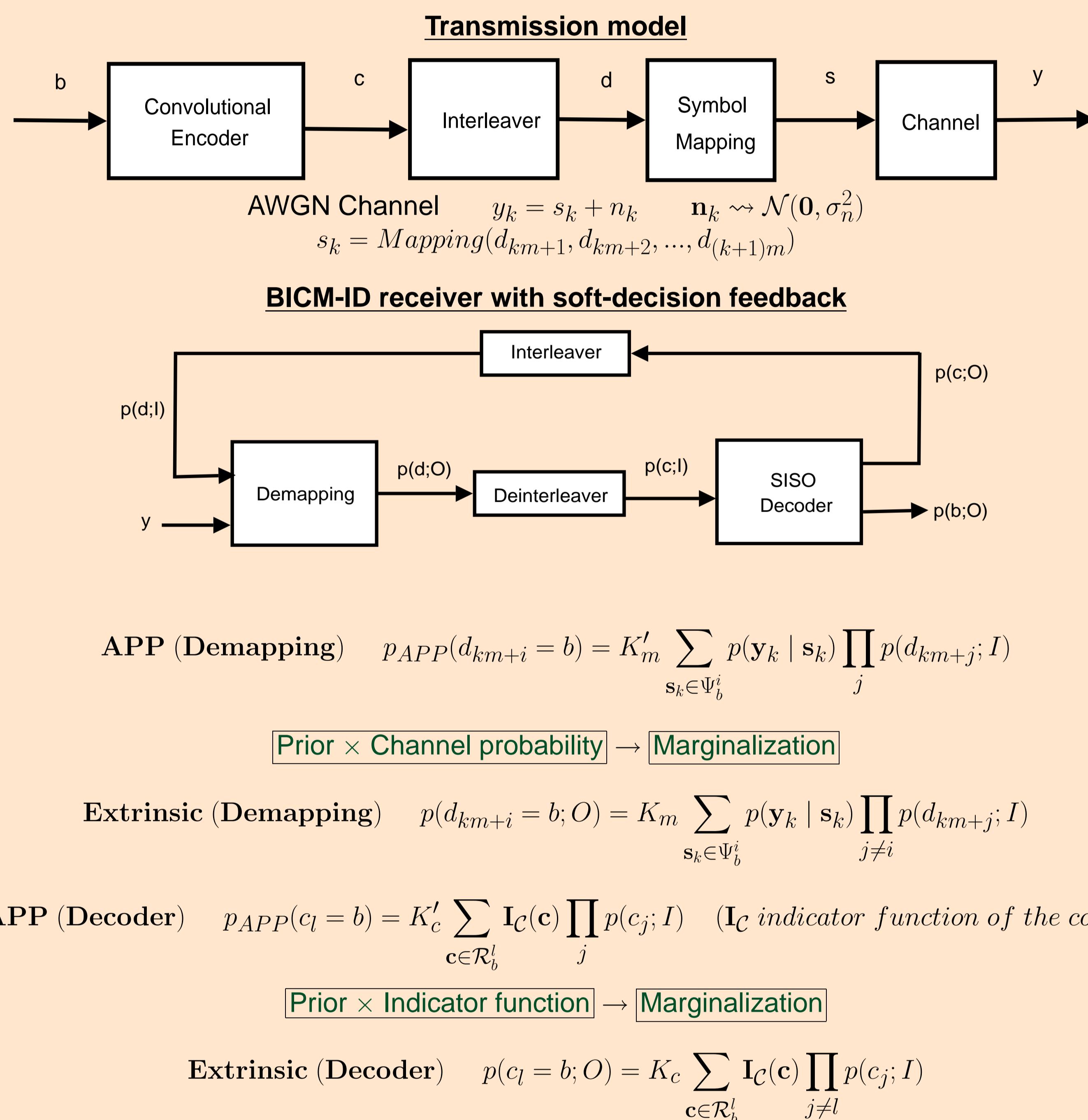
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**Context:** Turbo-like algorithms with iterative decoding

**Goal:** Make the link between iterative decoding and classical optimization techniques. Improve the performance of iterative decoding.

## System model (Bit Interleaved Coded Modulation)(Zehavi, 1992; Li, 2002)



## Information geometry and iterative decoding

### Basic tools and Notations (Walsh, 2006)

$(\mathbf{B}_i) \in \{0, 1\}^N$  binary representation of integer  $i$  gathered into matrix  $\mathbf{B} = (\mathbf{B}_0, \mathbf{B}_1, \dots, \mathbf{B}_{2^N-1})^T$ .

**PMF**  $\eta = (\Pr[\chi = \mathbf{B}_0], \Pr[\chi = \mathbf{B}_1], \dots, \Pr[\chi = \mathbf{B}_{2^N-1}])^T$

**Log-coordinates of PMF**  $\eta$   $[(\theta_i)_{0 \leq i \leq 2^N-1} = \ln(\Pr[\chi = \mathbf{B}_i]) - \ln(\Pr[\chi = \mathbf{B}_0])]$

**Bitwise log-probability ratio**  $(\lambda_j)_{0 \leq j \leq N-1} = \log\left(\frac{\Pr[\chi_j=1]}{\Pr[\chi_j=0]}\right)$

For factorisable probability measures, the log-coordinates take the form  $\theta = \mathbf{B}\lambda$ .

### Link with iterative decoding

#### • Demapping subblock

$$\begin{aligned} p_{APP}(d_{km+i} = b) &= K'_m \sum_{s_k \in \Psi_b^i} p(\mathbf{y}_k | \mathbf{s}_k) \prod_j p(d_{km+j}; I) \\ &\downarrow \theta_m \quad \downarrow \text{Log-coordinates} \\ p_{APP}(d_{km+i} = b) &= K''_m p(d_{km+i}; I) p(d_{km+i}; O) \\ &\downarrow \text{Log-coordinates} \\ &\downarrow \mathbf{B}\lambda_1 \quad \downarrow \mathbf{B}\lambda_2 \end{aligned}$$

The demapping sub-block solves, with respect to  $\lambda_2$ ,  $\bar{p}_{\mathbf{B}(\lambda_1+\lambda_2)} = \bar{p}_{\mathbf{B}\lambda_2+\theta_m}$

#### • Decoding sub-block

$$\begin{aligned} p_{APP}(c_l = b) &= K'_c \sum_{\mathbf{c} \in \mathcal{R}_b^l} \mathbf{I}_{\mathcal{C}}(\mathbf{c}) \prod_j p(c_j; I) \\ &\downarrow \theta_c \quad \downarrow \text{Log-coordinates} \\ &\downarrow \mathbf{B}\lambda_2 \end{aligned}$$

$$\begin{aligned} p_{APP}(c_l) &= K''_c p(c_l; I) p(c_l; O) \\ &\downarrow \mathbf{B}\lambda_2 \quad \downarrow \text{Log-coordinates} \\ &\downarrow \mathbf{B}\lambda_1 \end{aligned}$$

The decoding sub-block solves, with respect to  $\lambda_1$ ,  $\bar{p}_{\mathbf{B}(\lambda_1+\lambda_2)} = \bar{p}_{\mathbf{B}\lambda_2+\theta_c}$

#### • Global criterion

Let  $D_{FD}(\mathbf{p}, \mathbf{q}) = \sum_j p_j \ln\left(\frac{p_j}{q_j}\right) + \sum_j (1-p_j) \ln\left(\frac{1-p_j}{1-q_j}\right)$  denote the Fermi-Dirac entropy (Kullback-Leibler distance for bit probabilities).

The demapping sub-block solves the minimization problem  $\min_{\lambda_2} D_{FD}(\bar{p}_{\mathbf{B}\lambda_1+\theta_m}, \bar{p}_{\mathbf{B}(\lambda_1+\lambda_2)})$ .

The decoding sub-block solves the minimization problem  $\min_{\lambda_1} D_{FD}(\bar{p}_{\mathbf{B}\lambda_2+\theta_c}, \bar{p}_{\mathbf{B}(\lambda_1+\lambda_2)})$ .

## New criteria

### An hybrid proximal point algorithm

**Goal:** Link the two (independent) criteria using proximal point technique (Luque, 1984).

$$\begin{aligned} \text{Demapping} \quad \min_{\lambda_2} J_{\theta_m}(\lambda_1, \lambda_2) &= \min_{\lambda_2} D_{FD}(\bar{p}_{\mathbf{B}\lambda_1+\theta_m}, \bar{p}_{\mathbf{B}(\lambda_1+\lambda_2)}) + \mu_m D_{FD}(\bar{p}_{\mathbf{B}(\lambda_1^{(k)}+\lambda_2^{(k)})}, \bar{p}_{\mathbf{B}(\lambda_1+\lambda_2)}) \\ \text{Decoding} \quad \min_{\lambda_1} J_{\theta_c}(\lambda_1, \lambda_2) &= \min_{\lambda_1} D_{FD}(\bar{p}_{\mathbf{B}\lambda_2+\theta_c}, \bar{p}_{\mathbf{B}(\lambda_1+\lambda_2)}) + \mu_c D_{FD}(\bar{p}_{\mathbf{B}(\lambda_1^{(k)}+\lambda_2^{(k+1)})}, \bar{p}_{\mathbf{B}(\lambda_1+\lambda_2)}) \end{aligned}$$

Optimal choice of the step-size :

$$\begin{aligned} \text{Choose } \mu_m \text{ such that } J_{\theta_m}(\lambda_1^{(k)}, \lambda_2^{(k+1)}) &\leq J_{\theta_c}(\lambda_1^{(k)}, \lambda_2^{(k)}) \\ \text{Choose } \mu_c \text{ such that } J_{\theta_c}(\lambda_1^{(k+1)}, \lambda_2^{(k+1)}) &\leq J_{\theta_m}(\lambda_1^{(k)}, \lambda_2^{(k+1)}) \end{aligned}$$

Both criteria decrease with the iterations. Convergence towards the same stationary point than the classical iterative decoding.

### An hybrid minimum entropy algorithm

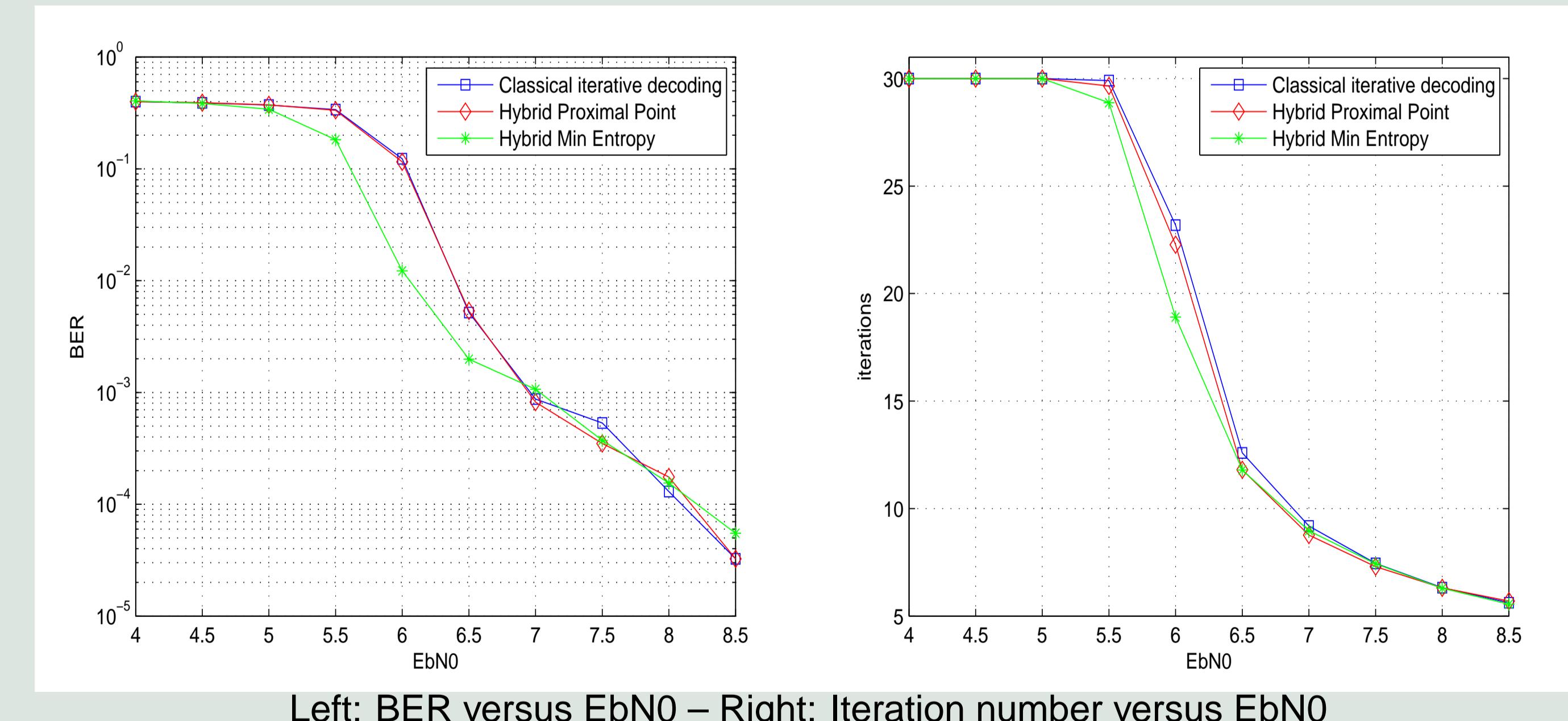
**Goal:** Improve the performance of the iterative decoding.

**Rationale:** The entropy of the APP gives a measure of the reliability of the decision :  $E_{B(\lambda_1+\lambda_2)} \rightarrow 0$  means that the iterative decoding is confident about its decisions (Kocarev, 2006).

$$\begin{aligned} \text{Demapping} \quad \min_{\lambda_2} D_{FD}(\bar{p}_{\mathbf{B}\lambda_1+\theta_m}, \bar{p}_{\mathbf{B}(\lambda_1+\lambda_2)}) + \eta_m E_{B(\lambda_1+\lambda_2)} \\ \text{Decoding} \quad \min_{\lambda_1} D_{FD}(\bar{p}_{\mathbf{B}\lambda_2+\theta_c}, \bar{p}_{\mathbf{B}(\lambda_1+\lambda_2)}) + \eta_c E_{B(\lambda_1+\lambda_2)} \end{aligned}$$

## Simulation

The generator polynomial of the encoder is  $g = [111; 001; 100]$ . The bits are mapped using subset partitioning to a 8-PSK modulation. The length of the coded bit sequence is  $L_c = 6000$ . The step-sizes  $\eta_m$  and  $\eta_c$  in the HMEA are both chosen equal to 0.05.



## References

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