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# New criteria for iterative decoding

Florence Alberge, Ziad Naja, Pierre Duhamel  
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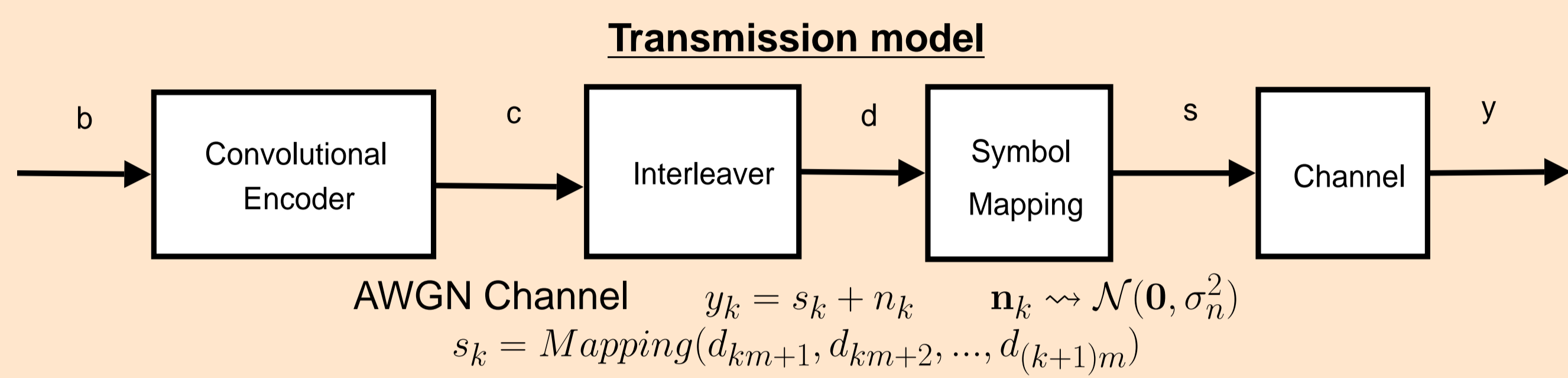
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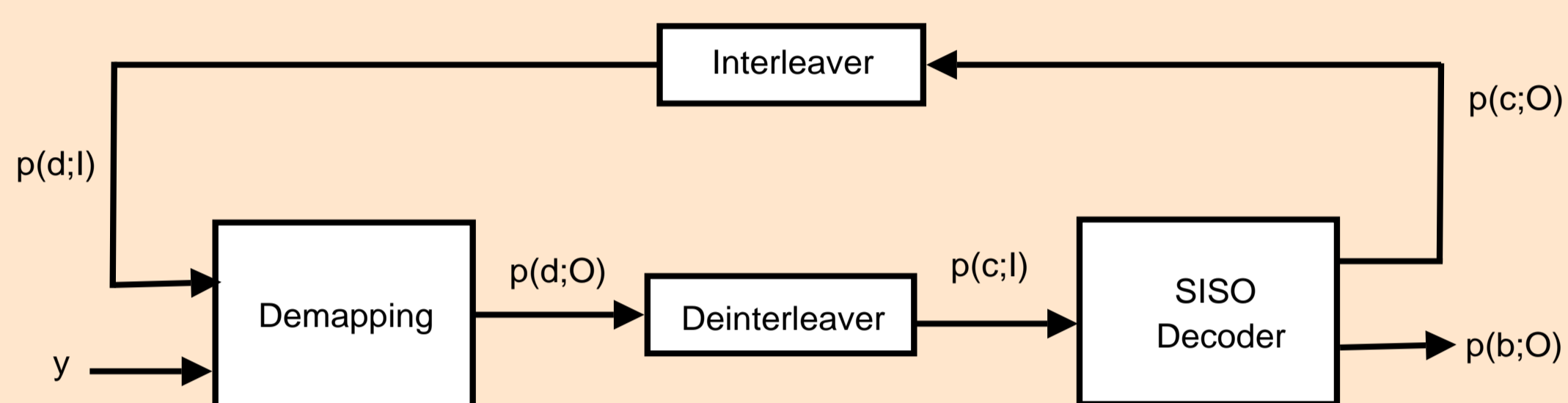
**Context:** Turbo-like algorithms with iterative decoding

**Goal:** Make the link between iterative decoding and classical optimization techniques. Improve the performance of iterative decoding.

## System model (Bit Interleaved Coded Modulation)(Zehavi, 1992; Li, 2002)



### BICM-ID receiver with soft-decision feedback



$$\text{APP (Demapping)} \quad p_{APP}(d_{km+i} = b) = K'_m \sum_{\mathbf{s}_k \in \Psi_b^i} p(\mathbf{y}_k | \mathbf{s}_k) \prod_j p(d_{km+j}; I)$$

Prior  $\times$  Channel probability  $\rightarrow$  Marginalization

$$\text{Extrinsic (Demapping)} \quad p(d_{km+i} = b; O) = K_m \sum_{\mathbf{s}_k \in \Psi_b^i} p(\mathbf{y}_k | \mathbf{s}_k) \prod_{j \neq i} p(d_{km+j}; I)$$

$$\text{APP (Decoder)} \quad p_{APP}(c_l = b) = K'_c \sum_{\mathbf{c} \in \mathcal{R}_b^l} \mathbf{I}_C(\mathbf{c}) \prod_j p(c_j; I) \quad (\mathbf{I}_C \text{ indicator function of the code})$$

Prior  $\times$  Indicator function  $\rightarrow$  Marginalization

$$\text{Extrinsic (Decoder)} \quad p(c_l = b; O) = K_c \sum_{\mathbf{c} \in \mathcal{R}_b^l} \mathbf{I}_C(\mathbf{c}) \prod_{j \neq l} p(c_j; I)$$

## Information geometry and iterative decoding

### Basic tools and Notations (Walsh, 2006)

$(\mathbf{B}_i) \in \{0, 1\}^N$  **binary representation** of integer  $i$  gathered into matrix  $\mathbf{B} = (\mathbf{B}_0, \mathbf{B}_1, \dots, \mathbf{B}_{2^N-1})^T$ .

$$\text{PMF} \quad \eta = (\Pr[\chi = \mathbf{B}_0], \Pr[\chi = \mathbf{B}_1], \dots, \Pr[\chi = \mathbf{B}_{2^N-1}])^T$$

$$\text{Log-coordinates of PMF } \eta \quad (\theta_i)_{0 \leq i \leq 2^N-1} = \ln(\Pr[\chi = \mathbf{B}_i]) - \ln(\Pr[\chi = \mathbf{B}_0])$$

$$\text{Bitwise log-probability ratio} \quad (\lambda_j)_{0 \leq j \leq N-1} = \log\left(\frac{\Pr[\chi_j=1]}{\Pr[\chi_j=0]}\right)$$

For **factorisable probability measures**, the log-coordinates take the form  $\theta = \mathbf{B}\lambda$ .

### Link with iterative decoding

#### Demapping sub-block

$$p_{APP}(d_{km+i} = b) = K'_m \sum_{\mathbf{s}_k \in \Psi_b^i} p(\mathbf{y}_k | \mathbf{s}_k) \prod_j p(d_{km+j}; I)$$

$$p_{APP}(d_{km+i} = b) = K''_m \underset{\mathbf{B}\lambda_1}{p(d_{km+i}; I)} \underset{\mathbf{B}\lambda_2}{p(d_{km+i}; O)}$$

The **demapping sub-block** solves, with respect to  $\lambda_2$ ,  $p_{\mathbf{B}(\lambda_1+\lambda_2)} = p_{\mathbf{B}\lambda_1+\theta_m}$

#### Decoding sub-block

$$p_{APP}(c_l = b) = K'_c \sum_{\mathbf{c} \in \mathcal{R}_b^l} \mathbf{I}_C(\mathbf{c}) \prod_j p(c_j; I)$$

$$p_{APP}(c_l = b) = K''_c \underset{\theta_c}{p(\mathbf{c}; I)} \underset{\mathbf{B}\lambda_2}{p(\mathbf{c}; O)}$$

$$p_{APP}(c_l) = K''_c \underset{\mathbf{B}\lambda_2}{p(c_l; I)} \underset{\mathbf{B}\lambda_1}{p(c_l; O)}$$

Log-coordinates

The **decoding sub-block** solves, with respect to  $\lambda_1$ ,  $p_{\mathbf{B}(\lambda_1+\lambda_2)} = p_{\mathbf{B}\lambda_2+\theta_c}$

#### Global criterion

Let  $D_{FD}(\mathbf{p}, \mathbf{q}) = \sum_j p_j \ln\left(\frac{p_j}{q_j}\right) + \sum_j (1-p_j) \ln\left(\frac{1-p_j}{1-q_j}\right)$  denote the **Fermi-Dirac entropy** (Kullback-Leibler distance for bit probabilities).

The demapping sub-block solves the minimization problem  $\min_{\lambda_2} D_{FD}(p_{\mathbf{B}\lambda_1+\theta_m}, p_{\mathbf{B}(\lambda_1+\lambda_2)})$ .

The decoding sub-block solves the minimization problem  $\min_{\lambda_1} D_{FD}(p_{\mathbf{B}\lambda_2+\theta_c}, p_{\mathbf{B}(\lambda_1+\lambda_2)})$ .

## New criteria

### An hybrid proximal point algorithm

**Goal:** Link the two (independent) criteria using **proximal point technique** (Luque, 1984).

$$\text{Demapping} \quad \min_{\lambda_2} J_{\theta_m}(\lambda_1, \lambda_2) = \min_{\lambda_2} D_{FD}(p_{\mathbf{B}\lambda_1+\theta_m}, p_{\mathbf{B}(\lambda_1+\lambda_2)}) + \mu_m D_{FD}(p_{\mathbf{B}(\lambda_1^{(k)}+\lambda_2^{(k)})}, p_{\mathbf{B}(\lambda_1+\lambda_2)})$$

$$\text{Decoding} \quad \min_{\lambda_1} J_{\theta_c}(\lambda_1, \lambda_2) = \min_{\lambda_1} D_{FD}(p_{\mathbf{B}\lambda_2+\theta_c}, p_{\mathbf{B}(\lambda_1+\lambda_2)}) + \mu_c D_{FD}(p_{\mathbf{B}(\lambda_1^{(k)}+\lambda_2^{(k+1)})}, p_{\mathbf{B}(\lambda_1+\lambda_2)})$$

Optimal choice of the step-size :

$$\text{Choose } \mu_m \text{ such that } J_{\theta_m}(\lambda_1^{(k)}, \lambda_2^{(k+1)}) \leq J_{\theta_m}(\lambda_1^{(k)}, \lambda_2^{(k)})$$

$$\text{Choose } \mu_c \text{ such that } J_{\theta_c}(\lambda_1^{(k+1)}, \lambda_2^{(k+1)}) \leq J_{\theta_c}(\lambda_1^{(k)}, \lambda_2^{(k+1)})$$

Both criteria **decrease with the iterations**. Convergence towards the same stationary point than the classical iterative decoding.

### An hybrid minimum entropy algorithm

**Goal:** Improve the performance of the iterative decoding.

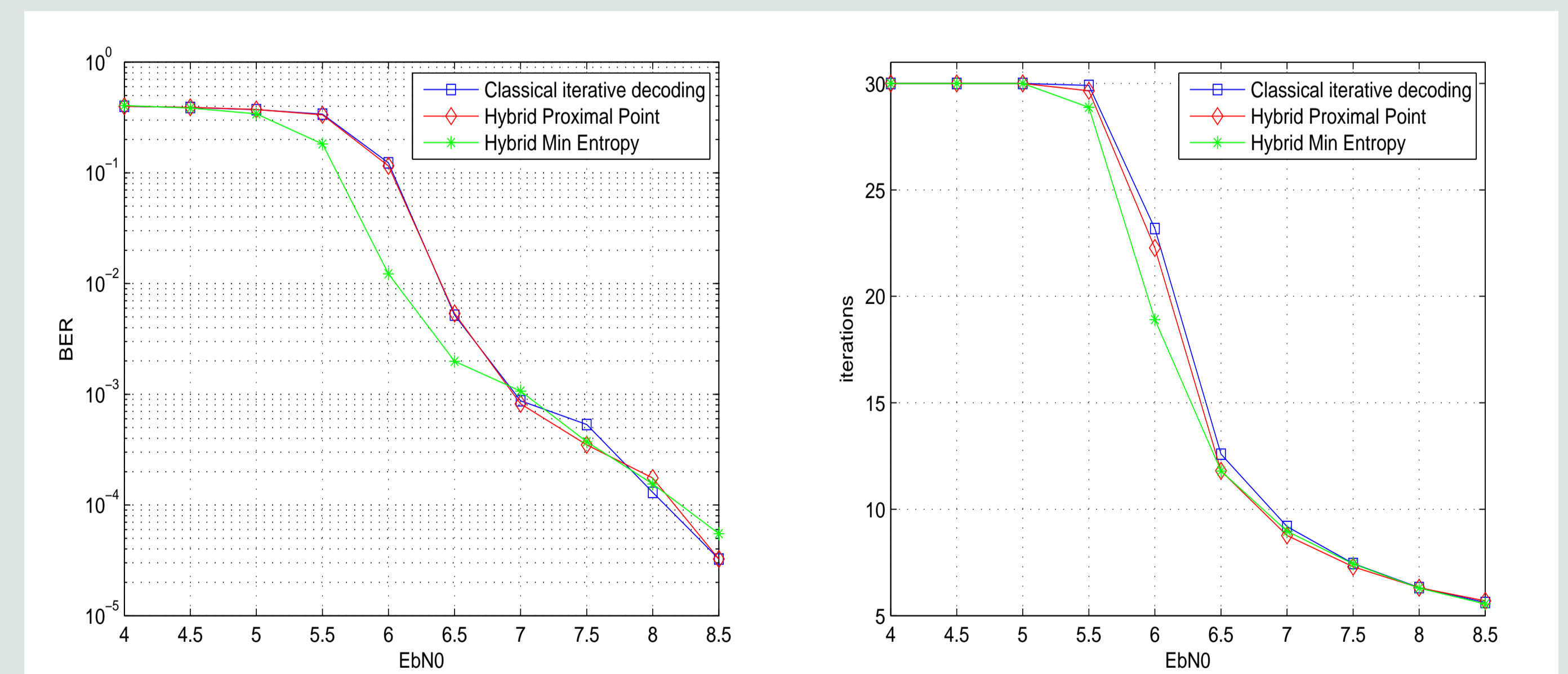
**Rationale:** The entropy of the APP gives a measure of the **reliability** of the decision :  $E_{B(\lambda_1+\lambda_2)} \rightarrow 0$  means that the iterative decoding is **confident about its decisions** (Kocarev, 2006).

$$\text{Demapping} \quad \min_{\lambda_2} D_{FD}(p_{\mathbf{B}\lambda_1+\theta_m}, p_{\mathbf{B}(\lambda_1+\lambda_2)}) + \eta_m E_{B(\lambda_1+\lambda_2)}$$

$$\text{Decoding} \quad \min_{\lambda_1} D_{FD}(p_{\mathbf{B}\lambda_2+\theta_c}, p_{\mathbf{B}(\lambda_1+\lambda_2)}) + \eta_c E_{B(\lambda_1+\lambda_2)}$$

## Simulation

The generator polynomial of the encoder is  $g = [111; 001; 100]$ . The bits are mapped using subset partitioning to a 8-PSK modulation. The length of the coded bit sequence is  $L_c = 6000$ . The step-sizes  $\eta_m$  and  $\eta_c$  in the HMEA are both chosen equal to 0.05.



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