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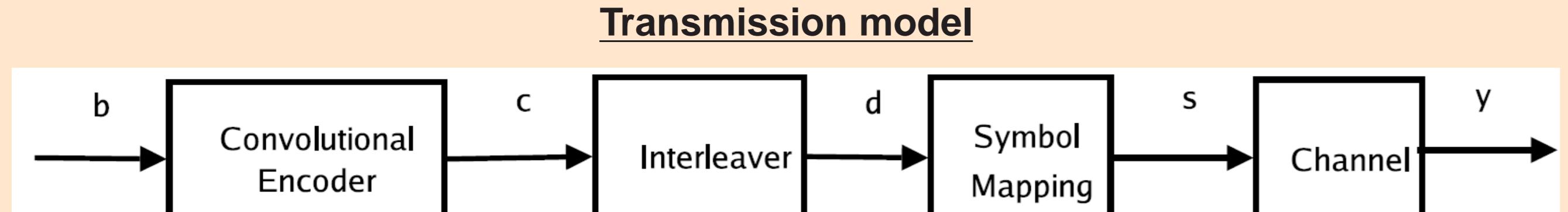
A Game-Theoretic Interpretation of Iterative Decoding

Florence Alberge

19th European Signal Processing Conference (EUSIPCO 2011)

Goal: Iterative Decoding as a distributed optimization strategy - Interpretation of turbo-like Iterative Decoding (Game) - Results on Convergence.

System model (Bit Interleaved Coded Modulation)



$$y_k = h_k s_k + n_k$$

$$s_k = \text{Mapping}(d_{kM+1}, d_{kM+2}, \dots, d_{(k+1)M})$$

$$n_b \text{ information bits, } n \text{ encoded bits}$$

Maximum Likelihood Decoding (MLD)

- Searching for optimal sequence of information bits $\hat{\mathbf{b}}_{MLD}$

$$\hat{\mathbf{b}}_{MLD} = \arg \max_{\mathbf{b} \in \{0,1\}^{n_b}} p(\mathbf{y} | \mathbf{b})$$

- Searching for optimal sequence of encoded/interleaved bits $\hat{\mathbf{d}}_{MLD}$

$$\hat{\mathbf{d}}_{MLD} = \arg \max_{\mathbf{d} \in \{0,1\}^n} \underbrace{p_{ch}(\mathbf{y} | \mathbf{d})}_{\text{Channel probability}} \underbrace{I_{co}(\mathbf{d})}_{\text{Indicator function}}$$

- Searching for optimal weighting $\hat{\mathbf{p}}_{MLD}$

$$\hat{\mathbf{p}}_{MLD}(\mathbf{d}) = \arg \max_{\mathbf{p} \in \mathcal{E}_s} \sum_{\mathbf{d}} I_{co}(\mathbf{d}) p_{ch}(\mathbf{y} | \mathbf{d}) \mathbf{p}(\mathbf{d}) \quad \mathcal{E}_s : \text{fully-factorized PMFs}$$

 Solution is $\hat{\mathbf{p}}_{MLD}(\mathbf{d}) = \delta_{\hat{\mathbf{d}}_{MLD}}(\mathbf{d})$

 Implementation untractable \Rightarrow Suboptimal optimization

Suboptimal Decoding

 Numerical value of $n \Rightarrow$ Solution 1: Compute bit-marginals

 Interleaver \Rightarrow Solution 2: Consider separately **decoding** and **demapping**

Leads to another expression for the MLD

$$(\hat{\mathbf{l}}_{MLD}(\mathbf{d}), \hat{\mathbf{q}}_{MLD}(\mathbf{d})) = \arg \max_{\mathbf{l}, \mathbf{q} \in \mathcal{E}_s} \sum_{d_k} \sum_{\mathbf{d}:d_k} \mathbf{I}_{co}(\mathbf{d}) \mathbf{q}(\mathbf{d}) p_{ch}(\mathbf{y} | \mathbf{d}) \mathbf{l}(\mathbf{d})$$

with optimal solution

$$\hat{\mathbf{l}}_{MLD}(\mathbf{d}) = \hat{\mathbf{q}}_{MLD}(\mathbf{d}) = \delta_{\hat{\mathbf{d}}_{MLD}}(\mathbf{d})$$

but still untractable...

n convenient APPROXIMATIONS: $\tilde{\mathcal{C}}_1, \dots, \tilde{\mathcal{C}}_k, \dots, \tilde{\mathcal{C}}_n$ specific for each bit-marginal

$$\sum_{d_k} \sum_{\mathbf{d}:d_k} \underbrace{\mathbf{I}_{co}(\mathbf{d}) \mathbf{q}(\mathbf{d}) p_{ch}(\mathbf{y} | \mathbf{d}) \mathbf{l}(\mathbf{d})}_{\text{bit-marginals of a product } (\mathcal{C}_{MLD}, \text{ optimal})} \approx \sum_{d_k} \left(\sum_{\mathbf{d}:d_k} \mathbf{I}_{co}(\mathbf{d}) \mathbf{q}(\mathbf{d}) \right) \left(\sum_{\mathbf{d}:d_k} \mathbf{p}_{ch}(\mathbf{y} | \mathbf{d}) \mathbf{l}(\mathbf{d}) \right)$$

$$\text{product of the bit-marginals } (\tilde{\mathcal{C}}_k, \text{ suboptimal})$$

- Suggests a distributed optimization strategy
- Average cost function: $\tilde{\mathcal{C}} = \sum_{i=1}^n \tilde{\mathcal{C}}_k \Rightarrow$ Score function for the distributed optimization strategy
- $\tilde{\mathcal{C}}$ is a weighted sum:

$$\tilde{\mathcal{C}}(\mathbf{l}, \mathbf{q}) = n \mathcal{C}_{MLD} + \sum_{r=1}^{n-1} (n-r) N_r(\mathbf{l}, \mathbf{q})$$

where $N_r(\mathbf{l}, \mathbf{q}) = \sum_{(\mathbf{d}, \mathbf{d}') \in \mathbb{S}_r} \mathbf{I}_{co}(\mathbf{d}) \mathbf{q}(\mathbf{d}) \mathbf{p}_{ch}(\mathbf{y} | \mathbf{d}') \mathbf{l}(\mathbf{d}')$ and \mathbb{S}_r denote the set of pairs $(\mathbf{d}, \mathbf{d}')$ such that $d_H(\mathbf{d}, \mathbf{d}') = r$ (Hamming distance)

Distributed Optimization

 Maximize separately the n cost functions $\tilde{\mathcal{C}}_k$

$$(\hat{l}_k, \hat{q}_k) = \arg \max_{l_k, q_k \in \mathcal{F}} \sum_{d_k} q_k(d_k) l_k(d_k) \left(\sum_{d:d_k} \mathbf{I}_{co}(\mathbf{d}) \Pi_{i \neq k} q_i(d_i) \right) \left(\sum_{d:d_k} \mathbf{p}_{ch}(\mathbf{y} | \mathbf{d}) \Pi_{i \neq k} l_i(d_i) \right)$$

 Linear in $q_k(d_k) l_k(d_k) \Rightarrow \hat{l}_k \hat{q}_k \in \{0, 1\}$: HARD decision

$$\text{SOFT decision preferred: } \Rightarrow \hat{q}_k \hat{l}_k \propto \left(\underbrace{\left(\sum_{d:d_k} \mathbf{I}_{co}(\mathbf{d}) \Pi_{i \neq k} q_i(d_i) \right)}_{\text{extrinsic (decoder)}} \right) \left(\underbrace{\sum_{d:d_k} \mathbf{p}_{ch}(\mathbf{y} | \mathbf{d}) \Pi_{i \neq k} l_i(d_i)}_{\text{extrinsic (demapper)}} \right)^\beta$$

Equivalently with LLR :

$$\lambda_{l,k} + \lambda_{q,k} = \beta \log \left(\frac{\sum_{d:d_k} \mathbf{I}_{co}(\mathbf{d}) \Pi_{i \neq k} q_j(d_j)}{\sum_{d:d_k} \mathbf{I}_{co}(\mathbf{d}) \Pi_{i \neq k} q_i(d_i)} \right) + \beta \log \left(\frac{\sum_{d:d_k} \mathbf{p}_{ch}(\mathbf{y} | \mathbf{d}) \Pi_{i \neq k} l_i(d_i)}{\sum_{d:d_k} \mathbf{p}_{ch}(\mathbf{y} | \mathbf{d}) \Pi_{i \neq k} l_j(d_j)} \right) \quad (1)$$

Can be recast as a distributed optimization process with strictly concave utility functions:

$$U_k(\lambda_{l,k}, \lambda_{q,k}, \lambda_{l,-k}, \lambda_{q,-k}) = - \| \lambda_{l,k} + \lambda_{q,k} - \beta \log \left(\frac{f_{d_k}(\mathbf{q}, I_{co}) f_{d_k}(1, p_{ch}(\mathbf{y} | \mathbf{d}))}{f_{d_k}(\mathbf{q}, I_{co}) f_{d_k}(1, p_{ch}(\mathbf{y} | \mathbf{d}))} \right) \|^2 \quad (2)$$

- Game $\mathcal{G}_{soft} \Rightarrow W_{soft} = 0$ for any NE (USELESS).
- Game $\mathcal{G}_{hard} \Rightarrow W_{hard} = \tilde{\mathcal{C}} \Rightarrow$ a posteriori evaluation of the optimization process and of the vicinity to the MLD solution.
- There always exists at least one NE (proved in (?) when $\beta = 1$).

Convergence

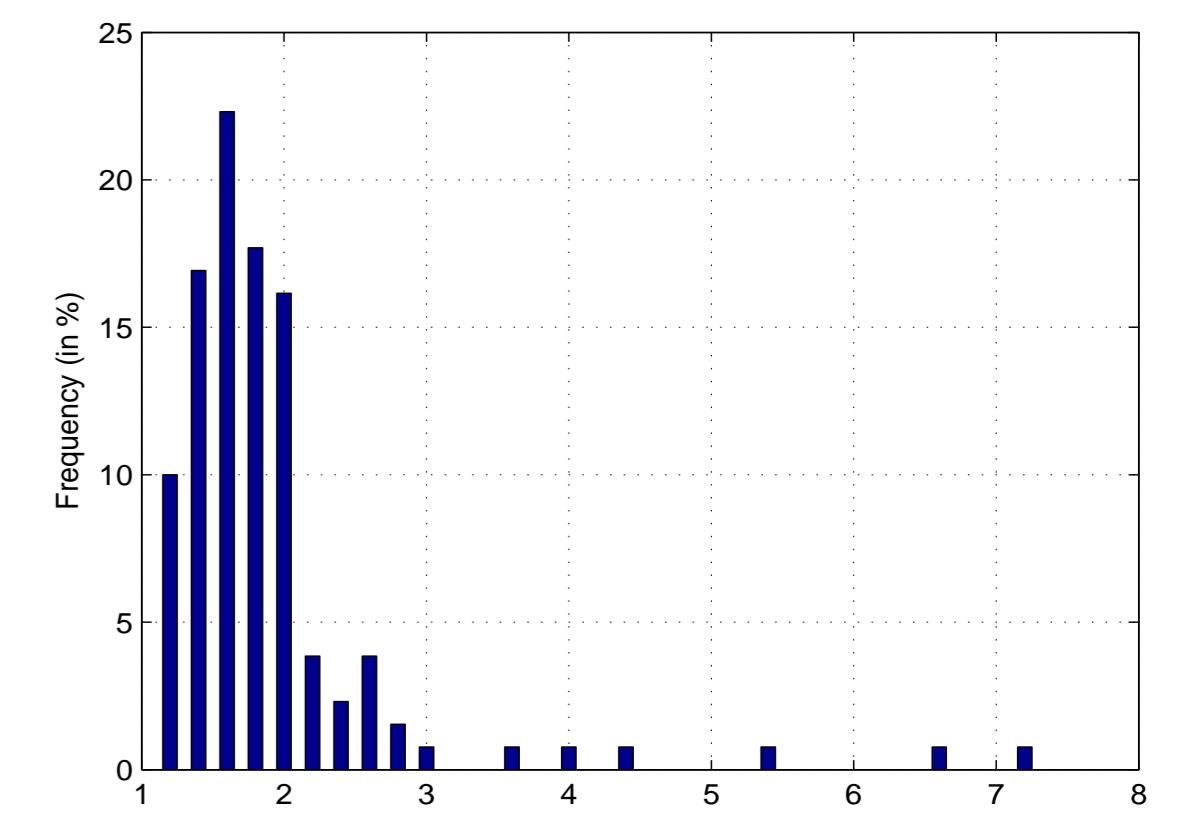
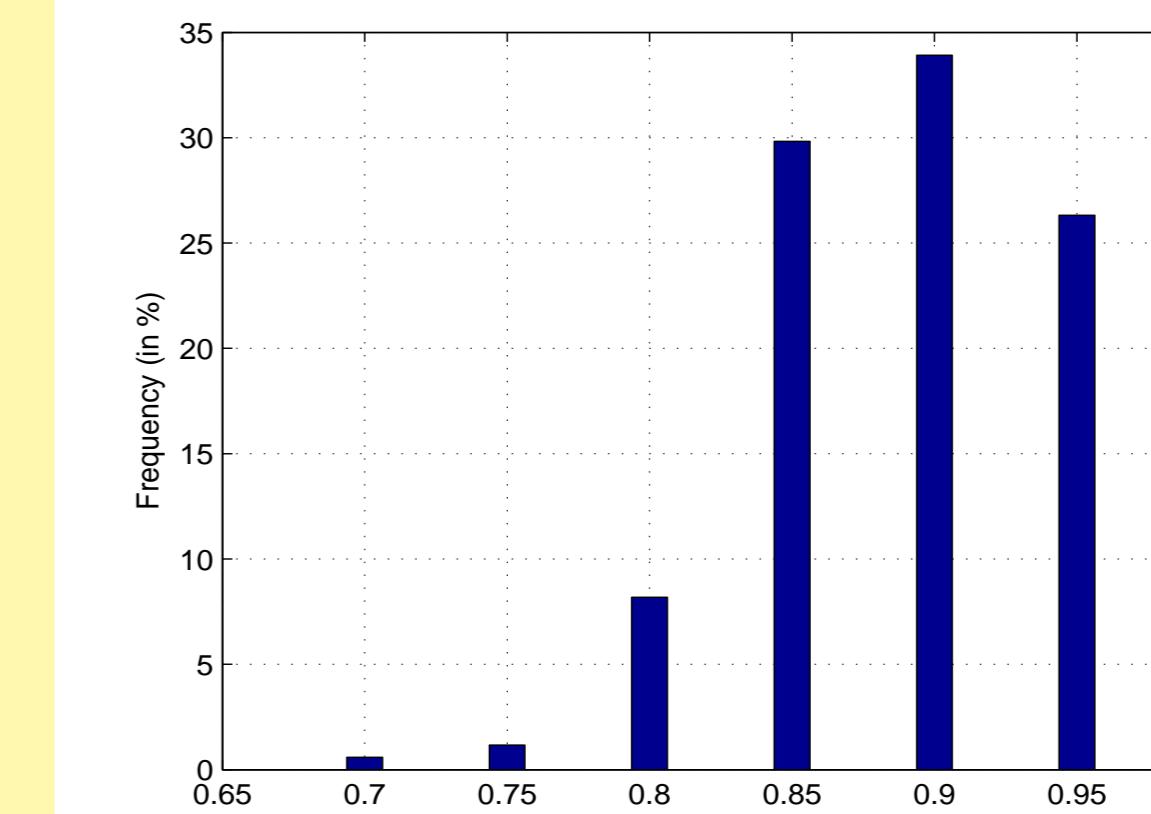
Iterative decoding = (?) + Jacobi/Gauss-Seidel scheduling (?).
 Convergence proofs based on the Jacobian ∇F (?) \rightarrow Key property for ∇F : STRICTLY DIAGONAL DOMINANCE.

Definition 4. Let \mathbf{A} denote a $n \times n$ matrix with elements a_{ij} in \mathbb{R} . \mathbf{A} is a strictly diagonal dominant matrix if

$$|a_{ii}| > \sum_{j \neq i} |a_{ij}| \quad \forall i \in \{1, \dots, n\}$$

Proposition 4. It always exist $\beta_0 > 0$ such that $\forall \beta \leq \beta_0, \nabla F(\lambda_l, \lambda_q)$ is a strictly diagonally dominant matrix for all $(\lambda_l, \lambda_q) \in \mathbb{R}^n \times \mathbb{R}^n$.

Illustration

 BICM - (5, 7) convolutional code of rate 1/2 - $n_b = 400$ (a frame). Set partitioning - 16-QAM.


(left) EbNo=5dB - β_0 distribution among non-converging sequences in classical BICM , (right) EbNo=7dB among converging sequences in classical BICM

Conclusion

- $\tilde{\mathcal{C}}$: a posteriori score function (optimization process / MLD solution)
- β useful to improve convergence rate
- Convergence is not an issue

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