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Analysing tasks for the Flipped Classroom from the perspective of Realistic Mathematical Education

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The advent of Flipped Classroom as a framework for organizing the teaching and learning of mathematics has the potential to revitalize the attention to tasks as a vehicle for meaningful learning in tertiary education. Flipped Classroom is based on the idea of student active learning under close guidance of the university teacher. Due to the possibility to engage the students in meaningful discovery of how mathematics can relate to real-life situations during in-class sessions, the tasks are seen to have a central role in a successful implementation of Flipped Classrooms. This paper explores the realistic mathematics education (RME) as a theoretical framework for task design and analysis in the context of in-class flipped engineering mathematics classrooms, and I seek evidence of knowledge construction through analysing students' work on modelling the height of a rider in a double Ferris wheel.

Keywords: Flipped Classroom, RME, Mathematics for engineers, the role of digital and other resources in university mathematics education

INTRODUCTION

One of the fundamental principles of Flipped Classroom (FC) approaches in education is the opportunity it provides for a more interactive and meaningful use of class time. (Bergmann & Sams, 2012). The idea is to use valuable classroom time for the more advanced learning processes like analysing, evaluating, and modelling in a cooperative environment consisting of peers and tutors. A contrast to this form of in-class student participation can be the traditional university teaching style where the lecturer is more or less seeking to “transfer” the mathematics in a monologue fashion (Sfard, 2014).

Moving the lecturing part out of the classroom as a preparatory activity can help remedy the problem lecturers often feel is present during the delivery of traditional lectures, the one of disengaged and passive students. Rather, the social setting of the classroom can be used to let the students discuss, evaluate, try out, and receive assistance with their ideas and conjectures. However, such learning processes need a structuring element, which can be the tasks that students work with, preferably in groups. The purpose of this paper is to advance knowledge on task design and analysis in connection with FC implementation, specifically considering the theoretical framework of RME. I attempt to do this through a case study of students' work with a modelling task that follows the heuristics of RME.

Previous research

FC is an innovative teaching approach, initially described by Lage, Platt, and Treglia (2000) that has many implementations. As the popularity of FC teaching has caught

up, interest has been considerable also for tertiary educational settings (Wasserman, Quint, Norris, & Carr, 2017).

Various authors have done research on general issues of FC pedagogy (Abeysekera & Dawson, 2015; Wan, 2015). However, little research has emerged on analysing how task design should be addressed in connection to FC implementation. In particular, I was not able to find any research considering RME as a framework for analysing tasks in connection with FC in-class activities. Several articles seem to call for more research on providing insights on FC pedagogic heuristics (Song, Jong, Chang, & Chen, 2017; Wan, 2015). Task design is of particular importance for the in-class FC component, since the model of FC considers the classroom activities to be the arena for attaining the highest levels of skills and knowledge (Bergmann & Sams, 2012). The challenge is to bridge the out-of-class videos with these in-class activities in a meaningful way.

The particular task analysis performed in this paper is based upon students' group work with mathematizing the double Ferris wheel movement. A study using a similar type of problem is described in Sweeney and Rasmussen (2014). They found that the students' bodily engagement using gestures and measurements by fingers facilitated a link between the movement of the rider and the mathematical model.

THEORETICAL FOUNDATIONS OF RME

According to Van den Heuvel-Panhuizen and Drijvers (2014), RME is a domain-specific instruction theory for mathematics that focuses on rich, "realistic" situations serving to initiate development of mathematical concepts, tools and procedures. Core teaching heuristics give these directions:

- The activity principle emphasises students' direct participation throughout the learning process. Mathematics is considered to be best learned by doing mathematics, which is in accordance with student active learning that is considered a widely accepted principle of FC.
- The reality principle expresses the importance of presenting students with real-life situations and problems that they can imagine and mathematize upon.
- The level principle highlights the idea that students pass through various cognitive development phases, from informal context-related descriptions of the problem towards the use of more formal mathematical language. This process sets the stage for bridging student understanding that the *model* of the context-related situation at hand can become a *model for* similar kinds of problems.
- The intertwinement principle emphasises that mathematical content domain should not be considered as isolated fragments, but rather seen as a connected whole. This principle supports task designs facilitating open problems that stimulate students' own thinking and reasoning about which mathematical solution techniques and mediating artefacts to employ.
- The interactivity principle relates to the idea that learning mathematics is not purely an individual pursuit, but rather a social activity, where group work and

whole-class discussions should be orchestrated by the teacher. This principle aligns well with socio-cultural learning theories, as also emphasised by Cobb, Jana, and Visnovska (2008). They consider semiotic mediation and cultural tools as important means of conveying mathematical meaning in an RME setting.

- The guidance principle refers to the idea of “guided re-invention” of mathematics. Instructional sequences in task design can involve historical evolutionary steps in mathematics as inspiration for rich context problems (Gravemeijer, 1999).

Further, RME makes a distinction between horizontal and vertical mathematization. According to Van den Heuvel-Panhuizen and Drijvers (2014), students use mathematics as a tool to understand and organize problems in a real-life context, typically during task solution. This is called horizontal mathematization. Vertical mathematization refers to mathematizing one’s own mathematical activity to reach a higher level of abstraction.

The research question explored in this paper can be framed in the theory of RME:

“To what extent does RME task design facilitate students’ modelling activities and knowledge construction in a FC context?”

METHODOLOGY

The data for this paper was collected from video filming the work of two groups of computer engineering students in their first semester of study. These student groups were part of a larger research project conducted at a university campus in Norway. FC teaching was performed throughout the whole year of study. I focus on one in-class session where students were modelling the double Ferris wheel.

The Flipped Classroom setting

Before this in-class session, the students had watched an out-of-class session of videos that introduced the unit circle and how the y-component would map to the sine function as the angle rotated (Figure 1). In addition to this, there were videos showing examples on how to make

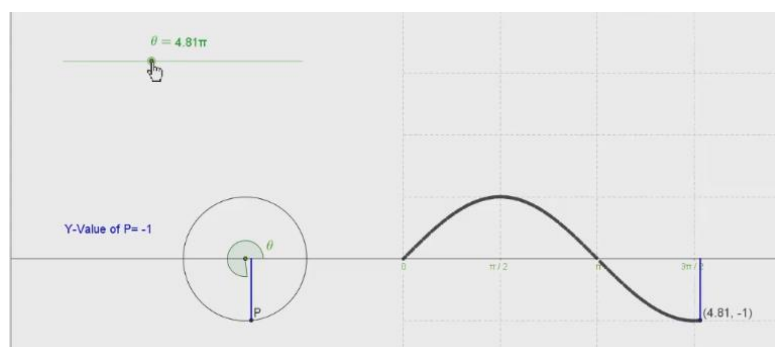


Figure 1: Mapping of unit circle y-component to the sine-function

mathematical models using this function, and how one could sketch a particular sine function based on textual information. Lastly, they were showed examples of how to determine the period, amplitude, baseline and phase shift by considering various graphs of this function. These videos were provided using English as the spoken and written language, while students were using Norwegian as working language in class.

The particular task that I refer to in this paper was given to the students in connection with the teaching of trigonometric functions during the second week of the semester. In this task, they were asked to create a model of the evolution of the height of a rider on a double Ferris wheel. The double Ferris wheel was represented in an applet available for the students to watch on their computer (Figure 2). The Ferris wheel consisted of two separate wheels rotating at the same speed and centred at the end of a larger rotating bar as seen on

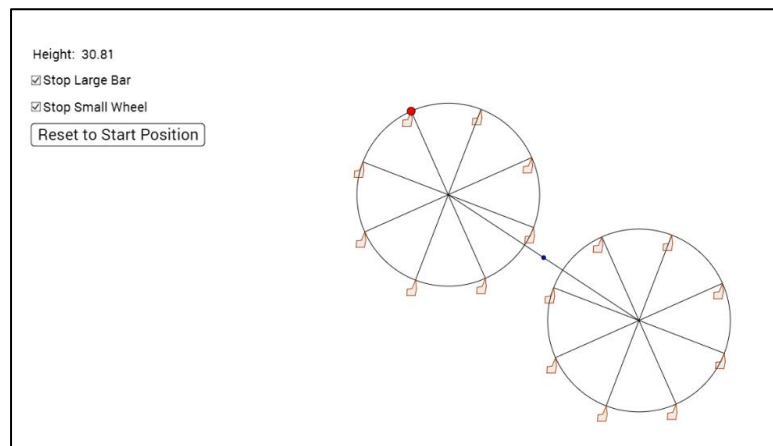


Figure 2: The double Ferris wheel simulation

Figure 2. The students had the possibility to study each rotation separately by starting/stopping each of them. The height measurement of an imagined rider called Marit (marked with a dot on one of the wheels), was available as a readout at all time. The interested reader can study the applet at <http://sigmaa.maa.org/rume/crume2017/Applet.html>. The task presented for the students was the following:

1. Create a representation of your choice that illustrates Marits' ride on the double-Ferris wheel
2. Create a sketch/graph of Marits' height versus time as she ride on the double Ferris-wheel
3. Create a function for Marits' height versus time as she travels on the Double-Ferris wheel.
4. Graph the functions you found in the previous problem using a graphing utility like geogebra. Does the graph of your function make sense? Is it similar to the graph you sketched?

The design is seen to follow the principles of RME. Firstly, the purpose of the task is to engage students in exploring the dynamics of a familiar situation; the Ferris wheel ride. This meets the demands of the activity and reality principles. Furthermore, the sequence of the task, asking students to initially make a representation of the movement, followed by sketching a graph, and finally creating a function, attempts at making the students move from more informal to formal mathematizing, respecting the level principle. In setting the stage for employing trigonometric functions for mathematical modelling, the intertwinement principle is considered. Further, the interactivity principle is kept through the group work context of the task. The guidance principle, however, will not apply to this task design. For most themes in engineering mathematics, a reinvention can be a considerably challenge to embed in task designs.

Students were placed in groups of 3 or 4 when working with the task, and a 90 minutes session was spent on the task. The author of the paper was the teacher throughout this

session¹. The study was performed under a naturalistic research paradigm to ensure as realistic settings as possible for the observation of students' collaboration (Moschkovich & Brenner, 2000). The students were filmed using a high definition camera that followed the group work for the whole session. Prior to the filming, students had acknowledged their participation in the study through signing a letter of consent. The selected students had watched the videos beforehand. Statistics about which videos and how long the students had spent watching them were available in the tool I used for video-distribution to the students. I analysed each recording using descriptive accounts (Miles & Huberman, 1994), where sessions of data are broken into separate entities of activity. Each such entity were subsequently analysed for content informing us on the research question, and I report on a variety of such episodes in what follows. In the aftermath of the last case, I performed an interview with two selected students to obtain deeper insights about certain episodes during the work with the task and general qualities about videos and the session.

Furthermore, the study was performed on the cohort of 2017/2018, a class consisting of 20 students. The two groups that I report from had highly varied backgrounds, most of them from a pre-calculus course comprising two years of mathematics from upper secondary into 1 year of study at the university. They were acquainted with trigonometric functions through this pre-calculus course before arriving at the engineering study. However, as one of the students mentioned in an interview afterwards, he only remembered sparsely what the various mathematical terms was named by, but claimed to be able to use them anyway. I filmed two of the groups to be capture some of the variation taking place inside the single classroom environment. I will refer to these groups as group 1 and group 2.

I specifically looked for traces of content from the videos, that is, the out-of-class component of FC when analysing students' discourse. Such content can be word use, theoretical elaborations like definitions and theorems, visually mediated ideas and specific examples.

RESULTS

The two groups seemed to develop a rather different focus in terms of how to approach the task. One group spent much more time on the formal mathematizing part of the task, compared to the other one that wanted to understand the movement from a more informal, empirical viewpoint.

Group 1

The students in this group followed the layout of the task in the sense that they attempted to sketch a graph of the movement before moving on towards formal mathematical modelling. This seemed to set the scene for a discussion among the group

¹ A pilot-study in a very similar setting was performed on the previous cohort of 2016. This was part of a cooperation project with San Diego State University, where the research fellow Matt Voigt was responsible for the teaching material in addition to conducting the in-class teaching.

members on which physical properties were influencing the model. They constantly argued about where the “x-line/dotted line” should be, meaning the baseline, in addition to discussing how the various lengths of the rods and rotation times were influencing the amplitude and period. However, synthesizing the two movements into one seemed to form the biggest challenge for the group. For instance, the teacher was consulted on how one could decide the amplitude on a combined sine. After about 15 minutes of trying to comprehend this, the group moved on to a more formal mathematical phase of modelling. One of the students, let us call him Finnegan, appeared to get a breakthrough at about 17:40 (timestamp corresponds to video recording). Figure 3 show this situation where he is pointing to the printed unit circle that expresses various exact values of sine and cosine, while he states:

“I suggest that we make two functions that are zero when... They are not zero, but $\frac{3\pi}{2}$ at 2π . Then you would have made a function that is zero when the period... Well you have to make two functions...”

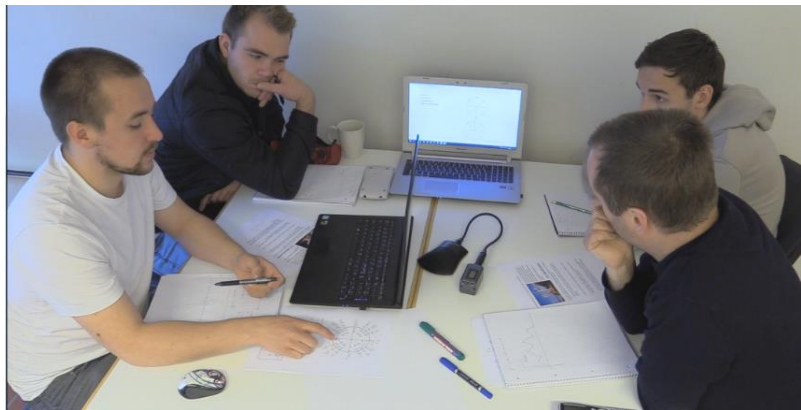


Figure 3: Phase shift of the sine mediated through the unit circle

After this episode, the students seemed to prefer working in two pairs to discuss further details of the functions. Both pairs of students agreed that the model should consist of a sum of two different sine functions.

Analysis

We can see that Finnegan struggles with the correct phrasing of what he is trying to express, but the visual connection between the Ferris wheel circle and the unit circle is guiding Finnegan towards an understanding of phase shifting. The printout of the unit circle became an important mediating artefact for Finnegan, although shared with students earlier for solving trigonometric equations, not for modelling. They spent much of the time getting accustomed to the mathematical terminology with amplitude, period and, as one sees, phase shifts. The informal part of the task where students were asked to elaborate on a sketch of the function seemed to facilitate these discussions, much in line with the interactivity and level principles of RME.

I was unable to make any clear connection to the videos from the discourse of these students. During the interview with one of the students in this group, it was pointed out that using English as the formal language in the videos was difficult to handle.

Group 2

The other group that was filmed during this session utilized a variety of means to depict the movement of the Ferris wheel. In Figure 4 we can see two of the students working in parallel, where one is considering using a graphing calculator, while the other is attempting to graph the model on GeoGebra. On the right PC the applet with the simulation is displayed.

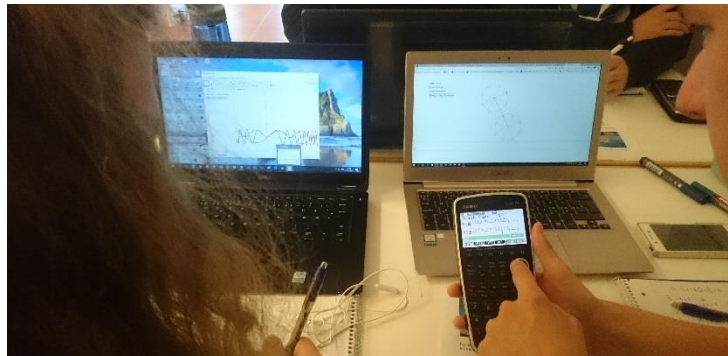


Figure 4: Utilizing various digital tools to model

Another member in this group even mapped the movement from the PC screen to a sheet of paper, letting the pen trace the movement of a point on the Ferris wheel, similar to what was reported in Sweeney and Rasmussen (2014). The group also attempted other practical means of tracing the movement like taking screenshots in timed intervals and putting the variable height values displayed in the applet in a function table for later analysis. It was even suggested that they would let GeoGebra use these data-points to produce an expression for them using interpolation. After working with various such “empirical” approaches to the problem, mostly on an individual basis, one student arrived at an expression for the model at 49:16. He gave this explanation to the others in the group on how he was able to arrive at a summation of two sinusoidal functions:

“Analyse the small circle, height over time, that became $f(x)$, then we did the same with the large one, called this $g(x)$, then we made a new one $h(x)$ consisting of the two of them together”

Analysis

The big surprise in this session was that the one student that had not prepared watching the videos seemed to be the leading the group discussions. It might seem that RME can act to remedy group dynamics where certain individuals come unprepared. The reality principle in RME stresses the importance of using real-life situations which is likely to provoke engagement, discussions and active participation during group work.

In this particular data set, it seemed hard to find a direct link between not being prepared by the out-of-class session and poor participation in the group work although this was a prevalent finding in other analyses of group work (Fredriksen, Hadjerrouit, Monaghan, & Rensaa, 2017).

An important observation that was made for both these groups was the physical arrangement of four students placed face to face, which made it harder to collaborate on the task. This was a setup originally meant to spur more discussion since the students would face each other. However, it turned out to be a poor design in this particular setting due to the importance of using the laptops for controlling the visual simulation in addition to the actual modelling in a digital graphical environment. It became

apparent that two students would look at one PC, while the adjacent students would look at an opposing one, effectively hindering much of the discussion across the group (Figure 3 and Figure 4) The teacher recognized this, trying to convince students to look at the same PC, but they quickly went back to work in pairs again. One of the students also confirmed this observation during the post-interview. Thus, it seems that this configuration interfered with the interactivity principle of RME for the groups.

DISCUSSION

In this study, I investigated to what extent RME task design facilitates students' modelling activities and knowledge construction in a FC context.

The analysis found students were making active use of computers, calculators, printouts of the unit circle, internet, and even their mobile phones to support their exploration of the task. This can be seen to support the *intertwinement* principle: students were actively using various tools, in addition to the mathematics introduced in the videos, to explore the problem.

The second group seemed to have problems progressing towards the formal mathematizing part of the task. An observation made from studying the group dynamics was a quite individualistic working attitude. There seemed to be little collaboration on working towards a common model for the group, effectively violating the *interactivity* principle of RME. During the post-interview with one of the students in the group, I asked him if he shared this impression, which he confirmed, stating that some kind of internal competition among the members evolved.

The *guidance* principle of RME refers to students' re-invention of mathematics. Although there are examples of RME facilitating such inductive discovery (Gravemeijer, 1999), we can conjecture that the FC out-of-class preparatory component turns this table. Through the videos, the students should have attained basic knowledge of mathematics that is to be articulated through the modelling activities in-class. Thus, instead of re-inventing the mathematics, they are rather re-employing it at a given situation and, through this, making conceptual ties towards it.

We can clearly see how the *level principle* of RME is at play in these observations. Initially, students are grappling with understanding the basics of the movement, using gestures, making sketches and talking to each other using informal language. The next stage of the task prompts students to express a formal mathematical description of the movement, which all students achieved to some extent. Although the students work was directed towards making a *model of* this particular movement, the whole-class discussion at the end of the session pointed towards similar situations that this could apply as a *model for*. This was also done in the videos prior to class.

The *reality* and *activity* principle seem to be well safeguarded in this task. This is also supported by the statements by one of the students during the informal interview conducted after the session:

“To see the task in a physical real framing was motivating”

Throughout the session, students used the familiar terms and concepts related to the sine function. Words like amplitude, phase shift, period and baseline were all referred to, and the students seemed to be able to connect these to the real-life situation of the Ferris wheel. If this really stems from watching the videos or originates from previous experience with the sine function, I have no direct evidence. This of course influences the capacity to answer the research question. Light on this may be shed by statements from the other student interviewed:

“It’s very nice that you can see the videos, think about it for some days, get it refreshed in the lesson and then start working with tasks related to it. This has helped me a lot”.

Although this is a statement about the general nature of the teaching layout, it nevertheless gives indication about FC as an instructional platform giving learners a potential for experiencing different motivations for mathematics.

CONCLUSION

When working with task design in mathematics, we employ various theoretical frameworks that has certain sets of design principles that the designer should adhere to (Kieran, Doorman, & Ohtani, 2015). We have seen that the Ferris wheel task design presented in this paper aligns well with the principles of RME for task design. However, designing such tasks for a FC pedagogic context will need an additional component, namely the adoption of the video preparation. Clearly, close attention to the pedagogical structure combining videos and tasks seem to be necessary for facilitating rich discussions, modelling activity and knowledge construction to take place in-class. This statement is supported by the cases that were analysed in this paper, where we saw students engaged in horizontal mathematization utilizing out-of-class preparation through videos and mathematical modelling of realistic situations.

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