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Comparison of two full-field identification methods for the wedge splitting test on a refractory

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Abstract

Two full-field identification methods are applied to the Wedge Splitting Test (WST) to obtain crack tip positions, stress intensity factors (SIFs) and T -stress. The first method is based on Finite Element Model Updating (FEMU), and the second is integrated digital image correlation (IDIC). Both are applied to a simplified virtual experiment and then to a cyclic WST. The gray level residuals are used to assess which results are more trustworthy. Fracture energy analyses are performed to validate the estimated R-curves.

Keywords: Crack tip position, digital image correlation, finite element model updating, stress intensity factors, virtual test, crack propagation

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1. Introduction

Refractory castables are ceramics that enable functional properties to be maintained in aggressive applications, generally at high temperatures [1]. Not only the chemical composition but also raw material characteristics should be considered when designing new formulations since they affect the resulting phases, microstructures and thermomechanical properties [2]. Different compositions may be needed for the same application because of several thermomechanical loading steps during industrial processes [3], which spur innovations to improve performance such as increasing reactivity during processing by using nanomaterials [2] or mimicking microstructures observed in nature [4]. In high risk applications, the material and mechanical properties should be well understood to better predict failures and thereby prevent accidents, while maximizing efficiency and lifetime [5].

The Wedge Splitting Test (WST) is a mechanical experiment allowing crack propagation to be assessed in (quasi)brittle materials [6, 7]. It leads to stable crack propagation by decreasing the elastic energy stored in the machine using a wedge and cylinders to apply the load [8]. A high fractured-area to volume ratio leads to achieving representative results for coarser microstructures at laboratory scales [7]. The classical goal of WSTs is to obtain the fracture energy, by integrating the load vs. displacement curve and dividing by the projected fractured area [7]. Although important, the fracture energy itself is not the only information that can be extracted from WSTs. Recently, it is becoming common practice to seek more data from each of these tests to better understand the fracture process. Relationships between WST measurements with microstructures can be obtained [9], and various works have shown how different compositions affect crack propagation (with the WST) for magnesia-spinel [10], $\text{Al}_2\text{O}_3\text{-C}$ [11], $\text{Al}_2\text{O}_3\text{-MgO-CaO}$ [12], and MgO-C [13, 14] systems. It is also possible to identify fracture properties using measured load data and compare them with numerical simulations of the WST [15].

More experimental data can be acquired from WSTs via full-field measure-

31 ments. Among them, Digital Image Correlation (DIC) enables displacement
32 fields to be measured [16], and has been successfully used in crack propagation
33 analyses [17, 18, 19, 20, 21, 22]. DIC was already applied to WSTs to ana-
34 lyze the strain fields for microcrack formation in magnesia refractories, when a
35 spinel phase was added [23]. Similar ideas were used to quantify the fracture
36 process zone of magnesia refractories, and highlight how microcracks tend to
37 decrease the strength but increase the fracture energy [24], and to estimate the
38 crack growth resistance [25]. Another interesting approach is to measure crack
39 propagation while checking the discontinuities in the displacement field [26]. An
40 integrated-DIC (IDIC) scheme to measure the R-curve behavior was also pro-
41 posed [27] by considering closed-form solutions of one propagating crack in an
42 elastic medium [28].

43 The aforementioned test [27] will be considered herein in order to compare
44 two different approaches for estimating fracture mechanics parameters using
45 full-field measurements. It is important to note that the hypothesis of one
46 straight crack is reasonable in experiments with the presence of a groove on the
47 propagating faces, as commonly used for the WST to ensure more straight crack
48 propagation [29]. However, crack branches may occur in the WST [25] and it
49 should be checked for each test. With both approaches investigated herein, this
50 check is part of the methodology.

51 In this paper, a procedure based on the methodology used in Ref. [30] is
52 applied to the WST. By using the outer measurements from DIC analyses as
53 Boundary Conditions (BC) for a Finite Element (FE) analysis and using internal
54 nodes for error estimators, it is possible to determine the crack tip position and
55 calculate Stress Intensity Factors (SIFs), *i.e.*, K_1 and K_2 , and the T-stress. It
56 will then be compared with IDIC. First, the experiment, the DIC principles and
57 both methodologies that will be compared are introduced. It is followed by an
58 analysis of a virtual experiment. Then an experimental study is performed to
59 compare both methods.

60 2. Methods

61 The two methods studied herein are summarized hereafter. Both of them
62 were used independently to analyze various experiments with cracks [30, 31,
63 20, 27]. However, they were never compared with the same data set, be they
64 synthetic or from an actual experiment. The first method couples FE analy-
65 ses and DIC measurements in order to determine the crack tip position, stress
66 intensity factors and T -stresses [30]. It belongs to the class of finite element
67 model updating techniques [32]. The second approach corresponds to inte-
68 grated DIC [20], which is a standalone technique in comparison with the pre-
69 vious framework when applied to the analysis of cracked samples. Augmented
70 Williams' series [28] are used, in particular, for the determination of the crack
71 tip position [33, 34].

72 2.1. Experiment

73 The WST analyzed herein was performed on a class C, anti-erosive com-
74 mercial refractory, with ultra low cement content, whose typical mineralogical
75 composition consists of quartz, mullite, kyanite, β -cristoballite and alumina [27].
76 Water was added to the mixture up to 8.5 wt% of concrete. Drying was per-
77 formed for 48h in humid environment at room temperature followed by 24h at
78 110°C. A heat treatment was performed with a 1°C/min rate and kept at 500°C
79 for 24h. The detailed chemical composition and the treatment of the material
80 are reported in Ref. [27]. Its processing and microstructure may lead to an in-
81 creasing R-curve behavior, with weakly bonded grains and initiated microcracks
82 due to anisotropic phases and differential thermal expansions.

83 The sample size is 100 mm in length, 100 mm in height and 72.5 mm in
84 thickness. The geometry is shown in Figure 1, in which it is possible to see the
85 contour of the sample and the loading devices (wedge, cylinders and blocks).
86 Two grooves (*i.e.*, lateral notches, see dashed line in Figure 1) are machined
87 on two opposite faces of the sample to reduce the local thickness and guide the
88 crack propagation vertically. The two zones where the splitting displacement is

89 evaluated via DIC are also shown in Figure 1 as yellow boxes. Considering δ as
 90 the initial distance between both zones, the horizontal displacements measured
 91 on these regions are averaged and their difference accounts for the splitting
 92 displacement $\Delta\delta$, which will be reported in Section 4.

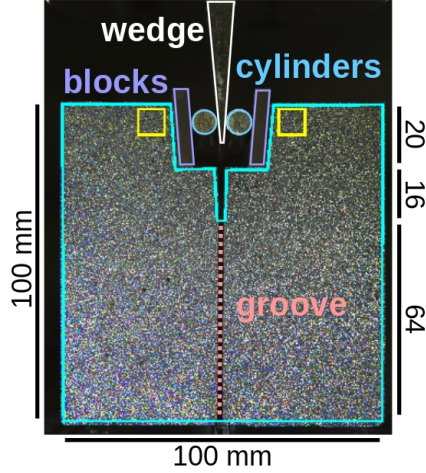


Figure 1: Detailed geometry of the wedge splitting test. The contour of the sample, including the pre-notch, is shown in cyan. The wedge, cylinders and blocks used to apply load are highlighted in white, blue and purple, respectively. The vertical groove in order to guide the crack vertically is shown with the dashed red line. The splitting displacement $\Delta\delta$ corresponds to the difference of the mean displacement of the two yellow boxes. The thickness of the specimen is 72.5 mm. All dimensions are expressed in mm.

93 The Young's modulus (E) and Poisson's ratio used for the investigated meth-
 94 ods are equal to 17 GPa (measured by the bar resonance method [35]) and 0.2,
 95 respectively. The test was driven by setting the velocity of the machine actuator
 96 to $1.3 \mu\text{m/s}$, and 313 pictures were taken for both faces of the specimen at a
 97 rate of one picture each 8 s. The images were simultaneously acquired with
 98 two Canon T5 cameras with 28-135 mm lenses, with the illumination provided
 99 by LEDs. The 16-bit picture definitions are 2601×1733 pixels, with a dy-
 100 namic range of approximately 60,000 gray levels. The imaged physical size of
 101 one pixel was $62 \mu\text{m}$. A random speckle pattern was sprayed onto the speci-
 102 men surfaces to increase the image contrast and improve the DIC resolutions.

103 The 5-cycle loading curve of the experiment, which corresponds to the vertical
 104 force F_v vs. vertical actuator displacement Δh , is shown in Figure 2. Further
 105 information on this test can be found in Ref. [27], and further characterization
 106 of the studied material, processing and microstructure in Refs. [36, 37].

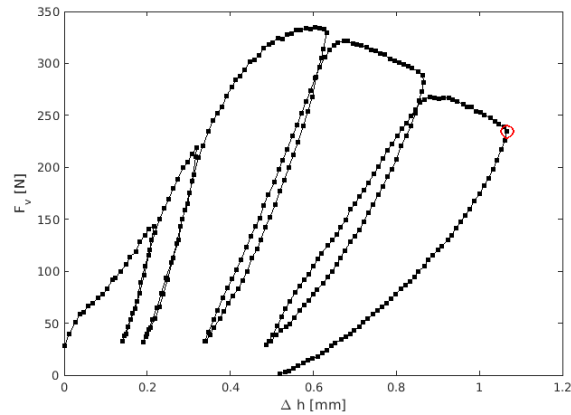


Figure 2: Loading history for the test analyzed in Section 4. Each point represents two image acquisitions (one per face) performed during the test for DIC analyses. The red circle depicts the picture acquisitions used to evaluate the strain fields shown in Figure 3.

107 2.2. Digital Image Correlation

108 In global DIC, the displacement fields $\mathbf{u}(\mathbf{x})$ are measured by minimizing the
 109 L2-norm of the gray level residuals, ϕ^2 , between the image of the reference state
 110 f , and at the deformed state g

$$\phi^2 = \sum_{\text{ROI}} [f(\mathbf{x}) - g(\mathbf{x} + \mathbf{u}(\mathbf{x}))]^2. \quad (1)$$

111 Searching for displacements at the pixel level is an ill-posed problem [38]. Pa-
 112 rameterizations of the sought displacement \mathbf{u} are used to regularize this problem
 113 and render the solution less affected by the acquisition noise

$$\mathbf{u}(\mathbf{x}) = \sum_{i=1}^N v_i \Psi_i(\mathbf{x}), \quad (2)$$

114 in which v_i are the degrees of freedom, and Ψ shape functions (*i.e.*, vector
 115 fields) that combine the displacement of a group of pixels in order to make the

116 problem well-posed. The solution becomes

$$\{\mathbf{v}_{\text{DIC}}\} = \arg \min_{\{\mathbf{v}\}} \phi^2(\{\mathbf{v}\}), \quad (3)$$

117 where $\{\mathbf{v}_{\text{DIC}}\}$ is the column vector gathering all amplitudes v_i . If the actual
 118 kinematics of the problem is not well known a priori, Ψ_i can be chosen as finite
 119 element shape functions [39]. In the present case, the DIC procedure is per-
 120 formed with 3-noded linear elements based on finite element discretization [40]
 121 and will be referred as T3DIC. In global approaches, the quality of the registra-
 122 tion can be evaluated pixel-wise by computing the gray level residuals

$$\rho_{\text{T3}} = f(\mathbf{x}) - g(\mathbf{x} + \Psi_{\text{T3}}(\mathbf{x}, \{\mathbf{v}_{\text{T3}}\})), \quad (4)$$

123 where Ψ_{T3} is the vector containing the shape functions converting nodal to
 124 pixel displacements, which depends linearly on the amplitude $\{\mathbf{v}_{\text{T3}}\}$. In the
 125 present case, the T3DIC mesh is composed of 3-noded elements whose average
 126 edge length is equal to 58 pixels (or 3.6 mm).

127 In the following, the global residual of DIC approaches will be compared. It
 128 is defined as the root-mean-square (RMS) gray level residual over the considered
 129 ROI

$$\bar{\rho}_{\text{T3}} = \frac{RMS(\rho_{\text{T3}})}{\Delta f}, \quad (5)$$

130 where Δf is the dynamic range of the picture of the reference configuration

$$\Delta f = \max_{\text{ROI}} f - \min_{\text{ROI}} f \approx 60,000 \text{ gray levels.} \quad (6)$$

131 The first step of any of the methods presented hereafter is to run T3DIC.
 132 It provides displacement fields that can be compared with FE analyses, but
 133 also allows the crack path to be chosen for integrated DIC [27]. The maximum
 134 *eigen* strain field is selected in order to check the validity of the straight crack
 135 propagation assumption and the presence of a single macro-crack. The two faces
 136 of the sample are analyzed with a very fine mesh of average element length of
 137 8.5 pixels (or 530 μm). Figure 3 shows the results for both faces for the last
 138 image before the final unloading (Figure 2). The standard uncertainty of the

139 maximum eigen strain is of the order of 3×10^{-4} and the minimum strain level
 140 in the color bars of Figure 3 is set to 3 times this value. It was determined by
 141 correlating the two pictures shot for the reference configuration on each face.
 142 Only one unique macro-crack is observed (guided by the groove).

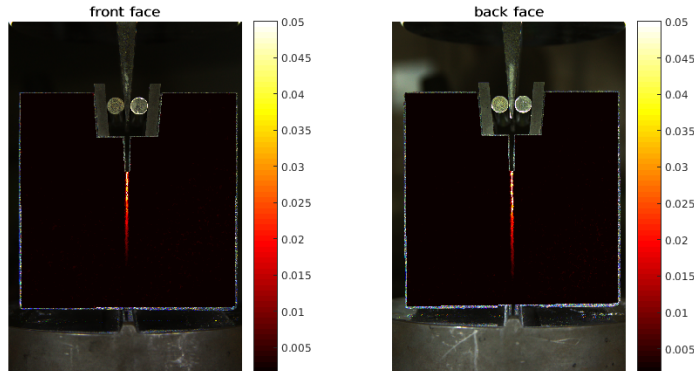


Figure 3: Maximum eigen strain fields for both analyzed faces for the image before the 5th unloading. No macro-crack branches are seen and consequently, the hypothesis of a straight crack propagation guided by the groove can be made.

143 2.3. Method 1: Finite Element Model Updating

144 The methodology described in this section is an adaptation of that proposed
 145 in Ref. [30] to the WST. DIC results are taken as Dirichlet boundary conditions
 146 and FE formulation computes the displacement field over the considered sur-
 147 face. The crack tip position is identified as the one that provides the best fit
 148 between the simulated and measured displacement fields. The main idea of the
 149 method is schematically shown in Figure 4, where the outer contour of T3DIC
 150 measurements, represented in green, are prescribed to the FE model, and inter-
 151 nal nodes (blue region) are used for comparison with FE analyses. Several crack
 152 tip positions are tested along the groove region (red dashed line), and the one
 153 that gives the least root mean squared displacement gap is considered the best
 154 estimate and thus, chosen as the crack tip position for the considered image.
 155 The method is then repeated for every image taken during the test. It will be
 156 referred to as FEMU henceforth.

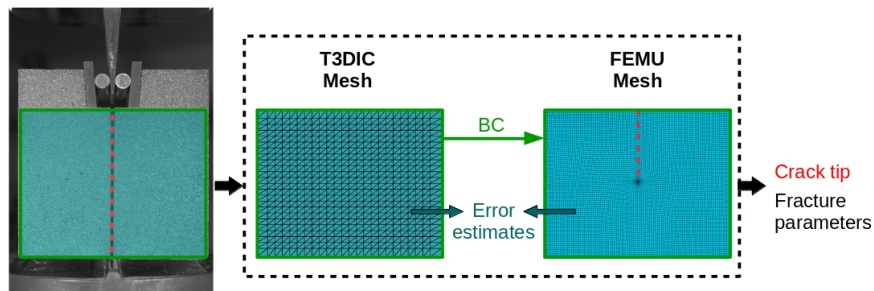


Figure 4: Adaptation of methodology introduced in Ref. [30] to the WST. An area around the propagation path is chosen and the displacements measured by T3DIC at the outer boundaries (green) are used as boundary conditions. The internal measured displacements (blue) are used for comparison with FE outputs, and the crack tip position is tested in the groove region (red).

157 It is important to highlight that such a procedure is run for *each* image
 158 taken during the experiment, so that one crack tip position is obtained for each
 159 considered step time. Within an FE code, it is generally possible to calculate
 160 quantities such as the mode I and II SIFs K_I , K_{II} , and the T -stress [41]. Thus
 161 the change of these fracture mechanics parameters is evaluated for each analyzed
 162 crack length. Given the thickness of the sample, a plane strain hypothesis is
 163 considered in this paper for all the reported analyses.

164 The FEMU displacements and mesh are exported and interpolated onto
 165 the T3DIC mesh. The nodal displacement difference between T3DIC and FE
 166 analyses is computed, and the displacement gap consists in the RMS difference

$$\phi_u^2 = \sum_{i=1}^N (v_i^{\text{FEMU}} - v_i^{\text{T3}})^2, \quad (7)$$

167 in which v_i are nodal displacements. The superscript indicates if the displace-
 168 ments are obtained from FEMU or T3DIC analyses. It is worth noting that the
 169 gap estimate proposed in Equation (7) considers the same influence from every
 170 considered node. This definition may be improved using the T3DIC Hessian
 171 to augment the weight on nodes with respect to lower measurement uncertain-
 172 ties [42]. Once different crack positions are tested, the one that provides the
 173 minimum ϕ_u is taken as the estimated crack tip position, and the fracture me-
 174 chanics properties are assessed with this configuration.

175 Although ϕ_u^2 is considered for crack tip identification, the gray level residual
 176 ρ_{FEMU} can be computed using the nodal displacements of the FE model $\{\mathbf{v}_{\text{FE}}\}$
 177

$$\rho_{\text{FEMU}} = f(\mathbf{x}) - g(\mathbf{x} + \mathbf{\Psi}_{\text{FE}}(\mathbf{x}, \{\mathbf{v}_{\text{FE}}\})), \quad (8)$$

178 where $\mathbf{\Psi}_{\text{FE}}$ is the vector containing the shape functions converting nodal to pixel
 179 displacements, which is linearly dependent on the measured degrees of freedom
 180 $\{\mathbf{v}_{\text{FE}}\}$. The corresponding global residual reads

$$\bar{\rho}_{\text{FEMU}} = \frac{RMS(\rho_{\text{FEMU}})}{\Delta f}. \quad (9)$$

181 It will be compared with T3DIC results and the following integrated method.

182 *2.4. Method 2: Integrated DIC*

183 This section summarizes the methodology used to analyze a wedge splitting
 184 test with Integrated-DIC [27] using a closed-form solution. Williams' series [28]
 185 describe the kinematics in this case, and the gray level residual is minimized
 186 (Equation 1), instead of the displacement gap for FEMU (Equation (7)). The
 187 sought displacement field reads

$$\mathbf{u}(\mathbf{z}) = \sum_{j=I}^{II} \sum_{n=p_i}^{p_f} \omega_n^j \boldsymbol{\psi}_n^j(\mathbf{z}), \quad (10)$$

188 where the vector fields are defined in the complex plane

$$\mathbf{z} = (x - x_c) + (y - y_c)i = r \exp(i\theta), \quad (11)$$

189 where (x_c, y_c) are the coordinates of the crack tip position, $j = I$ is related to the
 190 mode I (opening) regime and $j = II$ to mode II (shearing). The amplitudes ω_n^j
 191 become the unknown kinematic degrees of freedom of IDIC. The corresponding
 192 displacement fields are described by

$$\boldsymbol{\psi}_n^I = \frac{A(n)}{2\mu\sqrt{2\pi}} r^{n/2} \left[\kappa \exp\left(\frac{in\theta}{2}\right) - \frac{n}{2} \exp\left(\frac{i(4-n)\theta}{2}\right) + \left((-1)^n + \frac{n}{2}\right) \exp\left(-\frac{in\theta}{2}\right) \right], \quad (12)$$

193 and

$$\boldsymbol{\psi}_n^{II} = \frac{iA(n)}{2\mu\sqrt{2\pi}} r^{n/2} \left[\kappa \exp\left(\frac{in\theta}{2}\right) + \frac{n}{2} \exp\left(\frac{i(4-n)\theta}{2}\right) + \left((-1)^n - \frac{n}{2}\right) \exp\left(-\frac{in\theta}{2}\right) \right], \quad (13)$$

194 where κ is equal to $(3 - \nu)/(1 + \nu)$ for plane stress states or $3 - 4\nu$ for plane
 195 strain states, ν the Poisson's ratio and $A(n)$ is defined by

$$A(n) = \cos\left(\frac{n\pi}{2}\right)^2 + \sin\left(\frac{n\pi}{2}\right). \quad (14)$$

196 The amplitude ω_1^j gives access to Stress Intensity Factors (SIFs), the amplitude
 197 ω_2^I provides the T -stress (positive in compressive) and ω_2^{II} the rigid body rota-
 198 tion. Higher order fields account for deviations from the theoretical assumption
 199 of an infinite medium [34]. With the use of two additional terms in the series,
 200 namely using $p_i = 0$, ω_0^I and ω_0^{II} are related to rigid body translations.

201 Although not usual, negative values of p_i can help to account for nonlinearities
 202 ties [34, 30]. It can be seen from Equations (12) and (13) that for negative values
 203 of n , super-singular solutions arise near the crack tip ($r = 0$). They are also
 204 important to locate the crack tip position, especially ω_{-1}^I . With the assumption
 205 that the crack tip is perturbed by a small distance d , along with some recursive
 206 properties of the Williams' series [33], it is possible to derive the offset

$$d = \frac{2\omega_{-1}^I}{\omega_1^I}, \quad (15)$$

207 which provides an estimation of crack tip shift to find the correct solution. In
 208 the sequel, $p_i = -3$ is taken to account for nonlinearities [34]. The maximum
 209 value $p_f = 8$ is chosen after a convergence analysis [27]. A normalization of Ψ
 210 is also performed in order to decrease floating point rounding errors.

211 The pacman-like ROI used for IDIC¹ is shown in Figure 5. An opening of
 212 40 pixels (pacman mouth) is taken in order not to consider the cracked mouth.
 213 The Williams' series are projected onto an FE mesh to allow the use of the same
 214 FE-DIC code as used in the previous section. This mesh is chosen sufficiently
 215 fine not to influence the results. (A convergence study was performed to check
 216 this statement.)

¹This ROI is always centered about the evaluated crack tip position

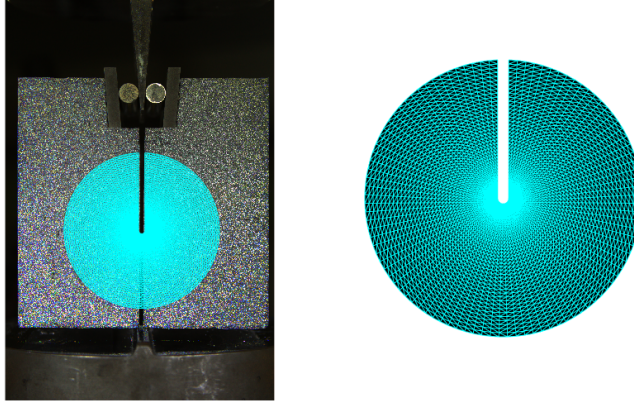


Figure 5: Example of the pacman-like mesh used in IDIC analyses.

217 The procedure to find the crack tip positions and the mechanical parameters
 218 of the fracture process can be summarized in the present steps:

- 219 1. A crack path is first defined as a straight line, in the groove region of
 220 the photographed sample face (Figure 4), as proved by the analysis of the
 221 maximum eigen strain fields (Figure 3);
- 222 2. The parameters defining the truncation of Williams' series are chosen, *i.e.*,
 223 $p_i = -3$, $p_f = 8$, the radius for normalization and mesh parameters;
- 224 3. The calculation is initialized for the crack position assumed to be located
 225 at the notch root;
- 226 4. An optimization algorithm evaluates the amplitudes $\{\omega\}$ that decrease
 227 the global gray level residual [20];
- 228 5. The crack tip position is updated using Equation (15);
- 229 6. Steps 4 and 5 are repeated until convergence (*i.e.*, $d < 0.1$ pixel).

230 After convergence, the corresponding gray level residual field is stored

$$\rho_{\text{IDIC}} = f(\mathbf{x}) - g(\mathbf{x} + \Psi_{\text{IDIC}}(\mathbf{x}, \{\omega\})), \quad (16)$$

231 where $\{\omega\}$ gathers all converged ω_n^j amplitudes (see Equation (10)), and Ψ_{IDIC}
 232 are the shape functions described in Equations (12) and (13) evaluated for each

233 pixel position \mathbf{x} . From this information, the global IDIC residual becomes

$$\bar{\rho}_{\text{IDIC}} = \frac{\text{RMS}(\rho_{\text{IDIC}})}{\Delta f}. \quad (17)$$

234 This methodology will be referred to as IDIC in Section 4.

235 **3. Analysis of a virtual experiment**

236 First, a virtual experiment is considered to test both approaches on a con-
237 figuration for which the exact solution is known. It is discussed how to deform a
238 reference image with displacements obtained from numerical simulations (*e.g.*,
239 *AbaqusTM*). In the sequel, this virtually deformed image is then analyzed using
240 the previous two methods. This virtual experiment will be referred to as VE in
241 Section 3.2

242 *3.1. Virtual experiment*

243 A sketch following the instructions from Ref. [9] (width = 100 mm and
244 height = 100 mm) is performed in *AbaqusTM*, with some adjustments such as
245 the depth of the extrusion set to 72.5 mm related to the sample geometry [27].
246 The numerical model is presented in Figure 6. Normal pressure was applied
247 to the vertical faces onto which the rollers would apply the load. The line at
248 the bottom of the sample does not move in the x and y -directions. In the z -
249 direction, one single point has no motion. A straight crack is added and its tip
250 is located in the middle of the crack propagation path of the sample (*i.e.*, only
251 one loading step is considered). The mesh is refined around the crack tip. A
252 2D mesh is extracted from the image plane presented in Figure 6 to consider a
253 plane strain state. K_I , K_{II} , and the T -stress measurements at this plane are
254 considered.

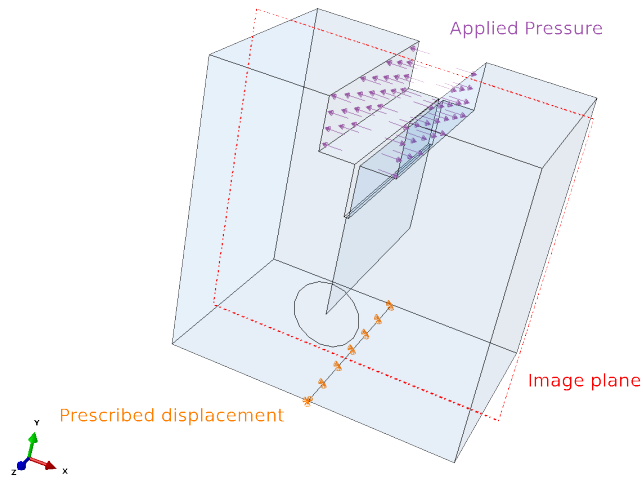


Figure 6: 3D *Abaqus*TM model showing the applied pressure and the prescribed displacements. It is also possible to see the partition lines in the middle of the crack propagation path. The used image plane is also shown.

255 For the present study, a reference image is also required. An *actual* image
 256 that shows the whole sample surface [27] is used in the sequel. The 2D mesh is
 257 extracted and interpolated onto pixel coordinates such that it fits the sample in
 258 the image, as shown in Figure 7.

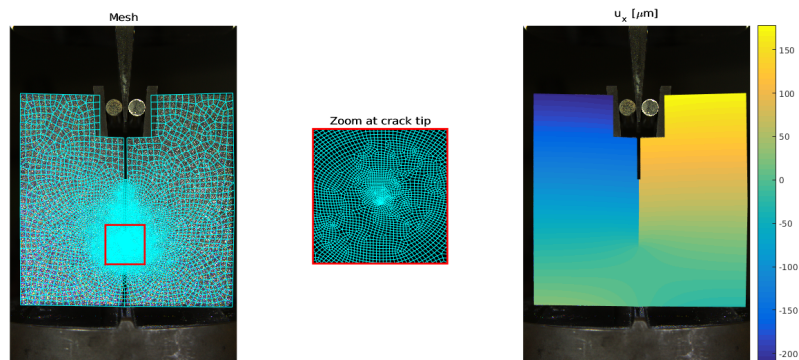


Figure 7: Superposition of 2D mesh extracted from the 3D *Abaqus*TM mesh for the reference image (on the left), with a zoom about the crack tip (on the middle). Horizontal displacements u_x (expressed in μm) (on the right) in the virtual experiment.

259 Once the displacement field for each pixel is known, the deformed image is
 260 created. In DIC the gray level conservation reads

$$f(\mathbf{x}) = g(\mathbf{x} + \mathbf{u}(\mathbf{x})), \quad (18)$$

261 in which \mathbf{x} contains integer pixel coordinates in the reference image. However the
 262 position $\mathbf{x} + \mathbf{u}(\mathbf{x}) = \boldsymbol{\theta}(\mathbf{x})$ is not necessarily an integer. The evaluation of $g(\boldsymbol{\theta}(\mathbf{x}))$,
 263 which corresponds to the picture of the deformed configuration corrected by
 264 the displacement field \mathbf{u} , requires an interpolation scheme of the gray levels.
 265 In the present case, an inverse mapping is required, namely, integer valued
 266 positions \mathbf{x}_g are sought to construct the picture in the deformed configuration
 267 g . Consequently, the position $\boldsymbol{\theta}^{-1}(\mathbf{x}_g)$ has to be determined for computing the
 268 gray level $f(\boldsymbol{\theta}^{-1}(\mathbf{x}_g))$, which also requires an interpolation scheme [43].

269 To be more representative of a real experiment, it is chosen to add real noise
 270 to the deformed image. The gray level residual from T3DIC results for the first
 271 deformed image of the front face is added to the artificially deformed image.
 272 The global residual is equal to 0.57%. This residual at the beginning of the test
 273 is considered to be a good approximation of the noise during the test since the
 274 load was small and no crack propagation had yet occurred.

275 3.2. Results

276 The results obtained from the VE analysis with different methods are gath-
 277 ered in Table 1. The global residuals, $\bar{\rho}_{\text{method}}$, in which *method* is replaced by
 278 VE, FEMU or IDIC, are assessed by correcting the image in the deformed con-
 279 figuration g with the corresponding displacement fields (see Section 3.1, Equa-
 280 tions (8) and (16), respectively). For consistency along the discussion, all the
 281 reported residuals are calculated in a pacman-shaped area centered about the
 282 crack tip position for the discussed *method* (see Figure 5).

Table 1: Virtual experiment results. Calibrated fracture mechanics parameters and corresponding global gray level residuals.

	Δa [mm]	K_I [MPa $\sqrt{\text{m}}$]	K_{II} [MPa $\sqrt{\text{m}}$]	T -stress [MPa]	$\bar{\rho}_{\text{method}}$ [% Δf]	$\bar{\rho}_{\text{method}}^{\text{T3}}$ [% Δf]	$\Delta\bar{\rho}_{\text{method}}^{\text{T3}}$ [% Δf]
VE (Sect. 3.1)	36.0	3.8	3×10^{-4}	6.4	1.68	2.45	-0.77
FEMU (Sect. 2.3)	38.9	3.1	3×10^{-3}	7.8	2.52	2.46	0.06
IDIC (Sect. 2.4)	36.4	3.9	-6×10^{-2}	8.8	2.48	2.45	0.03

283 The fact that the residuals for the exact solution are not vanishing is due
284 to the added noise to g and gray level interpolation inaccuracies. Consequently,
285 $\bar{\rho}_{\text{VE}}$ defines the minimum level that can be achieved. Both methodologies (*i.e.*,
286 FEMU and IDIC) are consistent with the exact (*i.e.*, VE) solution since the gray
287 level residuals ($\bar{\rho}_{\text{FEMU}}$ and $\bar{\rho}_{\text{IDIC}}$) are only 1.5 times higher than the minimum
288 level ($\bar{\rho}_{\text{VE}}$), and that their difference is very small with a value that is slightly
289 lower for IDIC. The later better predicts the crack tip position (Δa in Table 1)
290 and mode I SIF for an elastic body with a single crack, while the usage of
291 the FEMU procedures provides T -stress levels more accurately. Mode II SIF is
292 believed to be close to the resolution for both methods.

293 Since the exact solution is known in the present case, the discussion could
294 stop here. However, in an actual experiment, the exact solution is unknown.
295 Since the ROIs of each method are not located at the same position in the
296 reference image, the crack tip locations predicted by both methods are expected
297 to be different. T3DIC will thus be used to independently assess global residuals
298 computed over the same ROI as those in the considered *methods*. Since in T3DIC
299 no mechanics-based assumptions are made on the displacement fields apart from
300 their continuity,² the global residuals $\bar{\rho}_{\text{method}}^{\text{T3}}$ for the same ROI of each *method*
301 are also evaluated. The difference in global residuals

$$\Delta\bar{\rho}_{\text{method}}^{\text{T3}} = \bar{\rho}_{\text{method}} - \bar{\rho}_{\text{method}}^{\text{T3}} \quad (19)$$

²Note that the cracked area is masked by the pacman mouth (Figure 5)

302 then assesses the overall identification quality (*i.e.*, $\bar{\rho}_{\text{method}}^{\text{T3}}$ is thus taken as the
 303 reference) and the smaller $\bar{\rho}_{\text{method}}^{\text{T3}}$, the better the identification result.

304 Table 1 shows that $\bar{\rho}_{\text{method}}^{\text{T3}}$ is virtually identical for the three methods. This
 305 is expected since this virtual case only involves noise and gray level interpola-
 306 tion inaccuracies. The fact that $\bar{\rho}_{\text{VE}}^{\text{T3}}$ is greater than $\bar{\rho}_{\text{VE}}$ is due to the T3DIC
 307 mesh that is rather coarse. This choice was made since very small displacements
 308 are sought (see below) and a finer mesh would have induced higher measure-
 309 ment uncertainties [44]. This choice also explains why $\Delta\bar{\rho}_{\text{VE}}^{\text{T3}} < 0$. Had a finer
 310 mesh been used, T3DIC would be expected to be closer to the VE solution.
 311 Furthermore, $\bar{\rho}_{\text{FEMU}}^{\text{T3}}$ is slightly higher than $\bar{\rho}_{\text{IDIC}}^{\text{T3}}$, and more importantly, IDIC
 312 is closer to T3DIC than FEMU (*i.e.*, $\Delta\bar{\rho}_{\text{IDIC}}^{\text{T3}} = 0.03\%$ in comparison with
 313 $\Delta\bar{\rho}_{\text{FEMU}}^{\text{T3}} = 0.06\%$). The difference between both methods remains very small,
 314 which validates both procedures. However, IDIC slightly outperformed FEMU
 315 in the present analysis.

316 4. Experimental study

317 The two methodologies described in Sections 2.3 and 2.4 are now applied to
 318 one wedge splitting test. Figure 8 shows the crack tip position for the first part
 319 of the experiment during which the crack has propagated. $\Delta a = 0$ considers
 320 the crack tip to be located at the pre-notch root. Both methodologies have high
 321 uncertainties for the first two cycles, which are related to very small displace-
 322 ment ranges (*i.e.*, 0.15 pixel, or 9 μm at the most). It is observed that FEMU
 323 identifies smaller crack lengths than IDIC. It is worth noting that both method-
 324 ologies predict different crack propagation histories on the two analyzed faces
 325 and that, in the end, the crack propagated farther on the back face (Figure 3).

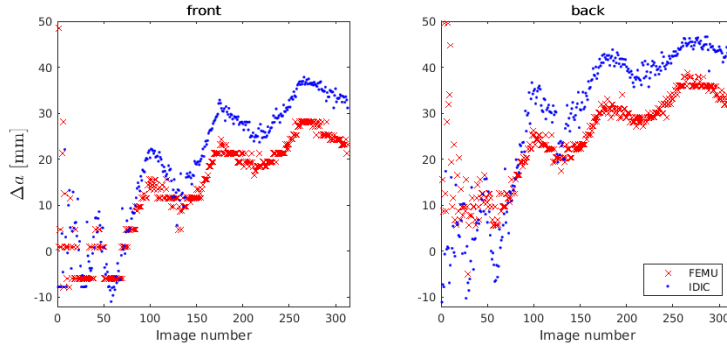


Figure 8: Crack tip position for front and back faces with both methodologies for the first 312 images acquired during the analyzed wedge splitting test.

326 Two features in Figure 8 need to be further commented. First, the negative
 327 values of Δa at the beginning of the experiment, namely, a crack tip position out
 328 of the sample, and second, the crack closure in the unloading phase of each cycle.
 329 These observations do not mean that crack healing occurs in the present case. It
 330 is believed that the negative values are due to the uncertainties associated with
 331 crack tip positioning when the displacement levels are very small (*i.e.*, at the
 332 beginning of the experiment). The crack tip itself is a feature associated with
 333 the considered fracture mechanics model (here defined with Williams' series
 334 or finite element simulations). Although the main conclusions of this work
 335 will not be affected, physically, crack propagation is believed to be stopped
 336 during unloading phases, and restarts once it reached a critical SIF level in the
 337 subsequent loading cycle.

338 The SIFs are reported in Figure 9. For this case in which a single macro-
 339 crack propagates guided by the groove (Figure 3), pure mode I is expected and is
 340 confirmed by both approaches. As K_{II} is close to zero, it can be used as an eval-
 341 uation of the resolution for SIF evaluations. The RMS of K_{II} values measured
 342 by both methodologies and at both faces is of the order of $3 \times 10^{-2} \text{ MPa}\sqrt{\text{m}}$.
 343 The general tendency observed for K_I is opposite in comparison with estimates
 344 of crack tip positions, namely, lower K_I levels and larger crack lengths are re-
 345 ported by IDIC in comparison to FEMU results. However, crack tip positions

346 and SIFs values obtained by both techniques are of the same order of magnitude.

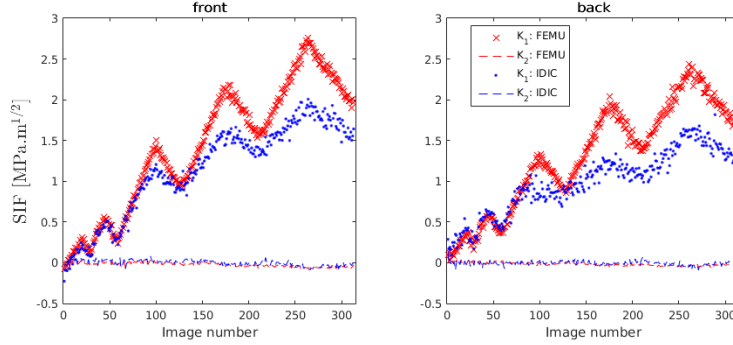


Figure 9: Mode I and II SIF histories for both faces with both approaches.

347 The T -stress histories are shown in Figure 10. The levels obtained via IDIC
 348 are generally higher, predominantly after the second cycle and for the front
 349 face. For the back face, the values are closer. The fluctuations could be related
 350 to mechanical features since some fluctuation were also observed on the load
 351 vs. crack mouth opening displacement curve for this test [27], but further studies
 352 are needed to confirm this hypothesis.

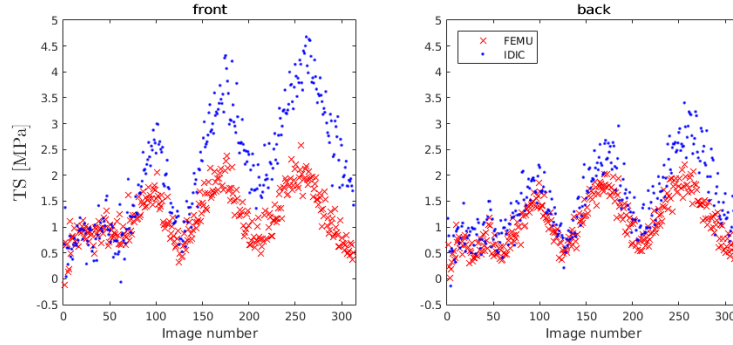


Figure 10: T -stresses measured with both methodologies for both faces.

353 In order to have an absolute evaluation of the quality of both approaches
 354 $\bar{\rho}_{\text{FEMU}}$ and $\bar{\rho}_{\text{IDIC}}$ are reported. This type of analysis is no longer an inter com-
 355 parison, but probes the individual merit of each technique with respect to the
 356 pictures acquired during the experiment with the same number of pixels. The

357 RMS gray level residuals are reported in Figure 11. The overall levels remain
 358 very small for the whole sequence, which validates the reported results. For both
 359 cases, increasing residuals occur with crack propagation. The longer the crack,
 360 the bigger the fracture process zone and the assumption of an elastic medium
 361 is presumably less true. Further, 3D effects may also be more pronounced [45],
 362 which make the 2D assumptions used in Williams' series and FE calculations
 363 less accurate.

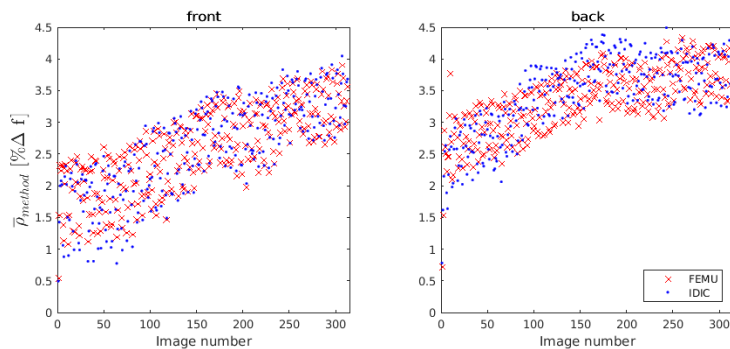


Figure 11: Dimensionless residuals for the two methods (FEMU and IDIC) and both faces.

364 To compare in a more quantitative way the two procedures the residual
 365 difference, $\Delta\bar{\rho}$, is computed

$$\Delta\bar{\rho} = \bar{\rho}_{\text{IDIC}} - \bar{\rho}_{\text{FEMU}} \quad (20)$$

366 Plots of $\Delta\bar{\rho}$ for both faces of the sample are shown in Figure 12. The differences
 367 are in 0.2% range, which is lower than the acquisition noise for these images
 368 (*i.e.*, $\approx 0.6\%$). However, IDIC shows lower residuals at the beginning the test.
 369 The crack tip position is better captured (Figure 8), even though the SIF levels
 370 are rather consistent with both approaches (Figure 9). The fact that IDIC
 371 becomes less accurate than FEMU at the end of the propagation step may be
 372 related to the Williams' series not describing the boundary effects as the crack
 373 tip approaches the sample edge.

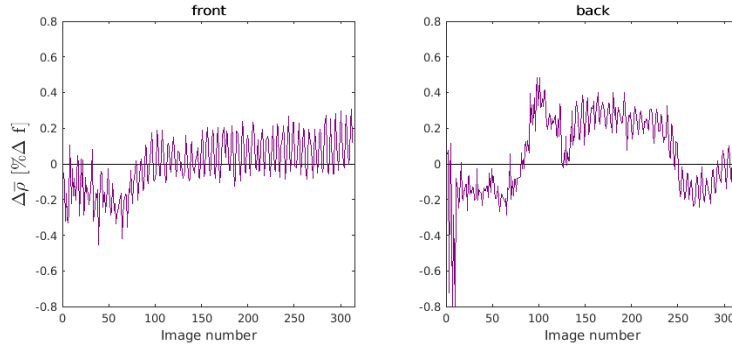


Figure 12: Residual difference for both faces. A negative (resp. positive) value corresponds to a more faithful evaluation via IDIC (resp. FEMU).

374 The residuals reported in Figure 11 and 12 were calculated for the same
 375 ROI size but centered about different crack tip positions (Figure 8). In a virtual
 376 experiment $\bar{\rho}_{\text{method}}^{\text{T3}}$ does not change considerably in different regions as reported
 377 in Table 1. In an actual experiment, not only slightly different image features at
 378 each ROI but also images artifacts such as lighting changes and lens focus may
 379 affect $\bar{\rho}_{\text{method}}^{\text{T3}}$. The residual difference $\Delta\bar{\rho}_{\text{method}}^{\text{T3}}$ (see Equation (19)) is shown
 380 in Figure 13 and indicates how close each *method* was close to T3DIC. IDIC
 381 residuals are closer to T3DIC than FEMU. Small negative values seen for IDIC
 382 and FEMU in some images indicate that they outperformed T3DIC, which can
 383 be explained by the non-optimal T3DIC mesh. Although the residuals for both
 384 methodologies are in the same range (Figure 12), the ROIs in which FEMU
 385 converged were presumably less affected by image artifacts and the residuals
 386 were farther from T3DIC residuals.

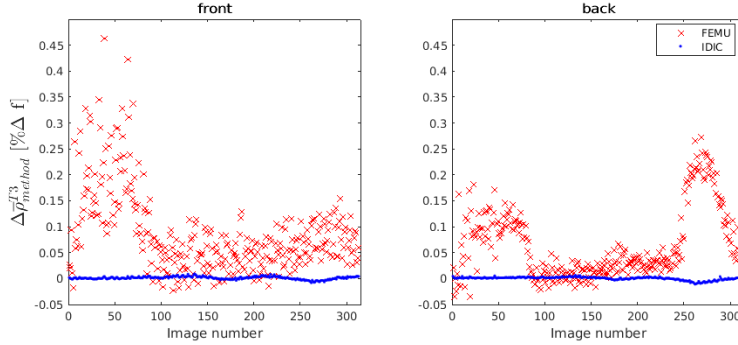


Figure 13: Differences from IDIC and FEMU residuals in comparison with T3DIC. Positive values are related to smaller residuals measured with T3DIC.

387 The main conclusion when considering different regions separately (Figure11
 388 and 12) is that IDIC was performing better at the very beginning and then
 389 FEMU would be preferred for the rest of the test. However, using $\bar{\rho}_{\text{method}}^{\text{T3}}$ as
 390 a reference (Figure 13) in order to account for image artifacts and textures
 391 on different regions, the conclusion is that, for the present case, IDIC is more
 392 accurate for the whole analysis.

393 Last, even though very small, K_{II} is considered to analyze the R-curve
 394 behavior defined as

$$R = \frac{K_I^2 + K_{II}^2}{E}(1 - \nu^2) \quad (21)$$

395 in a plane strain state, as considered in FEMU and IDIC. It is worth noting that
 396 although some deviation from linear elasticity may occur, the effective crack tip
 397 is measured with full-field approaches accounting for elasticity. In such cases,
 398 with the effective instead of visible crack tips, Equation (21) can be applied [46].

399 The R-curve is then plotted as a function of crack tip position in Figure 14. It
 400 shows that energy consumption increases as the crack propagates (with a steeper
 401 slope for FEMU), which is related to extrinsic toughening mechanisms such
 402 as crack branching, microcracking or bridges between aggregates [47]. In the
 403 present case, it was checked that crack branching was not occurring out of the
 404 groove in the photographed surface of sample, at the scale of DIC measurements
 405 (see Figure 3), namely, of the order of the element length (*i.e.*, ≈ 0.5 mm).

406 With the chosen magnification, the main purpose of DIC was to analyze the
 407 macroscopic fracture mechanics parameters, *i.e.*, R-curve behavior , and could
 408 resolve the effect of the toughening mechanisms occurring at lower scales. The
 409 observed R-curve behavior may indicate crack branching, and although not seen
 410 on the strain fields reported in Figure 3, it may take place in the bulk of the
 411 material. For checking such mechanisms, in situ tomographies analyzed with
 412 Digital Volume Correlation would be a suitable approach [48].

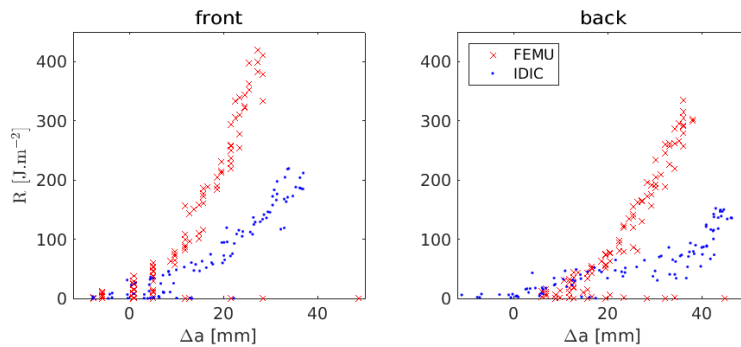


Figure 14: R vs. crack length for both faces with both approaches. Images taken during unloading steps are not accounted for the computation of R .

413 The present study enables to assess uncertainties associated with the use
 414 of two different identification techniques. The latter ones provided results that
 415 were both considered as realistic and may be used when comparing crack propa-
 416 gation in different materials compositions, for instance. However, they led to
 417 significant differences in the crack propagation resistance curve of the investi-
 418 gated material (Figure 14). Thanks to the use of gray level residuals, the merit
 419 of both techniques could be assessed, but more importantly it could be decided
 420 which one was more trustworthy (*i.e.*, IDIC in the present case).

421 In order to validate the R-curves reported in Figure 14, a last study is
 422 performed. First, the loading history shown in Figure 2 may be integrated to
 423 estimate the fracture energy, $\gamma_{wof(v)}$

$$\gamma_{wof(v)}^{\text{method}} = \frac{1}{2A^{\text{method}}} \int_{\Delta h_0}^{\Delta h_f} F_v d(\Delta h) \quad (22)$$

424 where the interval $[\Delta h_0, \Delta h_f]$ corresponds to the loading envelope and the final
 425 unloading, Δh is the vertical displacement of the actuator, F_v the vertical force,
 426 and method is substituted by IDIC or FEMU for the evaluation of the cracked
 427 area A . The latter is considered as the maximum crack length (Δa_{max} for
 428 last point before final unloading) obtained for the method (IDIC or FEMU)
 429 multiplied by the thickness t_g of the specimen inside the groove (*i.e.*, 65 mm).
 430 It is worth noting that $\gamma_{wof(v)}$ corresponds to an *upper bound* since it includes
 431 dissipated energy through friction of the loading parts.

432 The splitting force F_h vs. displacement $\Delta\delta$ curves are shown in Figure 15,
 433 in which the splitting displacement $\Delta\delta$ is measured from T3DIC results at the
 434 locations shown in Figure 1, namely, at the same height as the splitting forces
 435 are applied.

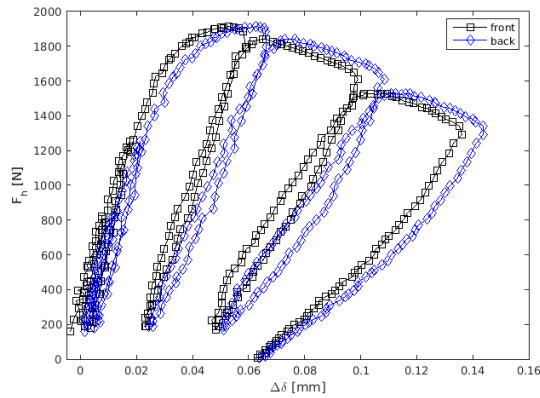


Figure 15: Splitting force (*i.e.*, 5.715 times the vertical force) versus splitting displacement $\Delta\delta$ measured at the locations shown in Figure 1.

436 By integrating the data shown in Figure 15, the fracture energy γ_{wof} reads

$$\gamma_{wof}^{\text{method}} = \frac{1}{2A^{\text{method}}} \int_{\Delta\delta_0}^{\Delta\delta_f} F_h d(\Delta\delta) \quad (23)$$

437 where the interval $[\Delta\delta_0, \Delta\delta_f]$ corresponds to the loading envelope and the final
 438 unloading. In the present case, the DIC extensometry only accounts for the
 439 work performed on the specimen itself, thus is more representative of the work

440 of fracture of the studied material.

441 To calculate the average of R , which is denoted by \bar{R} , for IDIC and FEMU
 442 the R-curves shown in Figure 14 are integrated

$$\bar{R}^{\text{method}} = \frac{1}{\Delta a_{max}} \int_{\Delta a_0}^{\Delta a_{max}} R^{\text{method}} d(\Delta a^{\text{method}}) \quad (24)$$

443 Since the dissipated energy is consumed to create two cracked surfaces, $\bar{R} =$
 444 $2\gamma_{wof(h)}$ [49].

445 The calculated energies are listed in Table 2. All reported values are expected
 446 to underestimate the fracture energy measured for the full crack propagation
 447 (with same methodology), which is not accessible in the experiment reported
 448 herein. The fracture energies have the same order of magnitude, which is a fur-
 449 ther validation of the R-curves predicted with IDIC and FEMU. However, IDIC
 450 predicts values that are closer to those obtained with DIC extensometry than
 451 FEMU. FEMU even provides estimates above the upper bound (*i.e.*, $\gamma_{wof(v)}$).

Table 2: Fracture energies (expressed in J/m²) calculated with different approaches.

	Fract. energy	front	back	mean
FEMU	$2\gamma_{wof(v)}$	114	83	99
	$2\gamma_{wof}$	92	71	82
	\bar{R}	162	97	130
IDIC	$2\gamma_{wof(v)}$	85	69	77
	$2\gamma_{wof}$	69	60	64
	\bar{R}	84	52	68

452 Even though FEMU and IDIC results are in the same range for the various
 453 quantities reported in the present section, IDIC results are more consistent with
 454 experimental data in terms of gray level residuals, but even more importantly
 455 when compared with independent estimates of fracture energies. For the latter
 456 ones, it is shown that about 20% is dissipated by friction when the upper bound
 457 estimate is compared with that derived from the splitting force vs. displacement
 458 data, or equivalently by R-curves provided by IDIC.

459 5. Conclusions

460 The FEMU methodology [30] was adapted to analyze WSTs. An automated
461 procedure was implemented to create an FE model with Dirichlet boundary
462 conditions measured via T3DIC analyses. The crack tip position was identified,
463 among several tested positions, as the one that provided the best kinematic fit.
464 Interaction integrals of the FE code were used to estimate fracture-related prop-
465 erties (*i.e.*, SIFs and T -stress). This approach was compared with IDIC when
466 applied to a WST [27]. First, a virtual experiment was analyzed, then followed
467 by experimental analyses for both faces of a refractory castable specimen.

468 FEMU allows for better T -stress estimates, while IDIC is more trustworthy
469 for estimating the crack tip position and mode I SIF in the virtual experiment.
470 Gray level residuals were used to check the merit of each technique directly with
471 experimental pictures. FEMU residuals were farther from T3DIC than IDIC
472 when considering T3DIC as the reference to check the quality of measurements,
473 proving that IDIC is slightly more trustworthy than FEMU in the present case.

474 IDIC provides longer crack lengths coupled with lower SIFs in comparison
475 with FEMU. Consequently, the R-curve behavior is less steep for the former.
476 Different levels of properties on each side and measured by each method high-
477 lights the importance of such analyses performed on *both* sample faces. These
478 differences are related to experimental imperfections such as the fine alignment
479 of the wedge and microstructural heterogeneities.

480 Accessing gray level residuals is a powerful tool to check analyses with the
481 experimental data when the actual solution is not known. It is of utmost im-
482 portance to consider the measurement regions in such analyzes. IDIC residuals
483 were closer to T3DIC resulting in more reliable measurements.

484 The evaluations of fracture energies were consistent with both discussed
485 methods, namely, the same order of magnitude of the R-curves was obtained via
486 IDIC and FEMU. When compared with FEMU, IDIC estimates were closer to
487 those based on DIC extensometry. While IDIC was comparable to conventional
488 methods, FEMU resulted in fracture energies greater than the experimental

489 upper bound. All these differences show that the estimation of the crack tip
490 location, which was different for both investigated approaches, has to be very
491 accurate. The use of the first supersingular field in Williams' series for estimat-
492 ing the crack tip location via IDIC is further validated thanks to the present
493 study.

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