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Growth and shape of a laboratory alluvial fan

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Using laboratory experiments, we investigate the influence of water and sediment discharges on the morphology of an alluvial fan. In our flume, a single-thread laminar river deposits corundum sand into a conical fan. We record the fan progradation with top-view images and measure its shape using the deformation of a Moiré pattern. The fan remains virtually self-affine as it grows, with a nearly constant slope. We find that, when the sediment discharge is small, the longitudinal slope of the fan remains close to that of a river at the threshold for sediment transport. Consequently the slope depends on the water discharge only. A higher sediment discharge causes the fan's slope to depart from the threshold value. Due to the downstream decrease of the sediment load, this slope gets shallower towards the fan's toe. This mechanism generates a concave fan profile. This suggests that we could infer the sediment flux that feeds a fan based on its proximal slope.

DOI: [10.1103/PhysRevE.00.002900](https://doi.org/10.1103/PhysRevE.00.002900)**I. INTRODUCTION**

Alluvial fans are sedimentary deposits which form at the outlet of mountain ranges. When a river flows through a mountain range, the flow-induced shear stress entrains sediment particles and carries them downstream. As it reaches the plain, the river loses its lateral confinement, and its slope drops. These changes induce sediment deposition [1–5]. As the river deposits its load, its bed rises until it overflows to find a new path. This phenomenon, called avulsion, maintains the radial symmetry of the deposit through time [6–8]. The geometry of the fan thus depends on the conditions under which it was built.

Three parameters are known to control the slope of an alluvial fan: the water discharge Q_w , the sediment discharge Q_s , and the sediment size d_s [9]. Field observations show that, in most cases, the slope of the fan increases with d_s and Q_s , whereas it decreases with Q_w [10–12]. There is no consensus yet about the physical origin of these relations or about their mathematical formulation [12].

Analogues of alluvial fans are easily produced in the laboratory, and they accord qualitatively with field observations [13–16]. Most authors represent the slope as a function of the discharge ratio Q_s/Q_w and find a positive correlation [17–21]. This function, however, appears to vary from experiment to experiment, suggesting that we investigate the influence of the water and the sediment discharges independently [20].

Guerit *et al.* [22] have produced an alluvial fan confined between two plexiglass plates. This experimental setup dictates the width of the river and precludes avulsion, thus simplifying the interpretation of the results. When the sediment discharge is vanishingly small, the fan adjusts its shape to keep the river's bed near the threshold of motion. Its profile is then virtually linear, and its slope depends only on water discharge and sediment size. To the contrary, a finite sediment load affects the

fan's shape—it steepens with sediment discharge. As sediment gets deposited along the fan, the slope shallows downstream, resulting in a concave upward profile [12,22].

Building upon this study, we remove the plexiglass plates that confine the flow and build a complete fan with a more realistic geometry. By doing so, we allow the river to (1) spontaneously select its own size and slope [23] and (2) distribute sediment over a self-formed lobe [14,24].

We design an experimental setup to generate an alluvial fan fed by a single-thread channel. We first impose a low sediment discharge, to remain near the threshold of sediment motion, and use the work of Seizilles *et al.* [25] to express the slope of our fan as a function of the water discharge (Sec. II). We then increase the sediment discharge to investigate its impact on the fan's profile (Sec. III). Finally, we show that we can infer the water and sediment discharges of a fan from its profile.

II. SINGLE-THREAD LABORATORY FAN**A. Experimental setup**

We produce our experimental fan in a 80-cm-wide and 50-cm-long tank. At the back of the tank, a 15-cm-high vertical wall simulates a mountain front. At the center of this wall, a 2.5-cm-wide tilted channel directs the sediment and fluid mixture toward the tank. The opposite side of the tank is bounded by a trench to evacuate the fluid. To prevent any fluid accumulation at the outlet, the fan lies on a 60-cm-wide and 30-cm-long shelf covered with a 200 μm sieve mesh. This setup allows us to build a fan with an opening angle of 180° (Fig. 1).

We use a viscous mixture of water (40 %) and glycerol (60 %) to maintain a laminar flow, at a Reynolds number of about 50 (density $\rho = 1150 \text{ kg m}^{-3}$ and viscosity $\nu = 7 \times 10^{-6} \text{ m}^2 \text{ s}$). A header tank provides a constant fluid discharge, which value we monitor using an electromagnetic flowmeter (Kobold, MIK-5NA10AE34R). Water evaporation concentrates the glycerol mixture. We measure its density

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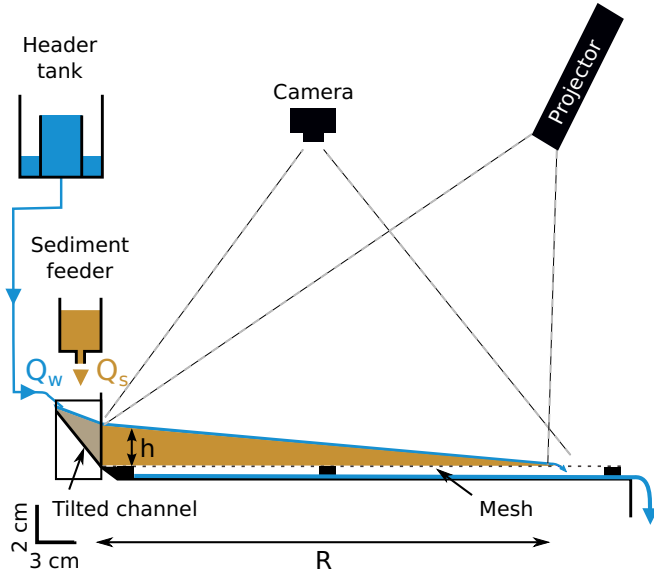


FIG. 1. Experimental setup. Blue: water-glycerol mixture. Gold: corundum sediment. Fluid and sediment discharges are Q_w and Q_s , respectively.

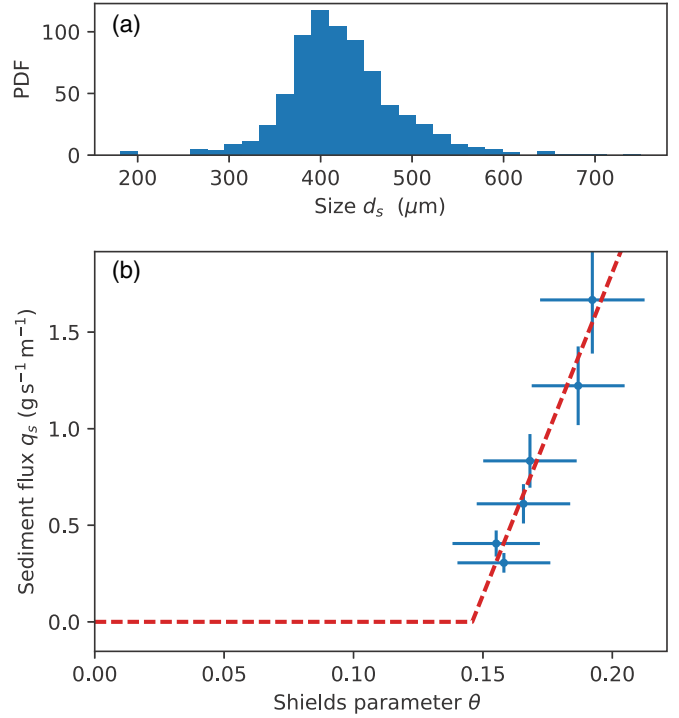


FIG. 2. (a) Sediment-size distribution. (b) Transport law. Blue dots and error bars: measurements. Red dashed line: Eq. (1), with $q_0 = 33.4 \pm 0.7 \text{ g s}^{-1} \text{ m}^{-1}$ and $\theta_c = 0.14 \pm 0.005$.

twice a day and compensate for evaporation with fresh water. As a result, the fluid discharge varies by less than 5% during an experiment, an indication that the viscosity of the fluid is also reasonably constant.

At the experiment inlet, a tilted channel mixes the fluid with a well-sorted sediment composed of corundum grains (crystalline aluminium oxide). We measure the grain size using image analysis [$d_{50} = 416 \pm 45 \mu\text{m}$; Fig. 2(a)] and its density using a pycnometer ($\rho_s = 3900 \pm 56 \text{ kg m}^{-3}$). At the macroscopic scale, the physical properties of the grain translate into a friction coefficient ($\mu = 0.7$) and a transport law relating the bedload intensity to the shear stress τ , namely,

$$q_s = q_0(\theta - \theta_c), \quad (1)$$

where $\theta = \tau / [(\rho_s - \rho)gd_s]$ is the Shields parameter with g the acceleration of gravity [26]. We measure the transport law of our corundum sediment with an independent experimental setup [22,27,28]. We find a critical Shields number $\theta_c = 0.14 \pm 0.005$ and a prefactor $q_0 = 33.4 \pm 0.7 \text{ g s}^{-1} \text{ m}^{-1}$ [Fig. 2(b)].

To produce a single-thread laboratory fan, the sediment discharge has to be low and constant throughout the fan growth. To achieve this, we have designed a conveyor-belt sediment dispenser. Sediments are stored in a hopper placed over a 3.5-cm-wide conveyor belt (Norcan). A stepper motor controlled by an Arduino motor shield drives the belt. To adjust the sediment discharge, we can adjust two parameters: the distance between the hopper and the belt, and the belt speed. To monitor the sediment discharge injected into the experiment, the entire device is placed on an electronic scale (Ohaus, Explorer 35), which measures the weight of the dispenser every minute.

B. Radially symmetric alluvial fan

At the beginning of an experiment, the surface of the experimental setup is bare. We start the fluid and sediment feed at the same time and keep them constant during the entire

experimental run. To monitor the fan evolution, we acquire top-view pictures using a camera fixed above the tank center (Canon EOS 100 D with a Canon 28-105 mm $f/3.5-4.5$ APO macro 0.5 m/1.6 ft Ultrasonic lens). We record a picture every 10 min (Fig. 3).

The growth of our fan is similar to the ones of Van Dijk *et al.* [19] and Reitz and Jerolmack [24]. The flow alternates between a single channel and a thin sheet of fluid covering a portion of the fan. In our case, however, the flow gets more and more channelized as the fan grows. The channel migrates either through the gradual erosion of its banks or through floodings and abrupt avulsion. This series of avulsions allows the channel to explore the entire fan surface, thus maintaining its radial symmetry (Fig. 3).

Using the top-view pictures, we measure the evolution of the fan radius. To do so, we develop an algorithm that automatically locates the fan toe. Because the fan color is well defined, we can detect its boundary by applying a threshold. We then measure the average radius of the fan by approximating its boundary by a half circle (red dashed lines on Fig. 3). We find that, typically, the distance from the apex to the boundary varies by less than 5% (Fig. 3). After a transient, which duration depends on the sediment discharge, the fan radius increases like the cubic root of time (Fig. 4). This experimental observation accords with the self-similar evolution of a semiconical deposit for which, based on mass balance, we expect $R \propto (Q_s t)^{1/3}$ [24,28].

Because of its radial symmetry, the fan's shape reduces to its downstream profile. Consequently, to describe the growth and shape of our laboratory fans, we need an accurate measurement of their profiles.

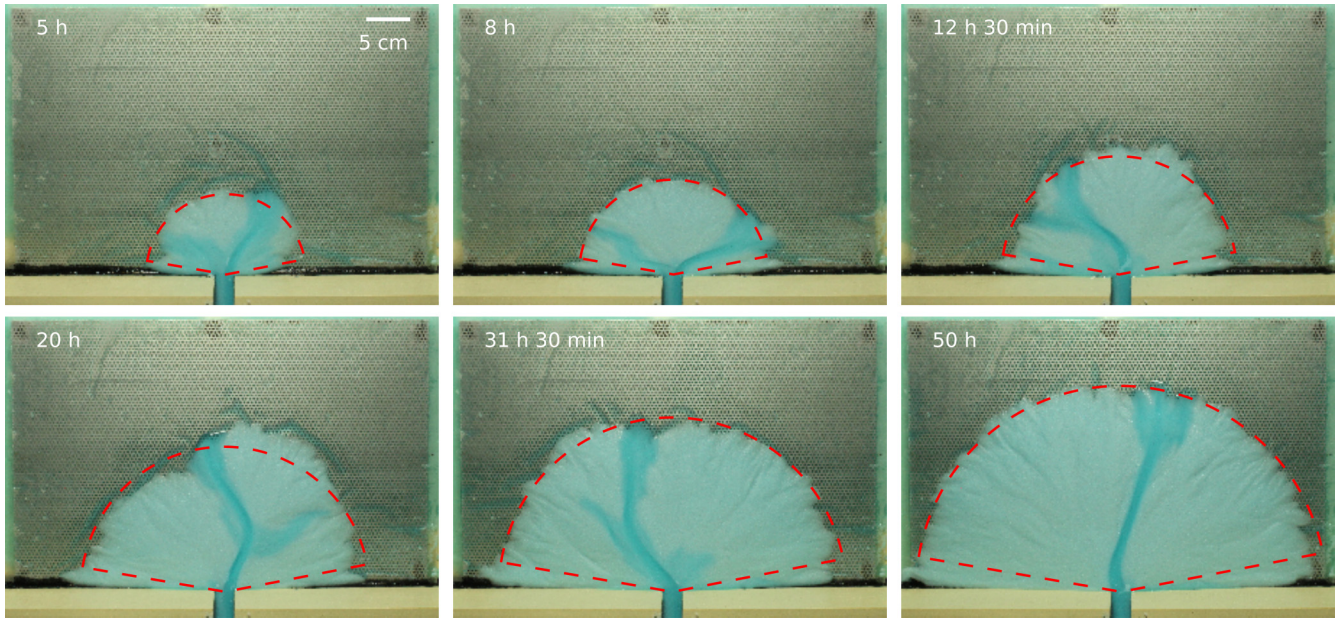


FIG. 3. Top-view pictures of an experimental fan during its growth, run 13 ($Q_w = 0.33 \text{ l min}^{-1}$ and $Q_s = 1.68 \text{ g min}^{-1}$). Fluid appears in blue, sediment appears in white. Red dashed line indicates the measured radius of the fan.

C. Measurement of the fan elevation

149 To measure the elevation of our experimental fan, we project
 150 a Moiré pattern onto its surface with a video projector [29,30].
 151 We then use the Light3D commercial software to calculate the
 152 digital elevation model (DEM) of the deposit [31] (Fig. 5).
 153 To evaluate the accuracy of our measurements, we scan
 154 the surface of a tray filled with milk and use it as horizontal
 155 reference. We find that the DEM of the milk surface is affected
 156 by lens distortion (raw data; Fig. 6). We thus measure the
 157 elevation of the milk surface at nine levels ranging from 7
 158 to 134 mm above the bottom of the tank.
 159

The lens-induced deformation depends on the coordinates
 of the pixel and on the distance from the lens (raw data; Fig. 6).
 We thus fit each DEM of the milk surface with a second-
 order, two-dimensional polynomial. We then evaluate the six
 coefficients of this polynomial for each elevation. By applying
 this correction to the measured DEM of the fan, we reduce the
 uncertainty of its elevation by a factor of about 10 (Fig. 6).
 After correction, the uncertainty on the measured elevation is
 less than $400 \mu\text{m}$, which corresponds approximately to the size
 of a sediment grain.

D. Self-similar growth

The elevation contours of the DEM are well approximated
 by concentric circles, thus confirming the radial symmetry of
 the fan (Fig. 5). This property suggests that we can compute
 the radially averaged profile of the fan with minimal loss of
 information [24,28] [Fig. 7(a)]. To evaluate how self-similar
 is the growth of the fan, we normalize its profile by rescaling
 horizontal and vertical dimensions with its radius, as measured

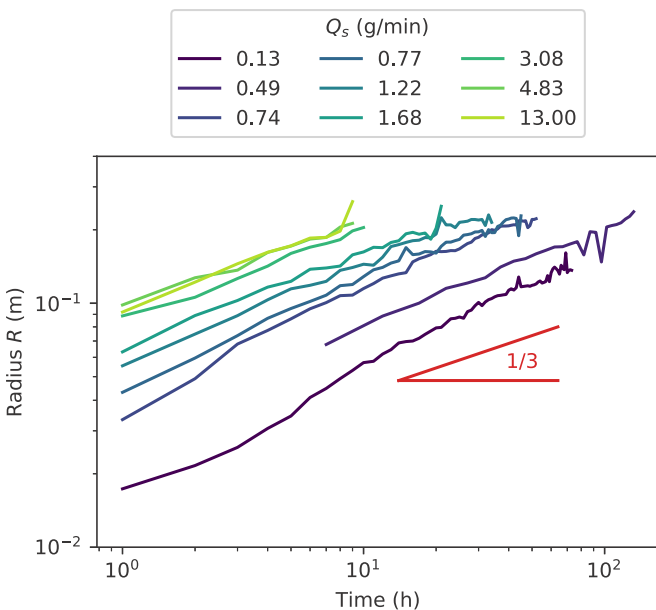


FIG. 4. Evolution of the fan radius ($Q_w = 0.33 \text{ l min}^{-1}$, Q_s from 0.13 to 13 g min^{-1}).

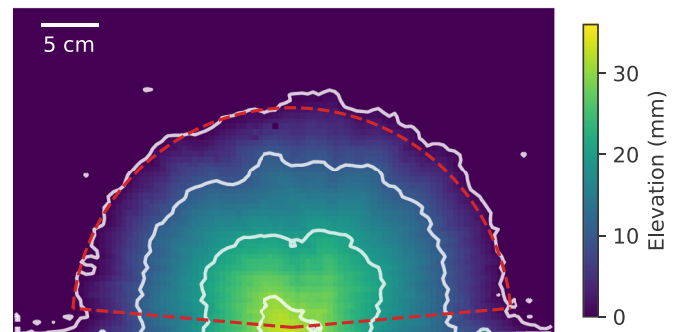


FIG. 5. Digital elevation model (DEM) of an experimental fan (run 13). Contours are 10 mm apart. Red dashed line indicates the measured radius of the fan.

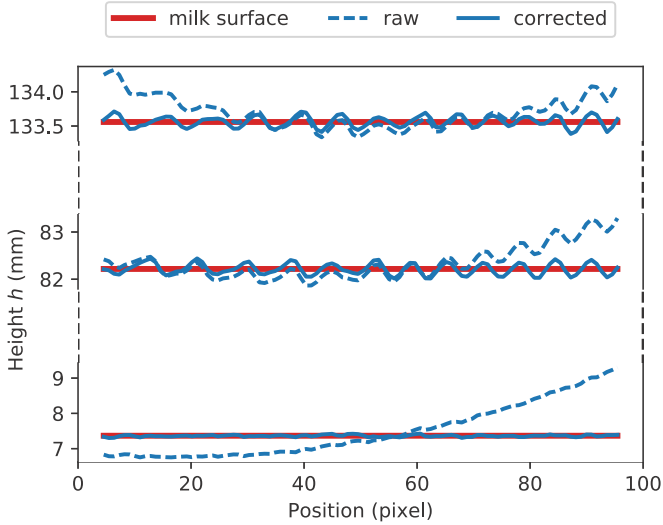


FIG. 6. Profile along the line of maximum deformation of the reference milk surface at different elevations. Dashed blue lines: raw data, solid blue lines: corrected data, red lines: actual elevation of the milk surface.

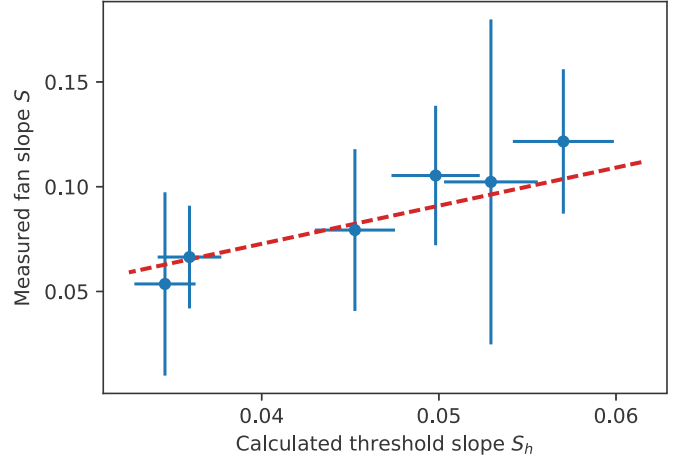


FIG. 8. Slope of the fan as a function of the theoretical threshold slope calculated using Eq. (2) (runs 1 to 6). The red dashed line is a linear fit with slope $A = 1.95$.

In the next section, we test this hypothesis using the work of Seizilles *et al.* [25].

E. Fan near threshold

The shape of an alluvial river results from the mechanical equilibrium of its bed [25,32,33]. When a sediment grain is immersed in a river, two forces act on it: gravity and the flow-induced shear stress. When a river transports a small amount of sediment, we expect its bed to remain near the threshold of sediment motion. At the threshold of motion, the normal component of these forces balances the tangential one. Using this assumption and the shallow-water approximation, Seizilles *et al.* [25] calculated that, for a laminar flow, the threshold slope of a river should read

$$S_h = \mu \left(\frac{4g}{9\nu} \right)^{1/3} \left(\frac{\theta_c \rho_s - \rho}{\mu \rho} d_s \right)^{4/3} \frac{1}{Q_w^{1/3}}. \quad (2)$$

We now compare this expression to the slope of our experimental fans. We perform six experiments where the sediment discharge, Q_s , remains fixed at a low value, whereas the fluid discharge, Q_w , varies between experiments (Table I). We then represent the average slope of our experimental fan as a function of the corresponding threshold slope, S_h [Eq. (2)] (Fig. 8). We find that the slope of our experimental fan is proportional to the threshold slope,

$$S = AS_h, \quad (3)$$

where A is a dimensionless coefficient, whose best-fit value is $A = 1.95 \pm 0.11$. If the shallow-water approximation were perfectly accurate, this coefficient would be one. The value of A we find by fitting our observations, however, depends on the parameters of Eq. (2) and therefore inherits their uncertainty. In Eq. (2) the value of the critical Shields parameter is the least reliable. Although we carefully estimated its value with a dedicated setup [Fig. 2(b)], this measurement remains a matter of debate [34]. Moreover, the critical Shields number is raised to the power 4/3 in Eq. (2). An error on its measurement has therefore a significant impact on the value of the threshold slope S_h .

from the top-view pictures. We find that individual profiles differ from their mean by less than 10 % [Fig. 7(b)].

Based on these observations, we propose to treat the fan's evolution as self-similar. Accordingly, relating the morphology of the fan to the input parameters reduces to understanding how these parameters control its dimensionless profile.

Figure 7 shows that we may approximate the fan profile with a straight line, at least when the sediment discharge is low. We further suggest that the river that feeds the fan controls its slope.

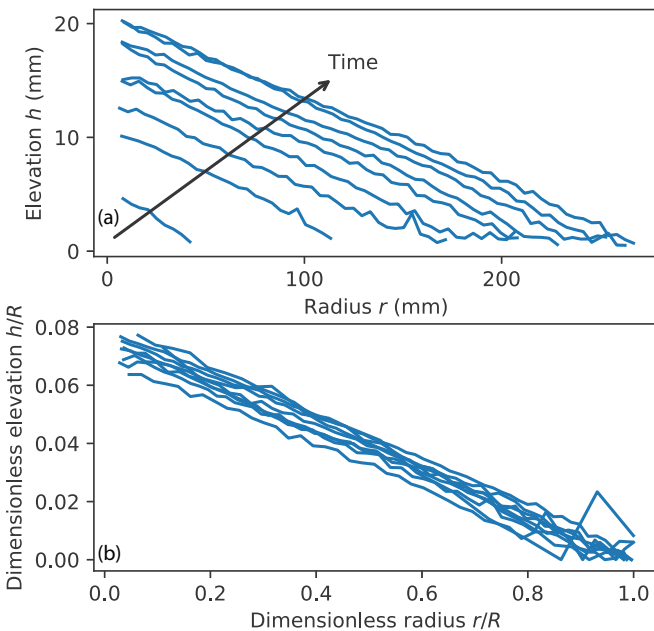


FIG. 7. Radially averaged profiles of an experimental fan (run 4, $Q_w = 0.3 \text{ l min}^{-1}$ and $Q_s = 0.2 \text{ g min}^{-1}$). (a) Six hours separate two lines. (b) Normalized fan profiles.

TABLE I. Experimental parameters for the experimental runs.

Constant sediment discharge		Constant fluid discharge					
$Q_s = 0.2 \pm 0.002 \text{ g min}^{-1}$		$Q_w = 0.33 \pm 0.02 \text{ l min}^{-1}$				$Q_w = 0.73 \pm 0.02 \text{ l min}^{-1}$	
Run	Fluid discharge (l min^{-1})	Run	Sediment discharge (g min^{-1})	Run	Sediment discharge (g min^{-1})	Run	Sediment discharge (g min^{-1})
1	0.9 ± 0.02	7	0.13 ± 0.0084	13	1.68 ± 0.048	18	0.82 ± 0.024
2	0.8 ± 0.02	8	0.49 ± 0.0056	14	3.08 ± 0.12	19	4.9 ± 0.15
3	0.4 ± 0.02	9	0.68 ± 0.013	15	4.83 ± 0.14	20	8.1 ± 0.28
4	0.3 ± 0.02	10	0.74 ± 0.016	16	6.18 ± 0.26		
5	0.25 ± 0.02	11	0.77 ± 0.019	17	13 ± 0.36		
6	0.2 ± 0.02	12	1.22 ± 0.028				

220 Regardless of the value of A , for a low sediment discharge,
 221 we find that the fan’s slope remains close to the threshold slope
 222 of an alluvial river, indicating that the fan inherits its slope from
 223 the river that builds it. We now investigate the influence of the
 224 sediment discharge on the fan’s profile.

225 **III. DEPARTURE FROM THRESHOLD: INFLUENCE OF**
 226 **SEDIMENT DISCHARGE**

227 **A. Sediment transport along the fan**

228 To evaluate the influence of the sediment discharge on the
 229 morphology of the fan, we perform 14 additional experiments
 230 with different values of Q_s (Table I). We evaluate the error on
 231 the sediment discharge by estimating geometrically the volume
 232 of sediment deposited in the inlet channel.

233 The observations of Sec. II B show that the deposit is radially
 234 symmetric, thus the spatially averaged profile of the fan suffices
 235 to represent its shape. Moreover, the fan grows self-similarly,
 236 which allows us to further average the resulting profiles
 237 (Sec. II D). To do so, we normalize each profile by dividing
 238 its horizontal and vertical coordinates by its radius. Figure 9
 239 shows essentially linear profiles, which steepen with sediment
 240 discharge. As the latter vanishes, the fan tends towards a cone
 241 at threshold, in accord with Sec. II E.

242 We now consider the influence of a finite sediment discharge
 243 on the fan’s slope. Because alluvial fans are depositional
 244 systems, only near the apex does the sediment discharge equals
 245 the input. Accordingly, we consider only the proximal slope
 246 S_p of our fans, where $r/R < 0.4$ (Fig. 9). However, the inlet
 247 disturbs the profile, which can be flat or even convex near
 248 the apex (Fig. 9). To avoid this disturbance, we measure the
 249 proximal slope, S_p , where the dimensionless radius, r/R , lies
 250 between 0.1 and 0.4 (proximal area on Fig. 9).

251 In our experiments, the water discharge takes two values
 252 (Table I, run 7 to 20) and so does the associated threshold slope,
 253 S_h (Sec. II). To investigate the departure from this threshold
 254 slope, we first normalize the proximal slope of our fans with the
 255 threshold slope, S_p/S_h . We then represent this dimensionless
 256 slope as a function of the dimensionless sediment discharge
 257 $Q_s/(q_0 d_s)$, where q_0 is the prefactor of the transport law
 258 [Fig. 2(b)] and d_s the sediment size (Fig. 10).

259 Our observations gather around a single curve the plane
 260 defined by the dimensionless sediment discharge and slope,
 261 suggesting that normalizing the fan’s slope with S_h removes the
 262 influence of water discharge. This finding accords with Guerit
 263 *et al.* [22], who were also able to separate the influence of water
 264 and sediment discharges, although with the different functional
 265 form that corresponds to the geometry of their experiment.

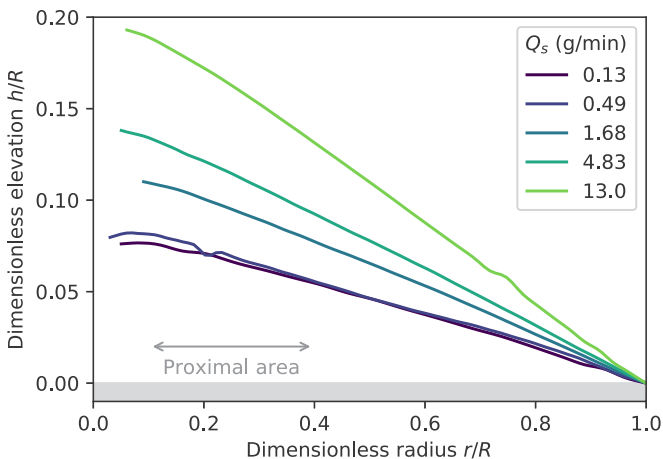


FIG. 9. Normalized fan profiles for five experiments ($Q_w = 0.33 \text{ l min}^{-1}$, Q_s from 0.13 to 13 g min^{-1}).

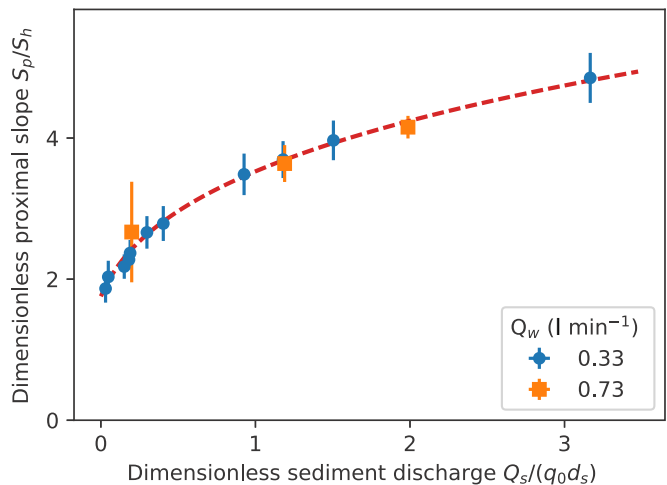


FIG. 10. Normalized proximal slope as a function of the dimensionless sediment discharge. Red dashed line: Eq. (4).

266 We now propose to fit our experimental data with the
267 following expression:

$$\frac{S}{S_h} = A \left(1 + B \frac{Q_s}{q_0 d_s} \right)^\alpha, \quad (4)$$

268 where B and α are dimensionless coefficients which we need
269 to fit to our data. By definition, A is the ratio of the measured
270 threshold slope to the theoretical threshold slope [Eq. (3)]. A
271 least-mean-square fit of the three parameters yields $A = 1.85$
272 ± 0.19 , $B = 0.18 \pm 0.09$, and $\alpha = 0.32 \pm 0.02$. This value of
273 A is close to the one of Sec. III E (Figs. 8 and 10).

274 Based on Eq. (4), we can now define rigorously what “a
275 small sediment discharge” means. When $Q_s/(q_0 d_s)$ is much
276 smaller than one, the influence of the sediment discharge on the
277 fan profile is negligible. We then recover the threshold slope
278 defined in Sec. III E.

279 Equation (4) formulates sediment transport at the scale the
280 river [as opposed to the local transport law of Fig. 2(b)]. It is a
281 function of the river’s slope, and therefore of the fan’s, which
282 results from the internal equilibrium of the river. We now inject
283 this empirical transport law into the sediment mass balance to
284 calculate the fan profile.

285 B. Fan profile

286 The fan grows from sediment deposition, a process formal-
287 ized by mass balance, which is often referred to as the “Exner
288 equation” in the context of sediment transport [35]:

$$\rho_s(1 - \lambda)\pi r \frac{\partial h}{\partial t} + \frac{\partial Q_s}{\partial r} = 0, \quad (5)$$

289 where h is the fan elevation and λ is the porosity of the deposit.
290 We estimate the value of λ by comparing the volume of the
291 deposit measured from its DEM to the volume of sediment
292 injected into the experiment $Q_s t$. We find $\lambda = 0.4 \pm 0.02$, a
293 common value for irregular grains.

294 The relation between the local slope of a river and its local
295 sediment discharge [Eq. (4)] expresses the sediment transport
296 along the fan:

$$\frac{B Q_s}{q_0 d_s} = \left(\frac{1}{A S_h} \frac{\partial h}{\partial r} \right)^{1/\alpha} - 1. \quad (6)$$

297 The system of Eqs. (5) and (6) is second-order in space and
298 first-order in time. It thus requires two boundary conditions
299 and an initial condition. The first boundary condition is set by
300 the sediment input at the fan’s apex:

$$Q_s(0, t) = Q_{s0}. \quad (7)$$

301 By definition, we also impose that the fan’s elevation vanishes
302 at its toe:

$$h(R, t) = 0. \quad (8)$$

303 As the fan grows, its radius increases and we do not know
304 R *a priori*—the domain over which we must solve the mass
305 balance has a free boundary. This Stefan problem thus requires
306 an additional boundary condition. In accordance with direct
307 observation of our experiments, we further assume that the fan
308 traps all the sediment it is fed with. Equivalently, the sediment
309 discharge vanishes at the fan toe:

$$Q_s(R, t) = 0. \quad (9)$$

310 We now seek a self-similar solution to Eqs. (5) and (6),
311 with the associated boundary conditions, Eqs. (7), (8), and (9).
312 We first define a similarity variable that takes into account the
313 growth of the fan:

$$X = \frac{r}{R}, \quad (10)$$

314 where the radius R is a function of time which needs to be
315 solved for. The dimensionless radius X varies between 0 and
316 1. Similarly, we define a dimensionless fan profile

$$H(X) = \frac{h(r, t)}{A S_h R} \quad (11)$$

317 and a dimensionless sediment discharge

$$\chi(X) = \frac{B Q_s}{q_0 d_s}. \quad (12)$$

318 Injecting these expressions into the sediment-transport equa-
319 tion [Eq. (6)] yields an ordinary differential equation:

$$H' = -(\chi + 1)^{1/\alpha}. \quad (13)$$

320 Next, we express the growth of the radius R as a function of the
321 fan’s shape and of the sediment input rate [Eq. (7)] based on the
322 total sediment balance. Namely, we integrate the elevation of
323 the fan over its domain, and differentiate the result with respect
324 to time. We find

$$Q_{s0} = \frac{\pi}{2} \mathcal{V} (1 - \lambda) \rho_s A S_h R^2 \frac{dR}{dt}, \quad (14)$$

325 where \mathcal{V} is the dimensionless volume of the fan:

$$\mathcal{V} = 6 \int_0^1 H X dX. \quad (15)$$

326 The coefficient 6 in the above definition ensure that a fan
327 at threshold has a dimensionless volume of one. Injecting
328 the dimensionless profile and sediment discharge [Eqs. (11)
329 and (12)] into the mass balance equation (5) and using the
330 radius-growth equation (14) yields another ordinary differential
331 equation:

$$\chi' = 2 \frac{\chi_0}{\mathcal{V}} X (X H' - H), \quad (16)$$

332 where χ_0 is the dimensionless sediment input:

$$\chi_0 = B Q_{s0} / q_0 d_s. \quad (17)$$

333 Finally, the two first-order, ordinary differential equations
334 (13) and (16) need boundary conditions. In dimensionless
335 variables, (8) and (9) become

$$\chi(1) = 0 \quad (18)$$

336 and

$$H(1) = 0, \quad (19)$$

337 respectively.

338 In a way, Eq. (16) is integro-differential, since it involves
339 the dimensionless volume \mathcal{V} . This oddity, however, does not
340 complicate much the numerical procedure we use to compute
341 the self-similar profile of the fan. We simply fix the ratio χ_0/\mathcal{V}
342 and solve Eq. (16) numerically with boundary conditions (18)
343 and (19). We then inject the resulting profile into Eq. (15) to get

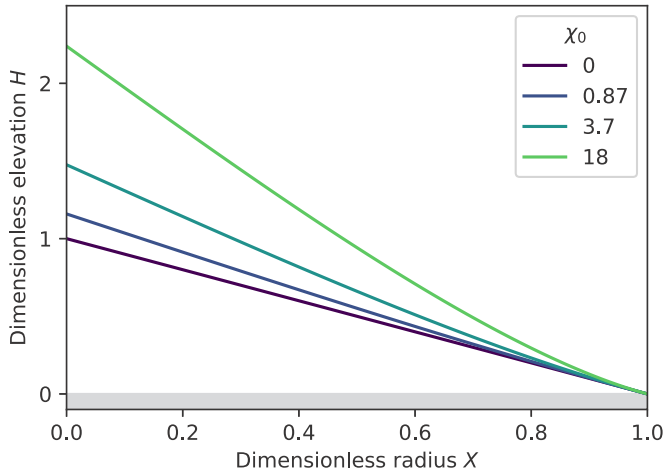


FIG. 11. Numerical fan profile for various values of the dimensionless sediment input χ_0 .

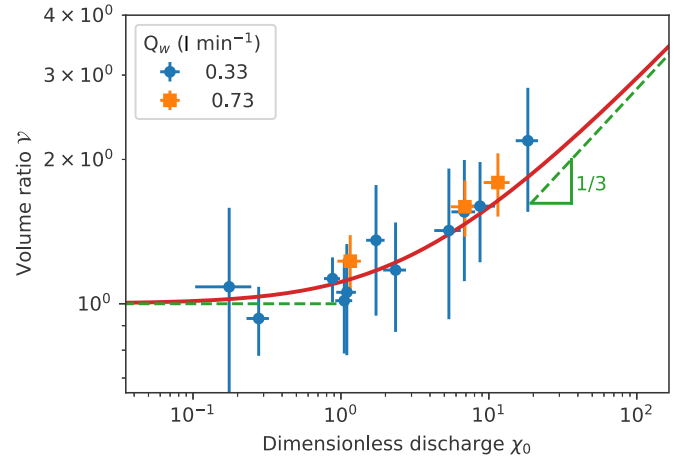


FIG. 12. Volume ratio as a function of sediment discharge. Blue and orange dots: experimental data ($Q_w = 0.33 \text{ l min}^{-1}$ and $Q_w = 0.73 \text{ l min}^{-1}$). Red line: numerical solution. Green dashed lines: asymptotes.

344 the dimensionless volume and thus the dimensionless sediment
345 input χ_0 (Fig. 11).

346 Like in our experiments, the numerical profiles steepen with
347 sediment discharge. Specifically, the proximal slope increases
348 with the sediment input rate and decreases downstream due
349 to sediment deposition. At the fan toe, all numerical profiles
350 merge, and the local slope reaches the threshold slope S_{th} , as
351 expected for a vanishing sediment flux. Sediment deposition
352 thus results in a concave upward profile.

353 When the sediment discharge vanishes, the entire profile
354 converges toward the linear profile associated to near-threshold
355 sediment transport.

356 In short, the slope near the fan toe is the threshold slope and
357 therefore records the water discharge. The proximal slope, on
358 the other hand, records the sediment discharge. In principle,
359 we can thus use the radial profile of a fan to infer both the
360 water and the sediment discharges that built it.

361 C. From morphology to discharge

362 The profiles of our experimental fans do not show any
363 concavity (Fig. 9), although their dependence on sediment
364 discharge is qualitatively similar to that of the numerical
365 profiles. A possible cause for this discrepancy between theory
366 and experiment is the divergence of the channel near the fan's
367 toe (Fig. 3).

368 Without measurable change in slope in our experiments, we
369 cannot infer the sediment discharge based on a local value of
370 the fan's slope. Instead, we may use an integral quantity, likely
371 to be less sensitive to measurement errors and natural variability.
372 One such quantity is the fan's dimensionless volume \mathcal{V} ,
373 which we expect to increase with sediment discharge.

374 Using the numerical procedure of Sec. III B, we compute the
375 dimensionless volume \mathcal{V} for a range of dimensionless sediment
376 discharge χ_0 (Fig. 12). The resulting curve transitions smoothly
377 between its two asymptotes. When the sediment discharge
378 vanishes, the fan returns to threshold, and its dimensionless
379 volume approaches one. On the other hand, a large sediment
380 discharge causes the fan to be so far from threshold that its

381 slope is essentially that of its apex, which grows as the sediment
382 discharge to the power 1/3—an unlikely regime in practice.

383 It is more convenient, and perhaps more telling, to express
384 the dimensionless volume as a ratio of volumes. Specifically,
385 \mathcal{V} is the ratio of the volume of the actual fan, which we refer
386 to as V_{fan} , to that of a hypothetical threshold fan with the same
387 radius:

$$\mathcal{V} = \frac{6 V_{fan}}{(1 - \lambda)\pi A S_{th} R^3}. \quad (20)$$

388 For illustration, we measure the volume of our laboratory
389 fans using their DEMs. We then calculate their volume ratio
390 \mathcal{V} according to the above formula and their dimensionless
391 discharge χ_0 according to Eq. (17). Plotting these quantities in
392 Fig. 12, we find that our experimental data accord reasonably
393 with the self-similar theory of Sec. III B.

394 IV. CONCLUSION

395 Using a simplified setup, we produced fans fed by a
396 single-thread river, which controls their slope, while avulsions
397 maintain their radial symmetry. The growth of the fan is
398 self-similar, and its radius grows like time to the power 1/3.
399 To assess the influence of the fluid and sediment discharges on
400 the fan profile, we combine these laboratory observations with
401 a first-order theoretical analysis.

402 For a low sediment discharge, the threshold of sediment
403 motion controls the slope of our experimental fans, as it
404 controlled the one-dimensional alluvial fan produced by Guerit
405 *et al.* [22]. When the sediment discharge increases, so does the
406 fan slope, especially near the fan apex, where the sediment
407 discharge is the highest. As a consequence, the fan profile
408 exhibits a slight curvature [22].

409 According to the semiempirical theory presented here, the
410 slope near the fan's toe is an indicator of fluid discharge.
411 Similarly, the ratio of apex slope to toe slope is a proxy for
412 sediment discharge. In this simplified framework, we can thus
413 use the present shape of an alluvial fan to infer the paleofluxes
414 that built it.

Before applying this theory to field observations, however, we will need to adapt it. First, the sediment discharge does not always vanish at the toe of a fan [36–38]. Consequently, we cannot use the distal slope as a straightforward proxy for water discharge; we first need to evaluate the sediment discharge that exits the fan. Given this measurement, we can probably extend the present theory to this new boundary condition.

In nature, alluvial fans are made of heterogeneous sediments. The river that transports them also sorts them according to mobility [28,39]. The coarser sediment gets deposited near the apex, whereas the finer one ends up at the toe or even exits the fan. This segregation causes the fan slope to decrease downstream [40]. Thus, both deposition and downstream fining translate into a concave upwards profile. To isolate these processes, we need reliable transport laws and a mass balance that can handle a distribution of grain sizes [12,28].

Sediment and water inputs are likely to vary during the fan history. If so, the self-similar theory presented here does not hold, as the shape of the deposit surface can record only the last increment of growth. However, the river probably adjusts

its profile to varying conditions quickly—at least with respect to the fan’s growth. If this is indeed true, the transport law and the mass-balance equation would still hold, and we could solve them numerically to follow varying inputs.

Finally, the sediment deposited at a specific period is often recognizable in the internal structure of the fan, in the form of a stratum. Interpreting these strata as proxies for paleoprofiles, geologists can reconstruct the shape of ancient fans [41,42]. Using these paleoprofiles as we have used the DEMs of our experiments, we might infer ancient mass fluxes and the tectonic and climatic forcings that induced them.

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