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## Growth and shape of a laboratory alluvial fan

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Using laboratory experiments, we investigate the influence of water and sediment discharges on the morphology of an alluvial fan. In our flume, a single-thread laminar river deposits corundum sand into a conical fan. We record the fan progradation with top-view images and measure its shape using the deformation of a Moiré pattern. The fan remains virtually self-affine as it grows, with a nearly constant slope. We find that, when the sediment discharge is small, the longitudinal slope of the fan remains close to that of a river at the threshold for sediment transport. Consequently the slope depends on the water discharge only. A higher sediment discharge causes the fan's slope to depart from the threshold value. Due to the downstream decrease of the sediment load, this slope gets shallower towards the fan's toe. This mechanism generates a concave fan profile. This suggests that we could infer the sediment flux that feeds a fan based on its proximal slope.

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### I. INTRODUCTION

Alluvial fans are sedimentary deposits which form at the outlet of mountain ranges. When a river flows through a mountain range, the flow-induced shear stress entrains sediment particles and carries them downstream. As it reaches the plain, the river loses its lateral confinement, and its slope drops. These changes induce sediment deposition [1–5]. As the river deposits its load, its bed rises until it overflows to find a new path. This phenomenon, called avulsion, maintains the radial symmetry of the deposit through time [6–8]. The geometry of the fan thus depends on the conditions under which it was built.

Three parameters are known to control the slope of an alluvial fan: the water discharge  $Q_w$ , the sediment discharge  $Q_s$ , and the sediment size  $d_s$  [9]. Field observations show that, in most cases, the slope of the fan increases with  $d_s$  and  $Q_s$ , whereas it decreases with  $Q_w$  [10–12]. There is no consensus yet about the physical origin of these relations or about their mathematical formulation [12].

Analogues of alluvial fans are easily produced in the laboratory, and they accord qualitatively with field observations [13–16]. Most authors represent the slope as a function of the discharge ratio  $Q_s/Q_w$  and find a positive correlation [17–21]. This function, however, appears to vary from experiment to experiment, suggesting that we investigate the influence of the water and the sediment discharges independently [20].

Guerit *et al.* [22] have produced an alluvial fan confined between two plexiglass plates. This experimental setup dictates the width of the river and precludes avulsion, thus simplifying the interpretation of the results. When the sediment discharge is vanishingly small, the fan adjusts its shape to keep the river's bed near the threshold of motion. Its profile is then virtually linear, and its slope depends only on water discharge and sediment size. To the contrary, a finite sediment load affects the

fan's shape—it steepens with sediment discharge. As sediment gets deposited along the fan, the slope shallows downstream, resulting in a concave upward profile [12,22].

Building upon this study, we remove the plexiglass plates that confine the flow and build a complete fan with a more realistic geometry. By doing so, we allow the river to (1) spontaneously select its own size and slope [23] and (2) distribute sediment over a self-formed lobe [14,24].

We design an experimental setup to generate an alluvial fan fed by a single-thread channel. We first impose a low sediment discharge, to remain near the threshold of sediment motion, and use the work of Seizilles *et al.* [25] to express the slope of our fan as a function of the water discharge (Sec. II). We then increase the sediment discharge to investigate its impact on the fan's profile (Sec. III). Finally, we show that we can infer the water and sediment discharges of a fan from its profile.

### II. SINGLE-THREAD LABORATORY FAN

#### A. Experimental setup

We produce our experimental fan in a 80-cm-wide and 50-cm-long tank. At the back of the tank, a 15-cm-high vertical wall simulates a mountain front. At the center of this wall, a 2.5-cm-wide tilted channel directs the sediment and fluid mixture toward the tank. The opposite side of the tank is bounded by a trench to evacuate the fluid. To prevent any fluid accumulation at the outlet, the fan lies on a 60-cm-wide and 30-cm-long shelf covered with a 200  $\mu\text{m}$  sieve mesh. This setup allows us to build a fan with an opening angle of 180° (Fig. 1).

We use a viscous mixture of water (40 %) and glycerol (60 %) to maintain a laminar flow, at a Reynolds number of about 50 (density  $\rho = 1150 \text{ kg m}^{-3}$  and viscosity  $\nu = 7 \times 10^{-6} \text{ m}^2 \text{ s}$ ). A header tank provides a constant fluid discharge, which value we monitor using an electromagnetic flowmeter (Kobold, MIK-5NA10AE34R). Water evaporation concentrates the glycerol mixture. We measure its density

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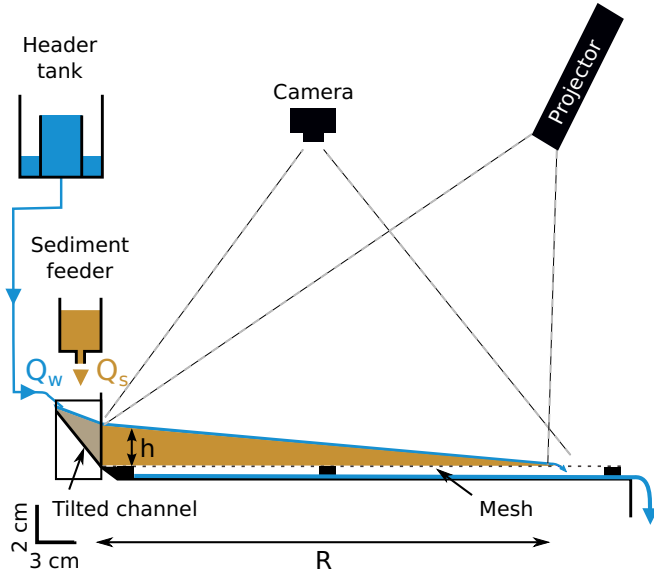


FIG. 1. Experimental setup. Blue: water-glycerol mixture. Gold: corundum sediment. Fluid and sediment discharges are  $Q_w$  and  $Q_s$ , respectively.

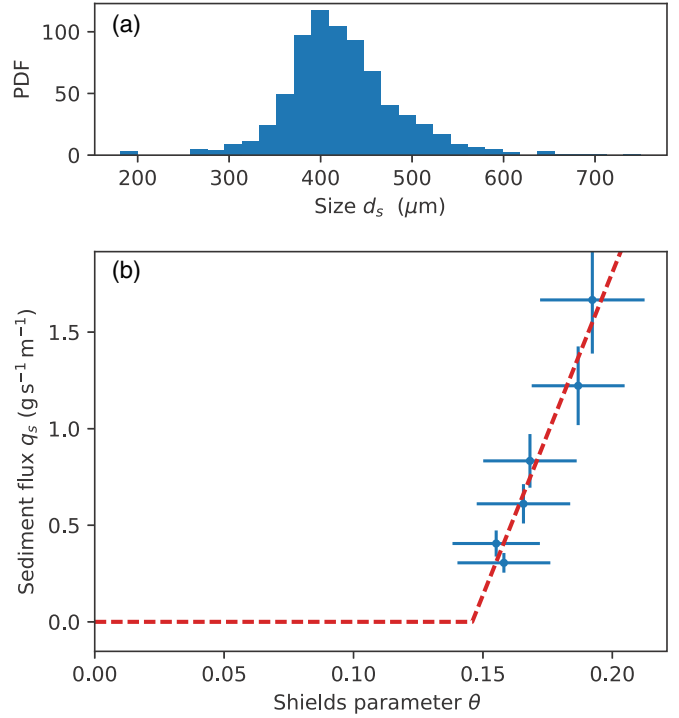


FIG. 2. (a) Sediment-size distribution. (b) Transport law. Blue dots and error bars: measurements. Red dashed line: Eq. (1), with  $q_0 = 33.4 \pm 0.7 \text{ g s}^{-1} \text{ m}^{-1}$  and  $\theta_c = 0.14 \pm 0.005$ .

86 twice a day and compensate for evaporation with fresh water.  
 87 As a result, the fluid discharge varies by less than 5% during  
 88 an experiment, an indication that the viscosity of the fluid is  
 89 also reasonably constant.

90 At the experiment inlet, a tilted channel mixes the fluid  
 91 with a well-sorted sediment composed of corundum grains  
 92 (crystalline aluminium oxide). We measure the grain size  
 93 using image analysis [ $d_{50} = 416 \pm 45 \text{ μm}$ ; Fig. 2(a)] and its  
 94 density using a pycnometer ( $\rho_s = 3900 \pm 56 \text{ kg m}^{-3}$ ). At the  
 95 macroscopic scale, the physical properties of the grain translate  
 96 into a friction coefficient ( $\mu = 0.7$ ) and a transport law relating  
 97 the bedload intensity to the shear stress  $\tau$ , namely,

$$q_s = q_0(\theta - \theta_c), \quad (1)$$

98 where  $\theta = \tau / [(\rho_s - \rho)gd_s]$  is the Shields parameter with  $g$   
 99 the acceleration of gravity [26]. We measure the transport law  
 100 of our corundum sediment with an independent experimental  
 101 setup [22,27,28]. We find a critical Shields number  $\theta_c = 0.14$   
 102  $\pm 0.005$  and a prefactor  $q_0 = 33.4 \pm 0.7 \text{ g s}^{-1} \text{ m}^{-1}$  [Fig. 2(b)].

103 To produce a single-thread laboratory fan, the sediment  
 104 discharge has to be low and constant throughout the fan growth.  
 105 To achieve this, we have designed a conveyor-belt sediment  
 106 dispenser. Sediments are stored in a hopper placed over a  
 107 3.5-cm-wide conveyor belt (Norcan). A stepper motor controlled  
 108 by an Arduino motor shield drives the belt. To adjust the  
 109 sediment discharge, we can adjust two parameters: the distance  
 110 between the hopper and the belt, and the belt speed. To monitor  
 111 the sediment discharge injected into the experiment, the entire  
 112 device is placed on an electronic scale (Ohaus, Explorer 35),  
 113 which measures the weight of the dispenser every minute.

## B. Radially symmetric alluvial fan

114  
 115 At the beginning of an experiment, the surface of the  
 116 experimental setup is bare. We start the fluid and sediment  
 117 feed at the same time and keep them constant during the entire

118 experimental run. To monitor the fan evolution, we acquire  
 119 top-view pictures using a camera fixed above the tank center  
 120 (Canon EOS 100 D with a Canon 28-105 mm  $f/3.5-4.5$  APO  
 121 macro 0.5 m/1.6 ft Ultrasonic lens). We record a picture every  
 122 10 min (Fig. 3).

123 The growth of our fan is similar to the ones of Van Dijk  
 124 *et al.* [19] and Reitz and Jerolmack [24]. The flow alternates  
 125 between a single channel and a thin sheet of fluid covering a  
 126 portion of the fan. In our case, however, the flow gets more and  
 127 more channelized as the fan grows. The channel migrates either  
 128 through the gradual erosion of its banks or through floodings  
 129 and abrupt avulsion. This series of avulsions allows the channel  
 130 to explore the entire fan surface, thus maintaining its radial  
 131 symmetry (Fig. 3).

132 Using the top-view pictures, we measure the evolution  
 133 of the fan radius. To do so, we develop an algorithm that  
 134 automatically locates the fan toe. Because the fan color is well  
 135 defined, we can detect its boundary by applying a threshold. We  
 136 then measure the average radius of the fan by approximating  
 137 its boundary by a half circle (red dashed lines on Fig. 3). We  
 138 find that, typically, the distance from the apex to the boundary  
 139 varies by less than 5% (Fig. 3). After a transient, which duration  
 140 depends on the sediment discharge, the fan radius increases like  
 141 the cubic root of time (Fig. 4). This experimental observation  
 142 accords with the self-similar evolution of a semiconical deposit  
 143 for which, based on mass balance, we expect  $R \propto (Q_s t)^{1/3}$   
 144 [24,28].

145 Because of its radial symmetry, the fan's shape reduces to its  
 146 downstream profile. Consequently, to describe the growth and  
 147 shape of our laboratory fans, we need an accurate measurement  
 148 of their profiles.

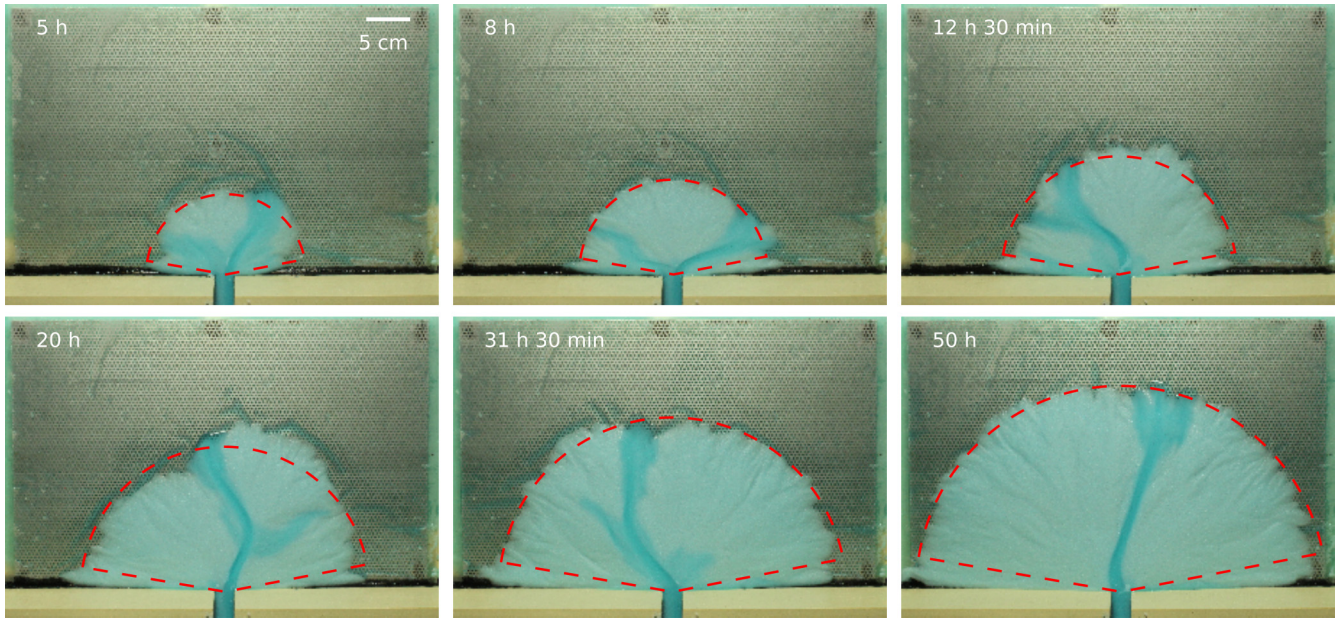


FIG. 3. Top-view pictures of an experimental fan during its growth, run 13 ( $Q_w = 0.33 \text{ l min}^{-1}$  and  $Q_s = 1.68 \text{ g min}^{-1}$ ). Fluid appears in blue, sediment appears in white. Red dashed line indicates the measured radius of the fan.

**C. Measurement of the fan elevation**

149 To measure the elevation of our experimental fan, we project  
 150 a Moiré pattern onto its surface with a video projector [29,30].  
 151 We then use the Light3D commercial software to calculate the  
 152 digital elevation model (DEM) of the deposit [31] (Fig. 5).  
 153 To evaluate the accuracy of our measurements, we scan  
 154 the surface of a tray filled with milk and use it as horizontal  
 155 reference. We find that the DEM of the milk surface is affected  
 156 by lens distortion (raw data; Fig. 6). We thus measure the  
 157 elevation of the milk surface at nine levels ranging from 7  
 158 to 134 mm above the bottom of the tank.  
 159

The lens-induced deformation depends on the coordinates  
 of the pixel and on the distance from the lens (raw data; Fig. 6).  
 We thus fit each DEM of the milk surface with a second-  
 order, two-dimensional polynomial. We then evaluate the six  
 coefficients of this polynomial for each elevation. By applying  
 this correction to the measured DEM of the fan, we reduce the  
 uncertainty of its elevation by a factor of about 10 (Fig. 6).  
 After correction, the uncertainty on the measured elevation is  
 less than 400  $\mu\text{m}$ , which corresponds approximately to the size  
 of a sediment grain.

**D. Self-similar growth**

The elevation contours of the DEM are well approximated  
 by concentric circles, thus confirming the radial symmetry of  
 the fan (Fig. 5). This property suggests that we can compute  
 the radially averaged profile of the fan with minimal loss of  
 information [24,28] [Fig. 7(a)]. To evaluate how self-similar  
 is the growth of the fan, we normalize its profile by rescaling  
 horizontal and vertical dimensions with its radius, as measured

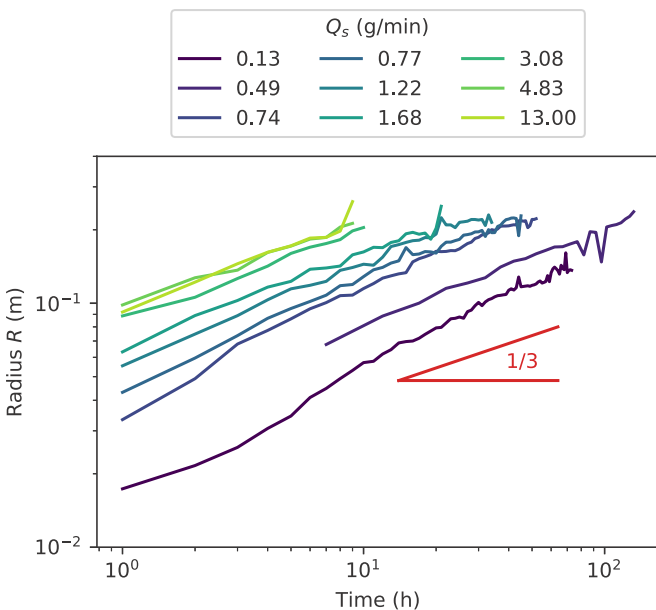


FIG. 4. Evolution of the fan radius ( $Q_w = 0.33 \text{ l min}^{-1}$ ,  $Q_s$  from 0.13 to  $13 \text{ g min}^{-1}$ ).

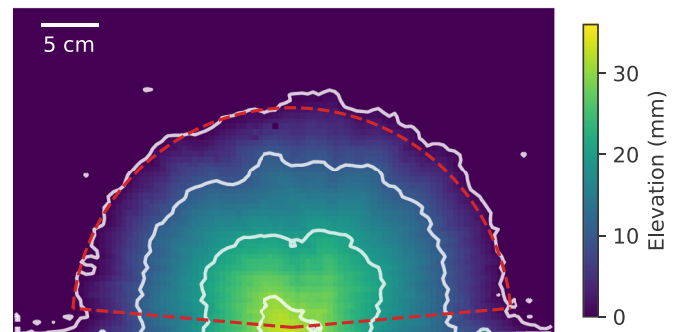


FIG. 5. Digital elevation model (DEM) of an experimental fan (run 13). Contours are 10 mm apart. Red dashed line indicates the measured radius of the fan.

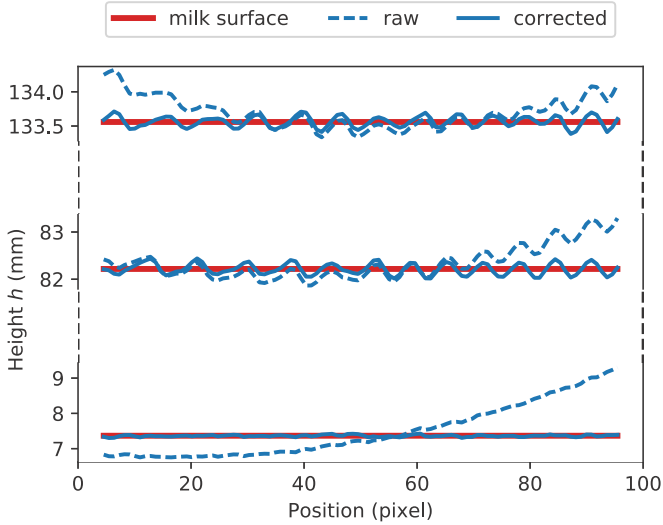


FIG. 6. Profile along the line of maximum deformation of the reference milk surface at different elevations. Dashed blue lines: raw data, solid blue lines: corrected data, red lines: actual elevation of the milk surface.

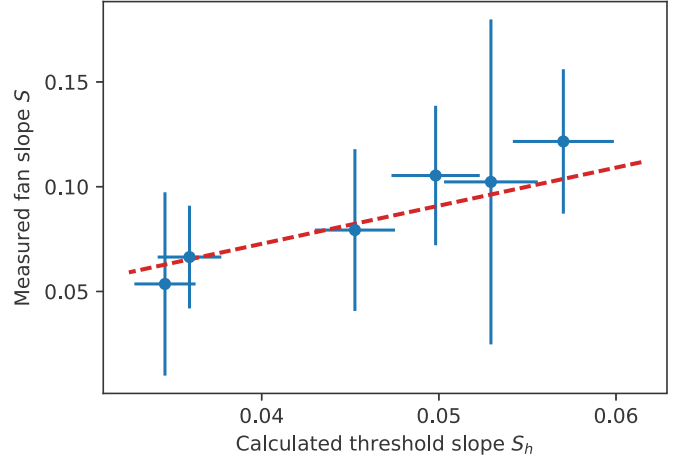


FIG. 8. Slope of the fan as a function of the theoretical threshold slope calculated using Eq. (2) (runs 1 to 6). The red dashed line is a linear fit with slope  $A = 1.95$ .

In the next section, we test this hypothesis using the work of Seizilles *et al.* [25].

### E. Fan near threshold

The shape of an alluvial river results from the mechanical equilibrium of its bed [25,32,33]. When a sediment grain is immersed in a river, two forces act on it: gravity and the flow-induced shear stress. When a river transports a small amount of sediment, we expect its bed to remain near the threshold of sediment motion. At the threshold of motion, the normal component of these forces balances the tangential one. Using this assumption and the shallow-water approximation, Seizilles *et al.* [25] calculated that, for a laminar flow, the threshold slope of a river should read

$$S_h = \mu \left( \frac{4g}{9\nu} \right)^{1/3} \left( \frac{\theta_c \rho_s - \rho}{\mu \rho} d_s \right)^{4/3} \frac{1}{Q_w^{1/3}}. \quad (2)$$

We now compare this expression to the slope of our experimental fans. We perform six experiments where the sediment discharge,  $Q_s$ , remains fixed at a low value, whereas the fluid discharge,  $Q_w$ , varies between experiments (Table I). We then represent the average slope of our experimental fan as a function of the corresponding threshold slope,  $S_h$  [Eq. (2)] (Fig. 8). We find that the slope of our experimental fan is proportional to the threshold slope,

$$S = AS_h, \quad (3)$$

where  $A$  is a dimensionless coefficient, whose best-fit value is  $A = 1.95 \pm 0.11$ . If the shallow-water approximation were perfectly accurate, this coefficient would be one. The value of  $A$  we find by fitting our observations, however, depends on the parameters of Eq. (2) and therefore inherits their uncertainty. In Eq. (2) the value of the critical Shields parameter is the least reliable. Although we carefully estimated its value with a dedicated setup [Fig. 2(b)], this measurement remains a matter of debate [34]. Moreover, the critical Shields number is raised to the power 4/3 in Eq. (2). An error on its measurement has therefore a significant impact on the value of the threshold slope  $S_h$ .

from the top-view pictures. We find that individual profiles differ from their mean by less than 10 % [Fig. 7(b)].

Based on these observations, we propose to treat the fan's evolution as self-similar. Accordingly, relating the morphology of the fan to the input parameters reduces to understanding how these parameters control its dimensionless profile.

Figure 7 shows that we may approximate the fan profile with a straight line, at least when the sediment discharge is low. We further suggest that the river that feeds the fan controls its slope.

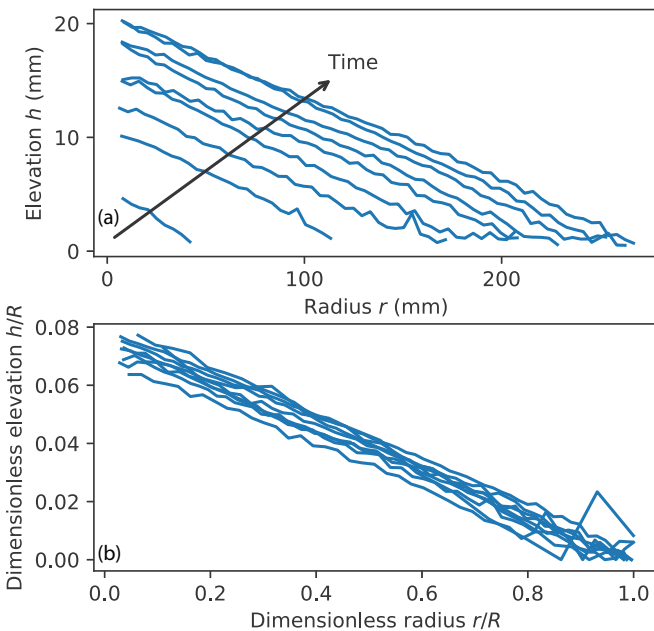


FIG. 7. Radially averaged profiles of an experimental fan (run 4,  $Q_w = 0.3 \text{ l min}^{-1}$  and  $Q_s = 0.2 \text{ g min}^{-1}$ ). (a) Six hours separate two lines. (b) Normalized fan profiles.

TABLE I. Experimental parameters for the experimental runs.

Constant sediment discharge		Constant fluid discharge					
$Q_s = 0.2 \pm 0.002 \text{ g min}^{-1}$		$Q_w = 0.33 \pm 0.02 \text{ l min}^{-1}$				$Q_w = 0.73 \pm 0.02 \text{ l min}^{-1}$	
Run	Fluid discharge ( $\text{l min}^{-1}$ )	Run	Sediment discharge ( $\text{g min}^{-1}$ )	Run	Sediment discharge ( $\text{g min}^{-1}$ )	Run	Sediment discharge ( $\text{g min}^{-1}$ )
1	$0.9 \pm 0.02$	7	$0.13 \pm 0.0084$	13	$1.68 \pm 0.048$	18	$0.82 \pm 0.024$
2	$0.8 \pm 0.02$	8	$0.49 \pm 0.0056$	14	$3.08 \pm 0.12$	19	$4.9 \pm 0.15$
3	$0.4 \pm 0.02$	9	$0.68 \pm 0.013$	15	$4.83 \pm 0.14$	20	$8.1 \pm 0.28$
4	$0.3 \pm 0.02$	10	$0.74 \pm 0.016$	16	$6.18 \pm 0.26$		
5	$0.25 \pm 0.02$	11	$0.77 \pm 0.019$	17	$13 \pm 0.36$		
6	$0.2 \pm 0.02$	12	$1.22 \pm 0.028$				

220 Regardless of the value of  $A$ , for a low sediment discharge,  
 221 we find that the fan's slope remains close to the threshold slope  
 222 of an alluvial river, indicating that the fan inherits its slope from  
 223 the river that builds it. We now investigate the influence of the  
 224 sediment discharge on the fan's profile.

### 225 III. DEPARTURE FROM THRESHOLD: INFLUENCE OF 226 SEDIMENT DISCHARGE

#### 227 A. Sediment transport along the fan

228 To evaluate the influence of the sediment discharge on the  
 229 morphology of the fan, we perform 14 additional experiments  
 230 with different values of  $Q_s$  (Table I). We evaluate the error on  
 231 the sediment discharge by estimating geometrically the volume  
 232 of sediment deposited in the inlet channel.

233 The observations of Sec. II B show that the deposit is radially  
 234 symmetric, thus the spatially averaged profile of the fan suffices  
 235 to represent its shape. Moreover, the fan grows self-similarly,  
 236 which allows us to further average the resulting profiles  
 237 (Sec. II D). To do so, we normalize each profile by dividing  
 238 its horizontal and vertical coordinates by its radius. Figure 9  
 239 shows essentially linear profiles, which steepen with sediment  
 240 discharge. As the latter vanishes, the fan tends towards a cone  
 241 at threshold, in accord with Sec. II E.

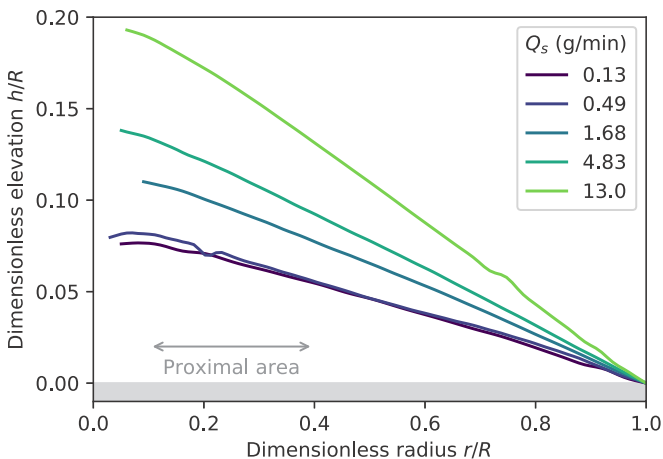


FIG. 9. Normalized fan profiles for five experiments ( $Q_w = 0.33 \text{ l min}^{-1}$ ,  $Q_s$  from  $0.13$  to  $13 \text{ g min}^{-1}$ ).

242 We now consider the influence of a finite sediment discharge  
 243 on the fan's slope. Because alluvial fans are depositional  
 244 systems, only near the apex does the sediment discharge equals  
 245 the input. Accordingly, we consider only the proximal slope  
 246  $S_p$  of our fans, where  $r/R < 0.4$  (Fig. 9). However, the inlet  
 247 disturbs the profile, which can be flat or even convex near  
 248 the apex (Fig. 9). To avoid this disturbance, we measure the  
 249 proximal slope,  $S_p$ , where the dimensionless radius,  $r/R$ , lies  
 250 between  $0.1$  and  $0.4$  (proximal area on Fig. 9).

251 In our experiments, the water discharge takes two values  
 252 (Table I, run 7 to 20) and so does the associated threshold slope,  
 253  $S_h$  (Sec. II). To investigate the departure from this threshold  
 254 slope, we first normalize the proximal slope of our fans with the  
 255 threshold slope,  $S_p/S_h$ . We then represent this dimensionless  
 256 slope as a function of the dimensionless sediment discharge  
 257  $Q_s/(q_0 d_s)$ , where  $q_0$  is the prefactor of the transport law  
 258 [Fig. 2(b)] and  $d_s$  the sediment size (Fig. 10).

259 Our observations gather around a single curve the plane  
 260 defined by the dimensionless sediment discharge and slope,  
 261 suggesting that normalizing the fan's slope with  $S_h$  removes the  
 262 influence of water discharge. This finding accords with Guerit  
 263 *et al.* [22], who were also able to separate the influence of water  
 264 and sediment discharges, although with the different functional  
 265 form that corresponds to the geometry of their experiment.

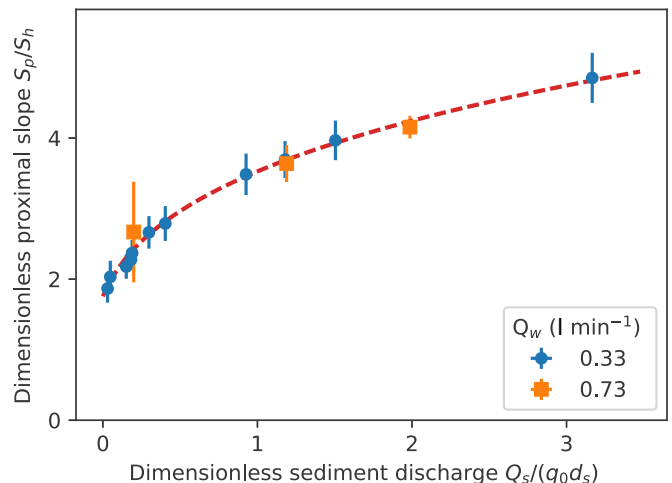


FIG. 10. Normalized proximal slope as a function of the dimensionless sediment discharge. Red dashed line: Eq. (4).

266 We now propose to fit our experimental data with the  
267 following expression:

$$\frac{S}{S_h} = A \left( 1 + B \frac{Q_s}{q_0 d_s} \right)^\alpha, \quad (4)$$

268 where  $B$  and  $\alpha$  are dimensionless coefficients which we need  
269 to fit to our data. By definition,  $A$  is the ratio of the measured  
270 threshold slope to the theoretical threshold slope [Eq. (3)]. A  
271 least-mean-square fit of the three parameters yields  $A = 1.85$   
272  $\pm 0.19$ ,  $B = 0.18 \pm 0.09$ , and  $\alpha = 0.32 \pm 0.02$ . This value of  
273  $A$  is close to the one of Sec. III E (Figs. 8 and 10).

274 Based on Eq. (4), we can now define rigorously what “a  
275 small sediment discharge” means. When  $Q_s/(q_0 d_s)$  is much  
276 smaller than one, the influence of the sediment discharge on the  
277 fan profile is negligible. We then recover the threshold slope  
278 defined in Sec. III E.

279 Equation (4) formulates sediment transport at the scale the  
280 river [as opposed to the local transport law of Fig. 2(b)]. It is a  
281 function of the river’s slope, and therefore of the fan’s, which  
282 results from the internal equilibrium of the river. We now inject  
283 this empirical transport law into the sediment mass balance to  
284 calculate the fan profile.

### 285 B. Fan profile

286 The fan grows from sediment deposition, a process formal-  
287 ized by mass balance, which is often referred to as the “Exner  
288 equation” in the context of sediment transport [35]:

$$\rho_s(1 - \lambda)\pi r \frac{\partial h}{\partial t} + \frac{\partial Q_s}{\partial r} = 0, \quad (5)$$

289 where  $h$  is the fan elevation and  $\lambda$  is the porosity of the deposit.  
290 We estimate the value of  $\lambda$  by comparing the volume of the  
291 deposit measured from its DEM to the volume of sediment  
292 injected into the experiment  $Q_s t$ . We find  $\lambda = 0.4 \pm 0.02$ , a  
293 common value for irregular grains.

294 The relation between the local slope of a river and its local  
295 sediment discharge [Eq. (4)] expresses the sediment transport  
296 along the fan:

$$\frac{B Q_s}{q_0 d_s} = \left( \frac{1}{A S_h} \frac{\partial h}{\partial r} \right)^{1/\alpha} - 1. \quad (6)$$

297 The system of Eqs. (5) and (6) is second-order in space and  
298 first-order in time. It thus requires two boundary conditions  
299 and an initial condition. The first boundary condition is set by  
300 the sediment input at the fan’s apex:

$$Q_s(0, t) = Q_{s0}. \quad (7)$$

301 By definition, we also impose that the fan’s elevation vanishes  
302 at its toe:

$$h(R, t) = 0. \quad (8)$$

303 As the fan grows, its radius increases and we do not know  
304  $R$  *a priori*—the domain over which we must solve the mass  
305 balance has a free boundary. This Stefan problem thus requires  
306 an additional boundary condition. In accordance with direct  
307 observation of our experiments, we further assume that the fan  
308 traps all the sediment it is fed with. Equivalently, the sediment  
309 discharge vanishes at the fan toe:

$$Q_s(R, t) = 0. \quad (9)$$

310 We now seek a self-similar solution to Eqs. (5) and (6),  
311 with the associated boundary conditions, Eqs. (7), (8), and (9).  
312 We first define a similarity variable that takes into account the  
313 growth of the fan:

$$X = \frac{r}{R}, \quad (10)$$

314 where the radius  $R$  is a function of time which needs to be  
315 solved for. The dimensionless radius  $X$  varies between 0 and  
316 1. Similarly, we define a dimensionless fan profile

$$H(X) = \frac{h(r, t)}{A S_h R} \quad (11)$$

317 and a dimensionless sediment discharge

$$\chi(X) = \frac{B Q_s}{q_0 d_s}. \quad (12)$$

318 Injecting these expressions into the sediment-transport equa-  
319 tion [Eq. (6)] yields an ordinary differential equation:

$$H' = -(\chi + 1)^{1/\alpha}. \quad (13)$$

320 Next, we express the growth of the radius  $R$  as a function of the  
321 fan’s shape and of the sediment input rate [Eq. (7)] based on the  
322 total sediment balance. Namely, we integrate the elevation of  
323 the fan over its domain, and differentiate the result with respect  
324 to time. We find

$$Q_{s0} = \frac{\pi}{2} \mathcal{V} (1 - \lambda) \rho_s A S_h R^2 \frac{dR}{dt}, \quad (14)$$

325 where  $\mathcal{V}$  is the dimensionless volume of the fan:

$$\mathcal{V} = 6 \int_0^1 H X dX. \quad (15)$$

326 The coefficient 6 in the above definition ensure that a fan  
327 at threshold has a dimensionless volume of one. Injecting  
328 the dimensionless profile and sediment discharge [Eqs. (11)  
329 and (12)] into the mass balance equation (5) and using the  
330 radius-growth equation (14) yields another ordinary differential  
331 equation:

$$\chi' = 2 \frac{\chi_0}{\mathcal{V}} X (X H' - H), \quad (16)$$

332 where  $\chi_0$  is the dimensionless sediment input:

$$\chi_0 = B Q_{s0} / q_0 d_s. \quad (17)$$

333 Finally, the two first-order, ordinary differential equations  
334 (13) and (16) need boundary conditions. In dimensionless  
335 variables, (8) and (9) become

$$\chi(1) = 0 \quad (18)$$

336 and

$$H(1) = 0, \quad (19)$$

337 respectively.

338 In a way, Eq. (16) is integro-differential, since it involves  
339 the dimensionless volume  $\mathcal{V}$ . This oddity, however, does not  
340 complicate much the numerical procedure we use to compute  
341 the self-similar profile of the fan. We simply fix the ratio  $\chi_0/\mathcal{V}$   
342 and solve Eq. (16) numerically with boundary conditions (18)  
343 and (19). We then inject the resulting profile into Eq. (15) to get

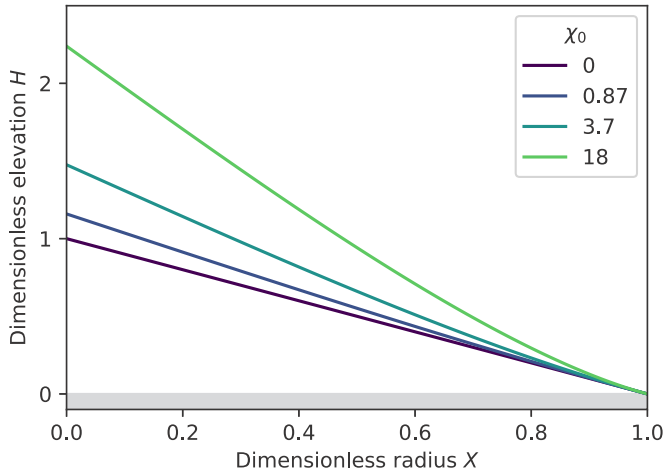


FIG. 11. Numerical fan profile for various values of the dimensionless sediment input  $\chi_0$ .

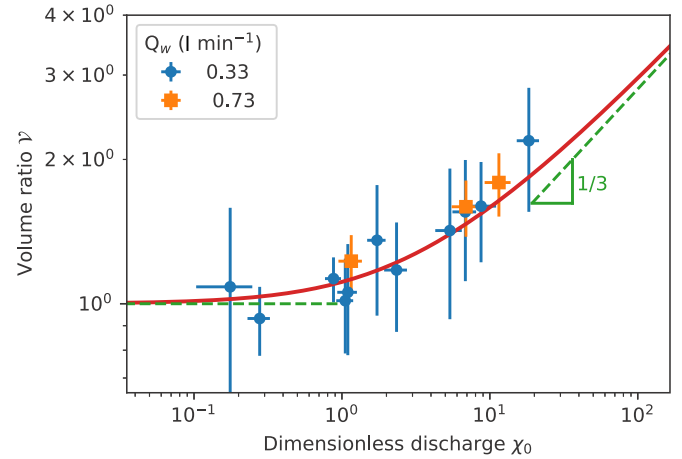


FIG. 12. Volume ratio as a function of sediment discharge. Blue and orange dots: experimental data ( $Q_w = 0.33 \text{ l min}^{-1}$  and  $Q_w = 0.73 \text{ l min}^{-1}$ ). Red line: numerical solution. Green dashed lines: asymptotes.

344 the dimensionless volume and thus the dimensionless sediment  
345 input  $\chi_0$  (Fig. 11).

346 Like in our experiments, the numerical profiles steepen with  
347 sediment discharge. Specifically, the proximal slope increases  
348 with the sediment input rate and decreases downstream due  
349 to sediment deposition. At the fan toe, all numerical profiles  
350 merge, and the local slope reaches the threshold slope  $S_{th}$ , as  
351 expected for a vanishing sediment flux. Sediment deposition  
352 thus results in a concave upward profile.

353 When the sediment discharge vanishes, the entire profile  
354 converges toward the linear profile associated to near-threshold  
355 sediment transport.

356 In short, the slope near the fan toe is the threshold slope and  
357 therefore records the water discharge. The proximal slope, on  
358 the other hand, records the sediment discharge. In principle,  
359 we can thus use the radial profile of a fan to infer both the  
360 water and the sediment discharges that built it.

### 361 C. From morphology to discharge

362 The profiles of our experimental fans do not show any  
363 concavity (Fig. 9), although their dependence on sediment  
364 discharge is qualitatively similar to that of the numerical  
365 profiles. A possible cause for this discrepancy between theory  
366 and experiment is the divergence of the channel near the fan's  
367 toe (Fig. 3).

368 Without measurable change in slope in our experiments, we  
369 cannot infer the sediment discharge based on a local value of  
370 the fan's slope. Instead, we may use an integral quantity, likely  
371 to be less sensitive to measurement errors and natural variability.  
372 One such quantity is the fan's dimensionless volume  $\mathcal{V}$ ,  
373 which we expect to increase with sediment discharge.

374 Using the numerical procedure of Sec. III B, we compute the  
375 dimensionless volume  $\mathcal{V}$  for a range of dimensionless sediment  
376 discharge  $\chi_0$  (Fig. 12). The resulting curve transitions smoothly  
377 between its two asymptotes. When the sediment discharge  
378 vanishes, the fan returns to threshold, and its dimensionless  
379 volume approaches one. On the other hand, a large sediment  
380 discharge causes the fan to be so far from threshold that its

381 slope is essentially that of its apex, which grows as the sediment  
382 discharge to the power 1/3—an unlikely regime in practice.

383 It is more convenient, and perhaps more telling, to express  
384 the dimensionless volume as a ratio of volumes. Specifically,  
385  $\mathcal{V}$  is the ratio of the volume of the actual fan, which we refer  
386 to as  $V_{fan}$ , to that of a hypothetical threshold fan with the same  
387 radius:

$$\mathcal{V} = \frac{6 V_{fan}}{(1 - \lambda)\pi A S_{th} R^3}. \quad (20)$$

388 For illustration, we measure the volume of our laboratory  
389 fans using their DEMs. We then calculate their volume ratio  
390  $\mathcal{V}$  according to the above formula and their dimensionless  
391 discharge  $\chi_0$  according to Eq. (17). Plotting these quantities in  
392 Fig. 12, we find that our experimental data accord reasonably  
393 with the self-similar theory of Sec. III B.

## 394 IV. CONCLUSION

395 Using a simplified setup, we produced fans fed by a  
396 single-thread river, which controls their slope, while avulsions  
397 maintain their radial symmetry. The growth of the fan is  
398 self-similar, and its radius grows like time to the power 1/3.  
399 To assess the influence of the fluid and sediment discharges on  
400 the fan profile, we combine these laboratory observations with  
401 a first-order theoretical analysis.

402 For a low sediment discharge, the threshold of sediment  
403 motion controls the slope of our experimental fans, as it  
404 controlled the one-dimensional alluvial fan produced by Guerit  
405 *et al.* [22]. When the sediment discharge increases, so does the  
406 fan slope, especially near the fan apex, where the sediment  
407 discharge is the highest. As a consequence, the fan profile  
408 exhibits a slight curvature [22].

409 According to the semiempirical theory presented here, the  
410 slope near the fan's toe is an indicator of fluid discharge.  
411 Similarly, the ratio of apex slope to toe slope is a proxy for  
412 sediment discharge. In this simplified framework, we can thus  
413 use the present shape of an alluvial fan to infer the paleofluxes  
414 that built it.



Before applying this theory to field observations, however, we will need to adapt it. First, the sediment discharge does not always vanish at the toe of a fan [36–38]. Consequently, we cannot use the distal slope as a straightforward proxy for water discharge; we first need to evaluate the sediment discharge that exits the fan. Given this measurement, we can probably extend the present theory to this new boundary condition.

In nature, alluvial fans are made of heterogeneous sediments. The river that transports them also sorts them according to mobility [28,39]. The coarser sediment gets deposited near the apex, whereas the finer one ends up at the toe or even exits the fan. This segregation causes the fan slope to decrease downstream [40]. Thus, both deposition and downstream fining translate into a concave upwards profile. To isolate these processes, we need reliable transport laws and a mass balance that can handle a distribution of grain sizes [12,28].

Sediment and water inputs are likely to vary during the fan history. If so, the self-similar theory presented here does not hold, as the shape of the deposit surface can record only the last increment of growth. However, the river probably adjusts

its profile to varying conditions quickly—at least with respect to the fan’s growth. If this is indeed true, the transport law and the mass-balance equation would still hold, and we could solve them numerically to follow varying inputs.

Finally, the sediment deposited at a specific period is often recognizable in the internal structure of the fan, in the form of a stratum. Interpreting these strata as proxies for paleoprofiles, geologists can reconstruct the shape of ancient fans [41,42]. Using these paleoprofiles as we have used the DEMs of our experiments, we might infer ancient mass fluxes and the tectonic and climatic forcings that induced them.

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- [1] W. B. Bull, *Progress Phys. Geog.* **1**, 222 (1977).  
 [2] A. Rachocki and M. A. Church, *Alluvial Fans: A Field Approach* (John Wiley & Sons, New York, 1990).  
 [3] T. C. Blair and J. G. McPherson, *J. Sediment. Res.* **64**, 450 (1994).  
 [4] A. M. Harvey, A. E. Mather, and M. Stokes, *Geol. Soc. Lond. Spec. Publ.* **251**, 1 (2005).  
 [5] T. C. Blair and J. G. McPherson, in *Geomorphology of Desert Environments* (Springer, New York, 2009), pp. 413–467.  
 [6] J. Field, *Geomorphology* **37**, 93 (2001).  
 [7] R. Slingerland and N. D. Smith, *Annu. Rev. Earth Planet. Sci.* **32**, 257 (2004).  
 [8] R. Sinha, *Curr. Sci.* **97**, 429 (2009).  
 [9] F. Drew, *Q. J. Geol. Soc.* **29**, 441 (1873).  
 [10] E. Blissenbach, *J. Sediment. Res.* **22**, 25 (1952).  
 [11] C. S. Denny, *Alluvial Fans in the Death Valley Region, California and Nevada*, Geological survey professional paper 466 (US Government Printing Office, Washington, DC, 1965).  
 [12] J. D. Stock, K. M. Schmidt, and D. M. Miller, *Geol. Soc. Am. Bull.* **120**, 619 (2008).  
 [13] S. A. Schumm, M. P. Mosley, and W. Weaver, *Experimental Fluvial Geomorphology* (John Wiley and Sons, New York, 1987).  
 [14] G. Parker, *J. Hydraul. Res.* **37**, 805 (1999).  
 [15] C. Paola, K. Straub, D. Mohrig, and L. Reinhardt, *Earth-Sci. Rev.* **97**, 1 (2009).  
 [16] L. E. Clarke, *Geomorphology* **244**, 135 (2015).  
 [17] K. X. Whipple, G. Parker, C. Paola, and D. Mohrig, *J. Geol.* **106**, 677 (1998).  
 [18] G. Parker, C. Paola, K. X. Whipple, D. Mohrig, C. M. Toro-Escobar, M. Halverson, and T. W. Skoglund, *J. Hydraul. Eng.* **124**, 996 (1998).  
 [19] M. Van Dijk, G. Postma, and M. G. Kleinhans, *Sedimentology* **56**, 1569 (2009).  
 [20] L. Clarke, T. A. Quine, and A. Nicholas, *Geomorphology* **115**, 278 (2010).  
 [21] E. J. Powell, W. Kim, and T. Muto, *J. Geophys. Res.: Earth Surf.* (2003–2012) **117**, 1 (2012).  
 [22] L. Guerit, F. Métivier, O. Devauchelle, E. Lajeunesse, and L. Barrier, *Phys. Rev. E* **90**, 022203 (2014).  
 [23] G. Lacey, in *Minutes of the Proceedings of the Institution of Civil Engineers*, Vol. 229 (Thomas Telford-ICE Virtual Library, London, 1930), pp. 259–292.  
 [24] M. D. Reitz and D. J. Jerolmack, *J. Geophys. Res.: Earth Surf.* (2003–2012) **117**, 2 (2012).  
 [25] G. Seizilles, O. Devauchelle, E. Lajeunesse, and F. Métivier, *Phys. Rev. E* **87**, 052204 (2013).  
 [26] A. Shields, *Preussische Versuchsanstalt Wasserbau Schiffbau* **26**, 524 (1936).  
 [27] G. Seizilles, E. Lajeunesse, O. Devauchelle, and M. Bak, *Phys. Fluids (1994–present)* **26**, 013302 (2014).  
 [28] P. Delorme, V. Voller, C. Paola, O. Devauchelle, E. Lajeunesse, L. Barrier, and F. Métivier, *Earth Surf. Dyn.* **5**, 239 (2017).  
 [29] J. H. Bruning, D. R. Herriott, J. Gallagher, D. Rosenfeld, A. White, and D. Brangaccio, *Appl. Opt.* **13**, 2693 (1974).  
 [30] F. Brémand, *Optics Lasers Eng.* **21**, 49 (1994).  
 [31] A. Limare, M. Tal, M. Reitz, E. Lajeunesse, and F. Métivier, *Solid Earth* **2**, 143 (2011).  
 [32] R. E. Glover and Q. Florey, *Stable Channel Profiles* (US Department of the Interior, Bureau of Reclamation, Hydr. Lab. Report, Washington, DC, 1951).  
 [33] F. M. Henderson, *J. Hydraul. Div.* **87**, 109 (1961).  
 [34] F. Charru, H. Mouilleron, and O. Eiff, *J. Fluid Mech.* **519**, 55 (2004).  
 [35] G. Parker, C. Paola, K. X. Whipple, and D. Mohrig, *J. Hydraul. Eng.* **124**, 985 (1998).  
 [36] E. Kiefer, M. J. Dorr, H. Ibbeken, and H.-J. Gotze, *Andean Geol.* **24**, 165 (1997).

- [37] T. Oguchi, *J. Quaternary Sci.* **12**, 381 (1997).
- [38] M. Jolivet, L. Barrier, S. Dominguez, L. Guerit, G. Heilbronn, and B. Fu, *Geomorphology* **214**, 168 (2014).
- [39] K. L. Miller, M. D. Reitz, and D. J. Jerolmack, *Geophys. Res. Lett.* **41**, 7191 (2014).
- [40] W. B. Bull, *Geomorphology of Segmented Alluvial Fans in Western Fresno County, California* (US Government Printing Office, Washington, DC, 1964).
- [41] J. Hornung, D. Pflanz, A. Hechler, A. Beer, M. Hinderer, M. Maisch, and U. Bieg, *Geomorphology* **115**, 202 (2010).
- [42] S. Rohais, S. Bonnet, and R. Eschard, *Basin Res.* **24**, 198 (2012).