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SuperIso Relic v4: A program for calculating dark matter and flavour physics observables in Supersymmetry

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Abstract

We describe **SuperIso Relic**, a public program for the calculation of dark matter relic density and direct and indirect detection rates, which includes in addition the **SuperIso** routines for the calculation of flavour physics observables. **SuperIso Relic v4** incorporates many new features, namely the possibility of parallel calculation of the relic density, new cosmological models, and the implementations of the calculation of the observables related to direct and indirect detection experiments. Furthermore, with the new version it is possible to evaluate straightforwardly the nuclear and astrophysical uncertainties, from namely nuclear form factors, dark matter density and velocity, as well as cosmic ray propagation through the galactic medium.

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Key words: Supersymmetry; MSSM scenarios; Relic density; Dark matter.

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NEW VERSION PROGRAM SUMMARY

Program Title: SuperIso Relic

Licensing provisions: GNU General Public License 3 (GPL)

Programming language: C and Fortran

Journal Reference of previous version: Comput. Phys. Commun. 181 (2010) 1277

Does the new version supersede the previous version: yes

Reasons for the new version: SuperIso Relic v4 features the calculation of observables related to dark matter direct and indirect detections in supersymmetry.

Summary of revisions:

- Improvement and parallelisation of the relic density calculation routines
- Implementation of dark matter direct detection constraints
- Implementation of dark matter indirect detection constraints for gamma-rays and antiprotons
- Addition of new cosmological scenarios

Nature of problem: Calculation of dark matter observables for the lightest supersymmetric particle, as well as flavour physics observables.

Solution method: SuperIso Relic computes the annihilation and co-annihilation amplitudes of two supersymmetric particles into Standard Model particles, which are used for the calculation of dark matter relic density as well as dark matter indirect detection. Different cosmological models are implemented, which can modify the Boltzmann equations. For indirect detection, the observational constraints from AMS-02 and Fermi-LAT are implemented. Dark matter direct detection observables, as well as constraints from PICO60, PANDAX-2 and XENON1T, are also implemented.

Unusual features: The relic density routines are parallelized and use the OpenMP library.

1 Introduction

`SuperIso Relic` is a mixed C / Fortran program devoted to the calculation of dark matter and flavour observables in different models. It is an extension of `SuperIso` [1–3] which contains the calculation of the flavour observables, and includes additional routines from `AlterBBN` [4,5]. The previous versions of `SuperIso Relic` were devoted to the calculation of the relic density in standard and modified cosmological scenarios [6,7].

Many new features have been added in the current version of `SuperIso Relic`, which justifies a major version number change. The main features are the possibility of parallel calculation of the relic density using the `OpenMP` library, new cosmological models, and the implementations of the calculation of observables related to direct and indirect detection experiments, with a focus on the possibility to alter the astrophysical description of dark matter and propagation of dark matter annihilation products. The user-friendly treatment of the astrophysical and cosmological uncertainties and the definition of default “conservative”, “standard” and “stringent” set-ups distinguish `SuperIso Relic` from the other existing public codes such as `DarkSusy` [8,9], `micrOMEGAs` [10], `MadDM` [11] and `DarkBit` [12,13].

`SuperIso Relic` is public and open-source, and is released with the GPL version 3 licence. A detailed description of the code is provided in the following; the content of the `SuperIso Relic` package is described in section 2, the compilation and installation instructions are provided in section 3, the input and output descriptions are given in section 4, and some illustrative results are presented in section 5. The appendices contain the description of the cosmological scenarios as well as the calculation of indirect and direct detection rates.

2 Content of the `SuperIso Relic` package

2.1 *General description*

The main directory of `SuperIso Relic` contains 23 main programs in C language. `sm.c`, `sm_chi2.c`, `flha.c`, `slha_chi2.c` and `thdm.c` are only relevant for `SuperIso` and do not involve dark matter observables. The other programs rely on the SUSY Les Houches Accord (SLHA) [14,15] as input format

for supersymmetric models. 10 main files are common to both `SuperIso` and `SuperIso Relic`:

- `amsb.c`: calculation of observables for an AMSB model point.
- `cmssm.c`: calculation of the observables for a CMSSM model point.
- `cnmssm.c`: calculation of the observables for a semi constrained NMSSM model point.
- `gmsb.c`: calculation of the observables for a GMSB model point.
- `hcamsb.c`: calculation of the observables for a hypercharged AMSB model point.
- `mmamsb.c`: calculation of the observables for a mixed moduli AMSB model point.
- `nnuhm.c`: calculation of the observables for a NMSSM NUHM model point.
- `ngmsb.c`: calculation of the observables for a NMSSM GMSB model point.
- `nuhm.c`: calculation of the observables for a NUHM model point.
- `slha.c`: scan of the user provided SLHA file and calculation of the observables.

And 8 main files are `SuperIso Relic`-specific:

- `create_propagation.c`: calculation of AMS-02 antiproton constraints [16] in a user-specified propagation model and profile.
- `direct.c`: calculation of the direct detection observables, and the limits by PANDAX-2 [17], XENON1T [18] and PICO60 [19].
- `indirect.c`: calculation of the indirect detection observables, and the gamma-ray results of FERMI-LAT [20] and AMS-02 antiproton limits [16].
- `test_phi.c`: reads the user provided SLHA file and calculates the relic density in a scenario with a decaying cosmological scalar field.
- `test_modeleff.c`: reads the user provided SLHA file and calculates the relic density for different radiation equations of state.
- `test_standmod.c`: reads the user provided SLHA file and calculates the relic density in a model with dark density and dark entropy in the early Universe. The BBN constraints are checked automatically with `AlterBBN`.
- `test_reheating.c`: reads the user provided SLHA file and calculates the relic density in a model with dark density and reheating in the early Universe. The BBN constraints are checked automatically with `AlterBBN`.
- `test_widthcalc.c`: reads the user provided SLHA file and calculates the relic density using the different width calculators which are included.

The files `README` and `README.superiso` describe `SuperIso Relic` and `SuperIso`, respectively. `example.lha` is an example of SLHA file. The files `configure` and `Makefile` allow the user to configure and compile the package. The `src/` directory contains the source files of the package, and in particular the C files:

- `bbn.c`: calculation of BBN constraints using `AlterBBN`.

- `bbnrate.c`: nuclear rates for `AlterBBN`.
- `DDetection.c`: calculation of dark matter direct detection observables.
- `DDhalo.c`: routines related to the dark matter halo profile for direct detection.
- `DDMSSM.c`: routines related to the calculation of the quark-neutralino effective couplings in the (N)MSSM for direct detection.
- `DDnuclear.c`: routines related to the nuclear models for direct detection.
- `feynhiggs.c`: procedures calling `FeynHiggs`.
- `hdecay.c`: procedures calling `Hdecay`.
- `IDetection.c`: calculation of indirect detection observables.
- `omega.c`: calculation of the dark matter relic density.
- `propagation.c` and `propagation.h`: routines for the proton/antiproton propagation model.
- `sdecay.c`: procedures calling `Sdecay/SUSYHIT`.
- `widthcalc.c`: routines related to the Higgs width calculation.
- `include.h`: definitions and prototypes.
- `include_dm.h`: definitions and prototypes specific to `SuperIso Relic`.

and five directories:

- `contrib/`: contains external programs useful for the relic density (`FeynHiggs`, `Hdecay`, `SUSYHIT`).
- `sgStar_heff/`: contains different tables of $g_{\text{eff}}(T)$ and $h_{\text{eff}}(T)$.
- `relic/`: contains all the Fortran routines used for the calculation of the (co-)annihilation amplitudes.
- `gammas/`: contains the data for interpreting the FERMI-LAT results.
- `antiprotons/`: contains the data for interpreting the AMS-02 results.

In addition, routines interfacing `SuperIso Relic` with `ISAJET` [21], `SOFTSUSY` [22], `SPheno` [23], `SuSpect` [24] and `NMSSMTools` [25] are provided. `FeynHiggs` [26] or `Hdecay`, `Sdecay` and `SUSYHIT` [27–29] are used to compute the decay widths of the Higgs bosons and supersymmetric particles useful for the relic density and neutralino annihilation cross section calculations. The propagation model included in `propagation.c` is the one used in [30,31].

The directory `relic/` includes the routines necessary for the relic density calculation. `Weff/` and `squared/` contain 3011 annihilation and coannihilation processes for the MSSM and `WeffN/` and `squaredn/` 4219 processes for the NMSSM in Fortran code. In comparison to the previous versions of `SuperIso Relic`, all these files have been re-generated with `FeynArts/FormCalc` [32–35] using the LANHEP [36,37] model files, in order to improve the compilation speed as well as to enable the use of the `OpenMP` library to compute the relic density. All the `COMMON` blocks which are automatically generated by `FormCalc` are defined as `THREADPRIVATE` with respect to `OpenMP`. The relic density Fortran libraries can be compiled as static or as dynamic, and the interface is

insured by `staticlibcalc.c/staticlibcalcN.c` and `dynlibcalc.c`, respectively. `model_superiso.F` enables the exchange of the supersymmetric parameters between the C and Fortran parts of the code.

2.2 Parameter structures

SuperIso Relic uses several C-structures to exchange data between the different subroutines, which are described in the following.

2.2.1 Internal parameters

The `parameters` structure contains the main parameters of SuperIso Relic, which are exchanged between most of the routines:

```
typedef struct parameters
/* structure containing all the scanned parameters from the SLHA file */
{
int SM;
int model; /* CMSSM=1, GMSB=2, AMSB=3 */
int generator; /* ISAJET=1, SOFTSUSY=3, SPHENO=4, SUSPECT=5, NMSSMTOOLS=6 */
double Q; /* Qmax ; default = M_EWSB = sqrt(m_stop1*mstop2) */

double m0,m12,tan_beta,sign_mu,A0; /* CMSSM parameters */
double Lambda,Mmess,N5,cgrav,m32; /* AMSB, GMSB parameters */
double mass_Z,mass_W,mass_b,mass_top_pole,mass_tau_pole; /* SM parameters */
double inv_alpha_em,alphas_MZ,Gfermi,GAUGE_Q; /* SM parameters */
double charg_Umix[3][3],charg_Vmix[3][3],stop_mix[3][3],sbot_mix[3][3],
stau_mix[3][3],neut_mix[6][6],mass_neut[6],alpha; /* mass mixing matrices */
double Min,M1_Min,M2_Min,M3_Min,At_Min,Ab_Min,Atau_Min,M2H1_Min,M2H2_Min,mu_Min,
M2A_Min,tb_Min,mA_Min; /* optional input parameters at scale Min */
double MeL_Min,MmuL_Min,MtauL_Min,MeR_Min,MmuR_Min,MtauR_Min; /* optional input
parameters at scale Min */
double MqL1_Min,MqL2_Min,MqL3_Min,MuR_Min,McR_Min,MtR_Min,MdR_Min,MsR_Min,
MbR_Min; /* optional input parameters at scale Min */
double N51,N52,N53,M2H1_Q,M2H2_Q; /* optional input parameters (N51...3: GMSB) */
double mass_d,mass_u,mass_s,mass_c,mass_t,mass_e,mass_nue,mass_mu,mass_num,
mass_tau,mass_nut; /* SM masses */
double mass_gluon,mass_photon,mass_Z0; /* SM masses */
double mass_h0,mass_H0,mass_A0,mass_H,mass_dnl,mass_upl,mass_stl,mass_chl,
mass_b1,mass_t1; /* Higgs & superparticle masses */
double mass_el,mass_nuel,mass_mul,mass_numl,mass_tau1,mass_nutl,mass_gluino,
mass_cha1,mass_cha2; /* superparticle masses */
```

```

double mass_dnr,mass_upr,mass_str,mass_chr,mass_b2,mass_t2,mass_er,mass_mur,
mass_tau2; /* superparticle masses */
double mass_nuer,mass_numr,mass_nutr,mass_graviton,
mass_gravitino; /* superparticle masses */
double gp,g2,gp_Q,g2_Q,g3_Q,YU_Q,yut[4],YD_Q,yub[4],YE_Q,yutau[4]; /* couplings */
double HMIX_Q,mu_Q,tanb_GUT,Higgs_VEV,mA2_Q,MSOFT_Q,M1_Q,M2_Q,M3_Q; /* parameters
at scale Q */
double MeL_Q,MmuL_Q,MtauL_Q,MeR_Q,MmuR_Q,MtauR_Q,MqL1_Q,MqL2_Q,MqL3_Q,MuR_Q,
McR_Q,MtR_Q,MdR_Q,MsR_Q,MbR_Q; /* masses at scale Q */
double AU_Q,A_u,A_c,A_t,AD_Q,A_d,A_s,A_b,AE_Q,A_e,A_mu,A_tau; /* trilinear
couplings */

/* SLHA2 */
int NMSSM,RV,CPV,FV;
double CKM_lambda,CKM_A,CKM_rhobar,CKM_etabar;
double PMNS_theta12,PMNS_theta23,PMNS_theta13,PMNS_delta13,PMNS_alpha1,
PMNS_alpha2;
double lambdaNMSSM_Min,kappaNMSSM_Min,AlambdaNMSSM_Min,AkappaNMSSM_Min,
lambdaSNMSSM_Min,xiFNMSSM_Min,xiSNMSSM_Min,mupNMSSM_Min,mSp2NMSSM_Min,
mS2NMSSM_Min,mass_H03,mass_A02,NMSSMRUN_Q,lambdaNMSSM,kappaNMSSM,
AlambdaNMSSM,AkappaNMSSM,lambdaSNMSSM,xiFNMSSM,xiSNMSSM,mupNMSSM,mSp2NMSSM,
mS2NMSSM; /* NMSSM parameters */
double PMNSU_Q,CKM_Q,IMCKM_Q,MSE2_Q,MSU2_Q,MSD2_Q,MSL2_Q,MSQ2_Q,TU_Q,TD_Q,TE_Q;
double CKM[4][4],IMCKM[4][4]; /* CKM matrix */
double HO_mix[4][4],AO_mix[4][4]; /* Higgs mixing matrices */
double sU_mix[7][7],sD_mix[7][7],sE_mix[7][7],sNU_mix[4][4]; /* mixing matrices */
double sCKM_msq2[4][4],sCKM_msl2[4][4],sCKM_msd2[4][4],sCKM_msu2[4][4],
sCKM_mse2[4][4]; /* super CKM matrices */
double PMNS_U[4][4]; /* PMNS mixing matrices */
double TU[4][4],TD[4][4],TE[4][4]; /* trilinear couplings */

/* non-SLHA*/
double mass_c_pole,mass_b_1S,mass_b_pole,mtmt;
int scheme_c_mass;
double Lambda3,Lambda4,Lambda5,Lambda6; /* Lambda QCD */
double alphasMZ_Lambda3,alphasMZ_Lambda4,alphasMZ_Lambda5,
alphasMZ_Lambda6; /* alpha_s */

:

/* Decay widths */
int widthcalc; /* 0=none, 1=hdecay, 2=feynhiggs */
double width_h0,width_H0,width_A0,width_H,width_Z,width_W,width_top,width_H03,
width_A02;
double width_gluino,width_t1,width_t2,width_b1,width_b2,width_ul,width_ur,
width_dl,width_dr;

```



```

double width_cl,width_cr,width_sl,width_sr,width_el,width_er,width_ml,width_mr,
width_tau1,width_tau2,width_gravitino;
double width_nuel,width_numl,width_nutaul,width_c1,width_c2,width_o1,width_o2,
width_o3,width_o4,width_o5;

/* 2HDM */
int THDM_model;
double lambda_u[4][4],lambda_d[4][4],lambda_l[4][4];

/* NMSSMTools */
int NMSSMcoll,NMSSMtheory,NMSSMups1S,NMSSMetab1S;

/* SDECAY */
double BRtbW,BRtbH,BRtt1o1,BRtt1o2,BRtt1o3,BRtt1o4,BRtt2o1,BRtt2o2,BRtt2o3,
BRtt2o4,BRgluino1tbar,BRgluino1bart,BRgluino1dbar,BRgluino1dbard,

                                :

/* HDECAY & FeynHiggs */
double mass_hOSM,width_hOSM;
double mass_HOSM,width_HOSM;
double mass_AOSM,width_AOSM;
double BRh0bb_SM,BRh0tautau_SM,BRh0WW_SM,BRh0gg_SM,BRh0gaga_SM,BRh0ZZ_SM;
double BRH0bb_SM,BRH0tautau_SM,BRH0WW_SM,BRH0gg_SM,BRH0gaga_SM,BRH0ZZ_SM;
double BRA0bb_SM,BRA0tautau_SM,BRA0WW_SM,BRA0gg_SM,BRA0gaga_SM,BRA0ZZ_SM;

double BRh0bb,BRh0tautau,BRh0WW,BRh0gg,BRh0gaga,BRh0ZZ;

                                :

}
parameters;

```

2.2.2 Relic density

The main relic density structure has been expanded since the previous versions of `SuperIso Relic`, to incorporate new cosmological scenarios and more parameters related to `AlterBBN`.

```

typedef struct relicparam
{
    int entropy_model,energy_model;
    double dd0,ndd,Tdend,Tddeq; /* dark density */
    double sd0,nsd,Tsend; /* dark entropy */

```

```

double Sigmad0,nSigmad,TSigmadend; /* dark entropy injection */
double Sigmarad0,nSigmarad,TSigmaradend; /* standard entropy injection */
double nt0,nnt,Tnend; /* non-thermal production of relics */

double quintn2,quintn3,quintn4,quintT12,quintT23,quintT34; /* quintessence */

int phi_model; /* decaying scalar field model switch */
double eta_phi,Gamma_phi,rhot_phi_Tmax,rho_phi; /* eta_phi = b / m_phi */
double rhot_phi0,Tphi0;
double T_RH;
double Sigmatildestar;
double Sigmatildestar_max;
double Tstdstar_max;

double mgravitino; /* gravitino mass */

double relicmass;
int scalar;

int solver; /* switch for linear or logarithmic differential equation solver */

double T; /* Temperature in GeV */
double Y; /* Y=n/s */
double Tfo,Tmax; /* Freeze-out and maximal temperatures */

int full_comput; /* Switch to deactivate the fast freeze-out temperature
determination */

double table_eff[276][3]; /* Reads values from the SgStar files */

int use_table_rhoPD;
double table_rhoPD[2][NTABMAX];
int size_table_rhoPD;

/*-----*/
/* AlterBBN parameters */
/*-----*/

int err;
int failsafe;
double eta0;
double Nnu;
double dNnu;
double life_neutron,life_neutron_error;
double xinu1,xinu2,xinu3;

```

```

    double m_chi;
    double g_chi;
    double Tinit;
    int wimp;
    int SMC_wimp;
    int selfConjugate;
    int fermion;
    int EM_coupled, neut_coupled, neuteq_coupled;
    double chi2;
    int nobs;
}
relicparam;

```

For the description of the specific `AlterBBN` parameters we refer the reader to [4,5].

2.2.3 Direct detection

The following structure describes the nuclear and astrophysical parameters for the calculation of direct detection constraints. The calculation of the nucleon-LSP scattering amplitudes is detailed in Appendix C.1. The uncertainties on the astrophysical parameters, namely the local DM density, and the escape and disk rotational velocities within the standard halo model are discussed in Appendix C. The nuclear uncertainties are discussed in Appendix C.2, together with the definition of the nucleon form factors and nuclear structure factors.

```

struct DDparameters
{
/* error option =1 to consider the underlying uncertainties, 0 otherwise */
    int nucleonSIerror; /* Spin-independent nucleon form factor */
    int nucleonSDerror; /* Spin-dependent nucleon form factor */
    int nuclearSDerror; /* Spin-dependent nuclear structure factor */
    int rho0error; /* local DM density */
    int vescerror; /* escape velocity */
    int vroterror; /* disk rotational velocity */
/* detector */
    int niso; /* isotope number */
    int A[NISOMAX]; /* proton+neutron number */
    int Z[NISOMAX]; /* proton number */
    double MA[NISOMAX]; /* isotope mass */
    double mu[NISOMAX]; /* isotope-LSP reduced mass */
    double massfract[NISOMAX]; /* isotope mass fraction */
    double J[NISOMAX]; /* spin */

```

```

/* LSP */
double relicmass;
double ASI[6]; /* quark contributions to the SI scattering amplitude */
double ASD[6]; /* quark contributions to the SD scattering amplitude */
double ASDp; /* proton-LSP SD scattering amplitude */
double ASDn; /* neutron-LSP SD scattering amplitude */
double ASIp; /* proton-LSP SI scattering amplitude */
double ASIn; /* neutron-LSP SI scattering amplitude */
double ddpSI; /* proton-LSP SI scattering cross section */
double ddnSI; /* neutron-LSP SI scattering cross section */
double ddpSD; /* proton-LSP SD scattering cross section */
double ddnSD; /* neutron-LSP SD scattering cross section */
/* Nucleon form factors */
/* Spin independent */
double sigmapin[2]; /* light quark content of nucleons */
double sigmas[2]; /* strange quark content of nucleons */
double z[2]; /* z parameter occurring in the form factor calculation */
double fdSIp; /* d-quark SI form factor in proton */
double fuSIp; /* u-quark SI form factor in proton */
double fsSIp; /* s-quark SI form factor in proton */
double fdSIn; /* d-quark SI form factor in neutron */
double fuSIn; /* u-quark SI form factor in neutron */
double fsSIn; /* s-quark SI form factor in neutron */
/* Spin dependent */
double a3[2]; /* fuSDp-fdSDp */
double a8[2]; /* fuSDp+fdSDp-2*fsSDp */
double fsSDp[2]; /* s-quark SD form factor in proton */
double fdSDp; /* d-quark SD form factor in proton */
double fuSDp; /* u-quark SD form factor in proton */
double fdSDn; /* d-quark SD form factor in neutron */
double fuSDn; /* u-quark SD form factor in neutron */
double fsSDn; /* s-quark SD form factor in neutron */
/* Nuclei form factors */
/* Spin independent */
double (*FFSI)(double , int ); /* SI nuclear form factor (Helm by default) */
/* Spin dependent */
double (*S00[NISOMAX])(double); /* SD nuclear structure factor S00 */
double (*S00min[NISOMAX])(double); /* S00 lower bound */
double (*S00max[NISOMAX])(double); /* S00 upper bound */
double (*S01[NISOMAX])(double); /* SD nuclear structure factor S01 */
double (*S01min[NISOMAX])(double); /* S01 lower bound */
double (*S01max[NISOMAX])(double); /* S01 upper bound */
double (*S11[NISOMAX])(double); /* SD nuclear structure factor S11 */
double (*S11min[NISOMAX])(double); /* S11 lower bound */
double (*S11max[NISOMAX])(double); /* S11 upper bound */

```

```

/* DM halo */
double rho0[2]; /* local DM density */
double vrot[2]; /* disk rotational velocity */
double vesc[2]; /* escape velocity */
double vearth[3]; /* peculiar velocity of Earth in the local standard of rest
double vearthmed; /* Earth velocity in the galactic frame */
double vearthmin; /* Minimal Earth velocity with regards to vrot uncertainties
double vearthmax; /* Maximal Earth velocity with regards to vrot uncertainties
double Maxwellnorm; /* Norm of the Maxwell distribution */
double Maxwellnormmin; /* Minimal value of Maxwellnorm
                        with regards to vrot and vesc uncertainties */
double Maxwellnormmax; /* Maximal value */
double(*eta)(struct DDparameters* ,double, char*); /* mean inverse speed */
};

```

2.2.4 Indirect detection

Several structures are used for indirect detection. The constraints from Fermi-LAT dwarf spheroidal galaxies and AMS-02 antiprotons are described in appendices B.2 and B.3, respectively.

2.2.4.1 Structure for the dwarf spheroidal galaxy list for FERMI-LAT This structure contains the main information of a particular dwarf spheroidal galaxy.

```

struct dSph{
    char name[500]; /* name of the galaxy */
    double J; /* J-factor */
    double deltaJ; /* J-factor uncertainty */
    double likelihood[600][3]; /* tabulated likelihood */
    double likelihood_noDM; /* likelihood in the background-only hypothesis */
    int sample; /* type of sample to which the galaxy belongs
                (= -1 for conservative, = 0 for nominal, = 1 for inclusive) */
};

```

2.2.4.2 Structure for FERMI-LAT: This structure contains the list of dwarf spheroidal galaxies, as well as the energy bins considered in the analysis of Fermi-LAT dSphs.

```

struct fermi
{
    int NdSphs; /* number of galaxies */
};

```

```

    struct dSph dSphs[50]; /* list of the galaxies */
    int Nenergybins; /* number of energy bins */
    double eflux[24]; /* photon flux from DM annihilation */
    double Emin[24]; /* lower bounds of the energy bins */
    double Emax[24]; /* upper bounds of the energy bins */
};

```

2.2.4.3 Structure for the dark matter halo: This structure contains the main properties of the DM halo density profile of the Milky Way used for AMS-02 antiproton analysis.

```

struct Structure_Halo
{
    char name[80]; /* name of the halo */
    double (*profile)(double, double); /* DM halo profile */
    double r_earth ; /* distance of the Earth from the galactic center */
    double rho_chi_solar; /* DM local density */
};

```

2.2.4.4 Structure for the propagation parameters: This structure contains the main quantities for AMS-02 antiproton analysis, in particular the DM halo density profile and the propagation parameters.

```

struct propagation_parameters
{
    struct Structure_Propagation Propagation; /* contains the propagation model */
    struct Structure_Halo Halo; /* contains the DM halo model */
    double chi2_noDM; /*  $\chi^2$  for the background only hypothesis */
    struct array bkg; /* secondary antiproton tabulated spectra */
}

```

2.2.4.5 Structure for the particles: This structure contains the main properties of particles and is used for the calculation of neutralino annihilation cross sections and for the calculation of antiproton and gamma-ray spectra.

```

struct particle
{
    char* name;
    double mass;
    int type; /* scalar, fermion or vector */
};

```

2.2.4.6 Structure for the processes related to indirect detection:

This structure defines $2 \rightarrow 2$ processes or decays and is used for the calculation of antiproton and gamma-ray spectra.

```
struct process
{
/* defines a 2->2 process particle1 particle2 -> particle3 particle4
   or a decay particle1 -> particle3 particle4 */

    char* name; /* name of the process */
    struct particle particle1;
    struct particle particle2;
    struct particle particle3;
    struct particle particle4;
    int sf34; /* =1 if part 3 and 4 are different, =2 if they are the same */
    int hel; /* defines the spin of the particles */
    double sigmav; /* thermally averaged annihilation cross section
                   or decay branching ratio */
};
```

2.2.4.7 Structure for the list of processes related to indirect de-

tection: This structure lists all the possible neutralino annihilation channels and the total thermally averaged annihilation cross section.

```
struct processes
{
    int nproc; /* number of processes */
    struct process list[100]; /* process list */
    int dof; /* number of processes contributing
             to > 1% to the total annihilation cross section */
    double relicmass;
    double sigmav; /* total annihilation cross section */
};
```

2.2.4.8 Main indirect detection structure: This structure is the main structure for indirect detection and contains all the parameters for the calculation of antiproton and gamma-ray spectra.

```
struct IDparameters
{
    struct processes o1o1; /* neutralino annihilation processes */
    struct processes h0; /* Light CP-even Higgs decays */
    struct processes H0; /* Heavy CP-even Higgs decays */
};
```

```

struct processes A0; /* CP-odd Higgs decays */
struct processes h3; /* Third CP-even Higgs decays in the NMSSM */
struct processes hb; /* Second CP-odd Higgs decays in the NMSSM */
struct processes hc; /* Third CP-odd Higgs decays in the NMSSM */
struct spectrum spectrum; /* summarises the successive annihilations and
                           decay processes in order to compute the photon
                           or antiproton spectra */

double relicmass;
int NMSSM; /* =0 in the MSSM, =1 in the NMSSM */
};

```

2.3 Main routines

We describe here the main `SuperIso Relic` routines. The descriptions of the `SuperIso` and `AlterBBN` routines are provided in their respective manuals, Refs. [1–3] and Refs. [4,5].

2.3.1 *SUSY Les Houches Accord routines*

- `int Les_Houches_Reader(char name[], struct parameters* param)`
This routine reads the SLHA file name and fills in the `param` structure.
- `int Higgs_Decays_Reader(char name[], struct parameters* param)`
`int SUSY_Decays_Reader(char name[], struct parameters* param)`
These two routines read the SLHA file name and fill in the Higgs and SUSY decay parts of the `param` structure.
- `int widthcalc(char name[], struct parameters* param)`
This routine computes the Higgs decay widths from the SLHA file and fills in the `param` structure. If `widthcalc=1` `Hdecay` is used, otherwise `FeynHiggs` is called.

2.3.2 *Relic density*

- `int Weff(double* res, double sqrtS, struct parameters* param)`
This routine computes the effective annihilation rate W_{eff} for a given \sqrt{S} value and stores the results in `res`. In case of failure, it returns 0, otherwise 1.
- `void Weff_table(double Wefftab[][] , int *nlines, double sqrtSmax, double maxenergy, struct parameters* param)`
This routine fills in a table of `nlines` values the effective annihilation rates W_{eff} for \sqrt{S} between 0 and `sqrtSmax` or `maxenergy`. It calls `Weff` a large number of times. The `OpenMP` library can be used to call them in parallel.

- `double sigmav(double T, double relicmass, double Wefftab[][], int nlines, struct parameters* param)`
This routine computes the effective cross section times velocity for a temperature T , using the relic mass `relicmass` and the W_{eff} table `Wefftab`.
- `void Tfo(double Wefftab[][], int nlines.Weff, struct parameters* param, double delta, struct relicparam* paramrelic)`
This routine determines the freeze-out temperature using the W_{eff} table `Wefftab`.
- `double relic_density(double Wefftab[][], int nlines.Weff, struct parameters* param, struct relicparam* paramrelic)`
This function returns the relic density using the W_{eff} table `Wefftab`. This is the central routine which solves the Boltzmann equations.
- `double relic_calculator(char name[])`
This routine is a container function which calls different subroutines to obtain the relic density after reading the SLHA file `name`.

For the usage of the other relic density routines, the user can refer to the main programs `test_phi.c`, `test_modeleff.c`, `test_standmod.c`, `test_reheating.c` and `test_widthcalc.c`.

2.3.3 Direct detection

- `int init_DDparamSLHA(struct DDparameters* DDparam, char* element, struct parameters* param)`
This routine fills in the `DDparam` structure which is used in the main direct detection routines, using the `param` parameters. `element` defines the target material and can be xenon "Xe", fluorine "F", germanium "Ge", argon "Ar", silicium "Si" or sodium iodide crystals "NaI".
After this routine is called, the parameters given below can be set to 1 or 0 in order to switch on or off the corresponding uncertainties in the calculation of the conservative, standard, and stringent constraints, as defined in Ref. [38]:
`DDparam.rho0error /* error on the local DM density */`
`DDparam.vescerror /* error on the escape velocity */`
`DDparam.vroterror /* error on the disk rotational velocity */`
`DDparam.nucleonSIerror /* error on the SI nucleon form factors */`
`DDparam.nucleonSDerror /* error on the SD nucleon form factors */`
`DDparam.nuclearSDerror /* error on the SD nuclear structure factors */`
- `void DDsigmaSINucleon(struct DDparameters* DDparam, double* ddpSI, double* ddnSI, char* option)`
`void DDsigmaSDNucleon(struct DDparameters* DDparam, double* ddpSD, double* ddnSD, char* option)`
These two routines compute the spin-independent and spin-dependent scattering cross sections of neutralinos with protons and neutrons from the

structure `DDparam`, and put them in `ddpSD` and `ddnSD`. The option can be "conservative", "standard" or "stringent", as defined in Ref. [38].

- `double XENON1Tlikelihood(struct DDparameters* DDparam, char *option)`
`double PANDAX2likelihood(struct DDparameters* DDparam, char *option)`
`double PICO60likelihood(struct DDparameters* DDparam, char* option)`

These routines compute the likelihoods from the `DDparam` structure with option "conservative", "standard" or "stringent" [38], for the XENON1T [18], PANDAX-2 [17] or PICO60 [19] experiments.

- `double scattering_SIp_calculator(char name[])`
`double scattering_SDp_calculator(char name[])`

These two routines compute the spin-independent and spin-dependent scattering cross sections of the neutralinos with protons, from an SLHA file name.

- `int direct_xenon1T_calculator(char name[], char* sigma)`
`int direct_pandax_calculator(char name[], char* sigma)`
`int direct_pico60_calculator(char name[], char* sigma)`

These routines check the exclusion by XENON1T [18], PANDAX-2 [17] or PICO60 [19] from an SLHA file name at a given confidence level: "1sigma", "80CL", "90CL", "95CL", "2sigma", "99CL", "3sigma", "99.9CL", "4sigma", or "5sigma". They return 1 if the model point is excluded and 0 if it is not.

- `void dRdE(struct DDparameters* DDparam, int n, double E[], double dRdE0[], char* option)`

This routine calculates the differential recoil rate per unit of target material mass for the particle model and target material defined in `DDparam` and for `n` recoil energy values in array `E`. The result is stored in `dRdE0`. option can be conservative, standard or stringent.

- `void set_nucleonFFSI(struct DDparameters* DDparam, double sigmapin[2], double sigmas[2], double z[2])`

This routine can be used to modify the main values and errors on the parameters $\Sigma_{\pi N}$, σ_s and z used in the calculation of SI nucleon form factors (see Appendix C.2.1.1). The first entry of `sigmapin`, `sigmas` and `z` must contain the central values and the second entry the errors. For instance, $\Sigma_{\pi N} = \text{sigmapin}[0] \pm \text{sigmapin}[1]$.

- `void set_nucleonFFSD(struct DDparameters* DDparam, double a3[2], double a8[2], double fsSDp[2])`

Similarly to `set_nucleonFFSI`, this routine allows the user to modify the values of the parameters a_3 , a_8 and Δs^p used in the calculation of SD nucleon form factors (see Appendix C.2.1.2).

- `void set_nuclei_Siformfactor(double (*FFSI)(double,int))`

This routine allows the user to define another SI nuclear form factor `FFSI` rather than the default Helm function. `FFSI` takes as input the momentum transfer q in keV and the nucleon number of the nucleus A .

- `void set_nuclei_SDformfactornoerr(struct DDparameters* DDparam, double (*S00[]) (double), double (*S01[]) (double), double (*S11[]) (double))`

This routine allows the user to define alternative SD structure factors S_{00} ,

S_{01} , and S_{11} . The inputs are arrays containing the structure factors of each isotope.

- `void set_Maxwell(struct DDparameters* DDparam, double rho0[], double vrot[], double vearth[], double vesc[])`

This routine allows the user to define alternative parameter values for the standard halo model. Each input contains the main value and error of the parameter. `rho0` is the local DM density, `vrot` is the galactic disk rotation velocity, `vearth` is the Earth velocity in the local standard of rest and `vesc` is the escape velocity.

- `void set_halo(struct DDparameters *DDparam, double rho0[], double (*eta)(struct DDparameters*, double, char*))`

This routine allows the user to specify alternative dark matter halo models by providing the local DM density `rho0` and a function `eta` giving the mean inverse speed. To be in agreement with the rest of the code, `eta` must take as inputs a structure `DDparameters`, a double `vmin` being the speed at which `eta` is evaluated, and a string of characters `option`.

For the usage of the other direct detection routines, the user can read the main program `direct.c`.

2.3.4 Indirect detection

2.3.4.1 Standard routines

- `int init_IDparameters(struct parameters* param, struct IDparameters* IDparam)`

This routine fills in the `IDparam` structure which is used in the main indirect detection routines, using the `param` parameters.

- `double deltalikelihood_fermi(struct fermi* fe, struct IDparameters* IDparam, char* option)`

This routine computes the likelihood from the `IDparam` structure with `option` "conservative", "standard" or "stringent" [38] for the FERMI-LAT experiment [20].

- `double deltachi2_AMS(struct IDparameters* IDparam, char* option)`

This routine computes the $\Delta\chi^2$ from the `IDparam` structure with `option` "conservative", "standard" or "stringent" [38] for the AMS-02 experiment [16].

- `double annihilation_sigmax_calculator(char name[])`

This routine computes the total thermally averaged annihilation cross section for a given SLHA file `name`.

- `int indirect_fermi_calculator(char name[], char* sigma)`

`int indirect_ams02_calculator(char name[], char* sigma)`

These routines check the exclusion by FERMI-LAT [20] or AMS-02 [16] from an SLHA file `name` at a given confidence level: "1sigma", "80CL", "90CL",

"95CL", "2sigma", "99CL", "3sigma", "99.9CL", "4sigma", or "5sigma". They return 1 if the model point is excluded and 0 if it is not.

For the usage of the other indirect detection routines, the user can read the main program `indirect.c`.

2.3.4.2 Propagation and dark matter halo models

- `void init_mypropagation(struct propagation_parameters* pparam, double kappa_0, double delta, double L, double v_conv, double v_a, char* propname)`
This routine is used for indirect detection to specify an antiproton propagation model. The ordered input parameters are the coefficients \mathcal{K}_0 , δ , L , V_{conv} and V_a , which are defined in Appendix B. An example of its usage is given in the main program `create_propagation.c`.
- `void init_myhalo(struct propagation_parameters* pparam, double r_earth, double rho_chi_solar, double (*profile)(double,double), char* name)`
This routine is used for indirect detection to specify a dark matter halo profile. An example of its usage is given in the main program `create_propagation.c`. `r_earth` is the distance of the Earth from the galactic center in kpc, `rho_chi_solar` the local DM density in GeV/cm^3 , `profile` is a function giving the DM density as a function of the distance from the galactic center r and the height z from the galactic plane.
- `void init_existing_model(struct propagation_parameters* pparam, char* halo, char* propagation)`
This routine is used to initialise a dark matter halo profile and the propagation model already provided in the package. `halo` can be `Einasto_CU10`, `NFW_M16` or `Burkert_NS13`, which are the DM halo density profiles defined in table B.2 and `propagation` can be `MIN`, `MED` or `MAX`, which are the benchmark sets of propagation parameters defined in table B.1.
- `void write_secondaries(struct propagation_parameters* pparam)`
This routine is used to write tabulated secondary antiproton spectra for the propagation model specified in `pparam`. The result can be found in the directory `src/antiprotons/models/halename/propname/secondaries/`, with `halename` and `propname` the names of the halo model and propagation model specified in `pparam`.
- `void write primaries(struct propagation_parameters* pparam, double mass_inf, double mass_sup)`
Similarly to `write_secondaries`, this routine writes primary antiproton spectra in the directory `src/antiprotons/models/halename/propname/primaries/` for LSP masses from `mass_inf` to `mass_sup`. `mass_inf` must be greater than 5 GeV and `mass_sup` must be smaller than 100000 GeV. Once the tabulated secondary and primary spectra are written, the calculation of AMS-02 an-

tiproton constraints is much faster, which can be useful when performing large scans.

For further descriptions of the propagation routines, we refer the user to the files `src/propagation.c` and `src/propagation.h`.

2.3.5 *Big-Bang Nucleosynthesis*

- `int bbn_excluded(struct relicparam* paramrelic)`
This routine belongs to `AlterBBN`. It returns 1 if the cosmological scenario defined in the `paramrelic` structure is excluded by the Big-Bang nucleosynthesis constraints, 0 if not, and -1 if the routine fails.

For the usage of the `AlterBBN` routines, we refer the user to the `AlterBBN` manual [4,5].

3 **Compilation and installation instructions**

The `SuperIso Relic` package and the latest version of the manual can be downloaded from:

<http://superiso.in2p3.fr/relic>

The package can be uncompressed with

```
tar xjvf superiso_relic.vX.X.tar.bz2
```

Before configuration, the user can choose the compilers with `BASH` commands such as

```
export CC=gcc
export FC=gfortran
export CXX=g++
```

The automatic configuration can be run with for example

```
./configure --with-mp
```

"`--with-mp`" is an option to activate the `OpenMP` multiprocessor compilation and calculation. The `configure` options can be accessed with `configure --help`.

At this step, it is possible to modify several configuration options in `Makefile`:

- `RELIC` can be set to 0 to deactivate the dark matter routines (this would correspond to the usage of `SuperIso` only for the calculation of the flavour

observables), 1 to activate them only for the MSSM, and 2 for the MSSM and NMSSM. This choice affects the compilation time and the size of the compiled library.

- The links to `SOFTSUSY`, `ISAJET`, `SuSpect`, `SPheno` and `NMSSMTools` have to be given here.
- The multiprocessor configuration can be forced by uncommenting `CFLAGS_MP`, `FFLAGS_MP` and `MAKE_MP`.

The next step is to configure the compilation mode with

```
make shared
```

which compiles the dark matter Fortran routines in shared dynamic libraries, or

```
make static
```

which compiles the routines in static libraries. The shared mode is faster to compile, but slower at execution, since the Fortran routines are compiled on-the-fly. The static mode is slower to compile, but easy to export.

The compilation of the libraries starts with

```
make
```

and the main programs are compiled with

```
make main_program.c
```

Examples of executions of main programs are given in the next section. In the static mode, if the `OpenMP` library is used, it is possible that a segmentation fault appears at execution, because of the large number of `COMMON` blocks used in the Fortran routines. In such a case, the `BASH` command

```
export OMP_STACKSIZE=16M
```

should be run before the execution.

4 Input and output description

4.1 *Standard cosmological and astrophysical scenarios*

The program `slha.x` calculates the observables using the parameters contained in a given SLHA file. The calculations are performed in the standard

cosmological and astrophysical scenarios. For example, the command

```
./slha.x example.lha
```

returns

SuperIso Relic v4.0 - A. Arbey, F. Mahmoudi & G. Robbins
SuperIso v4.0 - F. Mahmoudi

Observable	Value
BR(b->s gamma)	3.644e-04
delta0(B->K* gamma)	4.307e-02
	:
Relic density Oh2	1.254e+01
SI proton xsection	1.839e-10
SD proton xsection	5.242e-07
excluded_Xenon1T (standard)	0
excluded_PANDAX (standard)	0
excluded_PICO60 (standard)	0
Tot annihilation xsection	4.084e-30
excluded_Fermi (standard)	0
excluded_AMS02 (standard)	0

Here 0 means that the point is not excluded by the given constraint.

4.2 Modified relic density

SuperIso Relic incorporates different cosmological scenarios which affect the relic density calculation. The description of the scenarios is provided in the appendices, and their usage is explained below.

4.2.1 Alternative QCD equations of state

The program `test_modelreff.x` calculates the relic density using the parameters contained in a given SLHA file, for different QCD equations of state. For example, the command

```
./test_modeleff.x example.lha
```

returns

Dependence of the relic density on the calculation of h_{eff} and g_{eff}

For `model_eff=1` (model A): $\omega=1.254\text{e}+01$

For `model_eff=2` (model B (default)): $\omega=1.254\text{e}+01$

For `model_eff=3` (model B2): $\omega=1.262\text{e}+01$

For `model_eff=4` (model B3): $\omega=1.247\text{e}+01$

For `model_eff=5` (model C): $\omega=1.255\text{e}+01$

For `model_eff=6` (Bonn model): $\omega=1.231\text{e}+01$

For `model_eff=0` (old model): $\omega=1.229\text{e}+01$

The description of the different models are provided in Appendix A.1.

4.2.2 *Effective energy and entropy densities*

The program `test_standmod.x` calculates the relic density using the parameters contained in a given SLHA file, in a cosmological scenario where the energy and entropy contents are modified. In addition to the name of an SLHA file, this program needs four additional parameters and five optional ones, and is run with:

```
./test_standmod.x filename  $\kappa_\rho$   $n_\rho$   $\kappa_s$   $n_s$  ( $T_\rho$   $T_s$   $N$   $n_N$   $T_N$ )
```

where

- κ_ρ is the ratio of dark energy density to photon energy density at $T_{\text{BBN}} = 1$ MeV,
- n_ρ is the temperature exponent of the dark energy density,
- κ_s is the ratio of dark entropy density to photon entropy density at $T_{\text{BBN}} = 1$ MeV,
- n_s is the temperature exponent of the dark entropy density,

and the optional parameters (which, if not specified, are set to 0) are:

- T_ρ is a temperature cut in GeV below which the dark energy density is set to 0,
- T_s is a temperature cut in GeV below which the dark entropy density is set to 0,
- N is the non-thermal production rate of supersymmetric particles at $T_{\text{BBN}} = 1$ MeV,
- n_N is the temperature exponent of the non-thermal production rate of supersymmetric particles,
- T_N is a temperature cut in GeV below which the non-thermal production

is set to 0.

The parameters are further described in Appendix A.2.

For example, the command

```
./test_standmod.x example.lha 1 6 1 5
```

returns

```
SuperIso Relic v4.0 - A. Arbey, F. Mahmoudi & G. Robbins  
AlterBBN v2.0 - A. Arbey, J. Auffinger, K. Hickerson & E. Jenssen
```

For the cosmological standard model:

```
omega=1.254e+01
```

For the specified model with dark density/entropy/non thermal relics:

```
omega=9.924e+03
```

```
Model excluded by BBN constraints
```

4.2.3 Reheating

The program `test_reheating.x` calculates the relic density using the parameters contained in a given SLHA file, in a cosmological scenario with reheating. In addition to the name of an SLHA file, this program needs five additional parameters and six optional ones, and is run with:

```
./test_reheating.x filename  $\kappa_\rho$   $n_\rho$   $\kappa_{\Sigma_r}$   $n_{\Sigma_r}$   $T_{\text{cut}}$  ( $\kappa_s$   $n_s$   $N$   $n_N$   $\kappa_\Sigma$   $n_\Sigma$ )
```

- κ_ρ is the ratio of dark energy density to photon energy density at $T_{\text{BBN}} = 1$ MeV,
- n_ρ is the temperature exponent of the dark energy density,
- κ_{Σ_r} is the ratio of radiation entropy injection Σ_r to radiation entropy time-derivative at $T_{\text{BBN}} = 1$ MeV,
- n_{Σ_r} is the temperature exponent of the radiation entropy injection,
- T_{cut} is a temperature cut in GeV below which non-standard terms are set to 0,

and the optional parameters (which, if not specified, are set to 0) are:

- κ_s is the ratio of dark entropy density to photon entropy density at $T_{\text{BBN}} = 1$ MeV,
- n_s is the temperature exponent of the dark entropy density,
- N is a non-thermal production rate of supersymmetric particles at $T_{\text{BBN}} = 1$ MeV,
- n_N is the temperature exponent of the non-thermal production rate of su-

- persymmetric particles,
- κ_Σ is the ratio of dark entropy injection Σ to radiation entropy time-derivative at $T_{\text{BBN}} = 1$ MeV,
- n_Σ is the temperature exponent of the dark entropy injection.

The parameters are further described in Appendix A.2.

For example, the command

```
./test_reheating.x example.lha 1.e-6 6 4.75e-6 7 0.01
```

returns

For the cosmological standard model:

```
omega=1.254e+01
```

For the specified model with dark density/radiation entropy production:

```
omega=1.069e-01
```

```
Model compatible with BBN constraints
```

4.2.4 *Decaying scalar field*

The program `test_phi.x` calculates the relic density using the parameters contained in a given SLHA file, in a cosmological scenario with a decaying scalar field. In addition to the name of an SLHA file, this program needs two additional parameters and two optional ones, and is run with:

```
./test_phi.x filename  $\tilde{\rho}_\phi$   $T_{RH}$  ( $T_{\text{init}}$   $\eta_\phi$ )
```

where

- $\tilde{\rho}_\phi$ is the ratio of the scalar field density to photon energy density at $T = T_{\text{init}}$.
- T_{RH} is the reheating temperature in GeV,

and two optional ones:

- T_{init} is the initial temperature in GeV at which the calculation starts (100 GeV by default),
- η_ϕ is the non-thermal supersymmetric particle production parameter.

The parameters are further described in Appendix A.2.3.

For example, the command

```
./test_phi.x example.lha 100 10 50
```

returns

For the cosmological standard model:

$\omega=1.220e+01$

For the specified model with dark density/entropy/non thermal relics:

$\omega=3.989e+00$

Model compatible with BBN constraints

Note that for this scenario the parameter `full_comput` is set to 1 in order to compute the evolution of the scalar field before freeze-out. The calculation is therefore slower and the needed time increases with the initial temperature.

4.2.5 Width calculators

The program `test_widthcalc.x` calculates the relic density using the parameters contained in a given SLHA file, and the Higgs widths computed with different decay calculators. For example, the command

```
./test_widthcalc.x example.lha
```

returns

Dependence of the relic density on the width calculator

Widths in the SLHA file: $\omega=1.254e+01$

With Hdecay: $\omega=1.254e+01$

With FeynHiggs: $\omega=1.254e+01$

With FeynHiggs Tree: $\omega=1.254e+01$

which shows that for this specific point, the dependence on the Higgs widths is negligible.

4.3 Direct detection

The program `direct.x` calculates dark matter direct detection observables using the parameters contained in a given SLHA file. In addition to the name of an SLHA file, this program needs two additional parameters `nuclerr` and `haloerr`, which if set to 1, activate the uncertainties related to the nuclear form factors and halo profile, respectively, and if 0 deactivate them. The “standard” mode uses the central values of nuclear and astrophysical parameters, whereas the “conservative” and “stringent” modes scan over the parameter error ranges to calculate the weakest and strongest constraints, respectively. For example, the command

```
./direct.x example.lha 1 1
```

returns

```
/----WIMP-NUCLEON cross-section(pb) at 0-momentum transfer----/
```

```
 /-Spin-Independent-/
```

Conservative	Standard	Stringent
Proton 1.560368e-10	1.838746e-10	2.166668e-10
Neutron 1.601246e-10	1.907715e-10	2.242631e-10

```
 /-Spin-Dependent-/
```

Conservative	Standard	Stringent
Proton 4.702998e-07	5.242344e-07	5.810959e-07
Neutron 4.788661e-07	5.332762e-07	5.906135e-07

```
 /----PANDAX-2 2017 Poisson delta-loglikelihood
```

```
 (point excluded at 2 sigma if <-4.000000 with 1 d.o.f.)----/
```

Conservative	Standard	Stringent
-8.611648e-02	-2.265334e+00	-6.704124e+00

```
 /----XENON1T 2017 Poisson delta-loglikelihood
```

```
 (point excluded at 2 sigma if <-4.000000 with 1 d.o.f.)----/
```

Conservative	Standard	Stringent
-8.199753e-01	-2.076444e+00	-3.857909e+00

```
 /----PIC060 2017 Poisson delta-loglikelihood
```

```
 (point excluded at at 2 sigma if <-4.000000 with 1 d.o.f.)----/
```

Conservative	Standard	Stringent
-6.729574e-03	-2.261627e-02	-4.934238e-02

4.4 Indirect detection

4.4.1 Standard scenarios

The program `indirect.x` calculates dark matter indirect detection observables using the parameters contained in a given SLHA file, for standard astrophysical scenarios. For example, the command

```
./indirect.x example.lha
```

returns

neutralino mass 209.466537 GeV

channel <sigmav>[cm³/s]

```
-----  
o1 o1 -> ebar e | 6.50906e-38  
o1 o1 -> mbar m | 2.78280e-33  
o1 o1 -> lbar l | 5.06090e-31  
o1 o1 -> ubar u | 8.05216e-39  
o1 o1 -> dbar d | 7.72181e-36  
o1 o1 -> sbar s | 3.22156e-33  
o1 o1 -> cbar c | 5.86786e-33  
o1 o1 -> bbar b | 2.87770e-30  
o1 o1 -> tbar t | 1.73053e-31  
o1 o1 -> wbar w | 2.33753e-31  
o1 o1 -> z z | 7.59111e-32  
o1 o1 -> h z | 2.06015e-31  
-----
```

Total annihilation cross section 4.084403e-30 [cm³/s]

Fermi-LAT

delta-loglikelihood : excluded at 2 sigma if <-12.850000 with 6 d.o.f.

conservative standard stringent

2.907243e-04 2.729815e-04 -2.859356e-04

AMS-02

delta-loglikelihood : excluded at 2 sigma % CL if <-12.850000 with 6 d.o.f.

conservative standard stringent

1.656700e-02 2.215116e-02 7.300298e-02

4.4.2 Scenarios with modified propagation and halo models

The program `create_propagation.x` calculates dark matter indirect detection observables using the parameters contained in a given SLHA file, for the dark matter halo and propagation scenarios defined in `create_propagation.c`. For example, the command

```
./create_propagation.x example.lha
```

returns

```
neutralino mass 209.466537 GeV
```

channel	<sigmav>[cm ³ /s]
o1 o1 -> ebar e	6.50906e-38
o1 o1 -> mbar m	2.78280e-33
o1 o1 -> lbar l	5.06090e-31
o1 o1 -> ubar u	8.05216e-39
o1 o1 -> dbar d	7.72181e-36
o1 o1 -> sbar s	3.22156e-33
o1 o1 -> cbar c	5.86786e-33
o1 o1 -> bbar b	2.87770e-30
o1 o1 -> tbar t	1.73053e-31
o1 o1 -> wbar w	2.33753e-31
o1 o1 -> z z	7.59111e-32
o1 o1 -> h z	2.06015e-31

```
-----  
Total annihilation cross section 4.084403e-30 [cm3/s]
```

```
delta-loglikelihood 4.444281e-02
```

5 Results

The results of `SuperIso Relic` have been extensively compared with those of `micrOMEGAs` [39–41] and `DARKSUSY` [8,9], and show a very good global agreement. We refer the user to Refs. [38,42] for recent results obtained with `SuperIso Relic v4`.

The implementation of indirect and direct detection constraints is detailed in Appendices B and C and we show here the spin-independent (SI) upper limits from XENON1T [18] and PANDAX-2 [17], as well as the spin-dependent (SD) limit from PICO60 [19] in Figures 1 and 2, respectively. The official limits are also displayed for comparison. For the calculation of the upper limits, we used the standard parameter values of the dark matter halo $\rho_\chi = 0.3 \text{ GeV/cm}^3$, $v_{rot} = 220 \text{ km/s}$ and $v_{esc} = 544 \text{ km/s}$.

In addition, we show the impact of nuclear errors on these limits. To this end, we use the same pMSSM-19 sample of points as in Ref. [38] and calculate for each point the number of expected events μ which would be measured by XENON1T and PICO60 in the “conservative”, “standard” and “stringent” cases, according to the underlying uncertainties, and show the relative error

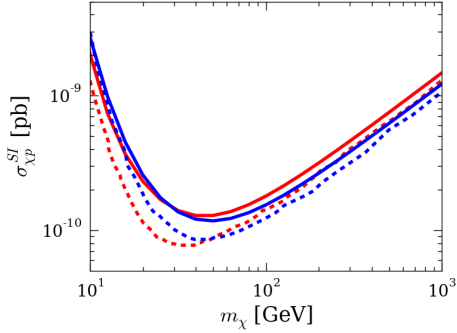


Fig. 1. 90% C.L. spin-independent upper limit for XENON1T (red) and PANDAX-2 (blue). Official limits published by the collaborations are shown in dashed lines while the limits calculated from this work are in solid lines.

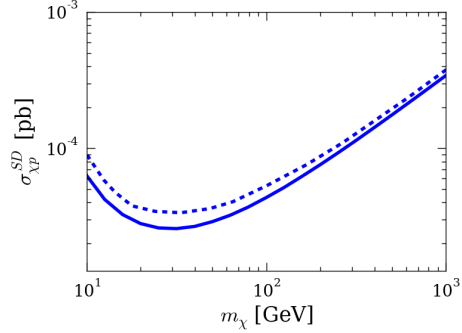


Fig. 2. 90% C.L. spin-dependent upper limit for PICO60. Official limit published by the collaboration is shown in dashed line while the limit calculated from this work is in solid line.

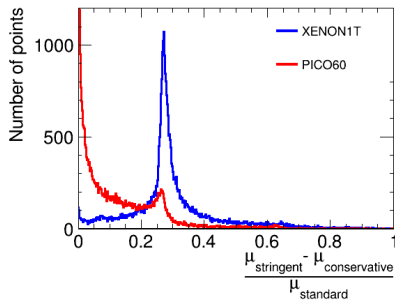


Fig. 3. Relative uncertainties from SI nucleon form factor errors on the number of events which would be measured by XENON1T (in blue) and PICO60 (in red) for our pMSSM sample of points.

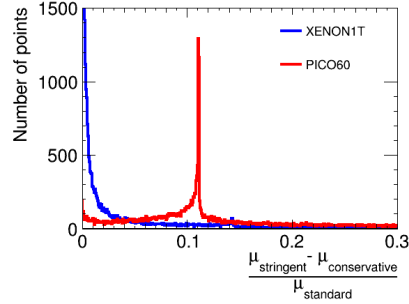


Fig. 4. Relative uncertainties from SD nucleon form factor errors on the number of events which would be measured by XENON1T (in blue) and PICO60 (in red) for our pMSSM sample of points.

$(\mu_{\text{stringent}} - \mu_{\text{conservative}})/\mu_{\text{standard}}$. The relative error coming from SI nucleon form factors is first shown in Figure 3. One can note a typical error of $\sim 27\%$ for the xenon experiment, whereas the error can be neglected in most cases for the fluorine experiment, which is more sensitive to SD interactions. By default, we will therefore consider the SI nucleon form factor uncertainties for xenon experiments but not for fluorine experiments. Then, we show in Figure 4 the relative error from SD nucleon form factor uncertainties. For PICO60, we obtain a typical error of $\sim 11\%$, whereas the uncertainties can be safely neglected for XENON1T. In this last case, SD nucleon form factor uncertainties are therefore, by default, not considered.

Finally, we show in figure 5, the impact of the uncertainties on the SD nuclear structure factors arising from two-body currents. While the effect on

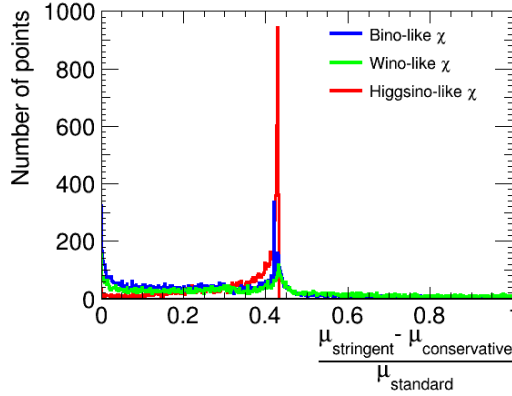


Fig. 5. Relative uncertainties from SD nuclear structure factor errors on the number of events which would be measured by PICO60 for our pMSSM sample of points. Wino-like neutralinos are shown in green, Higgsino-like neutralinos in red and bino-like neutralinos in blue.

XENON1T results can be safely disregarded, the relative error on PICO60 detection is of the order of 40%. Higgsino-like neutralinos suffer in particular from these uncertainties. Indeed, in the structure factor $S(q)$, the functions S_{01} and S_{11} , which have the uncertainties, are multiplied by $a_1 = A_p^{SD} - A_n^{SD}$ (see Appendix C). Spin-dependent nuclear uncertainties therefore have a significant effect in isospin violating models. As the effective SD neutralino-quark coupling through a Z boson exchange is proportional to the quark weak isospin t^3 and to the difference of the Higgsino mixing matrix elements $|N_{14}|^2 - |N_{13}|^2$, Higgsino-like neutralinos have thereby a particularly large a_1 and are therefore strongly affected by the structure function uncertainties.

Appendix A Relic density in alternative cosmology

We provide here the description of the modifications to the cosmological scenarios focussing in particular on the new aspects with respect to the original manual [6,7].

A.1 QCD equations of state

The number of relativistic effective radiation degrees of freedom of energy and entropy, g_{eff} and h_{eff} respectively, are important for the calculation of the relic density. The previous versions of `SuperIso Relic` incorporated the ideal gas scenarios as well as five more realistic models described in Ref. [43]. The current version incorporates in addition another scenario for which lattice QCD calculations are used to improve the description of the thermal plasma, which is defined in Ref. [44] and referred to as `Bonn model` in `SuperIso Relic`.

The models can be selected in the routine `Init_modeleff(int model_eff, struct relicparam* paramrelic)` by setting the value of `model_eff` as given below:

- Model A (`model_eff=1`): ignores hadrons completely.
- Model B (`model_eff=2`): models hadrons as a gas of free mesons and hadrons, with a sharp switch to the hadronic gas at a temperature $T = 154$ MeV.
- Model B2 (`model_eff=3`): variation of model B constructed by scaling the pressure and energy density lattice data by 0.9.
- Model B3 (`model_eff=4`): variation of model B constructed by scaling the pressure and energy density lattice data by 1.1.
- Model C (`model_eff=5`): models hadrons as a gas of free mesons and baryons, with a sharp switch to the hadronic gas at $T = 200$ MeV.
- Bonn model (`model_eff=6`): uses lattice QCD calculations to improve the description of the thermal plasma.
- Old Model (`model_eff=0`): models hadrons as an ideal gas.

An example main program is given in `test_modeleff.c`.

A.2 Modified Cosmological Models

The number density of supersymmetric particles can be determined by solving the Boltzmann equation:

$$\frac{dn}{dt} = -3Hn - \langle\sigma v\rangle(n^2 - n_{eq}^2) + N_{n.t.}, \quad (\text{A.1})$$

where n is the number density of supersymmetric particles, $\langle\sigma v\rangle$ the thermally averaged annihilation cross section, $H = \dot{a}/a$ the Hubble parameter, $a(t)$ the expansion factor, and n_{eq} the relic particle equilibrium number density. The term $N_{n.t.}$ corresponds to a possible non-thermal production of supersymmetric particles. The expansion rate H is related to the energy content of the Universe through the Friedmann equation:

$$H^2 = \frac{8\pi G}{3}(\rho_{rad} + \rho_D), \quad (\text{A.2})$$

where ρ_{rad} is the radiation energy density, considered as dominant before BBN in the standard cosmological model:

$$\rho_{rad} = g_{\text{eff}}(T) \frac{\pi^2}{30} T^4. \quad (\text{A.3})$$

ρ_D can be considered as an effective ‘‘dark energy’’ density which parametrises the modification of the expansion rate.

The relation between the time t and/or scale factor $a(t)$ and the temperature T is given by the entropy evolution equation:

$$\frac{ds}{dt} = -3Hs + \Sigma, \quad (\text{A.4})$$

where s is the entropy density and Σ an entropy injection. In the standard model of cosmology s contains the radiation entropy density only, which depends on the QCD equations of state, and $\Sigma = 0$. The radiation entropy density can be written as:

$$s_{rad} = h_{\text{eff}}(T) \frac{2\pi^2}{45} T^3. \quad (\text{A.5})$$

In a modified cosmological scenario, the entropy density can be composed of radiation entropy density and a ‘‘dark entropy’’ density s_D . Σ can therefore be an injection term either for radiation entropy or dark entropy.

In the following, we describe the different parametrisations implemented in `SuperIso Relic`.

A.2.1 Modified expansion rate

The expansion rate is modified through the addition of a “dark density” ρ_D , which is parametrised as [45]:

$$\rho_D(T) = \kappa_\rho \rho_\gamma(T_{\text{BBN}}) \left(\frac{T}{T_{\text{BBN}}} \right)^{n_\rho}, \quad (\text{A.6})$$

where $T_{\text{BBN}} = 1$ MeV is approximately the Big-Bang nucleosynthesis temperature. κ_ρ is therefore the ratio of the dark density to the photon density at this energy scale². n_ρ is the decrease exponent of this dark density (4 for radiation, 3 for matter, 0 for a cosmological constant, *etc.*). A temperature cut can be added below which the dark density is set to 0.

A.2.2 Modification of the entropy content

We consider two different cases for the entropy content, which can occur simultaneously or separately.

A.2.2.1 Dark entropy: If the “dark entropy” s_D is not linked to any reheating process of radiation, two different parametrisations are possible. The first one was described in Refs. [45,46]:

$$s_D = \kappa_s s_\gamma(T_{\text{BBN}}) \left(\frac{T}{T_{\text{BBN}}} \right)^{n_s}, \quad (\text{A.7})$$

where κ_s is the ratio of the dark entropy density to the photon entropy density at $T_{\text{BBN}} = 1$ MeV, and n_s is the decrease exponent of this component. Again, a temperature cut below which this density is strictly 0 can be set.

The second parametrisation of the “dark entropy” density is through the dark entropy injection Σ_D as described in Ref. [47]:

$$\Sigma_D(T) = \kappa_\Sigma \Sigma_{rad}^{\text{eff}}(T_{\text{BBN}}) \left(\frac{T}{T_{\text{BBN}}} \right)^{n_\Sigma}, \quad (\text{A.8})$$

where κ_Σ is the ratio of the dark entropy injection to the radiation entropy density time-derivative $\Sigma_{rad}^{\text{eff}}(T) \equiv \left| \frac{ds_{rad}}{dt} \right| = 3Hs_{rad}$ at $T_{\text{BBN}} = 1$ MeV and n_Σ is the decrease exponent of the entropy production. The corresponding “dark

² In the first versions of `SuperIso Relic`, κ_ρ was defined as the ratio of the dark density to the *radiation* density.

entropy” density is therefore the integral:

$$s_D(T) = 3\sqrt{\frac{5}{4\pi^3 G}} h_{\text{eff}}(T) T^3 \int_0^T dT' \frac{\sqrt{g_*(T) \Sigma_D(T')}}{h_{\text{eff}}^2(T') T'^6 \sqrt{1 + \rho_D(T)/\rho_{\text{rad}}(T)}}, \quad (\text{A.9})$$

where the values of $h_{\text{eff}}(T)$ and $g_{\text{eff}}(T)$ are tabulated.

A.2.2.2 Reheating: The radiation entropy density s_{rad} can receive contributions from a radiation entropy injection Σ_{rad} such as

$$\frac{ds_{\text{rad}}}{dt} = -3Hs_{\text{rad}} + \Sigma_{\text{rad}}, \quad (\text{A.10})$$

which can modify the relation between the temperature T and the time t , resulting in a “reheating” of the primordial plasma and a local increase of the radiation entropy density. In absence of Σ_{rad} , this relation corresponds to $d(s_{\text{rad}}a^3)/dt = 0$.

We use the following parametrisation:

$$\Sigma_{\text{rad}}(T) = \kappa_{\Sigma_r} \Sigma_{\text{rad}}^{\text{eff}}(T_{\text{BBN}}) \left(\frac{T}{T_{\text{BBN}}} \right)^{n_{\Sigma_r}}, \quad (\text{A.11})$$

where κ_{Σ_r} is the ratio of the radiation entropy injection to the radiation entropy density time-derivative $\Sigma_{\text{rad}}^{\text{eff}}(T) \equiv \left| \frac{ds_{\text{rad}}}{dt} \right| = 3Hs_{\text{rad}}$ at $T_{\text{BBN}} = 1$ MeV and n_{Σ_r} is the decrease exponent of the entropy production.

A.2.3 Decaying scalar field

We consider a scenario with a decaying pressureless primordial scalar field [48,49,42]. The scalar field density ρ_ϕ is driven by the Boltzmann equation

$$\frac{d\rho_\phi}{dt} = -3H\rho_\phi - \Gamma_\phi \rho_\phi, \quad (\text{A.12})$$

where Γ_ϕ is the decay width of the scalar field. The scalar field decay results in radiation entropy injection such as:

$$\frac{ds_{\text{rad}}}{dt} = -3Hs_{\text{rad}} + (1-b) \frac{\Gamma_\phi \rho_\phi}{T}, \quad (\text{A.13})$$

where b is the branching fraction of the decaying scalar field into supersymmetric particles. The non-thermal injection of supersymmetric particles affects

the Boltzmann equation such as:

$$\frac{dn}{dt} = -3Hn - \langle\sigma v\rangle(n^2 - n_{eq}^2) + \frac{b}{m_\phi}\Gamma_\phi\rho_\phi, \quad (\text{A.14})$$

with m_ϕ being the scalar field mass. One can define $\eta_\phi \equiv b/m_\phi$.

In general one can expect the relic density to be mainly generated thermally, so that b/m_ϕ remains small, and $b \ll 1$. The decay width can be related to the reheating temperature T_{RH} through

$$\Gamma_\phi = \sqrt{\frac{4\pi^3 g_{\text{eff}}(T_{RH})}{45}} \frac{T_{RH}^2}{M_P}, \quad (\text{A.15})$$

where g_{eff} is the effective relativistic energy degrees of freedom, which can be obtained from the tables contained in `sgStar_heff`.

This scenario requires three input parameters, the first one being $\tilde{\rho}_\phi$ the scalar field energy density divided by the photon energy density at the initial temperature (which can be either the freeze-out temperature or a user-defined temperature), the second one the reheating temperature T_{RH} and the third one the value of η_ϕ , which is expected to be small. T_{init} should be sufficiently larger than the reheating temperature and the freeze-out temperature in order to integrate properly the evolution equation of the scalar field and of the density of supersymmetric particles.

Appendix B Indirect detection

We turn to the calculation of the constraints from AMS-02 antiproton and Fermi-LAT gamma-ray data. These two types of constraints are derived from different kinds of analyses. However, they both require the calculation of the antiproton (or gamma-ray) flux produced from one dark matter particle annihilation process. These two fluxes at production are computed following the procedure described below. In both analyses, the lightest neutralino is assumed to account for the entire dark matter. There is no rescaling when the relic density calculated in the standard cosmological model is below the observed DM density. In this case, modified cosmological scenarios can generally be invoked to obtain the correct neutralino relic density.

B.1 Fluxes at production

Dark matter annihilates into pairs of Standard Model particles which subsequently hadronise into high-energetic cosmic rays. The flux at production of

antiprotons (or γ -rays) can be expressed as the sum over dark matter annihilation channels $\chi\chi \rightarrow p_3 p_4$ of the antiproton (γ -ray) flux resulting from the hadronisation of particles p_3 and p_4 with an energy of E_{p_3} and E_{p_4} respectively, weighted by the channel branching ratio:

$$\frac{dN_{prod}}{dK}(K) = \sum_{\chi\chi \rightarrow p_3 p_4} BR(\chi\chi \rightarrow p_3 p_4) \left(\frac{dN_{p_3}}{dK}(E_{p_3}, K) + \frac{dN_{p_4}}{dK}(E_{p_4}, K) \right), \quad (\text{B.1})$$

where K is the kinetic energy of antiprotons (γ -rays).

Keeping in mind that the center of mass energy of dark matter annihilation processes is $\sqrt{s} = 2m_\chi$, the energies E_{p_3} and E_{p_4} can be calculated from the energy and momentum conservation:

$$\begin{cases} E_{p_3} + E_{p_4} = \sqrt{s}, \\ \vec{p}_3 + \vec{p}_4 = \vec{0}, \end{cases} \quad (\text{B.2})$$

which gives

$$\begin{cases} E_{p_3} = \frac{\sqrt{s}}{2} + \frac{M_3^2 - M_4^2}{2\sqrt{s}}, \\ E_{p_4} = \frac{\sqrt{s}}{2} + \frac{M_4^2 - M_3^2}{2\sqrt{s}}, \end{cases} \quad (\text{B.3})$$

where M_3 and M_4 are the outgoing particle masses. In the case $M_3 = M_4$, we retrieve the simple relation $E_{p_3} = E_{p_4} = \frac{\sqrt{s}}{2} = m_\chi$.

We use the tabulated spectra at production $\frac{dN_{\chi\chi \rightarrow p\bar{p}}^{PPPC4DMID}}{dK}(m_\chi = E_p, K)$ from PPC4DMID [50,51], which gives the flux of antiprotons (γ rays) produced by one annihilation process of dark matter particles of a given mass m_χ annihilating via one of the following channels:

$$\chi\chi \rightarrow e^+e^-, \mu^+\mu^-, \tau^+\tau^-, q\bar{q}, c\bar{c}, b\bar{b}, t\bar{t}, W^+W^-, gg, \gamma\gamma, \nu_e\bar{\nu}_e, \nu_\mu\bar{\nu}_\mu, \nu_\tau\bar{\nu}_\tau,$$

where q stands for light quarks u, d, s . These channels do not cover all the possible annihilation processes in the MSSM. However, it is possible to deduce the fluxes produced by the hadronisation of a SM particle p with energy E_p from these tabulated spectra:

$$\frac{dN_p}{dK}(E_p, k) = \frac{1}{2} \frac{dN_{\chi\chi \rightarrow p\bar{p}}^{PPPC4DMID}}{dK}(m_\chi = E_p, K). \quad (\text{B.4})$$

The spectra for the annihilation into light SM Higgs bosons of mass 125 GeV are also provided, however Higgs branching ratios may receive significant corrections in the MSSM. The hadronisation spectra of the lightest Higgs boson is therefore re-calculated using the branching ratios computed with `HDECAY` [27] or `FeynHiggs` [26]

$$\frac{dN_h}{dK}(E_h, K) = \sum_{h \rightarrow p_3 p_4} BR(h \rightarrow p_3 p_4) \left(\frac{dN_{p_3}}{dK}(E_{p_3}, K) + \frac{dN_{p_4}}{dK}(E_{p_4}, K) \right), \quad (\text{B.5})$$

where E_{p_3} and E_{p_4} are calculated from Eq. (B.3) with $\sqrt{s} = E_h$. The hadronisation spectra of heavier Higgs bosons are then calculated in a similar way.

Finally, for the calculation of the neutralino annihilation branching ratios and cross sections in the MSSM and NMSSM, we use the same routines as for the relic density calculation to compute the neutralino annihilation amplitudes $\chi\chi \rightarrow p_3 p_4$.

For a Majorana dark matter particle, at small velocity limit, the annihilation cross sections can be calculated straightforwardly from these amplitudes, following:

$$\langle\sigma v\rangle_{\chi\chi \rightarrow p_3 p_4} = \frac{|\mathcal{A}|_{\chi\chi \rightarrow p_3 p_4}^2}{128\pi m_\chi^2} \left[1 - \frac{M_3^2 + M_4^2}{2m_\chi^2} + \frac{(M_3^2 - M_4^2)^2}{16m_\chi^4} \right]^{\frac{1}{2}}. \quad (\text{B.6})$$

B.2 Constraints from Fermi-LAT dwarf spheroidal galaxies

The analysis of Fermi-LAT dwarf spheroidal galaxies (dSphs) is based on [52]. Fermi-LAT collaboration performs a binned Poisson maximum-likelihood analysis in order to deduce the dark matter constraints. The energy range is separated into 24 bins, logarithmically spaced from 500 MeV to 500 GeV. Tabulated log-likelihoods are provided by the collaboration for each dSph and energy bin [53]. These tables allow us to estimate the log-likelihood \mathcal{L}_j^i for a dSph i and energy bin j as a function of the gamma-ray flux produced by dark matter annihilation.

The flux produced by the dark matter halo of a dSph i in the energy bin $[E_{\min}^j, E_{\max}^j]$ is calculated as

$$\Phi_j^i = \frac{1}{4\pi} \frac{\langle\sigma v\rangle}{2m_{DM}^2} \times J^i \times \int_{E_{\min}^j}^{E_{\max}^j} \left(\frac{dN_{\text{prod}}}{dE_\gamma} \right)_{\text{channel}} dE_\gamma, \quad (\text{B.7})$$

where $\frac{dN_{\text{prod}}}{dE_\gamma}$ is the gamma-ray flux at production calculated as described in

section B.1 and J^i is the J-factor of the dSph defined as $J = \int_{\Delta\Omega} \int_{l.o.s} \rho_{DM}^2(r(l)) dl d\Omega$ with $\Delta\Omega$ the solid angle under which the dSph is seen. The J-factor of each dSph is either deduced from their observed kinematics or, when no measurements are available, from an empirical law which states that the J-factor scales as the inverse square of the distance of the dSph.

In order to calculate the log-likelihood for a given dSph, we sum the log-likelihoods of every energy bins. Then, we add a corrective term to take into account the uncertainties on the J-factor:

$$\mathcal{L}^i(J_i) = \sum_j \mathcal{L}_j^i(J_i) - \frac{\left(\log_{10}(J_i) - \log_{10}(J_{obs,i})\right)^2}{2\sigma_i^2}, \quad (\text{B.8})$$

where J_i is the true value of the J-factor, considered as a nuisance parameter, and $J_{obs,i}$ is the measured J-factor with error σ_i . For each dSph, a maximum log-likelihood is then calculated according to the nuisance parameters. Finally, we sum the maximum log-likelihood of every dSphs:

$$\mathcal{L} = \sum_i \max_{J_i} \mathcal{L}^i(J_i). \quad (\text{B.9})$$

The statistical test that we use to derive constraints on dark matter is then calculated by subtracting from the maximum log-likelihood of every dSphs the log-likelihood in the case where no dark matter is assumed to participate to the spectrum:

$$\text{TS} = \mathcal{L}_{\text{DM}} - \mathcal{L}_{\text{noDM}}. \quad (\text{B.10})$$

This quantity follows a normal distribution and we will exclude points with a statistical test $\text{TS} < -\chi_0^2$, with χ_0^2 a critical value depending on the desired confidence level and on the number of degrees of freedom (d.o.f.). We choose the d.o.f. as the number of annihilation channels which contribute at least to 1% of the total annihilation cross section.

As already mentioned, the J-factor of some dSphs are calculated using an empirical relation. In order to assess the uncertainties on the log J-factor, we use 3 different values of $\sigma_i = 0.4, 0.6$ or 0.8 dex for the “stringent”, “standard” and “conservative” options.

In addition, three different samples of dSphs are defined in Fermi-LAT analysis: a “conservative”, a “nominal” and an “inclusive” sample, depending on the ambiguity of the kinematics of the galaxies. The “conservative” sample does not necessarily lead to a weaker limit compared to the “inclusive” one, as some dSphs in the “nominal” and “inclusive” samples show small but not significant enough excesses. Therefore, the delta log-likelihood is calculated for each sample. Our “conservative” option uses the largest delta log-likelihood, the “standard” option, the second largest one, and the “stringent” option, the smallest one.

B.3 Constraints from AMS-02 antiprotons

Contrary to gamma-rays, antiprotons are diffused by turbulent magnetic fields in the galaxy. Therefore, it is necessary to describe the propagation of cosmic rays in the galactic medium in order to deduce the antiproton flux reaching Earth. To this end, we embed in `SuperIso Relic` a code developed by Pierre Salati and Mathieu Boudaud, which is detailed in [30]. The propagation model used in this code is briefly described below.

B.3.1 Propagation Model

We use a two-zone diffusion model where the galactic medium is a thin disk of $h = 100$ pc height and $R = 20$ kpc radius and cosmic rays are diffused in a cylinder of half-height L and radius R .

The antiproton spectrum respects the differential equation of propagation:

$$\frac{\partial f}{\partial t} - \mathcal{K}(K) \cdot \nabla^2 f + \frac{\partial}{\partial z} \{ \text{sign}(z) f V_{\text{conv}} \} + \frac{\partial}{\partial E} \left\{ b(K, \vec{x}) f - \mathcal{K}_{\text{EE}}(K) \frac{\partial f}{\partial E} \right\} = Q, \quad (\text{B.11})$$

where $f = \frac{dN}{dK}(r, z, E)$ is the antiproton spectrum at radius r and height z . We assume a cylindrical symmetry in order to decompose f into Bessel transforms. This method allows us to solve semi-analytically the equation of transport, which reduces significantly the computation time compared to a full numerical approach.

The first term in the equation of transport (B.11) is set to zero since we only focus on steady-state solutions.

B.3.1.1 Space diffusion: The second term in Eq. (B.11) describes the antiproton space diffusion with a coefficient

$$\mathcal{K}(K) = \mathcal{K}_0 \beta p^\delta, \quad (\text{B.12})$$

where $\beta = v/c$ is the antiproton beta factor and p its momentum. \mathcal{K}_0 and δ are the free parameters of the model which set the normalisation and momentum dependence of the diffusion coefficient.

The third term in Eq. (B.11) corresponds to convection processes, with a characteristic velocity V_{conv} in the outside of the galactic disk. These processes tend to push the antiprotons vertically outside the disk.

Model	δ	\mathcal{K}_0 [kpc ² /Myr]	L [kpc]	V_{conv} [km/s]	V_a [km/s]
MIN	0.85	0.0016	1	13.5	22.4
MED	0.70	0.0112	4	12	52.9
MAX	0.46	0.0765	15	5	117.6

Table B.1

Benchmark MIN, MED, and MAX sets of propagation parameters [54].

B.3.1.2 Energy losses: The term in $b(K, \vec{x})$ accounts for energy losses. Antiprotons undergo energy losses according to three main processes: through the ionisation of the interstellar neutral medium, through the scattering off thermal electrons in interstellar ionised matter, and through convective processes. In addition, the inelastic but non-annihilating interactions of antiprotons with the interstellar medium (tertiary component) are treated following Ref. [30].

B.3.1.3 Diffusive re-acceleration: Finally, the last term on the left-hand side of Eq. (B.11) describes diffusive re-acceleration. The knots of the turbulent magnetic field can, in fact, drift the antiprotons with a characteristic velocity v_a , which results in a second order Fermi acceleration of antiprotons.

This model presents in total five free parameters that can be defined by the user. However, the three benchmark propagation models MIN, MED and MAX, which give a minimum, median and maximum antiproton flux at Earth, are directly provided (see Table B.1).

The term on the right-hand side of Eq. (B.11) corresponds to the sources of antiprotons and will be detailed below.

B.3.2 Source terms

B.3.2.1 Secondary antiprotons: The astrophysical antiproton background, so-called *secondary antiprotons*, is mostly created through the interaction of proton and helium cosmic rays produced by supernovae with hydrogen and helium atoms in the interstellar medium. Such antiprotons account for one part of the source term in the equation of transport (B.11):

$$Q_{\bar{p}}^{\text{II}}(\mathbf{x}, K_{\bar{p}}) = 4\pi (1 + N_{\text{IS}}(K_{\bar{p}})) \sum_{i=\text{p,He}} \sum_{j=\text{H,He}} \int_{K_i^0}^{+\infty} dT_i \frac{d\sigma_{ij \rightarrow \bar{p}X}}{dK_{\bar{p}}} (K_i \rightarrow K_{\bar{p}}) n_j(\mathbf{x}) \Phi_i(\mathbf{x}, K_i), \quad (\text{B.13})$$

where K_i is the kinetic energy of the nucleon i . The differential cross section $d\sigma_{ij\rightarrow\bar{p}X}/dK_{\bar{p}}$ is computed from the proton-proton differential cross section taken from Ref. [55] and the threshold K_p^0 of this reaction is set to $7m_p$.

The factor N_{IS} accounts for the fact that antineutrons are also produced, along with antiprotons, in the interaction of cosmic rays with the interstellar medium. Antineutrons subsequently decay into antiprotons and contribute to the source term. Most of the antiprotons are, however, produced in proton-proton reactions.

The fluxes of proton and helium cosmic rays at position \mathbf{x} , $\Phi_i(\mathbf{x})$, are deduced from the fluxes measured at the position of the Earth through a retropropagation technique.

B.3.2.2 Primary antiprotons: The production rate $Q_{\bar{p}}^{\text{I}}$ of primary antiprotons produced by the annihilation of two dark matter particles is given by:

$$Q_{\bar{p}}^{\text{I}}(\mathbf{x}, K_{\bar{p}}) = \eta \left(\frac{\rho(\mathbf{x})}{m_{\text{DM}}} \right)^2 \langle \sigma v \rangle \frac{dN_{\text{prod}}}{dK_{\bar{p}}}, \quad (\text{B.14})$$

where $\langle \sigma v \rangle$ is the thermal average annihilating cross section, and η is equal to 1/2 (1/4) for a Majorana (Dirac) type particle. $\frac{dN_{\text{prod}}}{dK_{\bar{p}}}$ is the flux at production of antiprotons which is calculated as described in section B.1.

B.3.3 Calculation of the constraints

We calculate the total antiproton spectrum at the Earth position as the sum of primary and secondary antiproton contributions:

$$\Phi_{\text{tot}}^{\oplus}(K, \phi_F, A) = \Phi_{\text{I}}^{\oplus}(K, \phi_F) + \Phi_{\text{II}}^{\oplus}(K, \phi_f, A).$$

The parameters A and ϕ_F are nuisance parameters over which we marginalise. They are related to the uncertainties on the antineutron production cross section and to the solar modulation, respectively.

B.3.3.1 Uncertainties on the antineutron production cross section: The parameter N_{IS} which accounts for antineutron production in Eq. (B.13) is energy-dependent and suffers from large uncertainties [55]. For this reason, we calculate the secondary antiproton spectra for the lower and upper bounds

of N_{IS} . The real secondary antiproton spectrum takes values between these two bounds, according to a nuisance parameter $A \in [0, 1]$

$$\Phi_{\text{II}}(A) = (1 - A) \times \Phi_{\text{II}}^{\text{lower}} + A \times \Phi_{\text{II}}^{\text{upper}}. \quad (\text{B.15})$$

B.3.3.2 Solar modulation: As antiprotons reach the Sun vicinity, they enter the sphere of influence of the Sun magnetic field and of its cosmic-ray wind. It has for effect to decrease the kinetic energy of antiprotons, especially for low-energetic particles ($\lesssim 10$ GeV). It is therefore necessary to modify the spectrum at the Earth position calculated from the equation of transport. A simple way to do so is to use a force-field approximation parametrised by the Fisk potential ϕ_F [56,57]:

$$\Phi^{\oplus}(K) = \Phi^0(K + |e|\phi_F) \times \frac{K(K + 2m_p)}{(K + m_p + |e|\phi_F)^2 - m_p^2}, \quad (\text{B.16})$$

where Φ^{\oplus} is the antiproton spectrum at Earth and Φ^0 the antiproton spectrum at the end of the propagation but before entering into the solar influence. For AMS-02 data, we take $\phi_F \in [0.1, 1]$.

In order to quantify the deviation of the theoretical spectrum from the antiproton spectrum measured by AMS-02, we calculate a χ^2 as:

$$\chi^2(\Phi_{\text{tot}}^{\oplus}) = \sum_i \left(\frac{\Phi_{\text{tot}}^{\oplus}(E_i) - \Phi_{\text{AMS-02}}^{\oplus}(E_i)}{\Delta\Phi_{\text{AMS-02}}^{\oplus}(E_i)} \right)^2, \quad (\text{B.17})$$

where we sum over AMS-02 energy bins, with central values E_i . AMS-02 flux $\Phi_{\text{AMS-02}}^{\oplus}$ is given with an error $\Delta\Phi_{\text{AMS-02}}^{\oplus}$ [58,59].

We minimise the χ^2 with respect to the nuisance parameters A and ϕ_F and we compare it to the χ^2 of the case without dark matter to obtain the statistical test:

$$\text{TS} = - \left(\min_{A, \phi_F} \{ \chi^2(\Phi_{\text{tot}}^{\oplus}) \} - \min_{A, \phi_F} \{ \chi^2(\Phi_{\text{noDM}}^{\oplus}) \} \right). \quad (\text{B.18})$$

The obtained TS follows a normal distribution. For instance, if only one annihilation channel is dominant (1 d.o.f.), a point will be excluded at 2σ if $\text{TS} < -4$.

The computation of antiproton primary and secondary spectra can be time-consuming when performing large scans. For this reason we also provide tabulated spectra for the benchmark sets of propagation parameters MIN, MED, MAX, and the three DM halo density profiles Burkert, Einasto and Navarro-Frenk-White (NFW), defined in Table B.2. Following our analysis of AMS-02 antiproton constraints in the MSSM [38], we define the limit derived by the

Halo profile	r_s [kpc]	ρ_s [GeV/cm ³]	R_\odot [kpc]	ρ_\odot [GeV/cm ³]
NFW	19.6	0.32	8.21	0.383
Einasto ($\alpha = 0.22$)	16.07	0.11	8.25	0.386
Burkert	9.26	1.57	7.94	0.487

Table B.2

Dark matter mass model parameters for NFW [60], Einasto [61] and Burkert [62] profiles.

Burkert profile and MED propagation model as the “conservative” constraint, whereas the Einasto MED and the Einasto MAX models provide “standard” and “stringent” constraints, respectively.

However, the users are welcome to define their own propagation parameters and DM density profile, as long as it respects axisymmetry. The constraints can, in this way, be calculated directly for the new sets of parameters. If the user wishes to perform large scans, we advise, nevertheless, to generate tabulated spectra of primary and secondary antiprotons using the routines `write_secondaries` and `write primaries`.

Appendix C Direct detection

We assume that the lightest neutralino accounts for the entire dark matter, as for the indirect detection constraints, and that there is no rescaling with the relic density calculated in the standard cosmological model.

The calculation of direct detection constraints requires the computation of the differential recoil rate per unit of target material mass $\frac{dR}{dE}$:

$$\frac{dR}{dE} = \frac{n_\chi}{M} \left\langle v \frac{d\sigma}{dE} \right\rangle = \frac{2\rho_\chi}{m_\chi} \int d^3v v f(\mathbf{v}, t) \frac{d\sigma_i}{dq^2}(q^2, v), \quad (\text{C.1})$$

where $n_\chi = \rho_\chi/m_\chi$ is the number density of WIMPs, with ρ_χ the local DM mass density. $f(\mathbf{v}, t)$ is the WIMP velocity distribution and $\frac{d\sigma_i}{dq^2}(q^2, v)$ is the differential WIMP/nucleus cross section, with $q^2 = 2ME$ the momentum exchanged in the scattering. Using the standard expression of the differential cross section for the most commonly assumed couplings, Eq. (C.1) can be simplified as

$$\frac{dR}{dE} = \underbrace{\frac{1}{2m_\chi\mu^2} \sigma(q)}_{\text{Particle physics}} \underbrace{\rho_\chi \eta(v_{\min}(E), t)}_{\text{Astrophysics}}, \quad (\text{C.2})$$

where $\sigma(q)$ is an effective scattering cross-section and

$$\eta(v_{min}, t) \equiv \int_{v>v_{min}} d^3v \frac{f(\mathbf{v}, t)}{v} \quad (\text{C.3})$$

is the mean inverse speed. v_{min} can be written as

$$v_{min} = \sqrt{\frac{ME}{2\mu^2}}, \quad (\text{C.4})$$

where μ is the reduced mass $\mu = m_\chi M / (m_\chi + M)$. In the general case where the target material is composed of more than one isotope, the total differential recoil rate is the sum of each isotope contribution weighted by the isotope mass fraction:

$$\frac{dR}{dE} = \sum_i \xi_i \frac{dR_i}{dE}. \quad (\text{C.5})$$

The first part in the right side of Eq. (C.2), in which the effective cross section appears, depends only on the particle model which is used. The main source of uncertainties in this term comes from nuclear form factors. The second part involves only astrophysical observables, namely dark matter local density and velocity profile. The calculation of each term is, therefore, to be done separately.

The calculation of $\eta(v_{min}, t)$ is performed with the Simpson method. We use the standard halo model which describes the dark matter halo as a non-rotating isothermal sphere [63,64]. It involves an isotropic Maxwellian distribution of the WIMP velocity $f(\mathbf{v})$, with the galactic disk rotation velocity v_{rot} being the most probable speed. We set the default v_{rot} central value and uncertainties to $v_{rot} = 220 \pm 20$ km/s [65–67]. This velocity distribution is truncated at the escape velocity v_{esc} at which a WIMP can escape the galaxy potential well, $v_{esc} = 544 \pm 50$ km/s [68]. DM direct detection constraints are, however, mostly influenced by the large uncertainties on the local DM density $\rho_0 = 0.4 \pm 2$ GeV/cm³ [69]. A “conservative”, “standard”, or “stringent” constraint can be calculated with respect to the uncertainties of these three parameters. We refer to Ref. [38] for the study of the impact of these uncertainties on direct detection constraints.

C.1 Scattering cross sections

The effective WIMP/nucleus scattering cross section is commonly decomposed into spin-dependent and spin-independent components $\sigma_i(q) = \sigma_i^{SI}(q) + \sigma_i^{SD}(q)$. Each component is subject to a special treatment, but both need the calculation of effective neutralino-quark couplings which will be detailed, in

the MSSM, in section C.1.2.

C.1.1 *SI and SD cross sections*

The **spin-independent** effective cross section can be written for a target nucleus composed of Z protons and $(A - Z)$ neutrons as:

$$\sigma_i^{SI}(q) = \frac{4\mu^2}{\pi} F_{SI}^2(q) \left[Z \times A_p^{SI} + (A - Z) \times A_n^{SI} \right]^2, \quad (\text{C.6})$$

where F_{SI} is a nuclear form factor which describes the nucleon content of the nucleus, and A_p^{SI} and A_n^{SI} are the proton/WIMP and neutron/WIMP effective scattering amplitudes, respectively. These amplitudes can be calculated from the WIMP/quark effective couplings λ_q^{SI} weighted by the quark form factors:

$$A_{p,n}^{SI} = \sum_{q=u,d,s} f_q^{p,n} \lambda_q^{SI}. \quad (\text{C.7})$$

The quark form factors $f_q^{p,n}$ is related to the mass content of the quarks in the nucleons:

$$f_q^N = \frac{m_q}{M_N} \langle N | \bar{\Psi}_q \Psi_q | N \rangle. \quad (\text{C.8})$$

As for the **spin-dependent** counterpart, the effective cross section can be written as:

$$\sigma_i^{SD}(q) = \frac{16\mu^2}{2J + 1} S(q), \quad (\text{C.9})$$

where J is the total spin of the nucleus and

$$S(q) = a_0^2 S_{00}(q) + a_1^2 S_{11}(q) + a_0 a_1 S_{01}(q). \quad (\text{C.10})$$

S_{00} , S_{01} and S_{11} are structure factors which depend on the nature of the target isotope and a_0 and a_1 are defined as

$$a_0 = A_p^{SD} + A_n^{SD}, \quad a_1 = A_p^{SD} - A_n^{SD}, \quad (\text{C.11})$$

where $A_{p,n}^{SD}$ are the WIMP/proton or neutron spin-dependent scattering amplitudes. In the same way as for the spin-independent amplitude, $A_{p,n}^{SD}$ are calculated by summing the WIMP/quark SD effective couplings λ_q^{SD} , weighted by quark form factors related to the spin content of the quarks in the nucleons $\Delta q^{p,n}$.

$$A_{p,n}^{SD} = \sum_{q=u,d,s} \Delta q^{p,n} \lambda_q^{SD}. \quad (\text{C.12})$$

$\Delta q^{p,n}$ is defined as:

$$\Delta q^N = \frac{1}{2s_\mu} \langle N | \bar{\Psi}_q \gamma_\mu \gamma^5 \Psi_q | N \rangle, \quad (\text{C.13})$$

where s_μ is the nucleon spin.

C.1.2 Neutralino/quark effective couplings

An important step in the calculation of the scattering cross sections consists in computing the effective couplings λ_q^{SI} and λ_q^{SD} . To this end, we use the explicit analytical formulas from Ref. [70] for the calculation of the SI and SD effective couplings in the MSSM and in the NMSSM. The implementation of the neutralino-nucleon scattering amplitudes in the NMSSM requires only a small modification of the Higgs couplings, as the additional CP-even Higgs also participates to the scattering process. These couplings were calculated using FormCalc [35]. We treat the twist-2 operator and the box diagrams as described in Ref. [70].

Additional higher-order corrections were implemented for the spin-independent amplitude. The neutralino can indeed interact with the gluon content of the nucleons via diagrams including heavy quark or squark loops. At first approximation, these diagrams can be treated as effective interactions between the neutralino and the heavy quarks and squarks, corrected by an appropriate form factor f_Q related to the gluon content of the nucleon. For heavy quarks, this form factor depends simply on the sum of the light quark form factors:

$$f_Q^{p,n} = \frac{2}{27} \left(1 - \sum_{q=u,d,s} f_q^{p,n} \right). \quad (\text{C.14})$$

This form factor is however enhanced in the case of a Higgs boson exchange due to QCD effects and we apply the corrections given in Ref. [71].

Finally, the bottom quark mass may receive significant corrections from gluino-squark loops in supersymmetric scenarios. While down-type quarks couple normally only to the Higgs doublet H_d , this correction allows also a small coupling between the b -quark and H_u . This SUSY-QCD correction modifies the coupling of the b -quark to CP-even Higgs as described in Ref. [71].

C.2 Uncertainties on the nucleon and nuclear form factors

C.2.1 Nucleon form factors

C.2.1.1 Spin-independent interaction Starting with the spin-independent nucleon form factors, $f_u^{p,n}$, $f_d^{p,n}$ and $f_s^{p,n}$ can be calculated from three parameters, namely the light and strange quark contents of the nucleon, defined as

$$\Sigma_{\pi N} \equiv \frac{m_u + m_d}{2} \langle N | \bar{u}u + \bar{d}d | N \rangle, \quad (\text{C.15})$$

$$\sigma_s \equiv m_s \langle N | \bar{s}s | N \rangle, \quad (\text{C.16})$$

and the parameter z defined as:

$$z = \frac{\langle N|\bar{u}u|N\rangle - \langle N|\bar{s}s|N\rangle}{\langle N|\bar{d}d|N\rangle - \langle N|\bar{s}s|N\rangle}. \quad (\text{C.17})$$

The light quark content of the nucleons is known from lattice QCD results and from the analysis of pion-nucleon scattering. As for the strange content of the nucleons, it is deduced from lattice QCD calculations. We use the ranges for the values of $\Sigma_{\pi N}$, σ_s and z given in Ref. [72]:

$$\Sigma_{\pi N} = 46 \pm 11 \text{ MeV} , \quad (\text{C.18})$$

$$\sigma_s = 35 \pm 16 \text{ MeV} , \quad (\text{C.19})$$

$$z = 1.5 \pm 0.5 . \quad (\text{C.20})$$

C.2.1.2 Spin-dependent interactions The nucleon SD form factors $\Delta_q^{(N)}$, can be computed from the combinations of two parameters:

$$a_3 = \Delta_u^{(p)} - \Delta_d^{(p)} , \quad (\text{C.21})$$

$$a_8 = \Delta_u^{(p)} + \Delta_d^{(p)} - 2\Delta_s^{(p)} , \quad (\text{C.22})$$

and $\Delta_s^{(p)}$ itself. a_3 is known from neutron β decay measurements $a_3 = 1.2723 \pm 0.0023$ [73] and $a_8 = 0.585 \pm 0.023$ from hyperon β decay results [74]. The uncertainties on these two parameters are rather small compared to the error on $\Delta_s^{(p)} = -0.09 \pm 0.03$, which is deduced from the measurement of the spin-dependent structure function of the deuteron from the COMPASS experiment [75].

C.2.2 Nuclear structure factors

C.2.2.1 Spin-independent interactions The SI form factor is essentially a Fourier transform of the mass distribution of the nucleus. A good approximation is the Helm form factor [76,77]:

$$F(q) = 3e^{-q^2 s^2/2} \frac{\sin(qr_n) - qr_n \cos(qr_n)}{(qr_n)^3} , \quad (\text{C.23})$$

where $s \simeq 0.9$ fm and $r_n^2 = c^2 + \frac{7}{3}\pi^2 a^2 - 5s^2$ is an effective nuclear radius with $a \simeq 0.52$ fm and $c \simeq 1.23A^{1/3} - 0.60$ fm. The uncertainties on the Helm form factors are expected to be rather small for small momentum exchanges. However, the users may define their own SI form factor if desired.

C.2.2.2 Spin-dependent interactions Large uncertainties on S_{01} and S_{11} structure factors exist from long-range two-body currents due to pion exchange [78]. There is a correlation between S_{01} and S_{11} errors, so one should be aware that we may slightly overestimate the uncertainties on $S(q)$ by taking independently the lower and upper bounds of S_{01} and S_{11} to estimate the impact of the uncertainties. However, this method allows us to stay conservative. The uncertainties are especially relevant for isospin violating models, since S_{01} and S_{11} are proportional to $A_p^{SD} - A_n^{SD}$.

C.3 Experimental limits

We implemented the constraints from the latest results of XENON1T [18], PANDAX-2 [17] and PICO60 [19]. For each constraint, we use a Poisson likelihood defined as:

$$\mathcal{L}(\mu|N_o) = \frac{(b + \mu)^{N_o} e^{-(b+\mu)}}{N_o!}, \quad (\text{C.24})$$

where μ is the total expected number of events from WIMP-nucleus scattering, N_o is the number of observed events and b is the expected background. We marginalise over b by choosing its value according to:

$$\begin{cases} b = N_o - \mu & \text{if } N_o > \mu, \\ b = 0 & \text{otherwise.} \end{cases} \quad (\text{C.25})$$

A point is excluded at 2 sigma if the difference of its log-likelihood with the background-only scenario respects:

$$\log \mathcal{L}(\mu) - \log \mathcal{L}(\mu = 0) < -4. \quad (\text{C.26})$$

We use the efficiencies provided in the GAMBIT package DDCalc [12]. They were calculated using the TPCMC Monte Carlo code [79] to model the detector response, which relies on NEST [80–82] for modelling the microphysics of a recoiling xenon atom.

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