

**Figure 1.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 1/4$ . The data have been obtained using Algo. 1 with 4 states per site.

## Improved Matrix Product Operator Renormalization Group: application to the 3-color random Ashkin-Teller chain

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**Figure 2.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = \sqrt{2}/4$ . The data have been obtained using Algo. 1 with 4 states per site.



**Figure 3.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 1/2$ . The data have been obtained using Algo. 1 with 4 states per site.



**Figure 4.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = \sqrt{2}/2$ . The data have been obtained using Algo. 1 with 4 states per site.



**Figure 5.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 1$ . The data have been obtained using Algo. 1 with 4 states per site.



Figure 6. Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = \sqrt{2}$ . The data have been obtained using Algo. 1 with 4 states per site.



**Figure 7.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 2$ . The data have been obtained using Algo. 1 with 4 states per site.



**Figure 8.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 2\sqrt{2}$ . The data have been obtained using Algo. 1 with 4 states per site.



**Figure 9.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 4$ . The data have been obtained using Algo. 1 with 4 states per site.



Figure 10. Phase diagram of the 2-color Ashkin-Teller model obtained from the crossings of the Binder cumulants  $U_m$  (crosses) and  $U_p$  (circles). The colors are associated to the pair of lattice sizes (see the legend) used to find the crossing of the cumulants. The data have been obtained using Algo. 1 with 4 states per site.



**Figure 11.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 1/4$ . The data have been obtained using Algo. 2 with 4 states per site.



Figure 12. Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = \sqrt{2}/4$ . The data have been obtained using Algo. 2 with 4 states per site.



**Figure 13.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 1/2$ . The data have been obtained using Algo. 2 with 4 states per site.



**Figure 14.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = \sqrt{2}/2$ . The data have been obtained using Algo. 2 with 4 states per site.



**Figure 15.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 1$ . The data have been obtained using Algo. 2 with 4 states per site.



Figure 16. Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = \sqrt{2}$ . The data have been obtained using Algo. 2 with 4 states per site.



**Figure 17.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 2$ . The data have been obtained using Algo. 2 with 4 states per site.



**Figure 18.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 2\sqrt{2}$ . The data have been obtained using Algo. 2 with 4 states per site.



**Figure 19.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 4$ . The data have been obtained using Algo. 2 with 4 states per site.



Figure 20. Phase diagram of the 2-color Ashkin-Teller model obtained from the crossings of the Binder cumulants  $U_m$  (crosses) and  $U_p$  (circles). The colors are associated to the pair of lattice sizes (see the legend) used to find the crossing of the cumulants. The data have been obtained using Algo. 2 with 4 states per site.



Figure 21. Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 1/4$ . The data have been obtained using Algo. 3 with 4 states per site.



Figure 22. Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = \sqrt{2}/4$ . The data have been obtained using Algo. 3 with 4 states per site.



**Figure 23.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 1/2$ . The data have been obtained using Algo. 3 with 4 states per site.



**Figure 24.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = \sqrt{2}/2$ . The data have been obtained using Algo. 3 with 4 states per site.



**Figure 25.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 1$ . The data have been obtained using Algo. 3 with 4 states per site.



Figure 26. Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = \sqrt{2}$ . The data have been obtained using Algo. 3 with 4 states per site.



**Figure 27.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 2$ . The data have been obtained using Algo. 3 with 4 states per site.



**Figure 28.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 2\sqrt{2}$ . The data have been obtained using Algo. 3 with 4 states per site.



**Figure 29.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 4$ . The data have been obtained using Algo. 3 with 4 states per site.



Figure 30. Phase diagram of the 2-color Ashkin-Teller model obtained from the crossings of the Binder cumulants  $U_m$  (crosses) and  $U_p$  (circles). The colors are associated to the pair of lattice sizes (see the legend) used to find the crossing of the cumulants. The data have been obtained using Algo. 3 with 4 states per site.



**Figure 31.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 1/4$ . The data have been obtained using Algo. 1 with 8 states per site.



**Figure 32.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = \sqrt{2}/4$ . The data have been obtained using Algo. 1 with 8 states per site.



**Figure 33.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 1/2$ . The data have been obtained using Algo. 1 with 8 states per site.



**Figure 34.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = \sqrt{2}/2$ . The data have been obtained using Algo. 1 with 8 states per site.



**Figure 35.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 1$ . The data have been obtained using Algo. 1 with 8 states per site.



**Figure 36.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = \sqrt{2}$ . The data have been obtained using Algo. 1 with 8 states per site.



**Figure 37.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 2$ . The data have been obtained using Algo. 1 with 8 states per site.



**Figure 38.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 2\sqrt{2}$ . The data have been obtained using Algo. 1 with 8 states per site.



**Figure 39.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 4$ . The data have been obtained using Algo. 1 with 8 states per site.



Figure 40. Phase diagram of the 2-color Ashkin-Teller model obtained from the crossings of the Binder cumulants  $U_m$  (crosses) and  $U_p$  (circles). The colors are associated to the pair of lattice sizes (see the legend) used to find the crossing of the cumulants. The data have been obtained using Algo. 1 with 8 states per site.



**Figure 41.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 1/4$ . The data have been obtained using Algo. 2 with 8 states per site.



**Figure 42.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = \sqrt{2}/4$ . The data have been obtained using Algo. 2 with 8 states per site.



**Figure 43.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 1/2$ . The data have been obtained using Algo. 2 with 8 states per site.



**Figure 44.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = \sqrt{2}/2$ . The data have been obtained using Algo. 2 with 8 states per site.



**Figure 45.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 1$ . The data have been obtained using Algo. 2 with 8 states per site.



**Figure 46.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = \sqrt{2}$ . The data have been obtained using Algo. 2 with 8 states per site.



**Figure 47.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 2$ . The data have been obtained using Algo. 2 with 8 states per site.



**Figure 48.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 2\sqrt{2}$ . The data have been obtained using Algo. 2 with 8 states per site.



**Figure 49.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 4$ . The data have been obtained using Algo. 2 with 8 states per site.



Figure 50. Phase diagram of the 2-color Ashkin-Teller model obtained from the crossings of the Binder cumulants  $U_m$  (crosses) and  $U_p$  (circles). The colors are associated to the pair of lattice sizes (see the legend) used to find the crossing of the cumulants. The data have been obtained using Algo. 2 with 8 states per site.



**Figure 51.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 1/4$ . The data have been obtained using Algo. 3 with 8 states per site.



**Figure 52.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = \sqrt{2}/4$ . The data have been obtained using Algo. 3 with 8 states per site.



**Figure 53.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 1/2$ . The data have been obtained using Algo. 3 with 8 states per site.



**Figure 54.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = \sqrt{2}/2$ . The data have been obtained using Algo. 3 with 8 states per site.



**Figure 55.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 1$ . The data have been obtained using Algo. 3 with 8 states per site.



**Figure 56.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = \sqrt{2}$ . The data have been obtained using Algo. 3 with 8 states per site.



**Figure 57.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 2$ . The data have been obtained using Algo. 3 with 8 states per site.



**Figure 58.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 2\sqrt{2}$ . The data have been obtained using Algo. 3 with 8 states per site.



**Figure 59.** Average Binder cumulant of magnetization (left) and polarization (right) for the 2-color Ashkin-Teller model with  $\epsilon = 4$ . The data have been obtained using Algo. 3 with 8 states per site.



Figure 60. Phase diagram of the 2-color Ashkin-Teller model obtained from the crossings of the Binder cumulants  $U_m$  (crosses) and  $U_p$  (circles). The colors are associated to the pair of lattice sizes (see the legend) used to find the crossing of the cumulants. The data have been obtained using Algo. 3 with 8 states per site.



Figure 61. Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 1/4$ . The data have been obtained using Algo. 1 with 8 states per site.



Figure 62. Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = \sqrt{2}/4$ . The data have been obtained using Algo. 1 with 8 states per site.



**Figure 63.** Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 1/2$ . The data have been obtained using Algo. 1 with 8 states per site.



**Figure 64.** Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = \sqrt{2}/2$ . The data have been obtained using Algo. 1 with 8 states per site.



**Figure 65.** Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 1$ . The data have been obtained using Algo. 1 with 8 states per site.



**Figure 66.** Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = \sqrt{2}$ . The data have been obtained using Algo. 1 with 8 states per site.



**Figure 67.** Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 2$ . The data have been obtained using Algo. 1 with 8 states per site.



**Figure 68.** Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 2\sqrt{2}$ . The data have been obtained using Algo. 1 with 8 states per site.



**Figure 69.** Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 4$ . The data have been obtained using Algo. 1 with 8 states per site.



Figure 70. Phase diagram of the 3-color Ashkin-Teller model obtained from the crossings of the Binder cumulants  $U_m$  (crosses) and  $U_p$  (circles). The colors are associated to the pair of lattice sizes (see the legend) used to find the crossing of the cumulants. The data have been obtained using Algo. 1 with 8 states per site.



Figure 71. Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 1/4$ . The data have been obtained using Algo. 2 with 8 states per site.



Figure 72. Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = \sqrt{2}/4$ . The data have been obtained using Algo. 2 with 8 states per site.



Figure 73. Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 1/2$ . The data have been obtained using Algo. 2 with 8 states per site.



**Figure 74.** Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = \sqrt{2}/2$ . The data have been obtained using Algo. 2 with 8 states per site.



**Figure 75.** Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 1$ . The data have been obtained using Algo. 2 with 8 states per site.



**Figure 76.** Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = \sqrt{2}$ . The data have been obtained using Algo. 2 with 8 states per site.



**Figure 77.** Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 2$ . The data have been obtained using Algo. 2 with 8 states per site.



Figure 78. Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 2\sqrt{2}$ . The data have been obtained using Algo. 2 with 8 states per site.



**Figure 79.** Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 4$ . The data have been obtained using Algo. 2 with 8 states per site.



Figure 80. Phase diagram of the 3-color Ashkin-Teller model obtained from the crossings of the Binder cumulants  $U_m$  (crosses) and  $U_p$  (circles). The colors are associated to the pair of lattice sizes (see the legend) used to find the crossing of the cumulants. The data have been obtained using Algo. 2 with 8 states per site.



Figure 81. Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 1/4$ . The data have been obtained using Algo. 3 with 8 states per site.



Figure 82. Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = \sqrt{2}/4$ . The data have been obtained using Algo. 3 with 8 states per site.



**Figure 83.** Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 1/2$ . The data have been obtained using Algo. 3 with 8 states per site.



**Figure 84.** Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = \sqrt{2}/2$ . The data have been obtained using Algo. 3 with 8 states per site.



**Figure 85.** Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 1$ . The data have been obtained using Algo. 3 with 8 states per site.



**Figure 86.** Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = \sqrt{2}$ . The data have been obtained using Algo. 3 with 8 states per site.



**Figure 87.** Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 2$ . The data have been obtained using Algo. 3 with 8 states per site.



**Figure 88.** Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 2\sqrt{2}$ . The data have been obtained using Algo. 3 with 8 states per site.



**Figure 89.** Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 4$ . The data have been obtained using Algo. 3 with 8 states per site.



Figure 90. Phase diagram of the 3-color Ashkin-Teller model obtained from the crossings of the Binder cumulants  $U_m$  (crosses) and  $U_p$  (circles). The colors are associated to the pair of lattice sizes (see the legend) used to find the crossing of the cumulants. The data have been obtained using Algo. 3 with 8 states per site.



Figure 91. Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 1/4$ . The data have been obtained using Algo. 1 with 16 states per site.



Figure 92. Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = \sqrt{2}/4$ . The data have been obtained using Algo. 1 with 16 states per site.



Figure 93. Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 1/2$ . The data have been obtained using Algo. 1 with 16 states per site.



Figure 94. Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = \sqrt{2}/2$ . The data have been obtained using Algo. 1 with 16 states per site.



**Figure 95.** Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 1$ . The data have been obtained using Algo. 1 with 16 states per site.



Figure 96. Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = \sqrt{2}$ . The data have been obtained using Algo. 1 with 16 states per site.



**Figure 97.** Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 2$ . The data have been obtained using Algo. 1 with 16 states per site.



**Figure 98.** Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 2\sqrt{2}$ . The data have been obtained using Algo. 1 with 16 states per site.



**Figure 99.** Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 4$ . The data have been obtained using Algo. 1 with 16 states per site.



**Figure 100.** Phase diagram of the 3-color Ashkin-Teller model obtained from the crossings of the Binder cumulants  $U_m$  (crosses) and  $U_p$  (circles). The colors are associated to the pair of lattice sizes (see the legend) used to find the crossing of the cumulants. The data have been obtained using Algo. 1 with 16 states per site.



Figure 101. Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 1/4$ . The data have been obtained using Algo. 2 with 16 states per site.



Figure 102. Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = \sqrt{2}/4$ . The data have been obtained using Algo. 2 with 16 states per site.



Figure 103. Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 1/2$ . The data have been obtained using Algo. 2 with 16 states per site.



Figure 104. Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = \sqrt{2}/2$ . The data have been obtained using Algo. 2 with 16 states per site.



**Figure 105.** Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 1$ . The data have been obtained using Algo. 2 with 16 states per site.



Figure 106. Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = \sqrt{2}$ . The data have been obtained using Algo. 2 with 16 states per site.



**Figure 107.** Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 2$ . The data have been obtained using Algo. 2 with 16 states per site.



Figure 108. Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 2\sqrt{2}$ . The data have been obtained using Algo. 2 with 16 states per site.



**Figure 109.** Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 4$ . The data have been obtained using Algo. 2 with 16 states per site.



**Figure 110.** Phase diagram of the 3-color Ashkin-Teller model obtained from the crossings of the Binder cumulants  $U_m$  (crosses) and  $U_p$  (circles). The colors are associated to the pair of lattice sizes (see the legend) used to find the crossing of the cumulants. The data have been obtained using Algo. 2 with 16 states per site.



**Figure 111.** Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 1/4$ . The data have been obtained using Algo. 3 with 16 states per site.



Figure 112. Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = \sqrt{2}/4$ . The data have been obtained using Algo. 3 with 16 states per site.



Figure 113. Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 1/2$ . The data have been obtained using Algo. 3 with 16 states per site.



Figure 114. Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = \sqrt{2}/2$ . The data have been obtained using Algo. 3 with 16 states per site.



**Figure 115.** Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 1$ . The data have been obtained using Algo. 3 with 16 states per site.



Figure 116. Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = \sqrt{2}$ . The data have been obtained using Algo. 3 with 16 states per site.



**Figure 117.** Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 2$ . The data have been obtained using Algo. 3 with 16 states per site.



Figure 118. Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 2\sqrt{2}$ . The data have been obtained using Algo. 3 with 16 states per site.



**Figure 119.** Average Binder cumulant of magnetization (left) and polarization (right) for the 3-color Ashkin-Teller model with  $\epsilon = 4$ . The data have been obtained using Algo. 3 with 16 states per site.



Figure 120. Phase diagram of the 3-color Ashkin-Teller model obtained from the crossings of the Binder cumulants  $U_m$  (crosses) and  $U_p$  (circles). The colors are associated to the pair of lattice sizes (see the legend) used to find the crossing of the cumulants. The data have been obtained using Algo. 3 with 16 states per site.