# How efficient are intercity railway prices and frequencies in Europe? Comparing a corridor in Belgium and in France 

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## Online appendices

## A. Detailed data

The specification of the gross utility function defined in Eq. (1) used in the empirical exercise in Sections 5 and 6 is:

$$
\begin{gathered}
u_{i, j}+\left[a_{i, j}^{C A R, P} x_{i, j}^{C A R, P}-0.5 b_{i, j}^{C A R, P}\left(x_{i, j}^{C A R, P}\right)^{2}\right] \\
+\left[a_{i, j}^{C A R, O P} x_{i, j}^{C A R, O P}-0.5 b_{i, j}^{C A R, O P}\left(x_{i, j}^{C A R, O P}\right)^{2}\right] \\
+\left[a_{i, j}^{R A I L, P} x_{i, j}^{R A I L, P}-0.5 b_{i, j}^{R A I L, P}\left(x_{i, j}^{C A R, P}\right)^{2}\right] \\
+\left[a_{i, j}^{R A I L, O P} x_{i, j}^{R A I L, O P}-0.5 b_{i, j}^{R A L L, O P}\left(x_{i, j}^{R A I L, O P}\right)^{2}\right] \\
-e_{i, j}^{C A R, P / C A R, O P} x_{i, j}^{C A R, P} x_{i, j}^{C A R, O P}-e_{i, j}^{C A R, P / R A I L, P} x_{i, j}^{C A R, P} x_{i, j}^{R A I L, P} \\
-e_{i, j}^{C A R, P / R A I L, O P} x_{i, j}^{C A R, P} x_{i, j}^{R A I L, O P}-e_{i, j}^{C A R, O P / R A I L, P} x_{i, j}^{C A R, O P} x_{i, j}^{R A I L, P} \\
-e_{i, j}^{C A R, O P / R A I L, O P} x_{i, j}^{C A R, O P} x_{i, j}^{R A I L, O P}-e_{i, j}^{R A I L, P / R A I L, O P} x_{i, j}^{R A I L, P} x_{i, j}^{R A I L, O P}
\end{gathered}
$$

$a_{i, j}^{m, k}, b_{i, j}^{m, k}$ and $e_{i, j}$ are parameters to be calibrated. This is the same specification as in in Börjesson, et al. (2017).

Data have been collected for year 2012. When data were not available for this year, we chose data whose publication date is close to this year.

Table 7 reports technological and rail cost data which are identical for the two corridors. Peak period is four hours long, two hours during the morning peak (7am to 9am) and two hours during evening peak ( 5 pm to 7 pm ). The total (fixed + variable) cost of a train has been computed by considering a cost of 10 M euros per carriage, a lifespan of 30 years, a discount rate of 4 per cent and 350 days of use per year. The total cost is equally split between fixed cost and variable cost in the baseline. The variable cost takes the form of a cost per seat per train. We assume a train operating cost of 12 euros per km, based on the Belgium data in Steer Davis Gleave (2015). The train fuel consumption is kept voluntarily low because in Belgium and France a non-negligible part of the lines are electrified. The environmental and accident externalities for train and car have been found in the Update of the Handbook of External Costs of Transport. ${ }^{1}$ The car fuel consumption corresponds to an average consumption of 7 liters per km.

| Parameters | Notation |  | Value |
| :---: | :---: | :---: | :---: |
| Peak period length (hours) | $h^{P}$ |  | 4 |
| Off-peak period length (hours) | $h^{0}$ |  | 15 |
| Cost of train ( $€ /$ train/day) | $\mu_{1}^{\text {RAIL }}$ |  | 1650 |
| Train operating cost ( $¢ / \mathrm{km}$ ) | $\mu_{2}^{\text {RAIL }}$ |  | 12 |
| Train fuel consumption (1/km) | $v^{\text {RAIL }}$ |  | 1 |
| Train environmental + accident externalities ( $€ / \mathrm{km}$ ) | $\iota^{\text {RAIL }}$ |  | 0.494 |
| Car fuel consumption (1/km) | $v^{C A R}$ |  | 0.07 |
| Car environmental + accident externalities ( $€ / \mathrm{km}$ ) | $\iota^{\text {CAR }}$ |  | 0.012 |
| Rail crowding cost (hours/users/m²) | $\eta^{\text {RAIL }}$ |  | 0.16 |
| Rail scheduling cost (hours/runs/hour) | $\delta$ |  | 0.24 |
| Rail delay cost (hours \times runs/hour) | $\theta^{\text {RAIL }}$ |  | 0.22 |
| Values of time (€ /hour) | $\alpha_{i}$ | Peak | Off-peak |
| Active + |  | 17.6 | 12.6 |
| Active - |  | 14 | 10 |
| Students |  | 14 | 10 |
| Retired |  | 9.5 | 6.8 |

Table 1: Parameters whose value is the same for all cases

[^0]The rail crowding cost coefficient has been obtained by a linear approximation of the travel time multipliers estimated by Kroes, et al. (2013). The scheduling cost coefficient has been computed in the $\alpha-\beta-\gamma$ framework by using the estimations produced by Wardman, et al. (2012). The rail delay cost efficient we use is from Pérez Herrero, et al. (2014).

The off-peak values of time are officially recommended values used in France (Quinet, 2013). The peak values of time have been obtained by using a factor 1.4 (Abrantes and Wardman, 2011).

We assume that the four categories are uniformly represented among the population of each of the city. Table 2 gives the modal shares per period distribution for the 4 types of users in the initial equilibrium. These numbers have been inferred from different sources (Cornelis, et al. (2012); DREAL (2014); Duabresse et al. (2015); AURG and SMTC (2015); Andries (2016)).

|  |  | Active + | Active - | Students | Retired | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Distribution trips from A |  |  |  |  |  |  |
| Train | Peak | 5.6\% | 4.6\% | 9.3\% | $3.1 \%$ | 22.6\% |
|  | Off-peak | 3.7\% | 3.1\% | 6.2\% | 7.2\% | 20.2\% |
| Car | Peak | 13\% | 10.2\% | 4.2\% | 3.5\% | 30.9\% |
|  | Off-peak | 8.7\% | 6.8\% | 2.8\% | 8.1\% | 26.4\% |
| Total |  | 30.9\% | 24.7\% | $22.4 \%$ | 21.9\% | 100\% |
| Distribution trips from B |  |  |  |  |  |  |
| Train | Peak | 3.5\% | 4.3\% | 8.6\% | 2.9\% | 19.3\% |
|  | Off-peak | 2.3\% | 2.9\% | 5.8\% | 6.7\% | 17.6\% |
| Car | Peak | 13.8\% | 11.1\% | 5.2\% | 3.9\% | 34\% |
|  | Off-peak | 9.2\% | 7.4\% | 3.5\% | 9.1\% | 29.1\% |
| Total |  | 28.8\% | 25.7\% | 23\% | 22.5\% | 100\% |

Table 2: Trips characteristics
Table 9 presents the geographical and physical characteristics of the two corridors. The road distance between cities as well as the road free travel times have been obtained through website

Google Maps. The rail distances between cities have been retrieved from Wikipedia pages dedicated to the railway stations. ${ }^{234}$ The rail average speeds have been retrieved from the timetables. The road congestion cost coefficient has been calibrated such that it reproduces the observed travel time while using a linear congestion function.

|  | Notation | Belgium | France |
| :--- | :---: | :---: | :---: |
| City A |  | Bruges | Grenoble |
| City B |  | Ghent | Bourgoin |
| City D | $N_{A}$ | 100000 | 700000 |
| Population living in A | $N_{B}$ | 200000 | 60000 |
| Population living in B |  |  |  |
| Number of daily trips | $X_{A}$ | 40000 | 20000 |
| $\quad$ From A | $X_{B}$ | 110000 | 20000 |
| $\quad$ From B | $d_{A B}^{\text {rail }}$ | 52 | 88 |
| Distance AB rail (km) | $d_{B D}^{\text {rail }}$ | 40 | 42 |
| Distance BD rail (km) | $d_{A B}^{\text {road }}$ | 44 | 50 |
| Distance AB road (km) | $d_{B D}^{\text {road }}$ | 56 | 64 |
| Distance BD road (km) |  | 69 | 87 |
| Average free train speed (km/hour) |  | 78 | 92 |
| Average free car speed (km/hour) | $\theta^{\text {road }}$ | 0.2 | 0.6 |
| Road congestion cost (hours /veh/road capacity) |  |  |  |

Table 3: General parameters whose value is corridor specific
Table 10 displays detailed transport data which are corridor specific. Comparable rail access (or infrastructure) charges excluding mark-ups for Belgium and France have been found in European Commission (2019). This source assures comparability. The marginal cost of rail access is much higher in Belgium than in France. Train capacities (number of seats per train) and frequencies have been computed from the description of the rolling stocks used on each of the lines and on the supplied service. ${ }^{5}$ Different materials are used, so these figures have to

[^1]be interpreted as average. Trains in Belgium are slightly larger than in France. Road capacity per hour corresponds to three lanes highway. Fuel prices and excises have been compiled from the Europe's Energy Portal. ${ }^{7}$ As there is no the same proportion of diesel and unleaded vehicles in both countries, we weighted prices and excise with respect to the shares of the vehicle types per country given by the European Automobile Manufacturers Association. ${ }^{8}$

|  | Notation | Belgium | France |
| :---: | :---: | :---: | :---: |
| Rail access charges ( $€ / \mathrm{km}$ ) |  | 8.1 | 2.9 |
| Train capacity (seats/train) | $S^{\text {RAIL }}$ | 650 | 600 |
| Rail frequency (train/hour) |  |  |  |
| Peak | $V^{\text {RAIL,P }} / h^{P}$ | 4 | 2 |
| Off-peak | $V^{\text {RAIL,O }} / h^{O}$ | 2 | 1 |
| Road capacity (veh/hour) |  |  |  |
| $\mathrm{A} \rightarrow \mathrm{B}$ | $s_{A B}^{\text {road }}$ | 6000 | 6000 |
| $\mathrm{B} \rightarrow \mathrm{D}$ | $s_{B D}^{\text {road }}$ | 6000 | 6000 |
| Fuel price ${ }^{9}(€ / 1)$ | $p^{\text {fuel }}$ | 1.28 | 1.2 |
| Fuel excise (€ /l) | $\kappa^{\text {fuel }}$ | 0.51 | 0.52 |
| Road tolls (€) |  |  |  |
| $\mathrm{A} \rightarrow \mathrm{B}$ | $\tau_{A B}^{C A R}$ | 0 | 6.7 |
| $\mathrm{B} \rightarrow \mathrm{D}$ | $\tau_{B D}^{C A R}$ | 0 | 3.5 |
| Train fares (€) |  |  |  |
| Peak $\quad$ A $\rightarrow$ B | $p_{A}^{\text {RAIL, } P}$ | 2.5 | 5.0 |
| $\mathrm{B} \rightarrow \mathrm{D}$ | $p_{B}^{\text {RAIL, } P}$ | 1.9 | 2.3 |
| Off-peak A $\rightarrow$ B | $p_{A}^{R A I L, O}$ | 2.1 | 7.5 |
| $\mathrm{B} \rightarrow \mathrm{D}$ | $p_{B}^{\text {RAIL,O }}$ | 1.9 | 3.3 |

Table 4: Transport parameters whose value is corridor specific
Road tolls for France have been retrieved from a local newspaper article. ${ }^{10}$ We met difficulties in estimating the average fare paid by train users: some users buy an annual or season ticket, some others pay full price for a one way trip... Consequently, we chose to retain

[^2]the price of an annual ticket. ${ }^{11}{ }^{12}$ This price is spread on 250 working days and 2 trips a day. As the annual ticket is usually the cheapest one, we apply on it a multiplicative coefficient of two.

## B. Elasticities and cross elasticities

Tables 11, 12, 13 and 14 present the price elasticities and diversion factors $\chi$ to other modes and this for the 4 categories of users. These data are consistent with the literature (Mayeres (2000); Litman (2004); Oum, et al. (2008); Dargay and Clark (2012)). In the absence of more locally differentiated data, we use for the two corridors the same demand and price elasticity data. This makes the corridors more comparable but also loses some realism.

| Active + |  | Peak |  | Off-peak |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | Rail | Car | Rail | Car |  |
| Elasticities |  |  |  |  |  |
| Elasticity of demand wrt generalized cost | -0.3 | -0.3 | -0.6 | -0.6 |  |
| Fraction of increased transit coming from |  |  |  |  |  |
| Peak | Rail | - | End. | End. | End. |
|  | Car | 0.5 | - | End. | End. |
| Off-peak | Rail | 0.15 | 0.15 | - | End. |
|  | Car | 0.15 | 0.15 | 0.5 | - |
| Increased overall travel demand | End. | End. | End. | End. |  |

Note: "End." means that the number is endogenously determined.
Table 5: Cost-elasticities and origins of increased transit of the representative individual of the active + population

[^3]| Active - | Peak |  | Off-peak |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | Rail | Car | Rail | Car |  |
| Elasticities |  |  |  |  |  |
| Elasticity of demand wrt generalized cost | -0.2 | -0.2 | -0.6 | -0.6 |  |
| Fraction of increased transit coming from |  |  |  |  |  |
| Peak $\quad$ Rail | - | End. | End. | End. |  |
|  | Car | 0.7 | - | End. | End. |
| Off-peak | Rail | 0.05 | 0.05 | - | End. |
|  | Car | 0.05 | 0.05 | 0.7 | - |
| Increased overall travel demand | End. | End. | End. | End. |  |

Note: "End." means that the number is endogenously determined.
Table 6: Cost-elasticities and origins of increased transit of the representative individual of the active population

| Students | Peak |  | Off-peak |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | Rail | Car | Rail | Car |  |
| Elasticities |  |  |  |  |  |
| Elasticity of demand wrt generalized cost | -0.3 | -0.3 | -0.7 | -0.7 |  |
| Fraction of increased transit coming from |  |  |  |  |  |
| Peak | Rail | - | End. | End. | End. |
|  | Car | 0.2 | - | End. | End. |
| Off-peak | Rail | 0.1 | 0.1 | - | End. |
|  | Car | 0.1 | 0.1 | 0.3 | - |
| Increased overall travel demand |  | End. | End. | End. | End. |

Note: "End." means that the number is endogenously determined.
Table 7: Cost-elasticities and origins of increased transit of the representative individual of the students population

| Retired | Peak |  | Off-peak |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | Rail | Car | Rail | Car |  |
| Elasticities |  |  |  |  |  |
| Elasticity of demand wrt generalized cost | -0.5 | -0.5 | -0.7 | -0.7 |  |
| Fraction of increased transit coming from |  |  |  |  |  |
| Peak | Rail | - | End. | End. | End. |
|  | Car | 0.2 | - | End. | End. |
| Off-peak | Rail | 0.3 | 0.3 | - | End. |
|  | Car | 0.3 | 0.3 | 0.6 | - |
| Increased overall travel demand |  | End. | End. | End. | End. |

Note: "End." means that the number is endogenously determined.
Table 8: Cost-elasticities and origins of increased transit of the representative individual of the retired population

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## C. First-best optimum

## a. Objective function

The Lagrangian function is as follows

$$
\begin{aligned}
& \mathcal{L}=\sum_{i} \sum_{j} N_{i, j} \times[\underbrace{U_{i, j}}_{\text {Users srplus }}+\underbrace{\sum_{m} \sum_{k} x_{i, j}^{m, k} \times N M U C_{i, j}^{m, k}}_{\text {Users non-monetary costs }}] \\
& +\underbrace{\Pi_{\text {Pollution and accident externalities }}^{\text {RAIL }}-\sum_{m}^{\sum_{k} E X T^{m, k}}}_{\text {Rail company profit }} \\
& +\underbrace{\kappa^{\text {fuel }} \times\left[\left(V^{R A I L, P}+V^{\text {RAIL,OP }}\right) \times v^{\text {RAIL }} \times d_{A D}^{\text {rail }}+\sum_{k} \sum_{j} V_{j}^{C A R, k} \times v^{C A R} \times d_{j D}^{\text {road }}\right]}_{\text {Revenue of the fuel tax }} \\
& +\underbrace{\sum_{k} \sum_{j} V_{j}^{C A R, k} \times \tau_{j}^{C A R, k}}_{\text {Revenue of the road tolls }} \\
& +\underbrace{\sum_{i} \sum_{j} \lambda_{i, j} \times\left(R_{i, j}-q_{i, j}-\sum_{m} \sum_{k}^{m, j} \times M U C_{j}^{m, k}\right)}_{\text {Revenue of the road tolls }}
\end{aligned}
$$

We derive $\mathcal{L}$ with respect to $\lambda_{i, j}$ :

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \lambda_{i, j}} & =R_{i, j}-q_{i, j}-\sum_{m} \sum_{k} M U C_{j}^{m, k} x_{i, j}^{m, k} \\
\Leftrightarrow q_{i, j} & =R_{i, j}-\sum_{m} \sum_{k} M U C_{j}^{m, k} x_{i, j}^{m, k}
\end{aligned}
$$

The revenue is fully spent.

## b. Optimal quantity of the numeraire good consumed

Optimal quantities of the numeraire goods is given by:
$\frac{\partial \mathcal{L}}{\partial q_{i, j}}=N_{i, j} \frac{\partial U_{i, j}}{\partial q_{i, j}}-\lambda_{i, j}$.

Recalling that $\frac{\partial U_{i, j}}{\partial q_{i, j}}=1$, it implies
$\lambda_{i, j}=N_{i, j}$.

## c. Optimal train fares

To find the optimal level of train fares, we derive $\mathcal{L}$ with respect to $x_{i, j}^{R A L L, k}$.

$$
\begin{aligned}
& \frac{\partial \mathcal{L}}{\partial x_{i, B}^{R A I L, k}}=\quad N_{i, B}\left(\frac{\partial U_{i, B}}{\partial x_{i, B}^{\text {RALL,k }}}-N M U C_{i, B}^{R A L L, k}\right) \\
& -\sum_{n} \sum_{r} N_{n, r} \cdot x_{n, r}^{R A L L k} \cdot\left[\frac{\partial X_{B}^{R A I L, k}}{\partial x_{i, B}^{R A L L, k}} \frac{\partial o_{B}^{R A L L, k}}{\partial X_{B}^{\text {RALL,k}}} \cdot \frac{\partial \operatorname{discom}_{n, B}^{R A L L, k}}{\partial o_{B}^{R A I L, k}}\right] \\
& +p_{B}^{R A I L, k} \frac{\partial X_{B}^{R A I L, k}}{\partial x_{i, B}^{R A L L, k}}-\lambda_{i, B} M U C_{B}^{R A I L, k} \\
& \frac{\partial \mathcal{L}}{\partial x_{i, A}^{\text {RAlL }, k}}= \\
& N_{i, A}\left(\frac{\partial U_{i, A}}{\partial x_{i, A}^{R A L L, k}}-N M U C_{i, A}^{R A I L, k}\right) \\
& -\sum_{n} \sum_{r} N_{n, r} \cdot x_{n, r}^{R A I L, k} \cdot\left[\begin{array}{c}
\frac{\partial X_{A}^{R A L L, k}}{\partial x_{i, A}^{R A L L, k}} \frac{\partial o_{A B}^{R A L L, k}}{\partial X_{A}^{R A I L, k}} \cdot \frac{\partial \operatorname{discom}_{n, A}^{R A L L, k}}{\partial o_{A}^{R A L L, k}} \\
\left.+\frac{\partial X_{B}^{R A L L, k}}{\partial x_{i, A}^{R A L L, k}} \frac{\partial o_{B}^{R A I L, k}}{\partial X_{B}^{R A L L, k}} \cdot \frac{\partial \operatorname{discom}_{n, B}^{R A L L, k}}{}\right] \\
\partial o_{B}^{R A I L, k}
\end{array}\right. \\
& +p_{A}^{R A I L, k} \frac{\partial X_{A}^{R A I L, k}}{\partial x_{i, A}^{R A L L, k}}-\lambda_{i, A} M U C_{A}^{R A I L, k}
\end{aligned}
$$

Recall that $\lambda_{i, j}=N_{i, j}$ from Equation (App. 1), that $M U C_{j}^{R A I L, k}=p_{j}^{R A I L, k}$, that $\frac{\partial U_{i, j}}{\partial x_{i, j}^{R A L, k}}-$ $N M U C_{i, j}^{R A L L, k}=M U C_{i, j}^{R A I L, k}$ from Equation (11) and that $\frac{\partial X_{j}^{R A I L, k}}{\partial x_{i, j}^{R A L, k}}=N_{i, j}^{R A I L, k}$. Setting Equations above equal to zero and rearranging, we find the optimal fares of a journey by train
$p_{B}^{R A I L, k}=\quad \sum_{n} \sum_{r} N_{n, r} \cdot x_{n, r}^{R A I L, k} \cdot\left[\frac{\partial o_{B}^{R A L L, k}}{\partial X_{B}^{R A L L, k}} \cdot \frac{\partial \operatorname{discom}_{n, B}^{R A I L, k}}{\partial o_{B}^{R A I L, k}}\right] ;$
$p_{A}^{R A L L, k}=\sum_{n} \sum_{r} N_{n, r} \cdot x_{n, r}^{R A I L, k} \cdot\left[\frac{\partial o_{A}^{R A L L, k}}{\partial X_{A}^{\text {RAIL,k }}} \cdot \frac{\partial \operatorname{discom}_{n, A}^{R A L L, k}}{\partial o_{A}^{R A I L, k}}+\frac{\partial o_{B}^{R A I L, k}}{\partial X_{A}^{R A I L, k}} \cdot \frac{\partial \operatorname{discom_{n,B}^{RAIL,k}}}{\partial o_{B}^{R A I L, k}}\right]$.

This can be simply rewritten as
$p_{j}^{R A I L, k}=\frac{\partial o_{j}^{R A L L, k}}{\partial X_{j}^{R A I L, k}} \operatorname{DISCO} M_{o_{j}^{m, k}}^{R A L L, k}, \quad \forall k, j$,
where $\operatorname{DISCOM} \underset{o_{j}^{m, k}}{R A I L, k}$ is the variation in aggregated discomfort cost in rail during period $k$ due to a change in the occupancy rate from $j$ to $D$ during period $k$.

## d. Optimal train supply

To find the optimal train supply, we derive $\mathcal{L}$ with respect to the number of runs operated, $V^{R A I L, k}$, and to the optimal number of seats per train, $S^{R A I L}$.

$$
\begin{array}{r}
\frac{\partial \mathcal{L}}{\partial V^{R A L L, k}=-\sum_{i} \sum_{j} N_{i, j} \cdot x_{i, j}^{R A I L, k} \cdot\binom{\frac{\partial u n r e l_{i}^{R A I L, k}}{\partial V^{R A I L, k}}+\frac{\partial o_{A}^{R A I L, k}}{\partial V^{R A I L, k}} \frac{\partial d i s c o m_{i, j}^{R A L L, k}}{\partial o_{A}^{R A L L, k}}}{+\frac{\partial o_{B}^{R A I L, k}}{\partial V^{R A I L, k}} \frac{\partial d i s c o m_{i, j}^{R A L L, k}}{\partial o_{B}^{R A L L, k}}+\frac{\partial s c h e d_{i}^{R A I L, k}}{\partial V^{R A I L, k}}}} \\
-\frac{\partial T C^{R A I L}}{\partial V^{R A L L, k}}-\frac{\partial E X T^{R A I L, k}}{\partial V^{R A L L, k}}+\kappa^{\text {fuel }} \times v^{R A I L} \times d_{A D}^{\text {rail },}
\end{array}
$$

Setting Equations above equal to zero and rearranging, we find

$$
\begin{aligned}
& -E X T_{V^{R A L L, k}}^{R A L L, k}+F U E L_{V^{R A L L, k}}^{R A L}, \forall k, \\
& T C_{S^{m}}^{R A I L}=\quad-\sum_{k} \sum_{j} \frac{\partial o_{j}^{R A I L, k}}{\partial S^{R A I L}} \operatorname{DISCOM}_{o_{j}^{R A L L, k}}^{R A I L, k} .
\end{aligned}
$$

## e. Optimal road tolls

To find the optimal level of travel quantities, we derive $\mathcal{L}$ with respect to $x_{i, j}^{C A R, k}$

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial x_{i, j}^{C A R, k}}=N_{i, j}\left(\frac{\partial U_{i, j}}{\partial x_{i, j}^{C A R, k}}-\text { time }_{i, j}^{C A R}-\operatorname{cong}_{i, j}^{C A R, k}\right)-\sum_{n} \sum_{r} N_{n, r} \cdot x_{n, r}^{C A R, k} \cdot \frac{\partial c o n g_{n, r}^{C A R, k}}{\partial V_{j}^{C A R, k}} \\
-\frac{\partial E X T^{C A R, k}}{\partial V_{j}^{C A R, k}}+\left(\kappa^{g a s} \times v^{m} \times d_{j D}^{\text {road }}+\tau_{j}^{C A R, k}\right)-\lambda_{i, j} M U C_{j}^{C A R, k}
\end{aligned}
$$

Recall that $\lambda_{i, j}=N_{i, j}$ from Equation (App. 1), that $M U C_{j}^{C A R, k}=\kappa^{g a s}+v^{m} \times d_{j D}^{\text {road }}+$ $\tau_{j}^{C a r, k}$, and that $\frac{\partial U_{i, j}}{\partial x_{i, j}^{C R R, k}}-N M U C_{i, j}^{C A R, k}=M U C_{i, j}^{C A R, k}$ from Equation (11)). Setting Equations above equal to zero and rearranging, we find the optimal tolls on road
$\tau_{j}^{C A R, k}=\operatorname{CONG} G_{V_{j}^{C A R, k}}^{C A R, k}+E X T V_{V_{j}^{C A R, k}}^{C A R, k}-F U E L_{V_{j}^{C A R, k}}^{C A R}, \quad \forall k, j$.

## Appendices references

Abrantes, P. A. L. and M. R. Wardman (2011): ‘Meta-analysis of UK values of travel time: An update', Transportation Research Part A: Policy and Practice, 45, 1-17.

Andries, P. (2016): ‘Diagnostic des déplacements domicile-lieu de travail 2014’, Bruxelles: Service public fédéral Mobilité et Transport.

AURG and SMTC (2015): Enquête ménages déplacements 2010 - Vieillissement et perte de mobilité, un enjeu pour les politiques de déplacement.

Börjesson, M., Fung, C. M. and S. Proost (2017): ‘Optimal prices and frequencies for buses in Stockholm', Economics of transportation, 9, 20-36.

Cornelis, E., Hubert, M., Huynen, P., Lebrun, K., Patriarche, G., De Witte, A., Creemers, L., Declercq, K., Janssens, D., Castaigne, M., Hollaert, L. and F. Walle (2012): La mobilité en Belgique en 2010: résultats de l'enquête BELDAM.

Dargay, J. M. and S. Clark (2012): ‘The determinants of long distance travel in Great Britain', Transportation Research Part A: Policy and Practice, 46, 576-587.

Duabresse, C., Geurts, K., Gusbin, D., Hertveldt, B., Hoornaert, B., and A. Van Steenbergen (2015): Perspectives de l'évolution de la demande de transport en Belgique à l'horizon 2030, Bureau fédéral du Plan; Bruxelles.

European Commission (2019): Sixth Report on Monitoring Development of the Rail Market. COM/2019/51 final.

Gleave, S. D. (2015): Study on the Cost and Contribution of the Rail Sector, Final Report prepared for the European Commission.

Ivaldi, M. and P. Seabright (2003): ‘The Economics of Passenger Rail Transport: A Survey’, IDEI Working Paper, $\mathrm{n}^{\circ} 163$.

Kroes, E., Kouwenhoven, M., Debrincat, L. and N. Pauget (2013): 'On the value of crowding in public transport for Ile-de-France', International Transport Forum Discussion Papers, No. 2013/18.

Litman, T (2004): 'Transit price elasticities and cross-elasticities', Journal of Public Transportation, 7, 37-58.

Mayeres, I. (2000): ‘The efficiency effects of transport policies in the presence of externalities and distortionary taxes', Journal of Transport Economics and Policy, 34, 233-259.

Oum, T. H., Waters, I. I. a. W. G. and X. Fu (2008): ‘Transport Demand Elasticities’, in D. A. Hensher and K. J. Button (ed) Handbook of Transport Modelling, 2nd edition. Elsevier, Oxford, United Kingdom, ch. 12.

Pérez Herrero, M., Brunel, J. and G. Marlot (2014): ‘Rail externalities: assessing the social cost of rail congestion', Traffic Management, 3, 331-344.

Quinet, E. (2013): ‘L'évaluation socio-économique des investissements publics’, Rapport du Commissariat Général à la Stratégie et à la Prospective, La Documentation Française, Paris.

DREAL (2014): Atlas régional de la mobilité en Rhône-Alpes.

Wardman, M., Chintakayala, P., de Jong, G. and D. Ferrer (2012): ‘European wide metaanalysis of values of travel time', ITS, University of Leeds, Paper prepared for EIB.


[^0]:    ${ }^{1}$ https://ec.europa.eu/transport/sites/transport/files/handbook_on_external_costs_of transport 2014_0.pdf

[^1]:    ${ }^{2}$ https://fr.wikipedia.org/wiki/Ligne_50A_(Infrabel)
    ${ }^{3}$ https://fr.wikipedia.org/wiki/Gare de_Bourgoin-Jallieu
    ${ }^{4}$ https://fr.wikipedia.org/wiki/Gare de_Grenoble
    ${ }^{5}$ https://fr.wikipedia.org/wiki/InterCity_(Belgique)
    ${ }^{6}$ https://fr.wikipedia.org/wiki/Ligne_Lyon _-_Grenoble

[^2]:    ${ }^{7}$ https://www.energy.eu/fuelprices/
    ${ }^{8}$ http://www.acea.be/statistics/tag/category/passenger-car-fleet-by-fuel-type
    ${ }^{9}$ Includes VAT and excise.
    ${ }^{10} \mathrm{http}: / / \mathrm{www} . l e d a u p h i n e . c o m / i s e r e-n o r d / 2012 / 01 / 31 /$ si-vous-entrez-a-villefontaine

[^3]:    ${ }^{11} \mathrm{http}: / / \mathrm{www}$. belgianrail.be/en/tickets-railcards/age/adults-seniors/frequent/section-season-ticket.aspx
    ${ }^{12} \mathrm{https}: / / \mathrm{www} . t e r . s n c f . c o m / a u v e r g n e-r h o n e-a l p e s / t a r i f s / d e v i s / r e c h e r c h e ? o l d R e g i o n=R A L ~$

