



HAL
open science

Enriched finite elements for high-frequency vibrations of geometrically heterogeneous bars and Timoshenko beams

Rémi Cornaggia, Eric Darrigrand, Loïc Le Marrec, Fabrice Mahé

► To cite this version:

Rémi Cornaggia, Eric Darrigrand, Loïc Le Marrec, Fabrice Mahé. Enriched finite elements for high-frequency vibrations of geometrically heterogeneous bars and Timoshenko beams. Congrès National d'Analyse Numérique 2018, May 2018, Cap d'Agde, France. hal-01835428

HAL Id: hal-01835428

<https://hal.science/hal-01835428>

Submitted on 11 Jul 2018

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Enriched finite elements for high-frequency vibrations of geometrically heterogeneous bars and Timoshenko beams

Rémi Cornaggia, Université Rennes 1

Éric Darrigrand, Université Rennes 1

Loïc Le Marrec, Université Rennes 1

Fabrice Mahé, Université Rennes 1

Mots-clés : Enriched finite elements, high frequency vibrations, bar, Timoshenko beam

Introduction This study proposes a new enriched finite element method (FEM) to handle high-frequency vibrations of bars (traction-compression) and beams (bending) with varying cross-section. Indeed, analytical solutions are available for a limited number of geometries, especially for Timoshenko beams [2], and traditional h - and h - p -FEM may become costly at increasing frequencies as they require a minimal *resolution* (number of elements per wavelength) to reach the convergence regime. Hereinafter, the focus is on bars, but numerical illustrations will be provided for both bars and beams.

Problem and approximation space Consider a bar of length L made of an homogeneous material, such that the wavespeed c is constant, and submitted to time-harmonic excitations with circular frequency ω . In a non-dimensional setting, the amplitude $u(x)$ of the longitudinal displacement then obeys the wave equation $(\mathcal{A}u')' + \mathcal{A}k^2u = f$, where $\mathcal{A}(x)$ is the profile of the cross-section, k is the non-dimensional counterpart of ω/c and f is the amplitude of time-harmonic distributed forces. Boundary conditions, possibly including boundary excitations, complete the problem.

Let $\Omega_h = \{x_0, \dots, x_N\}$ be a mesh of the domain $[0, L]$. Following the ideas of the partition of unity method [1], we define the following enriched FE space:

$$V_h^\Psi = \text{Span} \left\{ \left\{ \varphi_n^0 \right\}_{n=0 \dots N} \cup \left\{ \left\{ \varphi_n^- \psi_n^m \right\} \cup \left\{ \varphi_n^+ \psi_n^m \right\} \right\}_{n=1 \dots N, m=1,2} \right\},$$

where the φ_n^0 are the traditional “hat” functions (the basis of the space of continuous and piecewise affine functions), the φ_n^- (resp. φ_n^+) are “half-hat” functions, i.e. the restriction of φ_{n+1}^0 (resp. φ_n^0) to the n -th element, and $\Psi_n = \{\psi_n^1, \psi_n^2\}$ is the enrichment family associated with this element. Two of these families are considered.

Global and local enrichments First, we use the family $\Psi_n^k = \{\sin(kx), \cos(kx)\}$ for all elements, i.e. the basis of solutions to the wave equation for constant profiles. We prove that using the corresponding enriched space provides the same convergence order than using a fifth-order polynomial basis, and show that *static condensation* is easily implemented thanks of the choice of “half-hat” functions as partition of unity, resulting in a well-conditioned linear system. On numerical tests, we see that (i) the resolution needed to reach convergence is much lower than for the polynomial basis, and (ii) the error decreases with the frequency for a given resolution, making our proposal well adapted to high-frequency problems. Finally, we consider the following element-dependent family:

$$\Psi_n^{k,\delta} = \left\{ e^{-\delta_n x} \sin(\tilde{k}_n x), e^{-\delta_n x} \cos(\tilde{k}_n x) \right\}, \quad \text{with } \delta_n = (\mathcal{A}'/2\mathcal{A})(x_{n-1/2}) \quad \text{and} \quad \tilde{k}_n = \sqrt{k^2 - \delta_n^2},$$

which corresponds to the basis of solutions for exponential profiles $\mathcal{A}_\delta(x) = e^{-2\delta_n x}$, the value of δ_n being determined using a Taylor expansion of the wave equation about the middle $x_{n-1/2}$ of the n -th element. This second family enable to keep all the properties described above, while reducing again the FEM error by a factor up to 2 (depending on the profile \mathcal{A}) compared to the first family.

Acknowledgements The authors thank the Centre Henri Lebesgue ANR-11-LABX-0020-01 and the program “Défis émergents” of Université Rennes-1 for financial support.

Références

- [1] J.M. Melenk and I. Babuška. The partition of unity finite element method: Basic theory and applications. *Computer Methods in Applied Mechanics and Engineering*, 139(1):289 – 314, 1996.
- [2] J. Yuan, Y.-H. Pao, and W. Chen. Exact solutions for free vibrations of axially inhomogeneous Timoshenko beams with variable cross section. *Acta Mechanica*, 227(9):2625–2643, 2016.