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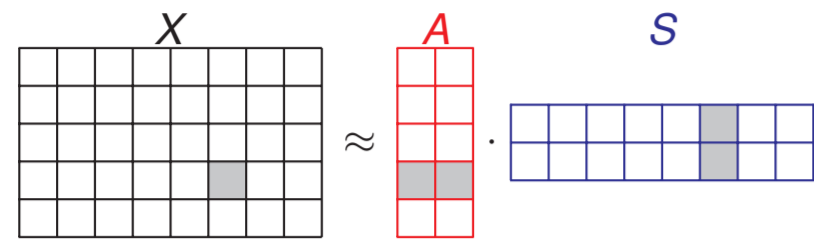
# Do random projections fasten an already fast NMF technique using Nesterov optimal gradient?

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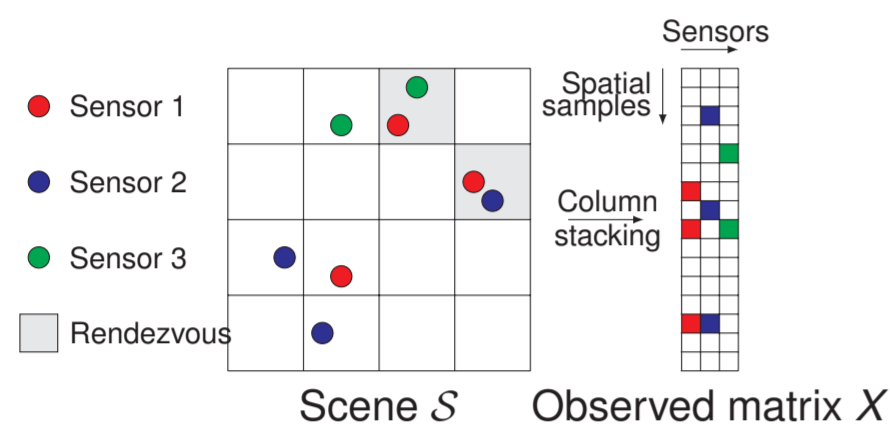
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## Context and problem statement



► **Nonnegative Matrix Factorization (NMF)** is a popular tool in Signal/Image Processing and Machine Learning

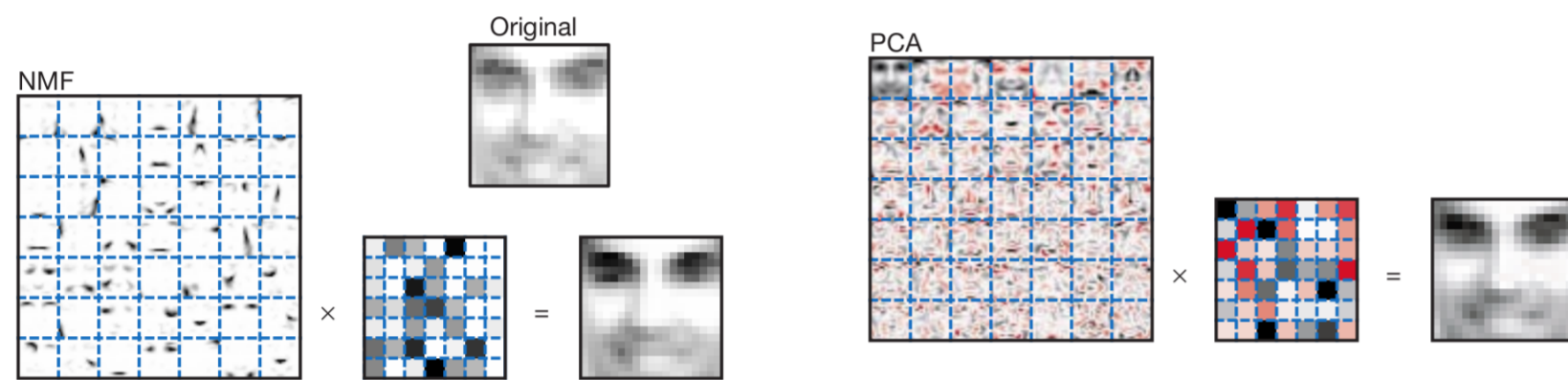
- Goal: estimate two nonnegative  $n \times p$  and  $p \times m$  matrices  $A$  and  $S$  such that an observed low-rank nonnegative  $n \times m$  matrix  $X$  can be written as  $X \approx A \cdot S$  :
- Some applications:
  - **Source separation**, dictionary learning, graph analysis, topic modelling, collaborative filtering, **low-rank nonnegative matrix completion**, **sensor calibration**...
- (informed) NMF for blind sensor calibration



Blind calibration revisited as a weighted NMF problem (affine model) [1]

$$W_0 \begin{bmatrix} x(1,1) & \dots & x(1,m-1) & y(1) \\ \vdots & & \vdots & \vdots \\ x(n,1) & \dots & x(n,m-1) & y(n) \end{bmatrix} \approx W_0 \begin{bmatrix} y(1) & 1 \\ \vdots & \vdots \\ y(n) & 1 \end{bmatrix} \cdot \begin{bmatrix} \alpha_1 & \alpha_2 & \dots & \alpha_{m-1} & 1 \\ \beta_1 & \beta_2 & \dots & \beta_{m-1} & 0 \end{bmatrix}$$

- Why is NMF so popular? **Better interpretability** than no-sign-constrained approaches



NMF and PCA applied to face dataset (source: [2])

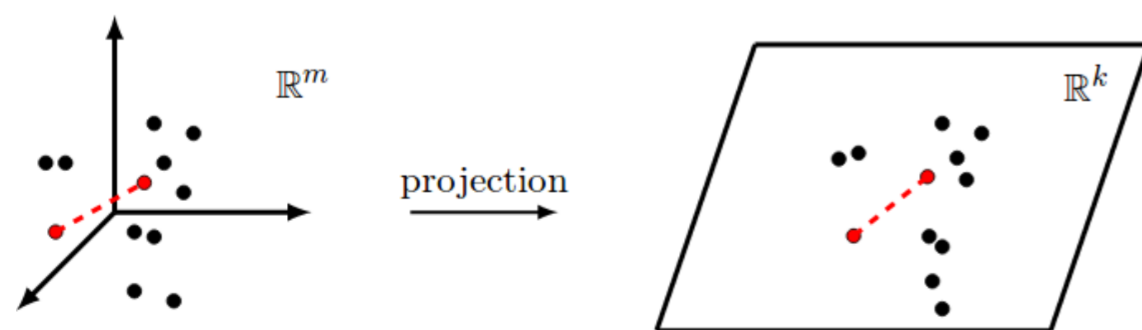
- How is NMF working?
  - Iterative procedure where  $A$  and  $S$  are alternately updated
  - Historical techniques known to be slow (multiplicative updates, projected gradients, nonnegative least squares, etc)
- NMF and Big Data: How to face the data deluge?
  - Distributed computing (e.g., [3])
  - Online factorization (e.g., [4])
  - Fast solver (e.g., [5], [6])
  - Randomized strategies (e.g., [7, 8, 9, 10])
- Most randomized NMF techniques are combined with a slow solver (except for separable NMF, which is out of the scope of this poster).
- ◊ **Is there any interest to combine a fast solver with randomization?**

## NMF with Nesterov iterations

- In 1983, Nesterov fastens the plain Gradient Descent by replacing the line search for optimal stepsize by its proposed iterative Optimal Gradient Descent
- ◊ NeNMF [5] much faster than MU-NMF [2] or PG-NMF [11]
- The NeNMF algorithm consists of:
  - an outer loop
    - initialize  $t = 1, A^1$ , and  $S^1$
    - loop
      - $t \leftarrow t+1$ ;
      - $S^{t+1} \leftarrow \text{NeInnerLoopF}(X, A^t, S^t)$ ;
      - $A^{t+1} \leftarrow \text{NeInnerLoopG}(X, A^t, S^{t+1})$ ;
    - end loop
  - an inner loop (here detailed for an update of  $F$ )
    - define  $\alpha_0 = 1, \alpha_{k+1} = \frac{1 + \sqrt{4\alpha_k^2 + 1}}{2}, \forall k \in \mathbb{N}$
    - initialize  $Y_0 \triangleq S^t, k = 0$
    - repeat
      - $S_k = (Y_k - \frac{1}{2} \nabla_{S^t} \mathcal{J}(A, Y_k))^+$  with  $\mathcal{L} \triangleq \|A\|_2^2$  Lipschitz constant
      - $Y_{k+1} = S_k + \frac{\alpha_k - 1}{\alpha_{k+1}} (S_k - S_{k-1})$ ,
    - until Stopping criterion reached at Iteration  $K$
    - return  $S^{t+1} = Y_K$
    - Stopping criterion:  $k = \text{MaxIter}$  or  $\left\| \nabla_{S^t}^+ \mathcal{J}(A, Y_k) \right\|_{\mathcal{F}} \leq \epsilon \cdot \left\| \left[ \nabla_{A^t}^+ \mathcal{J}(A^t, S^t), \nabla_{S^t}^+ \mathcal{J}(A^t, S^t) \right] \right\|_{\mathcal{F}}$
    - +  $k > \text{MinIter}$

## Principles of random projections

- In an *ideal world* (i.e., noiseless and errorless), only two rows and columns of  $X$  are needed to update  $S$  and  $A$  on the top figure, respectively
- But we are not living in an ideal world...
  - **Random projections** is a popular tool in machine learning to speed-up computations while preserving pairwise structure



Principles of the random projections (source: [12])

- Design of the compression scheme for NMF (e.g., using power iteration)

**Require:** a target rank  $\nu$  (with  $p \leq \nu \ll \min(n, m)$ ) and an integer  $q$  (e.g.,  $q = 4$  in [9])  
 Draw Gaussian random matrices  $\Omega_L \in \mathbb{R}^{m \times \nu}$  and  $\Omega_R \in \mathbb{R}^{\nu \times n}$   
 Define  $B_L \triangleq (X X^T)^q \cdot X \cdot \Omega_L$  and  $B_R \triangleq \Omega_R \cdot X \cdot (X^T X)^q$   
 Derive  $L \in \mathbb{R}^{\nu \times n}$  and  $R \in \mathbb{R}^{m \times \nu}$  by QR decompositions of  $B_L$  and  $B_R$ , respectively.

- NMF with random projections:

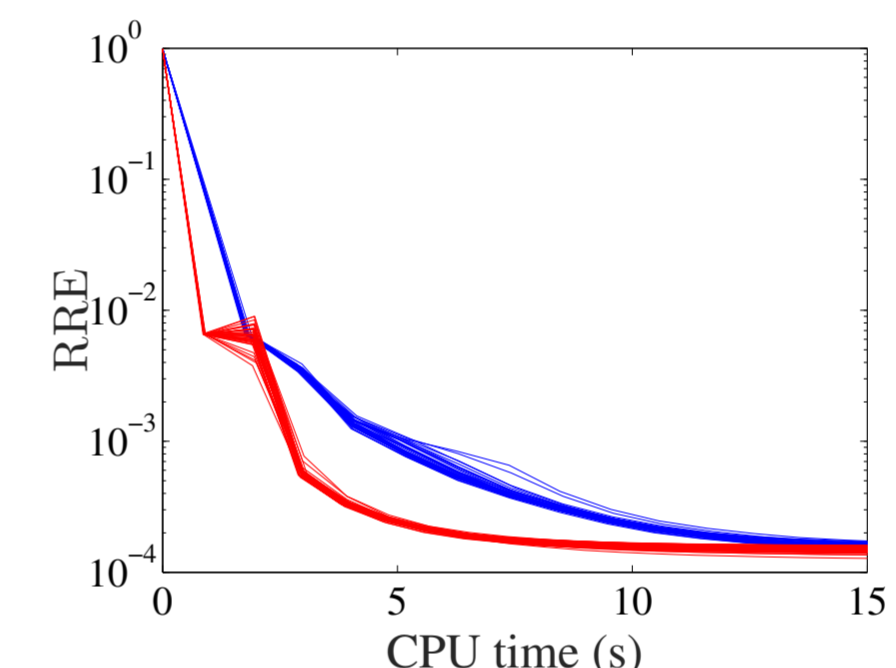
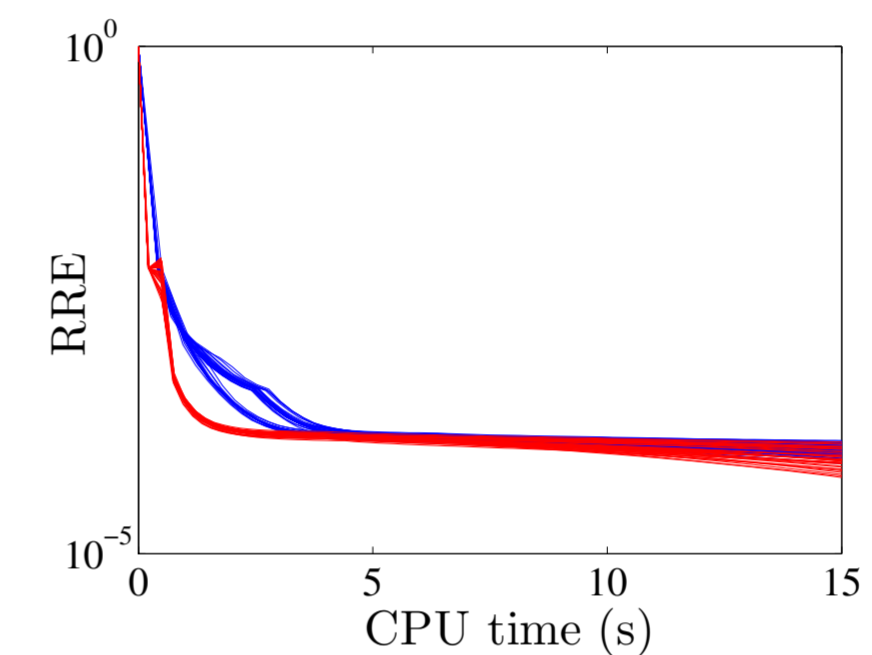
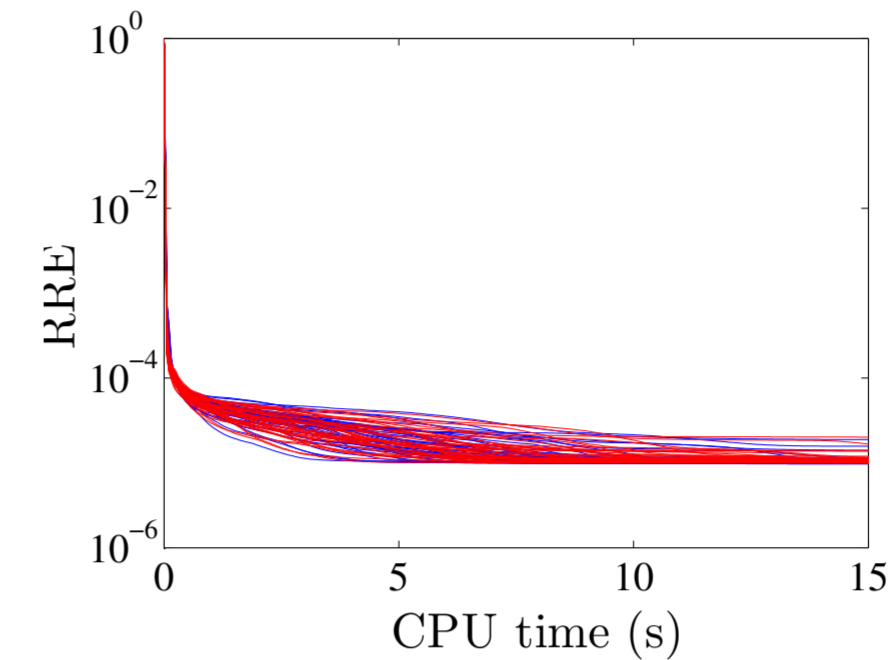
**Require:** initial matrices  $A$  and  $S$ , and compression matrices  $L$  and  $R$ .  
 Define  $X_L \triangleq L \cdot X$  and  $X_R \triangleq X \cdot R$   
**repeat**  
 Define  $S_R \triangleq S \cdot R$   
 Update  $A$  by resp. replacing  $X$  and  $S$  by  $X_R$  and  $S_R$   
 Define  $A_L \triangleq L \cdot A$   
 Update  $S$  by resp. replacing  $X$  and  $A$  by  $X_L$  and  $A_L$   
**until** a stopping criterion

## Experiments

- Simulations with rank-15 matrices  $X$  of size  $m = n = 500, 5000$ , and  $10000$ .
  - Theoretical matrices  $A$  and  $S$  are known
  - Input SNR  $\approx 30$  dB
  - Methods are run 15 s and their performance is regularly computed
  - Performance criterion:

$$\text{RRE} \triangleq \frac{\|X - A \cdot S\|_{\mathcal{F}}}{\|X\|_{\mathcal{F}}}$$

- Randomized NeNMF vs (already fast) vanilla NeNMF



Performance of the randomized NeNMF (in red) vs the vanilla one (in blue) with respect to time.  
 Top:  $n = 500$ . Middle:  $n = 5000$ . Bottom:  $n = 10000$ .

## Conclusion and perspectives

- An experimental study of the combination of random projections with an optimal solver for NMF
- The proposed approach is shown to be really fast (much more than vanilla NeNMF for very large matrices)
- In future work, we will extend the approach to weighted NMF (matrix completion, blind sensor calibration)

## References and acknowledgments

- [1] C. Dorffer, M. Puigt, G. Delmaire, and G. Roussel. "Informed Nonnegative Matrix Factorization Methods for Mobile Sensor Network Calibration". In: *IEEE Trans. Sig. Inf. Proc. Networks* (to appear).
- [2] D.D. Lee and H.S. Seung. "Learning the parts of objects by non negative matrix factorization". In: *Nature* 401.6755 (1999), pp. 788–791.
- [3] C. Liu, H.-C. Yang, J. Fan, L.-W. He, and Y.-M. Wang. "Distributed Nonnegative Matrix Factorization for Web-Scale Dyadic Data Analysis on MapReduce". In: *Proc. WWW Conf.'10. Apr. 2010*.
- [4] Julien Mairal, Francis Bach, Jean Ponce, and Guillermo Sapiro. "Online learning for matrix factorization and sparse coding". In: *Journal of Machine Learning Research* 11. Jan (2010), pp. 19–60.
- [5] Naiyang Guan, Dacheng Tao, Zhigang Luo, and Bo Yuan. "NeNMF: An optimal gradient method for nonnegative matrix factorization". In: 60.6 (2012), pp. 2882–2898.
- [6] C. Dorffer, M. Puigt, G. Delmaire, and G. Roussel. "Fast nonnegative matrix factorization and completion using Nesterov iterations". In: *Proc. LVA/ICA'17. Vol. LNCS 10179. 2017*, pp. 26–35.
- [7] Fei Wang and Ping Li. "Efficient nonnegative matrix factorization with random projections". In: *Proc. SIAM ICDM'10. SIAM. 2010*, pp. 281–292.
- [8] G. Zhou, A. Cichocki, and S. Xie. "Fast Nonnegative Matrix/Tensor Factorization Based on Low-Rank Approximation". In: 60.6 (June 2012), pp. 2928–2940.
- [9] M. Tepper and G. Sapiro. "Compressed Nonnegative Matrix Factorization Is Fast and Accurate". In: 64.9 (May 2016), pp. 2269–2283.
- [10] N Benjamin Erichson, Ariana Mendible, Sophie Wihlborn, and J Nathan Kutz. "Randomized Nonnegative Matrix Factorization". In: *Pattern Recognition Letters* (2018).
- [11] C. Lin. "Projected Gradient Methods for Nonnegative Matrix Factorization". In: *Neural Computation* 19.10 (Oct. 2007), pp. 2756–2779.
- [12] N Benjamin Erichson, Sergey Voronin, Steven L Brunton, and J Nathan Kutz. "Randomized matrix decompositions using R". In: *arXiv preprint arXiv:1608.02148* (2016).



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