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# Robust $\mathcal{H}_\infty$ Proportional-Integral Observer for Fault Diagnosis: Application to Vehicle Suspension. <sup>\*</sup>

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**Abstract:** The main contribution of this paper is the design of robust  $\mathcal{H}_\infty$  proportional-integral (PI) observer for fault detection, isolation and magnitude estimation, applied for uncertain linear time-invariant (LTI) system. It is based on a combination of PI observer and  $\mathcal{H}_\infty$  norm in order to deal with system uncertainty and disturbance attenuation. This multi-objective problem is tackled by the solution of linear matrix inequality (LMI) feasibility problem, obtained from the analysis of estimation errors and the use of majoration lemma. The application to a real suspension system is then presented to highlight the performances of the proposed observer.

*Keywords:* Observers, Fault diagnosis, Fault detection, Uncertain linear systems, Semi-active suspension, Robust estimation.

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## 1. INTRODUCTION

In the observer design, the parametric uncertainty, which has negative effects on system stability and observation performance, is always an attractive topic for research (see Jabbari and Benson (1992)). To solve this kind of time-varying and bounded uncertainty, most methods are based on the linear matrix inequality (LMI) optimization, obtained from the Lyapunov stability function. In Nouailletas et al. (2007), the observer design problem was addressed by applying the projection lemma and the Schur complement to linear discrete-time linear switched system with uncertainties.

As system is also adversely affected by the system disturbance,  $\mathcal{H}_\infty$  observer has been developed to attenuate the perturbation effect on the estimation. In Lu and Ho (2004), a robust  $\mathcal{H}_\infty$  observer design for nonlinear discrete systems with time-delay and parameter uncertainties was proposed. Another nonlinear  $\mathcal{H}_\infty$  observer design was then introduced by using LMI optimization for a class of Lipschitz nonlinear uncertain systems in Abbaszadeh and Marquez (2007), in which the majoration lemma (see Wang et al. (1992)) was applied to deal with system uncertainties. Afterwards, extensive work has been carried out for nonlinear discrete-time uncertain systems in Abbaszadeh and Marquez (2008) and uncertain discrete-time nonlinear delay systems in Delshad et al. (2012).

In addition, there is always a static error between the system state and its estimation. To overcome this drawback, proportional-integral (PI) observer was firstly introduced by Wojciechowski (1978) for the single-input-single-output system. The synthesis of robustness performance, parametric uncertainty and perturbation was proposed for PI observer by Shafai and Carroll (1985). The application of PI observer to estimate the system states and faults was then presented in Marx et al. (2003) for the descriptor system. In Farhat and Koenig (2015), the authors introduced a robust PI observer in order to detect the faults of switched system by applying the LMI optimization; however, their work can only be applied to the uncertainties existing in state-matrix A.

As a result, there is a need for the reconciliation of previous methods to overcome fault estimation problems. Hence, the robust  $\mathcal{H}_\infty$  PI observer design is introduced in this study with the following contributions:

- The observer stability is solved base on only one LMI optimization problem by using the majoration lemma, and thus promoting a flexible solution for observer design by modifying the majoration factor;
- Fault estimation result is robust against the system uncertainties existing in the matrices of states and outputs.

The application of robust  $\mathcal{H}_\infty$  PI observer to suspension system is also presented. To cope with system uncertainties and disturbances, the proposed observer design is applied to fault estimation of chassis-tire displacement position.

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The paper is organized as follows. Firstly, the problem formulation is presented in Section 2. In Section 3, under fault occurrence, the observer design problem against system uncertainties is solved by using LMI optimization. The diagnosis system for suspension is presented in Section 4. In Section 5, the modelling for suspension system is introduced and an experimental validation is realized with the existence of actuator and sensor faults. Finally, the conclusion with remarks and future work are presented in Section 6.

*Notations:*  $\mathbb{R}^n$  and  $\mathbb{R}^{m \times n}$  respectively represent the  $n$ -dimensional Euclidean space and the set of all  $m \times n$  real matrices;  $X^T$  is the transpose of matrix  $X$ ;  $X > 0$  is the real symmetric positive definite matrix;  $0$  and  $I$  denote the zeros and identity matrix with appropriate dimensions; the symbol  $(*)$  denotes the transposed block in the symmetric position; and we denote  $He\{A\} = A + A^T$ .

## 2. PROBLEM FORMULATION

Consider the following uncertain faulty LTI system:

$$\begin{cases} \dot{x} = (A + \Delta A)x + Bu + E_d d + E_f f \\ y = (C + \Delta C)x + Du + F_d d + F_f f \end{cases} \quad (1)$$

where:  $x \in \mathbb{R}^{n_x}$  is the state vector;  $y \in \mathbb{R}^{n_y}$  is the measurement output vector;  $u \in \mathbb{R}^{n_u}$  is the input vector;  $d \in \mathbb{R}^{n_d}$  is the disturbance vector;  $f \in \mathbb{R}^{n_f}$  is the fault vector to detect and estimate. Matrices  $A$ ,  $B$ ,  $C$ ,  $D$ ,  $E_d$ ,  $E_f$ ,  $F_d$ , and  $F_f$  are all known constant matrices with appropriate dimensions, which correspond to the nominal system. The terms  $\Delta A$  and  $\Delta C$  are time-varying parameter matrices corresponding to uncertainty of nominal system. They can be represented as:

$$\begin{cases} \Delta A = M_a \Delta_a N_a \\ \Delta C = M_c \Delta_c N_c \end{cases} \quad (2)$$

In which,  $M_a \in \mathbb{R}^{n_x \times n_{1a}}$ ,  $N_a \in \mathbb{R}^{n_{2a} \times n_x}$ ,  $M_c \in \mathbb{R}^{n_y \times n_{1c}}$ , and  $N_c \in \mathbb{R}^{n_{2c} \times n_x}$  are known real constant matrices.  $\Delta_a \in \mathbb{R}^{n_{1a} \times n_{2a}}$ ,  $\Delta_c \in \mathbb{R}^{n_{1c} \times n_{2c}}$  are unknown real-valued matrices satisfying that  $\Delta_a^T \Delta_a \leq I_{n_{2a}}$  and  $\Delta_c^T \Delta_c \leq I_{n_{2c}}$ .

The observer design for (1) is presented in the section 3.

## 3. ROBUST PI $\mathcal{H}_\infty$ OBSERVER

*Assumption 1.* As discussed in Marx et al. (2003) and Hamdi et al. (2012), the faults are considered to be bounded and supposed to be in low frequency domain, i.e.,  $\dot{f} \simeq 0$ . In fact, most system faults, such as an actuator jam, a hardover, and offsets in sensor outputs, exist in this zone (see Isermann (2006), and Alwi et al. (2011)).

To estimate the fault, an augmented state is considered, so the PI  $\mathcal{H}_\infty$  observer has the representation:

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + Bu + L_P(y - \hat{y}) + E_f \hat{f} \\ \dot{\hat{f}} = L_I(y - \hat{y}) \\ \hat{y} = C\hat{x} + Du + F_f \hat{f} \end{cases} \quad (3)$$

where  $L_P$  and  $L_I$  are the proportional and integral gains of the PI observer, respectively.

The state estimation error  $e_x$  and fault estimation error  $e_f$  are defined as:

$$\begin{cases} e_x = x - \hat{x} \\ e_f = f - \hat{f} \end{cases} \quad (4)$$

Using (1) and (3), the dynamics of estimation errors are given by:

$$\begin{aligned} \dot{e}_x &= \dot{x} - \dot{\hat{x}} = (A - L_P C)e_x + (E_f - L_P F_f)e_f \\ &\quad + (E_d - L_P F_d)d + (\Delta A x - L_P \Delta C)x \end{aligned} \quad (5)$$

$$\dot{e}_f = \dot{f} - \dot{\hat{f}} = -L_I C e_x - L_I F_f e_f - L_I F_d d - L_I \Delta C x \quad (6)$$

In matrix form:

$$\begin{aligned} \begin{bmatrix} \dot{e}_x \\ \dot{e}_f \end{bmatrix} &= \left( \begin{bmatrix} A & E_f \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} L_P \\ L_I \end{bmatrix} \begin{bmatrix} C & F_f \end{bmatrix} \right) \begin{bmatrix} e_x \\ e_f \end{bmatrix} + \\ &\left( \begin{bmatrix} E_d \\ 0 \end{bmatrix} - \begin{bmatrix} L_P \\ L_I \end{bmatrix} F_d \right) d + \left( \begin{bmatrix} \Delta A \\ 0 \end{bmatrix} - \begin{bmatrix} L_P \\ L_I \end{bmatrix} \Delta C \right) x \end{aligned} \quad (7)$$

By defining that:  $e = \begin{bmatrix} e_x \\ e_f \end{bmatrix}$ ,  $\bar{I} = [I_{n_x} \ 0_{n_x \times n_f}]$ ,  $\hat{x}_a = \begin{bmatrix} \hat{x} \\ \hat{f} \end{bmatrix}$ ,

$$\begin{aligned} M_{ap} &= \begin{bmatrix} M_a \\ 0_{n_f \times n_{1a}} \end{bmatrix}, A_a = \begin{bmatrix} A & E_f \\ 0 & 0 \end{bmatrix}, L_a = \begin{bmatrix} L_P \\ L_I \end{bmatrix}, B_a = \begin{bmatrix} B \\ 0 \end{bmatrix} \\ E_{da} &= \begin{bmatrix} E_d \\ 0_{n_f \times n_d} \end{bmatrix}, C_a = [C \ F_f], C_{af} = [0_{n_x} \ I_{n_f}], \end{aligned}$$

and using the transformation:  $\begin{bmatrix} \Delta A \\ 0_{n_f \times n_x} \end{bmatrix} = M_{ap} \Delta_a N_a$ .

The dynamics of estimated errors can be rewritten as:

$$\begin{aligned} \dot{e} &= (A_a - L_a C_a)e + (E_{da} - L_a F_d)d \\ &\quad + (M_{ap} \Delta_a N_a \bar{I} - L_a M_c \Delta_c N_c \bar{I})(e + \hat{x}_a) \end{aligned} \quad (8)$$

*Remark 1:*  $x = e_x + \hat{x} = \bar{I}(e + \hat{x}_a)$

From (3), the observer states can be represented as:

$$\begin{cases} \dot{\hat{x}} = (A\hat{x} + E_f \hat{f}) + (L_P C e_x + L_P F_f e_f) + Bu \\ \quad + L_P \Delta C x + L_P F_d d \\ \dot{\hat{f}} = (L_I C e_x + L_I F_f e_f) + L_I F_d d + L_I \Delta C x \end{cases} \quad (9)$$

In other words,

$$\dot{\hat{x}}_a = A_a \hat{x}_a + L_a C_a e + L_a F_d d + B_a u + L_a \Delta C \bar{I}(e + \hat{x}_a) \quad (10)$$

The dynamic of observer is illustrated by:

$$\begin{cases} \dot{\hat{x}}_a = (A_a + L_a M_c \Delta_c N_c \bar{I})\hat{x}_a \\ \quad + (L_a C_a + L_a M_c \Delta_c N_c \bar{I})e + B_a u + L_a F_d d \\ \dot{e} = (A_a - L_a C_a + M_{ap} \Delta_a N_a \bar{I} - L_a M_c \Delta_c N_c \bar{I})e \\ \quad + (M_{ap} \Delta_a N_a \bar{I} - L_a M_c \Delta_c N_c \bar{I})\hat{x}_a \\ \quad + (E_{da} - L_a F_d)d \\ e_f = C_{af} e \end{cases} \quad (11)$$

Some remarks on LMI and inequality problems are presented by the following lemmas:

*Lemma 1.* (Majoration lemma) (see Wang et al. (1992))

If there exists  $F^T F \leq I$ , for given matrices  $X$  and  $Y$  with appropriate dimensions, the following statement is always true with an arbitrary scalar  $\sigma > 0$ :

$$X F Y + Y^T F^T X^T \leq \sigma X X^T + \sigma^{-1} Y^T Y \quad (12)$$

In other words,  $He\{XFY\} \leq \sigma XX^T + \sigma^{-1}Y^TY$

*Lemma 2.* (Schur complement) (see Boyd et al. (1994))

Let  $Q \leq 0$ , S and R be given matrices, the following statements are equivalents:

$$\begin{aligned} \begin{bmatrix} Q & R \\ R^T & S \end{bmatrix} \leq 0 \\ Q - RS^{-1}R^T \leq 0 \end{aligned} \quad (13)$$

As usual (see Marx et al. (2003)), the main objective of robust PI  $\mathcal{H}_\infty$  observer is to minimize the effect of disturbances on the fault estimation error  $e_f$ , which can be rewritten as:

$$\min_{\gamma > 0} \|S_{e_f d}\|_\infty = \min_{\gamma > 0} \frac{\|e_f\|_2}{\|d\|_2} \leq \gamma \quad (14)$$

The sufficient stability for the PI observer and the disturbance attenuation objective (14) can only be achieved if the following condition is satisfied:

$$\dot{V} + e_f^T e_f - \gamma^2 d^T d \leq 0 \quad (15)$$

where the candidate Lyapunov function is chosen as:  $V = V_1 + V_2 = e^T P e + \hat{x}_a^T P \hat{x}_a$  ( $P > 0$ ) (see Farhat and Koenig (2015)).

*Theorem 1.* System (11) is robust stable, if there exist a symmetric positive definite matrix P, a matrix Q, a scalar  $\gamma$  and given positive scalars  $\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5$  and  $\sigma_6$  such that the following LMI is satisfied:

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & 0 & \Omega_{14} & PM_{ap} & QM_c & 0 \\ (*) & \Omega_{22} & \Omega_{23} & \Omega_{24} & 0 & 0 & QM_c \\ (*) & (*) & 0 & 0 & 0 & 0 & 0 \\ (*) & (*) & (*) & \Omega_{44} & 0 & 0 & 0 \\ (*) & (*) & (*) & (*) & \Omega_{55} & 0 & 0 \\ (*) & (*) & (*) & (*) & (*) & \Omega_{66} & 0 \\ (*) & (*) & (*) & (*) & (*) & (*) & \Omega_{77} \end{bmatrix} \leq 0 \quad (16)$$

where  $\Omega_{11} = He\{PA_a + QC_a\} + C_{af}^T C_{af} + \sigma_1^{-1} \bar{I}^T N_a^T N_a \bar{I} + (\sigma_2^{-1} + \sigma_6^{-1}) \bar{I}^T N_c^T N_c \bar{I}$ ,  $\Omega_{12} = -C_{af}^T Q^T$ ,  $\Omega_{14} = PE_{da} + QF_d$ ,  $\Omega_{22} = PA_a + A_a^T P + \sigma_3^{-1} \bar{I}^T N_a^T N_a \bar{I} + (\sigma_4^{-1} + \sigma_5^{-1}) \bar{I}^T N_c^T N_c \bar{I}$ ,  $\Omega_{23} = PB_a$ ,  $\Omega_{24} = -QF_d$ ,  $\Omega_{44} = -\gamma^2 I$ ,  $\Omega_{55} = -(\sigma_1 + \sigma_3)^{-1}$ ,  $\Omega_{66} = -(\sigma_2 + \sigma_4)^{-1}$ , and  $\Omega_{77} = -(\sigma_5 + \sigma_6)^{-1}$ .

The gains of the PI observer (3) can be calculated by:

$$\begin{bmatrix} L_P \\ L_I \end{bmatrix} = L_a = -P^{-1}Q \quad (17)$$

*Remark 2:* Without the system uncertainty and disturbance,  $\Omega_{11}$  presents the stability of matrix  $(A_a - L_a C_a)$  in the sense of Lyapunov, and therefore the estimation errors  $e \rightarrow 0$  when  $t \rightarrow \infty$ .

**Proof.** Using the new variable  $Q = -PL_a$  and Lyapunov function  $V = V_1 + V_2 = e^T P e + \hat{x}_a^T P \hat{x}_a$ , we obtain:

$$\begin{aligned} \dot{V}_1 &= He\{e^T(PA_a + QC_a)e\} + He\{e^T(PE_{da} + QF_d)d\} \\ &+ He\{(e^T PM_{ap})\Delta_a(N_a \bar{I}e)\} + He\{(e^T QM_c)\Delta_c(N_c \bar{I}e)\} \\ &+ He\{(e^T PM_{ap})\Delta_a(N_a \bar{I}\hat{x}_a)\} \\ &+ He\{(e^T QM_c)\Delta_c(N_c \bar{I}\hat{x}_a)\} \end{aligned} \quad (18)$$

Using the lemma 1 (12),  $\forall \sigma_1, \sigma_2, \sigma_3, \sigma_4 > 0$ , the LMI (18) can be majorated by the following inequality:

$$\begin{aligned} \dot{V}_1 &\leq He\{e^T(PA_a + QC_a)e\} + He\{e^T(PE_{da} + QF_d)d\} \\ &+ e^T(\sigma_1 PM_{ap} M_{ap}^T P + \sigma_1^{-1} \bar{I}^T N_a^T N_a \bar{I})e \\ &+ e^T(\sigma_2 QM_c M_c^T Q^T + \sigma_2^{-1} \bar{I}^T N_c^T N_c \bar{I})e \\ &+ e^T(\sigma_3 PM_{ap} M_{ap}^T P)e + \hat{x}_a^T(\sigma_3^{-1} \bar{I}^T N_a^T N_a \bar{I})\hat{x}_a \\ &+ e^T(\sigma_4 QM_c M_c^T Q^T)e + \hat{x}_a^T(\sigma_4^{-1} \bar{I}^T N_c^T N_c \bar{I})\hat{x}_a \end{aligned} \quad (19)$$

On the other hand,

$$\begin{aligned} \dot{V}_2 &= He\{\hat{x}_a^T PA_a \hat{x}_a - \hat{x}_a^T QC_a e - \hat{x}_a^T QF_d d + \hat{x}_a^T PB_a u\} \\ &+ He\{\hat{x}_a^T (-Q)M_c \Delta_c N_c \bar{I} \hat{x}_a\} + He\{\hat{x}_a^T (-Q)M_c \Delta_c N_c \bar{I} e\} \end{aligned} \quad (20)$$

Using the lemma 1 (12),  $\forall \sigma_5, \sigma_6 > 0$ , the LMI (20) can be majorated by the following inequality:

$$\begin{aligned} \dot{V}_2 &\leq He\{\hat{x}_a^T PA_a \hat{x}_a - \hat{x}_a^T QC_a e - \hat{x}_a^T QF_d d + \hat{x}_a^T PB_a u\} \\ &+ \hat{x}_a^T(\sigma_5 QM_c M_c^T Q^T + \sigma_5^{-1} \bar{I}^T N_c^T N_c \bar{I})\hat{x}_a \\ &+ \hat{x}_a^T(\sigma_6 QM_c M_c^T Q^T)\hat{x}_a + e^T \sigma_6^{-1} \bar{I}^T N_c^T N_c \bar{I} e \end{aligned} \quad (21)$$

Combining (19) and (21), the left side of (15) can be expressed as:

$$\dot{V} + e_f^T e_f - \gamma^2 d^T d \leq [e^T \hat{x}_a^T u^T d^T] \Omega \begin{bmatrix} e \\ \hat{x}_a \\ u \\ d \end{bmatrix} \quad (22)$$

In which,

$$\Omega = \begin{bmatrix} \Omega'_{11} & \Omega_{12} & 0 & \Omega_{14} \\ (*) & \Omega_{22} & \Omega_{23} & \Omega_{24} \\ (*) & (*) & 0 & 0 \\ (*) & (*) & (*) & \Omega_{44} \end{bmatrix} \quad (23)$$

$$\begin{aligned} \Omega'_{11} &= He\{PA_a + QC_a\} + (\sigma_1 + \sigma_3)PM_{ap}M_{ap}^T P \\ &+ (\sigma_2 + \sigma_4)QM_c M_c^T Q^T + \sigma_1^{-1} \bar{I}^T N_a^T N_a \bar{I} \\ &+ (\sigma_2^{-1} + \sigma_6^{-1}) \bar{I}^T N_c^T N_c \bar{I} + C_{af}^T C_{af} \end{aligned} \quad (24)$$

$$\begin{aligned} \Omega'_{22} &= PA_a + A_a^T P + (\sigma_5 + \sigma_6)QM_c M_c^T Q^T \\ &+ \sigma_3^{-1} \bar{I}^T N_a^T N_a \bar{I} + (\sigma_4^{-1} + \sigma_5^{-1}) \bar{I}^T N_c^T N_c \bar{I} \end{aligned} \quad (25)$$

The sufficient condition, for both the stability of observer (3) and disturbance attenuation (14), holds if the right-hand-side of (22)  $\leq 0 \forall [e^T \hat{x}_a^T u^T d^T]^T \neq 0$ . In other words:

$$\Omega = \begin{bmatrix} \Omega'_{11} & \Omega_{12} & 0 & \Omega_{14} \\ (*) & \Omega'_{22} & \Omega_{23} & \Omega_{24} \\ (*) & (*) & 0 & 0 \\ (*) & (*) & (*) & \Omega_{44} \end{bmatrix} \leq 0 \quad (26)$$

*Remark 3:* If there exists numerical computation problem due to zero element (3,3) in (26), the feedback control  $u = -K\hat{x}_a$  can be applied to avoid this problem.

Applying Schur complement (lemma 2) to  $\Omega'_{11}$  and  $\Omega'_{22}$ , the following LMI is obtained:

$$\begin{bmatrix} \begin{bmatrix} \Omega_{11} & \Omega_{12} & 0 & \Omega_{14} \\ (*) & \Omega_{22} & \Omega_{23} & \Omega_{24} \\ (*) & (*) & 0 & 0 \\ (*) & (*) & (*) & \Omega_{44} \end{bmatrix} & \begin{bmatrix} PM_{ap} & QM_c \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 \\ QM_c \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ (*) & \begin{bmatrix} \Omega_{55} & 0 \\ 0 & \Omega_{66} \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ (*) & (*) & \Omega_{77} \end{bmatrix} \leq 0 \quad (27)$$

From the above result, the sufficient condition for (15) is that above LMI (27) must hold, which completes the proof.

Finding the solution for the LMI (16) allows the estimation the fault vector  $f$ . However, to improve  $\mathcal{H}_\infty$  performance, there is a need for a diagnosis system, where each fault can be detected and diagnosed separately.

#### 4. DIAGNOSIS SYSTEM

To isolate and estimate the existing faults, a bank of observers, each of which is sensitive to a certain fault and capable of attenuating the influence of disturbance and other faults' existence, is proposed (see Chen et al. (1996)). This procedure can be illustrated by Fig. 1 :

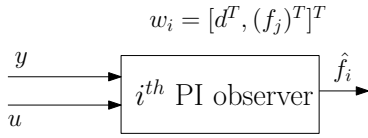


Fig. 1. Fault estimation observer

where  $f_i$  is the fault to detect corresponding to the  $i^{th}$  observer;  $\hat{f}_i$  is the fault estimation;  $d$  is the vector of disturbance;  $f_j$  is the vector of non-estimated faults and  $w_i = [d^T \ (f_j)^T]^T$  is the exogenous input.

The representation  $\sum_i$  of the system LTI (1) corresponds to the conception of the  $i^{th}$  PI observer:

$$\sum_i \begin{cases} \dot{x} &= (A + \Delta A)x + Bu + [E_d \ E_{f_j}] w_i + E_{f_i} f_i \\ y &= (C + \Delta C)x + Du + [F_d \ F_{f_j}] w_i + F_{f_i} f_i \end{cases} \quad (28)$$

where  $F_{f_i}$  and  $F_{f_j}$  are known matrices corresponding to the faults  $f_i$  and  $f_j$ , respectively.

Based on the synthesis in (28), the  $\mathcal{H}_\infty$  objective in (14) can be rewritten for the  $i^{th}$  observer as:

$$\min_{\gamma > 0} \|S_{e_{f_i} w_i}\|_\infty = \min_{\gamma > 0} \frac{\|e_{f_i}\|_2}{\|w_i\|_2} \leq \gamma \quad (29)$$

where  $e_{f_i} = f_i - \hat{f}_i$  is fault estimation error of the fault  $f_i$ .

In addition, it is worth noting that those are not perfect decoupling because of the  $\mathcal{H}_\infty$ -norm use. The relation between the  $\mathcal{H}_\infty$  objective and each observer can be illustrated by Table 1 which is based on the dedicated observer scheme (DOS) in fault detection and isolation (FDI) process (see Clark (1978)). The difference is that the aim of each observer is to estimate a certain fault (actuator or sensor fault) and attenuate its disturbances, whereas in DOS, a certain fault is only detected and its residual needs to decouple perfectly from its perturbation.

Table 1. Fault estimation and disturbance attenuation table

Fault	PI Observer 1	PI Observer $j = 2 \dots n$
$f_1$	$ T_{\hat{f}_1 f_1} _{w=0:bw_1} = 1(0 \text{ dB})$	$\ S_{e_{f_j} f_1}\ _\infty \leq \gamma_j$
$f_j$	$\ S_{e_{f_1} f_j}\ _\infty \leq \gamma_1$	$ T_{\hat{f}_j f_j} _{w=0:bw_j} = 1(0 \text{ dB})$
d	$\ S_{e_{f_1} d}\ _\infty \leq \gamma_1$	$\ S_{e_{f_j} d}\ _\infty \leq \gamma_j$

where  $T_{\hat{f}_i f_i} = \hat{f}_i / f_i$  is the complementary sensibility function of  $\hat{f}_i$  and  $f_i$ ;  $S_{e_{f_i} f_j} = e_{f_i} / f_j$  is the sensibility function of  $e_{f_i}$  and  $f_j$ ;  $S_{e_{f_i} d} = e_{f_i} / d$  is the sensibility function of  $e_{f_i}$  and  $d$ ; and  $bw_i$  is the bandwidth of fault  $f_i$ .

Moreover, the  $\mathcal{H}_\infty$  performance can be improved for specific zones of frequency by applying specific filters to the disturbance  $w_i = [d^T \ (f_j)^T]^T$ . More details on filter design are presented in 5.4 and Koenig et al. (2016).

$$W_d : \begin{cases} \dot{x}_d = A_d x_d + B_d \bar{d} \\ d = C_d x_d + D_d \bar{d} \end{cases} \quad (30)$$

$$W_{f_j} : \begin{cases} \dot{x}_{f_j} = A_{f_j} x_{f_j} + B_{f_j} \bar{f}_j \\ f_j = C_{f_j} x_{f_j} + D_{f_j} \bar{f}_j \end{cases} \quad (31)$$

where  $\bar{d}$  is the fictive disturbance of  $d$ ; and  $\bar{f}_j$  is the fictive non-estimated fault of  $f_j$ .

The faulty system of (28) can be represented by  $\bar{d}$  and  $\bar{f}_j$  as:

$$\sum_i \begin{cases} \dot{x}_s &= (A_s + \Delta A_s)x_s + B_s u + E_{ds} \begin{bmatrix} \bar{d} \\ \bar{f}_i \end{bmatrix} + E_{f_s} f_i \\ y &= (C_s + \Delta C_s)x_s + D u + F_{ds} \begin{bmatrix} \bar{d} \\ \bar{f}_i \end{bmatrix} + F_{f_s} f_i \end{cases} \quad (32)$$

$$\text{where } x_s = \begin{bmatrix} x \\ x_d \\ x_{f_j} \end{bmatrix}, A_s = \begin{bmatrix} A & E_d C_d & E_{f_j} C_{f_j} \\ 0 & A_d & 0 \\ 0 & 0 & A_{f_j} \end{bmatrix},$$

$$\Delta A_s = \begin{bmatrix} \Delta A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, B_s = \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix}, E_{ds} = \begin{bmatrix} E_d D_d & E_{f_j} D_{f_j} \\ B_d & 0 \\ 0 & B_{f_j} \end{bmatrix}^T,$$

$$E_{f_s} = [E_{f_i}^T \ 0 \ 0]^T, C_s = [C \ F_d C_d \ F_{f_j} C_{f_j}], \Delta C_s = [\Delta C \ 0 \ 0], F_{ds} = [F_d D_d \ F_{f_j} D_{f_j}], \text{ and } F_{f_s} = F_{f_i}.$$

The relation between the fault estimation and the disturbance is bounded by:

$$\frac{\|e_{f_i}\|_2}{\|\bar{w}_i\|_2} \leq \gamma_i \iff \begin{cases} \frac{\|e_{f_i}\|_2}{\|\bar{d}\|_2} \leq \gamma_i |W_d|^{-1} \\ \frac{\|e_{f_i}\|_2}{\|\bar{f}_j\|_2} \leq \gamma_i |W_{f_j}|^{-1} \end{cases} \quad (33)$$

$$\text{where } \bar{w}_i = [(\bar{d})^T \ (\bar{f}_j)^T]^T$$

The proposed diagnosis system (32) will be applied to vehicle suspension system in the next section.

## 5. APPLICATION TO VERTICAL CAR SYSTEM

### 5.1 Platform INOVE

In the GIPSA laboratory, Saint-Martin-d'Hres, France, there exists platform INOVE (see Fig. 2), which is a 1:5-scaled racing car including wheels, engine, steering and breaking systems, semi-active suspensions and a system of sensors (for instance position sensors, force sensors, and accelerometers) for observation and control purposes. Ground vibrations (road profiles) are controlled thanks to 4 linear brushless motors applying vertical displacement to each wheel. The system is operated with the sampling time of 5 ms (see Tudón-Martínez et al. (2015)).



Fig. 2. SOBEN car of platform INOVE

### 5.2 Suspension modelling

The quarter-car, or the semi-active suspension in the platform, can be modeled by a mass-spring-damper system (see Fig. 3). In which: the sprung mass  $m_s$  represents a quarter of the chassis body and  $z_s$  is the vertical displacement around the equilibrium point of  $m_s$ ; the sprung mass  $m_{us}$  represents wheel/tire of the vehicle and  $z_{us}$  is the vertical displacement around the equilibrium point of  $m_{us}$ ; the semi-active suspension is composed of a spring with the stiffness coefficient  $k_s$  and a controllable damper with the damping coefficient  $c$ , in which  $c_{min} \leq c \leq c_{max}$ ; the tire is modeled by a spring with the stiffness coefficient  $k_t$ ; and the road profile  $z_r$  is considered as unknown input  $d$  for the suspension.

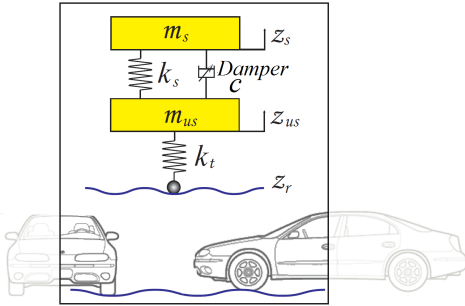


Fig. 3. The quarter-car model

The suspension dynamics are described by following equations (Savaresi et al. (2010)):

$$\begin{cases} m_s \ddot{z}_s &= -k_s(z_s - z_{us}) - F_c \\ m_{us} \ddot{z}_{us} &= k_s(z_s - z_{us}) + F_c - k_t(z_{us} - z_r) \end{cases} \quad (34)$$

where  $F_c = c\dot{z}_{def}$  is the damper force;  $z_{def} = z_s - z_{us}$  is the displacement (deflection) between the chassis and the tire position; and  $\dot{z}_{def}$  is the deflection speed.

In order to obtain LTI model of suspension system, the damper force  $F_c$  is decomposed into 2 components:

$$F_c = c\dot{z}_{def} = c_0\dot{z}_{def} + u \quad (35)$$

In which,  $u$  is the model input corresponding to the varying part of semi-active damper force  $F_c$ ;  $c_0$  is the nominal value of damper, which corresponds to a passive damper when there is no control input  $u$ .

*Remark 4:* In this paper, the linear modelling for suspension damper (35) is applied. Some discussions on nonlin-

earity of the damper can be found in Pletschen and Badur (2014) and Pletschen and Diepold (2017).

According to Do et al. (2011), in case of semi-active suspension, the authors chose  $c_0 = (c_{min} + c_{max})/2$  as the nominal damping value, so the control input  $u$  in (35) is supposed to be limited in symmetric region  $[-u_{max}^*, u_{max}^*]$ , where  $u_{max}^* = (c_{max} - c_{min})\dot{z}_{def}/2$ .

In the application section, 2 available outputs are used in platform INOVE:  $z_{def}$  is the displacement between  $z_s$  and  $z_{us}$ ; and  $\ddot{z}_{us}$  is the tire acceleration. As a result, there may exist the following faults in LTI model (36):  $f_{z_{def}}$  in the displacement sensor  $z_{def}$ ,  $f_{\ddot{z}_{us}}$  in the tire acceleration sensor  $\ddot{z}_{us}$  and  $f_a$  in the damper actuator input  $u$ .

The faulty LTI model has been considered for the study as (Savaresi et al. (2010)):

$$\begin{cases} \dot{x} &= Ax + Bu + E_d d + E_f f \\ y &= Cx + Du + F_d d + F_f f \end{cases} \quad (36)$$

In which,  $x = [z_s \ \dot{z}_s \ z_{us} \ \dot{z}_{us}]^T$  is the state vector;  $y = [z_{def} \ \ddot{z}_{us}]^T$  is the output vector;  $d$  is the road profile  $z_r$  considered as unknown input;  $u$  is the control input;

$f = [f_a^T \ f_{z_{def}}^T \ f_{\ddot{z}_{us}}^T]^T$  is vector of faults;

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_s}{m_s} & -\frac{c_0}{m_s} & \frac{k_s}{m_s} & \frac{c_0}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_{us}} & \frac{c_0}{m_{us}} & -\frac{kt + k_s}{m_{us}} & -\frac{c_0}{m_{us}} \end{bmatrix}; E_d = \begin{bmatrix} 0 & 0 & 0 & \frac{k_t}{m_{us}} \end{bmatrix}^T;$$

$$C = \begin{bmatrix} 1 & 0 & -1 & 0 \\ \frac{k_s}{m_{us}} & \frac{c_0}{m_{us}} & -\frac{kt + k_s}{m_{us}} & -\frac{c_0}{m_{us}} \end{bmatrix}; F_d = \begin{bmatrix} 0 & \frac{k_t}{m_{us}} \end{bmatrix}^T;$$

$$B = \begin{bmatrix} 0 & -\frac{1}{m_s} & 0 & \frac{1}{m_{us}} \end{bmatrix}^T; D = \begin{bmatrix} 0 & \frac{1}{m_{us}} \end{bmatrix}^T;$$

$$[B \ 0_{nx \times ny}]; F_f = [D \ I_{ny}].$$

The parameters of SOBEN car for a corner are presented in the following table:

Table 2. Quarter-car parameters

Parameters	Unit	Value	Description
$m_s$	kg	2.64	A quarter-car chassis mass
$m_{us}$	kg	0.485	Rear tire mass
$k_s$	N/m	1396	Suspension stiffness
$c_{min}$	N.s/m	17.59	Minimum damping coefficient
$c_{max}$	N.s/m	1028	Maximum damping coefficient
$k_t$	N/m	12270	Tire stiffness

### 5.3 System uncertainty

The damping coefficient  $c$  is varying from  $c_{min}$  to  $c_{max}$ , so it has the uncertainty of  $\Delta c_0 = (c_{max} - c_{min})/2$  comparing to the nominal value  $c_0$ .

The effect of uncertainty on system dynamics are presented by the eigenvalues of  $(A + \Delta A)$  with  $\Delta A = 0$  and with both  $c_{min}$  and  $c_{max}$  cases, as mentioned in Fig. 4 which proves an important influence of damping coefficient's variation on the system dynamics. Therefore, the application of robust observer is necessary to deal with this uncertainty.

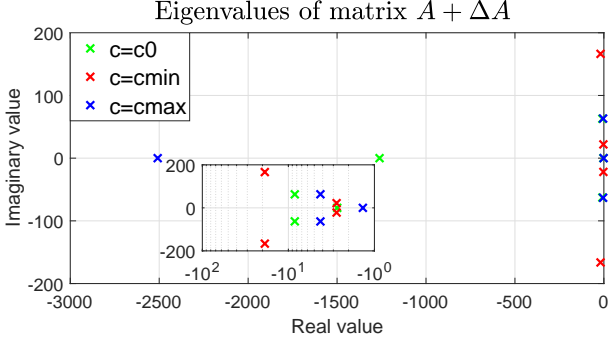


Fig. 4. Dynamics of uncertain system

#### 5.4 Frequency Analysis

Due to space limitation, only the solution to  $f_{z_{def}}$  PI observer, whose disturbances include the road profile  $d$  and the non-estimated faults  $f_a$  and  $f_{z_{us}}$ , is presented.

As previously defined in (30) and (31), the  $\mathcal{H}_\infty$  performance can be improved by using filters (37) and (38), which are based on idea of low-pass filter attenuating input signal since a specific frequency. Therefore, the  $\mathcal{H}_\infty$  norm can be optimized for an active zone of frequency. For example, the filter  $W_d$  allows the fictive road profile  $\bar{d}$  to be attenuated since its cut-off frequency  $w_d$  until it achieves  $20\log(K_d)$  (dB) of attenuation, so the  $\mathcal{H}_\infty$  performance can be optimized in the domain from 0 to  $w_d$  (the same principle for filter  $W_f$ ).

$$W_d = \frac{\frac{s}{\omega_d K_d} + 1}{\frac{s}{\omega_d} + 1} \quad (37)$$

$$W_f = W_{f_a} = W_{f_{z_{us}}} = \frac{\frac{s}{\omega_f K_f} + 1}{\frac{s}{\omega_f} + 1} \quad (38)$$

According to assumption 1, the faults  $f_a$  and  $f_{z_{us}}$  are supposed to exist in low frequency, given the domain from 0 to 2 Hz, and the working frequency of road profile  $d$  is from 0 to 20 Hz (see Savaresi et al. (2010)). As a result,  $w_d = 20$  Hz and  $w_f = 2$  Hz are chosen as the cut-off frequencies of the filter  $W_d$  and filter  $W_f$ , respectively. The gains  $K_d$  and  $K_f$  are chosen in order to obtain sufficient attenuation, particularly in this study:  $K_d = K_f = 40$  (32.04 dB).

*Note:* The dynamics of the PI observer can be modified by the pole placement method. (see Boyd et al. (1994)).

By using Yalmip toolbox (Lofberg (2004)) and Sedumi solver (Sturm (1999)), the optimal  $\mathcal{H}_\infty$  performance of the PI observer is calculated:  $\gamma_{z_{def}} = 0.01$ . The gains of PI observer are presented as followings:

$$L_P = \begin{bmatrix} 0.1027 & -2.2802e-5 \\ 27.9654 & 0,0716 \\ 0.0054 & -1.7345e-5 \\ -150.3090 & -0,3731 \\ -8.9604e-4 & 5.5338e-6 \\ -1.0946e-7 & 2,1441e-10 \\ 2.2390e-8 & 9.7757e-11 \end{bmatrix} \quad (39)$$

$$L_I = [2.4588 \quad -2.1770e-4] \quad (40)$$

The eigenvalues of  $(A_a - L_a C_a)$  are :  $\{-1.74e3; -1.26e2; -8.84 \pm 62.51i; -3.43; -1.9575; -12.57; -12.57\}$  present a

stable dynamic. In other words, matrix  $(A_a - L_a C_a)$  is a Hurwitz matrix as discussed in *Remark 2*.

The frequency analysis is illustrated below to evaluate the sensitivity of the fault estimation error  $e_{f_{z_{def}}}$  to its disturbances. Fig. 5 shows efficient attenuation of the road profile  $d$  (cm) impact on the fault estimation error  $e_{f_{z_{def}}}$ . It has a maximum at 9.85 Hz, which emphasizes the worst case of  $-65.7$  dB in its frequency range. In other words, it reflects an impact of 0.06 cm on the fault estimation error if road profile amplitude is 1 cm. The influence of the actuator fault  $f_a$  on fault estimation error decreases in the range of 0 to 2 Hz, with great attenuation less than  $-100$  dB (see Fig. 6). With the magnitude less than  $-110$  dB, bode diagram in Fig. 7 shows that the  $f_{z_{us}}$  fault impact is significantly attenuated. The higher the frequency, the less fault impact on the estimation error. In general, all above sensibilities satisfy the condition (33).

In Fig. 8,  $|T_{f_{z_{def}} f_{z_{def}}}| = 0$  from 0 to 0.05 Hz, so the fault  $f_{z_{def}}$  can be well estimated if its bandwidth is less than 0.05 Hz.

#### 5.5 Validation condition

The open-loop validation process is done during 30 seconds.

**Road profile  $d$ :** is modeled as a smooth highway (road profile type B) (see Fig. 9), according to International

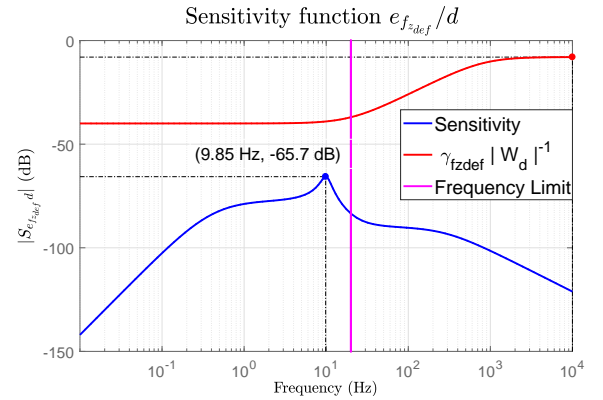


Fig. 5. Sensitivity function  $e_{f_{z_{def}}}/d$

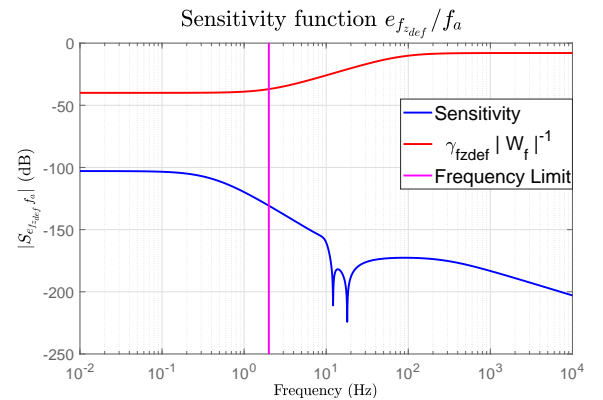


Fig. 6. Sensitivity function  $e_{f_{z_{def}}}/f_a$



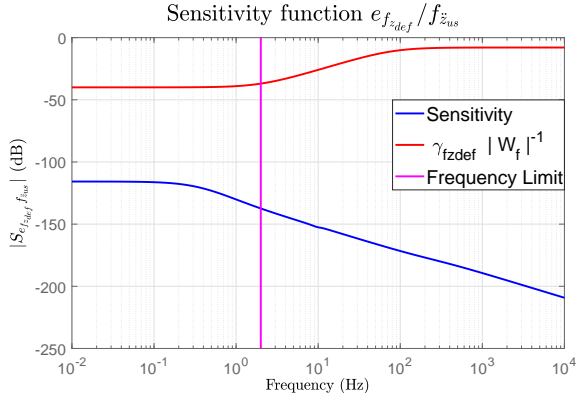


Fig. 7. Sensitivity function  $e_{f_{z_{def}}}/f_{z_{us}}$

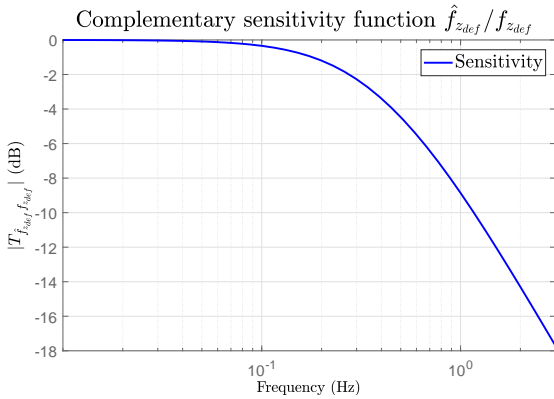


Fig. 8. Complementary sensitivity function  $\hat{f}_{z_{def}}/f_{z_{def}}$

Organization for Standardization (ISO) 8608 classification (see ISO (1995), and Tudón-Martínez et al. (2015)).

**Fault Scenario:** Abrupt faults (stepwise) have been considered in the test. In terms of the sensor faults  $f_{z_{def}}$  and  $f_{z_{us}}$ , step up signals are applied in order to model the instant offsets.

$$f_{z_{def}} = \begin{cases} 0.03 & \text{if } (1s \leq t \leq 7s) \\ 0 & \text{otherwise} \end{cases} \quad (m) \quad (41)$$

$$f_{z_{us}} = \begin{cases} 2 & \text{if } (11s \leq t \leq 18s) \\ 0 & \text{otherwise} \end{cases} \quad (ms^{-2}) \quad (42)$$

In practice, the damper force  $F_c$  of semi-active suspension is controlled by duty cycle  $f_{pwm}$ , which is expressed as (Do et al. (2010)):

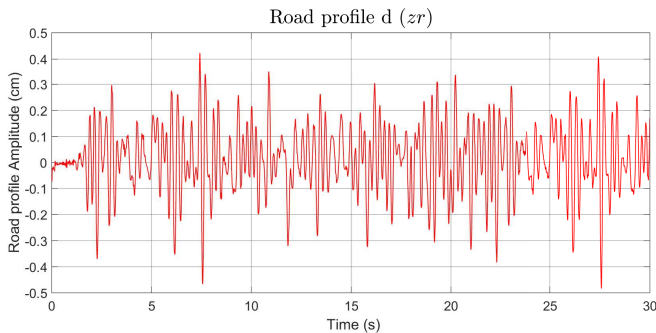


Fig. 9. Road profile: smooth highway

$$F_c = c_1 \dot{z}_{def} + k_1 z_{def} + G_c f_{pwm} \tanh(c_2 \dot{z}_{def} + k_2 z_{def}) \quad (43)$$

$$= c_0 \dot{z}_{def} + u \quad (44)$$

where the parameters:  $G_c = 24.95[Ns/m]$ ,  $c_1 = 20.78[Ns/m]$ ,  $k_1 = 186.33[N/m]$ ,  $c_2 = 21.45[Ns/m]$ ,  $k_2 = 13.30[N/m]$ , and  $0.1 \leq f_{pwm} \leq 0.35$ .

When actuator fault exists, i.e  $F_c = (c_0)\dot{z}_{def} + u + f_a$ , the relation between  $f_a$  and non-faulty duty cycle  $f_{pwm}$  is adjusted by a gain  $\kappa$ :

$$G_c \kappa f_{pwm} \tanh(c_2 \dot{z}_{def} + k_2 z_{def}) = (c_0)\dot{z}_{def} + u + f_a - c_1 \dot{z}_{def} - k_1 z_{def} \quad (45)$$

As a result, the modification of  $\kappa$  allows the simulation of actuator fault  $f_a$ , which is defined as:

$$f_a = \begin{cases} -0.6 & \text{if } (22s \leq t \leq 27s) \\ 0 & \text{otherwise} \end{cases} \quad (N) \quad (46)$$

## 5.6 Experimental result

The fault estimation of  $f_{z_{def}}$  is presented in Fig. 10. With the road profile disturbance, Fig. 10 shows that the displacement fault  $f_{z_{def}}$  is well estimated with the rising time about 2 seconds. Moreover, the coupling problem with other faults has negligible effects on fault estimation thanks to  $\mathcal{H}_\infty$  application. The overshoot in fault estimation can be modified by changing the dynamics of the PI observer.

## 6. CONCLUSION

By applying the majoration lemma to the  $\mathcal{H}_\infty$  PI observer, robust fault estimation problems have been structurally formulated for LMI optimization whose solution can be easily obtained by LMI solvers. Moreover, through frequency analysis and the real application to the semi-active suspension system, the proposed observer design has proven its capacity to efficiently attenuate the disturbances and deal with the system uncertainties, while estimating the faults with satisfactory convergence properties. For future work, a far-reaching solution can be offered to linear parameter-varying system, where the same idea of boundedness can be applied. In addition, the fault estimation results can provide useful information for fault tolerant control application.

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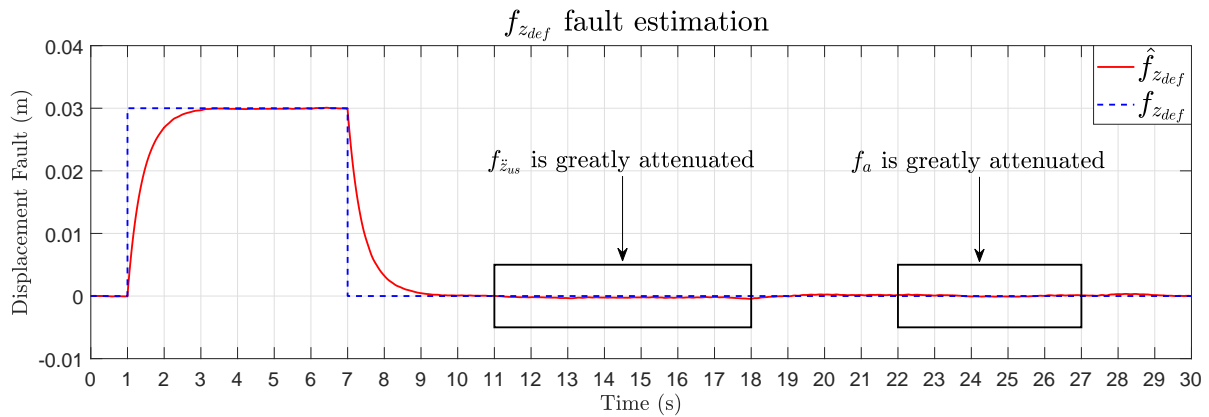


Fig. 10. Fault estimation result from 0 to 30 seconds

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