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Kriging-based spatial interpolation from measurements for sound level mapping in urban areas

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Abstract

Network-based sound monitoring systems are deployed in various cities over the world and mobile applications allowing participatory sensing are now common. Nevertheless, the sparseness of the collected measurements, either in space or in time, complicates the production of sound maps. This paper describes the results of a measurement campaign that has been conducted in order to test different spatial interpolation strategies for producing sound maps. Mobile measurements have been performed while walking multiple times in every street of the XIIIth district of Paris. By adaptively constructing a noise map on the basis of these measurements, the role of the density of observations and the performance of four different interpolation strategies is investigated. Ordinary and universal Kriging methods are assessed, as well as the effect of using an alternative definition of the distance between observation locations, which takes the topology of the road network into account. The results show that a high density of observation points is necessary to obtain an interpolated sound map close to the reference map.

Keywords: Kriging; Spatial interpolation; Sound map; Opportunistic measurements

I. Introduction

In this work, we investigated the use of a large amount of in situ measurements and the interpolation of these to construct a sound level map.

The Directive 2002/CE/49 contributed to the development and harmonization of noise prediction models (EC 2002). For making urban sound maps, model-based numerical engineering methods are currently widely used and these methods provide a good compromise between accuracy and computation time (Kephalopoulos et al. 2014). Nevertheless, they have many limitations, and the resulting sound maps neglect the diversity of urban sound environments in terms of both sound sources and sound environment dynamics.

Sound maps based on measurements can help to improve sound mapping (Zambon et al. 2017; Asensio 2017; Hong and Jeon 2017; Harman, Koseoglu, and Yigit 2016). The recent development of small and autonomous acoustic sensors contributes to this movement, and network-based sound monitoring systems are being deployed in an increasing number of cities over the world, using either high-quality or low-cost microphones (Mydlarz, Salamon, and Bello 2017; Asensio 2017; Sevillano et al. 2016). Also, smartphone applications, allowing participatory sensing, are now common, which multiplies the amount of available

40 data to potentially map the sound environment of a city based on measurements (Aspuru et al. 2016;
41 Guillaume et al. 2016; Maisonneuve et al. 2009; Issarny et al. 2016).

42 Nevertheless, the production of sound maps based on measurements is complicated by metrological
43 issues inherent to the typical microphones used in consumer electronics, the time sparseness of the
44 measurements collected through mobile monitoring applications, and the space sparseness of the
45 measurements collected through fixed sound monitoring networks. Therefore, it is fundamental to know
46 the time and space representativeness of such measurements, as this knowledge is required to be able to
47 propose relevant interpolation methods that can be used to produce sound maps that cover the full
48 temporal and spatial extent of the study area.

49 The temporal structure of urban sound levels (highly correlated day or week patterns, seasonal trends)
50 can be exploited to restrict the number of sampled days, or to rely only on measurements performed at
51 selected periods of the day, in order to estimate Lden values or Daily Average Noise Patterns (Hong and
52 Jeon 2017; Geraghty and O'Mahony 2016; J. M. Barrigón Morillas and Prieto Gajardo 2014; Zuo et al.
53 2014). Previous studies also revealed that a 10 or 15-minute measurement is representative of a 1h-period
54 in an urban context, as the majority of the 10 or 15 minute-measurements are in the same sound level
55 range during homogeneous periods (Brocolini et al. 2013; Prieto Gajardo and Barrigón Morillas 2015).
56 Other studies that rely on short-term recording methods proved the relevance of 15-minute sampling
57 periods (Morillas et al. 2005; Arnaud Can et al. 2011). Even shorter measurement periods can be found in
58 the literature, especially in those cases where the opportunistic measurement context offered by
59 smartphone applications is used. In this case, the short measurement duration is compensated by the large
60 number of measurements (Guillaume et al. 2016), shifting the focus from the duration of each
61 measurement episode to the number of sampling episodes, as recommended in (Mateus, Dias Carrilho,
62 and Gameiro da Silva 2015).

63 Zuo et al. (Zuo et al. 2014) showed that the sound level variability in urban environments can be
64 explained for a large part by the spatial characteristics of the environment. Also, the space
65 representativeness and the spatial interpolation of the measurements is an important issue when computing
66 sound maps based on measurements. Maps interpolated from the data obtained through fixed sound
67 measurement stations have recently been produced (Liu et al. 2013; Harman, Koseoglu, and Yigit 2016;
68 Segura Garcia et al. 2016; Huang et al. 2017), and form a useful tool to estimate the noisiness of a
69 neighborhood or to give a global overview of the city sound levels. However, the large distance between
70 measurement stations often does not allow one to map sound levels in each street, which is offered by
71 maps calculated using model-based numerical methods. A study by Can et al. (A. Can, Dekoninck, and
72 Botteldooren 2014) that involved mobile measurements performed using sound level meters attached to
73 backpacks permitted one to compare an interpolated map with a reference map, but only for a very small
74 area (four streets). More studies are therefore needed to investigate the density of measurements that is
75 required to have an acceptable accuracy at the street resolution.

76 Another parameter to take into account is the method of interpolation. Several methods have been
77 tested for urban sound level interpolation: Inverse Distance Weighting (IDW) methods (A. Can,
78 Dekoninck, and Botteldooren 2014; Harman, Koseoglu, and Yigit 2016; Segura Garcia et al. 2016; Hong
79 and Jeon 2017), Kriging methods (Harman, Koseoglu, and Yigit 2016; A. Can, Dekoninck, and
80 Botteldooren 2014; Segura Garcia et al. 2016) and multiquadratic interpolation (Harman, Koseoglu, and
81 Yigit 2016). Alternative interpolation methods that involve a modified definition of the distance between

82 measurement locations, in order to account for the city geometry or the road network, have recently been
83 proposed (A. Can, Dekoninck, and Botteldooren 2014; López-Quílez and Muñoz 2009; Hachem et al.
84 2015). Nevertheless, these interpolation methods have only been tested on small measurement samples,
85 and larger studies are needed to validate the conclusions of these works. It is worth mentioning that the
86 density of measurements appears to be more important than the method of interpolation (Harman,
87 Koseoglu, and Yigit 2016).

88 Recently, a number of data fusion techniques have been proposed to correct model-based numerically
89 computed sound maps with measurements (Hachem et al. 2015; Wei et al. 2016; Zambon et al. 2017;
90 Ventura et al. 2017). These techniques are promising, but require a pre-calculated sound map, which can
91 be expensive, and give priority to the potential indicators available in this sound map. Most of the existing
92 sound maps involve the energy-equivalent sound level, whereas in situ measurements allow calculating a
93 wide range of acoustical indicators, which may include information about the temporal dynamics of the
94 sound environment; for example, percentile or emergence indicators can also be interesting to interpolate.
95 The spatial interpolation methods can even be based on perceptual assessments (Aletta and Kang 2015).

96 In this study, a large measurement campaign has been conducted in the XIIIth district of Paris, with the
97 goal to test different spatial interpolation strategies. Mobile measurements have been performed with
98 sound measurement stations attached to backpacks that were carried by researchers when walking in every
99 street of the district between 1 and 15 times. The measurements are aggregated over a grid of locations in
100 the study area, and are used to compute a reference map of the district. An analysis of the sensitivity of the
101 sound level values with respect to the radius of the integration and the number of measurements is done
102 via a bootstrap method. From the reference sound map, four Kriging methods for interpolation between a
103 set of measurement locations are tested, based on a combination of two strategies: (i) Ordinary Kriging
104 and universal Kriging which consists of adding a linear trend, defined from the distance between an
105 observation location of the domain and its closest categorized road, and (ii) a variation of the distance
106 definition between observation locations, which can be Euclidian or computed from the road network to
107 take into account the influence of the city geometry. By progressively decreasing the number of
108 observation locations, the impact of the density of observation locations and the performance of different
109 spatial interpolation methods is investigated.

110 II. Method

111 A. Study area

112 Figure 1 presents the study area, which corresponds to the XIIIth District of Paris. This district includes
113 a large variety of urban sound environments: large avenues with high traffic density, lively streets with
114 bars and restaurants, schools, small and large parks, quiet streets. The size of the study area is
115 approximately 2.8 km² with a maximum extent of 2 km west to east and a maximum extent of 1.7 km
116 north to south.

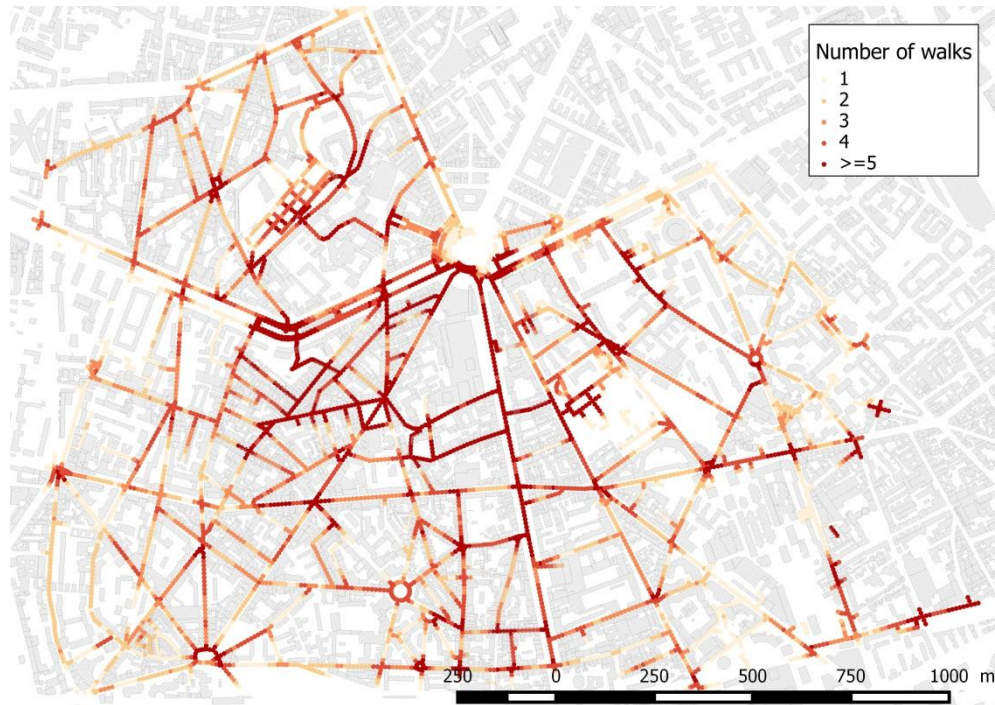
117 B. Measurement set-up

118 All measurements were carried out using dedicated mobile sound measurement stations developed by
119 ASAsense (De Coensel et al. 2015). These stations record the instantaneous 1/3-octave band spectrum
120 with a 125-ms temporal resolution as well as the instantaneous GPS position with a 1-s temporal

121 resolution. Both sound and GPS data are synchronized, such that the spatio-temporal evolution of the
122 sound spectrum during each measurement session can be reconstructed afterwards. In order to fully
123 capture the characteristics of the sound environment, a large set of indicators is calculated on the basis of
124 this 1/3-octave band spectrum data, including the A-weighted energy-equivalent sound level at each
125 second, $L_{eq,1s}$, which is used throughout this study.

126 C. *Mobile measurements*

127 The sound measurement stations were mounted in backpacks with power provided by a battery pack,
128 and, subsequently, mobile measurements were carried out between October 22th 2014 and May 26th 2015.
129 Five operators participated in the measurements. In order to minimize the variation between measurement
130 sessions and to be able to calculate sound levels that are representative for homogeneous sound
131 environments, measurements were only carried out on weekdays, either between 10 a.m. and 12 a.m., or
132 between 2 p.m. and 4 p.m. As shown in (Prieto Gajardo and Barrigón Morillas 2015; Zuo et al. 2014;
133 Brocolini et al. 2013), these periods, which exclude rush hour traffic and lunch times, provide a similar
134 sound environment. Depending on the variability of the sound environment, the number of walks in each
135 street was varied. As shown in (Prieto Gajardo and Barrigón Morillas 2015), the sound environment of a
136 calm street is more sensitive to single sound events than that of a large boulevard, thus a higher number of
137 measurements is required to calculate a sound level value that is representative of the sound environment
138 of a calm street, as compared to a large boulevard. After each day of measurements, the variance of the
139 sound level was computed for each street, providing feedback on those streets that would benefit from
140 more measurements to get a stable estimate. The number of passages per street ranged from 1 to 15 times,
141 with an average of 4 and a standard deviation of 2.7 passages. Figure 1 shows the number of walks (only
142 validated measurements were kept to plot this figure).



144

145

Figure 1 (color online) Number of passages at each location (only validated measurements).

146 D. Map matching

147 A GPS track is associated with each measurement session. However, the accuracy of the GPS data
 148 depends on many factors such as the quality of the GPS receiver, the characteristics of the surroundings
 149 such as the presence of high buildings, or the weather. In this study, the median standard deviation
 150 associated with the GPS locations was about 10 meters. Although rarely considered in studies dealing with
 151 geo-referenced mobile measurements, this can be problematic for the present analysis, because some
 152 measurements can be associated with an erroneous street.

153 Therefore, it was necessary to preprocess the GPS data, mapping each measurement to a location on
 154 the road, a problem commonly known as map-matching. For this study, a point-to-point method was
 155 developed, on the basis of the following conditions: (i) all measurements were performed on sidewalks or
 156 on roads, and the sound levels measured on both sides of each street are considered to be equivalent and
 157 are snapped to the middle of the street; (ii) all GPS locations are snapped to the center of the closest street
 158 under the conditions that the operator walked with a maximum speed of 5 km/h and that the map-matched
 159 point conserves the same direction of displacement (with a direction tolerance of 60°) as the original
 160 point; and (iii) the map-matched points are located at a maximum distance from the original GPS points
 161 equal to twice the standard deviation of the GPS tracker.

162 E. From mobile measurements to observation locations

163 In a previous study (Aumond et al. 2017), it was shown that the median sound level L_{50} is well
 164 correlated with the perceived loudness of an urban sound environment. The median sound level presents

165 the advantage that it is less sensitive to peaks in the measurement than the energy-equivalent level.
166 Exceptionally, peaks can be generated by the operators themselves, or can occur when extremely noisy
167 vehicles pass by in the vicinity of the operators. In addition, the median sound level L_{50} does not include
168 A-weighting, which is known to reduce too much the influence of low frequencies [63-500 Hz] at sound
169 levels encountered in urban environments, and thus the influence of road traffic sound on overall
170 perceived loudness. The first step in carrying out the spatial interpolation between measurement locations,
171 was to aggregate, at each location, all mobile measurements that are within a radius r . This aggregation
172 step is performed using the median sound level of all the 1-s values $L_{eq,1s}$ considered as independent
173 observations. The associated value is assumed to be representative of the L_{50} sound level at the
174 observation location on weekdays and during the measurement periods [10-12h; 14-16h]. Every
175 aggregated observation location is situated on the road network, because mobile measurements were only
176 taken on sidewalks and pedestrian walkways in the public space. The road network that was used for map-
177 matching was based on OpenStreetMap (“OpenStreetMap” n.d.).

178 *F. Variogram and Kriging*

179 *1. Kriging method*

180 Ordinary Kriging method is a well-known interpolation method that has been used in various
181 applications, especially in environmental applications. It also bears similarities with classical data
182 assimilation methods that have helped environmental forecasting, including at urban scale for air pollution
183 (Tilloy et al. 2013) and noise pollution (Ventura et al. 2017; Harman, Koseoglu, and Yigit 2016). The
184 approach is likely to succeed when some meaningful function can fit the empirical variogram, which is the
185 case in this study. Nonetheless, at urban scale, the sound levels exhibit a special distribution due to the city
186 geometry, and this is a difference with typical applications of Kriging (López-Quílez and Muñoz 2009).
187 This is the reason why, in the construction of the variogram, we made use of the distance along the road
188 network instead of a raw Euclidean distance (see Section II.F.3). Also, since it is known that sound levels
189 can be approximated from external data (Juan Miguel Barrigón Morillas et al. 2011) and can provide some
190 sort of prior for the spatial distribution of the sound levels, we also tested universal Kriging, which carries
191 out the interpolation of the measurements on top of a prior sound level map (see Section II.F.4).

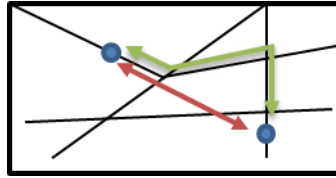
192 *2. Implementations and parameters*

193 The variogram and Kriging algorithms presented in this study are applied using the functions `variog`
194 (computation of the variogram), `variofit` (best fit of the variogram) and `krige.conv` (Kriging function) of
195 the packages `GeoR` (“GeoR: Analysis of Geostatistical Data Version 1.7-5.2 from CRAN” n.d.) and
196 `GeoRcb` (López-Quílez and Muñoz 2009). The empirical variogram is computed over a distance of 1000
197 meters. The classical estimator is chosen to compute the empirical variogram as defined in (Cressie 2015).
198 The Matérn covariance model, as defined in (Diggle and Ribeiro 2007), is used to compute the best fit of
199 the experimental variogram with a fixed value for the shape parameter $\kappa=0.5$ and ϕ the value of the range
200 parameter to estimate.

201 *3. Euclidian vs cost-based distance*

202 The package `GeoR` has been used to interpolate using the Euclidian distance (EUCL). The package
203 `GeoRcb` permits the use of alternative definitions of distance. In this study, the distance between two
204 observation locations has also been defined along the road network as presented in Figure 2 The error
205 correlations in the traffic flow are assumed to be better modeled as a function of the distance along the

206 road than the Euclidean distance. As a consequence, the use of the distance along the road network
207 presumably allows one to better model errors that come from the traffic. The distance between all the
208 possible observation locations of the domain has been calculated with the Johnson's algorithm of the
209 “distances” function of the package “igraph” of the R software described in (West 2001).



210

211 Figure 2(color online) Schema of the road network (shown in black), with the Euclidian Distance (shown in red) as well as the Road Network
212 Distance (shown in green).

213

214 4. Ordinary and universal Kriging

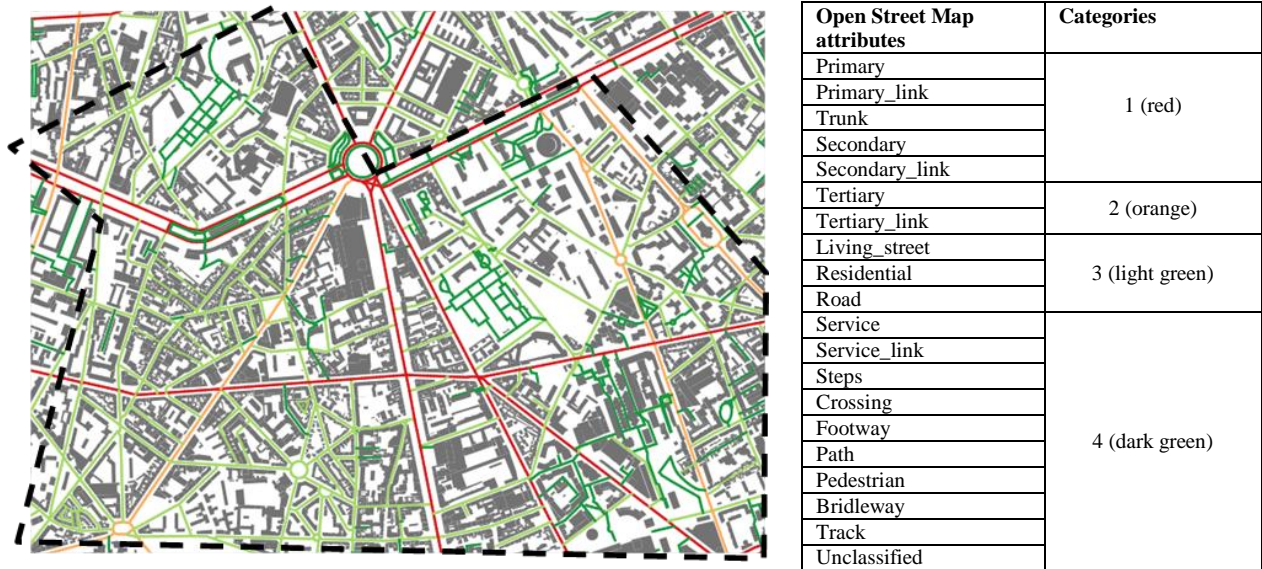
215 Two Kriging methods are compared: Ordinary Kriging (OK) and Universal Kriging (UK). Universal
216 Kriging is a variant of the ordinary Kriging operation that includes a linear trend. Barrigon et al. showed
217 that urban sound is strongly stratified (Morillas et al. 2005). Based on a similar statement, the linear trend
218 T in this study is defined as a linear regression based on 4 explanatory variables which are the distances D_i
219 between the observation locations of the study area and its closest roads belonging to 4 categories i .
220 Equation 1 presents the equation of the trend:

$$221 T \sim a.D_1 + b.D_2 + c.D_3 + d.D_4 + e$$

222 in which a , b , c , d and e are adjusted constants.

223 Road categories have been defined based on the OpenStreetMap attributes as shown in Table 1. Figure
224 3 presents a map of the study area, in which the roads are shown in different colors according to their
225 category. The first three categories correspond to streets with vehicular traffic, the last one to pedestrian
226 streets.

227



228 Figure 3 (color online) Outline of the study area (black dashed line) and, in color, the 4 roads categories.

229 *G. Performance metrics*

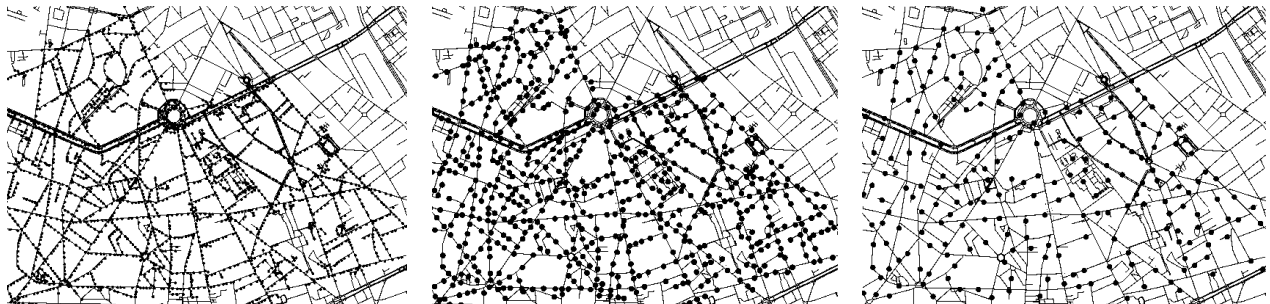
230 The performance of the interpolation methods is assessed with two indicators: the Root Mean Square
 231 Error (RMSE) and the Pearson correlation coefficient (r) between the interpolated and the reference map.
 232 The geospatial interpolations are proposed following four methods: (a) ordinary Kriging (OK + EUCL),
 233 (b) ordinary Kriging using the distance along the road network (OK + ROAD), (c) universal Kriging (UK
 234 + EUCL), (d) universal Kriging using the distance along the road network (UK + ROAD).

235 **III. Results**

236 *A. Reference sound map*

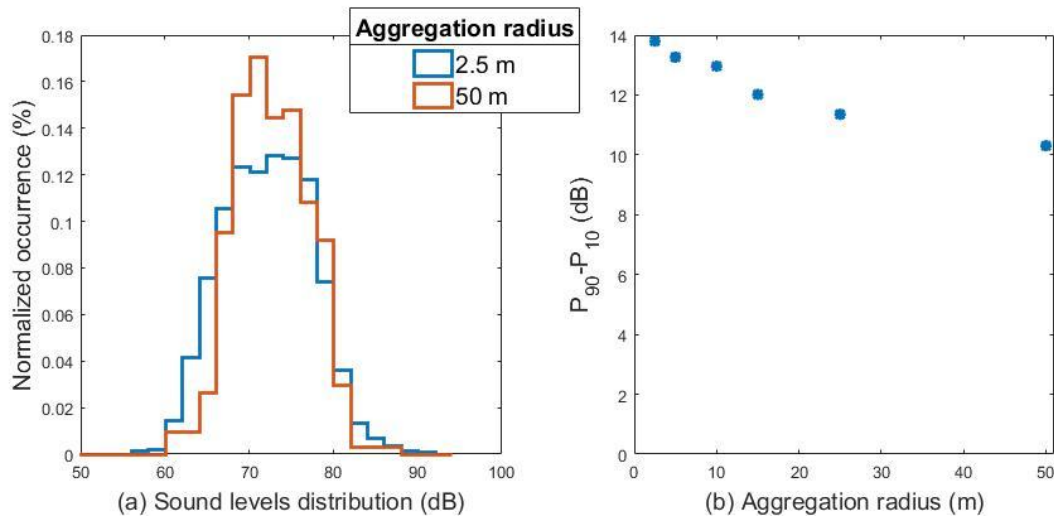
237 A reference sound map based on all mobile measurements is first computed. For each observation
 238 location, the mobile measurements that are within a specific integration radius are mapped to this location.
 239 A sensitivity analysis of the spatial representativeness and the expected accuracy of the sound level at the
 240 aggregation radius and the aggregated number of 1-s samples is then performed.

241 Six values for the aggregation radius have been tested: 2.5 m, 5 m, 10 m, 15 m, 25 m and 50 m. For
 242 each radius value, a set of observation locations has been selected, uniformly distributed. For this
 243 statistical analysis, the observation locations are spaced apart by at least two times the radius value, to
 244 avoid redundancy. In Figure 4, three of those subsets of observation locations are shown.



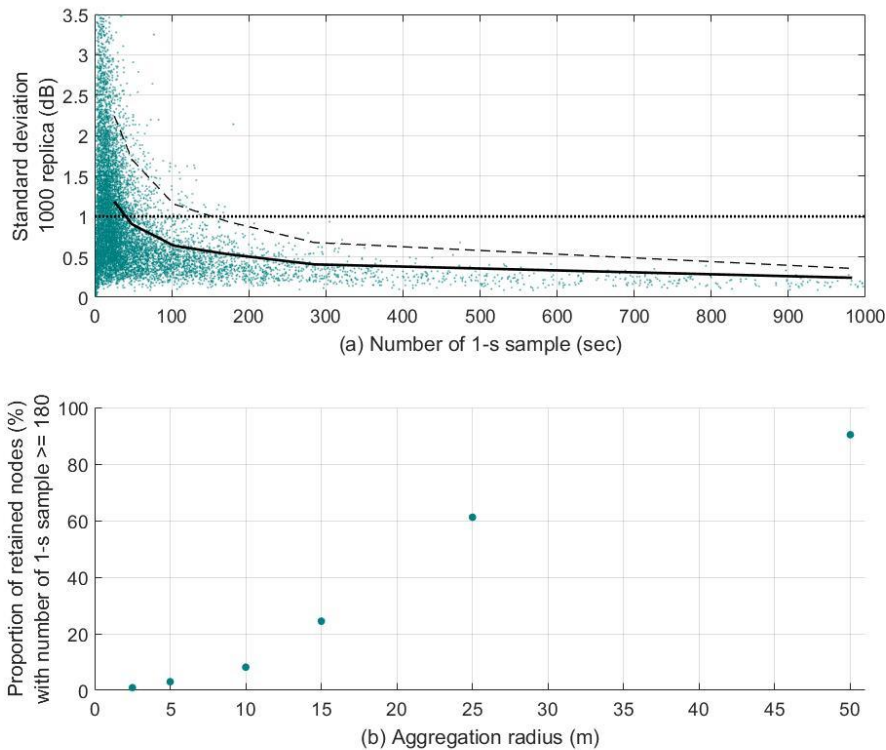
245 (a) (b) (c)
 246 Figure 4 Example of three selected sets of observation locations (dots) for radii: (a) 2.5m, (b) 15 m and (c) 50m. Observation locations are located on the study area road network (solid lines).

247 Figure 5(a) shows the distribution of the aggregated sound levels L_{50} over the study area for the
 248 smallest and largest aggregation radii (2.5m and 50m) as presented in Section II.E. Figure 5 (b) shows the
 249 $P_{90}-P_{10}$ indicator (where P_{10} and P_{90} are the percentiles 10 and 90 of the sound level distributions), which
 250 reflects the width of these distributions for all the studied aggregation radii. As expected, the range of the
 251 aggregated sound levels L_{50} over the study area decreases as the value of the radius increases. More
 252 prosaically, the reference sound map will appear to be blurred. A small radius will give rise to a more
 253 detailed reference sound map, but comes with a decreased number of measurements at each location that
 254 may therefore no longer be representative. The results show that the influence of the aggregation radius
 255 over the sound level distribution is relatively small, with $P_{90}-P_{10}$ decreasing from 13.5 dB to 10.5 dB.



256 (a) Sound levels distribution (dB) (b) Aggregation radius (m)
 257 Figure 5 (color online) (a) Distribution of the aggregated sound levels L_{50} over the study area for two values of the aggregation radius. (b) Width
 258 of the distribution of the sound levels ($P_{90}-P_{10}$) for 6 different aggregation radii
 259

260 A bootstrapping method (bootstrp function, statistical toolbox, Matlab) is proposed to analyze the
 261 sensitivity of the L_{50} value to the aggregation radius and the number of measurements. This method relies
 262 on random sampling, with replacement, of the 1-s measurements for each location within the study area
 263 (Efron 1979). Multiple replications of the method permits the computation of the variance associated with
 264 the average L_{50} value at each observation location due to the sample characteristics. Figure 6 presents (a)
 265 the standard deviation of 1000 bootstrap replications of the calculated L_{50} varying with the number of 1-s
 266 samples and (b) the relationship between the aggregation radius and the proportion of retained locations,
 267 considering a minimum number of 1-second samples of 180. The correlation between these two
 268 parameters is statistically significant ($r=0.53$, $p<.05$), but a large aggregation radius does not always imply
 269 a large number of measurements (e.g., along the borders of the study area) and vice versa (e.g., if the
 270 operator measured a few minutes at one specific location).



271

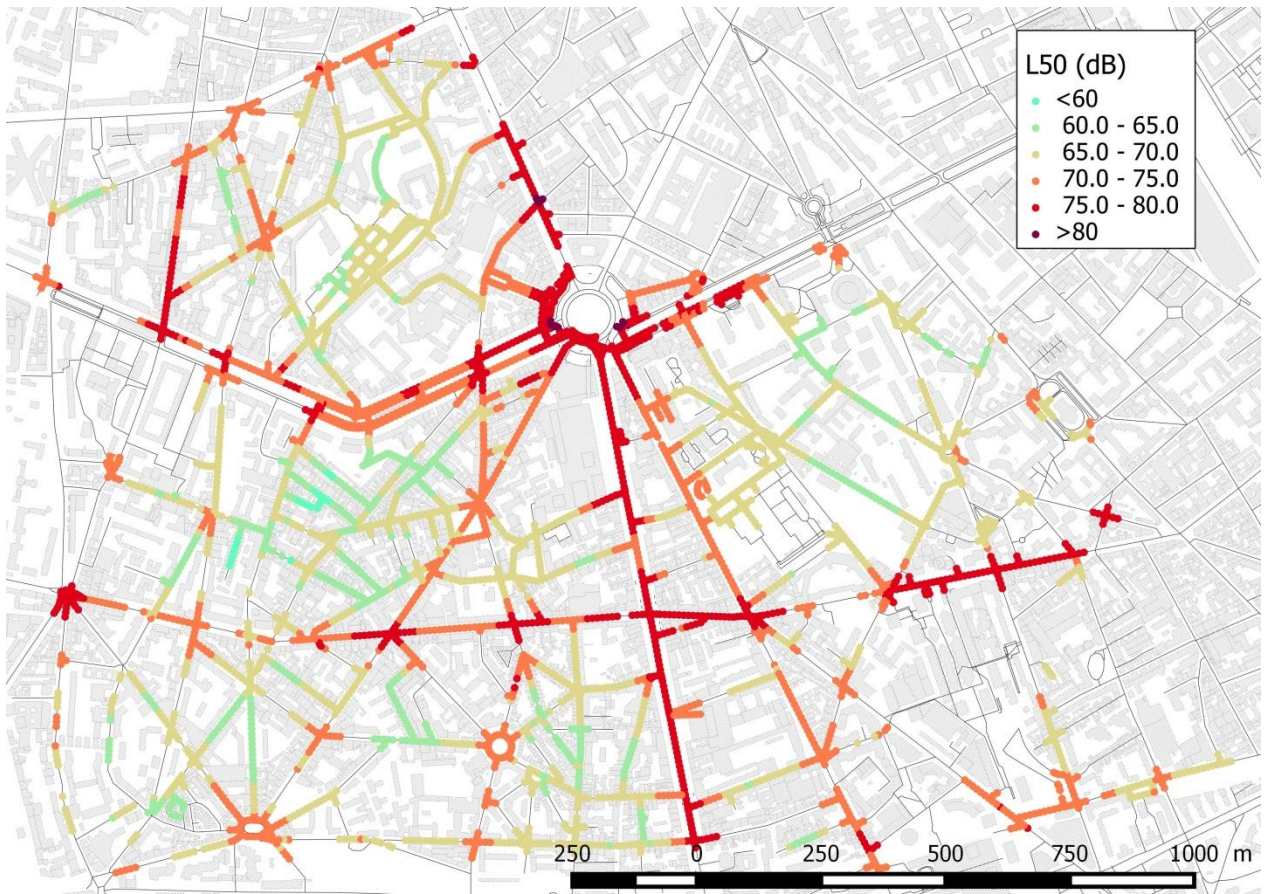
272 Figure 6 (color online) The upper plot shows the relationship between the standard deviation of 1000 replications (subsets) of the underlying
 273 reference data set and the number of 1-s samples of the subsets. The lower plot shows the relationship between the aggregation radius and the
 274 proportion of retained measurement locations, considering a minimum number of 1-second samples of 180. Each dot represents a replication, the
 275 solid line denotes the mean value, the dashed line denotes the percentile-90.

276

277 The variation specific to each location is found to be considerably smaller than the global variation
 278 between the locations over the whole study area, which is about 5 dB (see Figure 5). If we consider a
 279 standard deviation smaller than 1 dB as acceptable, 90% of the calculated standard deviations are below
 280 this threshold when a sample is composed by a minimum of 180 1-s measurements. The chosen threshold
 281 also guarantees to have enough data to carry out the spatial analysis. An aggregation radius of 25 meters is
 282 chosen as the threshold value because it permits one to retain more than 60% of the computed locations.
 283 This aggregation radius also has the advantage that it corresponds to the longitudinal spatial
 284 representativeness of sound level measurements found in literature, perceptually (Brocolini et al. 2009) or
 285 physically (A. Can, Dekoninck, and Botteldooren 2014), thus suggesting that the resulting sound level
 286 map will be consistent with observed spatio-temporal variations.

287 On the basis of the above results, it can be concluded that, at least for the urban study area considered,
 288 three-minute measurements provide sufficient confidence in the aggregated measurement value, even if its
 289 representativeness is not guaranteed. As the purpose of the present study was to perform a comparison of
 290 interpolation techniques, the calculated sound maps only need to respect the spatial variation of the sound
 291 level, but do not necessarily need to be representative of a homogeneous time period.

292 Figure 7 presents the resulting reference map of the median sound level (L_{50} , dB). For the calculation
293 of this map, a distance of 10 m between each observation location was used, resulting in 4360 locations.



294

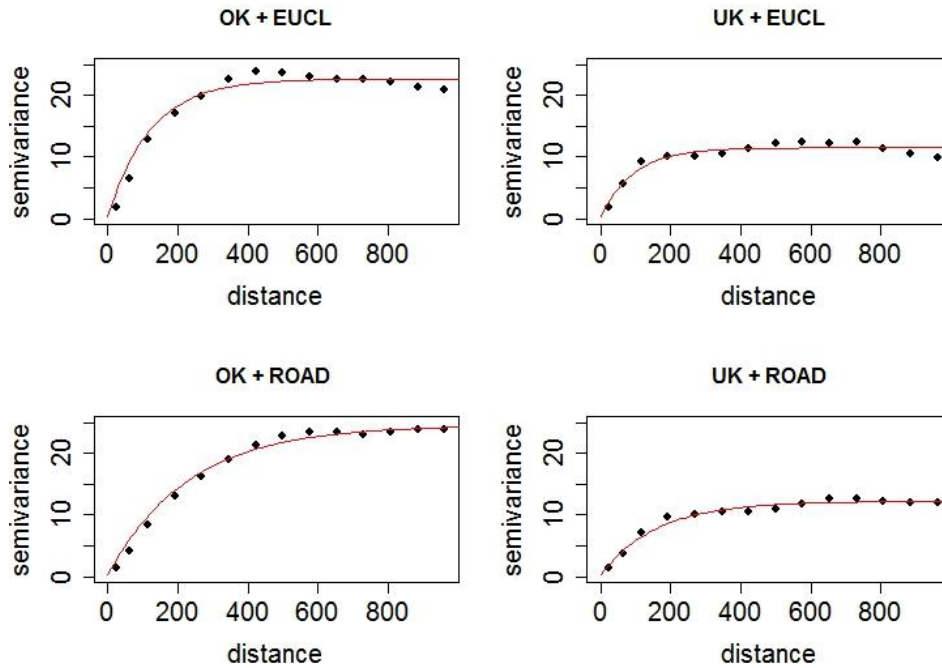
295

Figure 7 (color online) Reference sound map, integration radius 25 meters, integration time > 180 sec. (n= 4360)

296 *B. Spatial dependence of the data*

297 The spatial dependence of the data is highlighted through the calculation of variograms, which express
298 the semivariance between L_{50} values for a couple of locations according to their distance. On Figure 8,
299 four fitted variograms derived from the reference sound map are presented: (a) an ordinary variogram with
300 the Euclidean distance (OK+EUCL), (b) an ordinary variogram using the distance along the road network
301 (OK+ROAD) (c) an universal variogram which accounts for the trend with the Euclidean distance
302 (UK+EUCL), (d) an universal variogram which accounts for the trend and also uses the distance along the
303 road network (UK+ROAD).

304



305

306
307
308

Figure 8 Empirical variograms (dots) and best fitted parametrical models (red line) computed using the ordinary Kriging (OK) and universal Kriging (UK) methods. The printed distance is computed using the Euclidian distance (EUCL) or the distance along the road network (ROAD) (distance in meters, and semivariance in dB²).

309

310 The parameters of the best fitted covariance models are presented in Table 1. The practical ranges of
 311 the variograms, defined as the value for which the correlation function decays to 5% of its value at 0, are
 312 (a) 366 m for OK+EUCL (b) 285 m for UK+EUCL (c) 691 m for OK+ ROAD (d) 481 m for UK+ROAD.
 313 For OK+EUCL, hardly any information is given by an observation to an estimated value located in a
 314 radius superior to 366 m. As the nugget variance τ^2 is null, the variogram asymptote, or sill value,
 315 corresponds to the signal variance σ^2 . This value at 1000 meters is 22.4 dB² and 24.2 dB² for OK + EUCL
 316 and OK + ROAD methods, and 11.2 dB² and 11.9 dB² for UK + EUCL and UK + ROAD. Thus, adding
 317 the trend permits one to considerably reduce the signal variance and illustrates the strong correlation
 318 between the urban sound levels and the proximity to different types of roads. Also, the practical range of
 319 the variograms increases when the alternative definition of distance from the road network is used (from
 320 366 to 691 m and from 285 to 481 m).

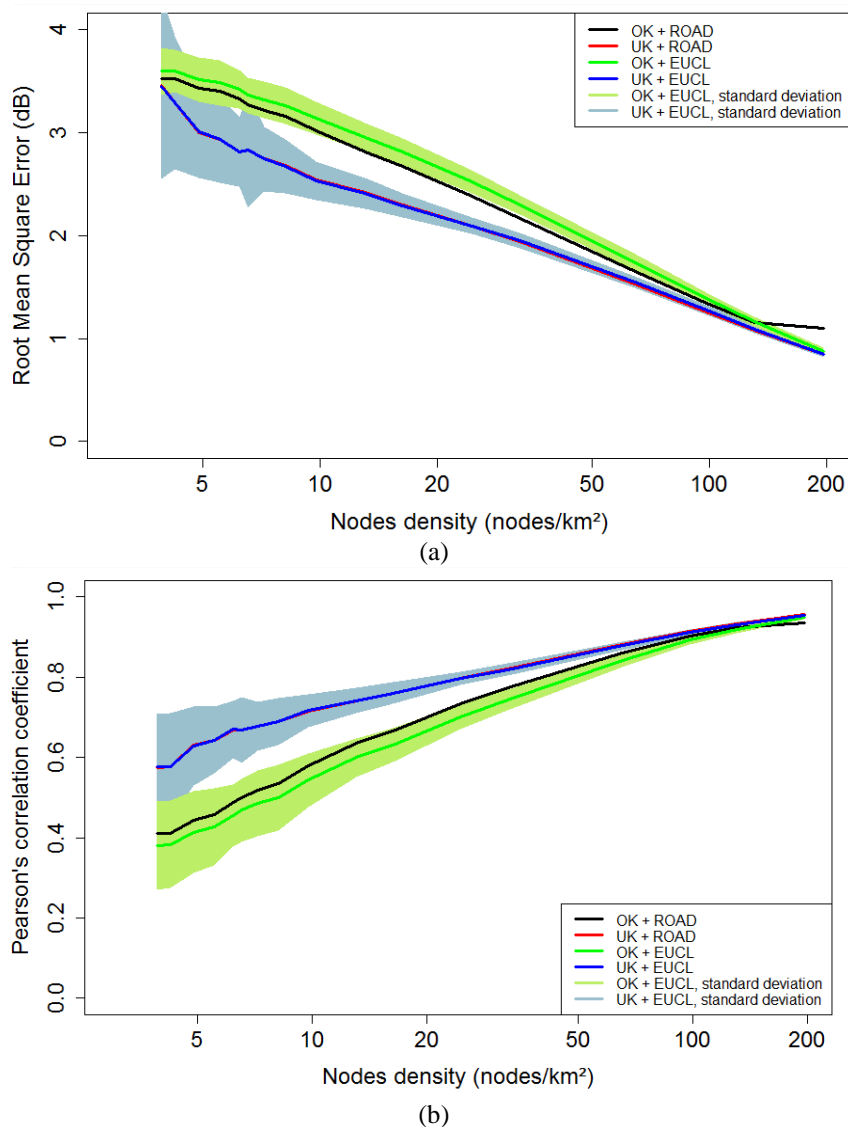
321

Table 1 – Parameters of the Kriging methods

Kriging method	Covariance model	Nugget variance (dB ²)	Sill (dB ²)	Range parameter (m)	Practical Range (m)
		τ^2	σ^2	φ	
OK+EUCL	Matérn with fixed $\kappa = 0.5$	0	22.4	122.2	366
UK+EUCL	Matérn with fixed $\kappa = 0.5$	0	11.2	95.1	285
OK+ROAD	Matérn with fixed $\kappa = 0.5$	0	24.2	230.7	691
UK+ROAD	Matérn with fixed $\kappa = 0.5$	0	11.9	160.6	481

322 C. *Spatial interpolation and performance analysis*

323 A subset of the observation locations is selected from the reference map and is interpolated over the
 324 whole study area using the four tested strategies. As described in Section III.A, the median sound level
 325 values at the observation locations are computed from at least 180 $L_{eq,1s}$ measurements included in a 25 m
 326 radius around each observation location. The subset of observation locations for evaluation is randomly
 327 selected and 1000 replications are performed for each subset configuration. The replications give
 328 information about the variability due to a chosen set of observation nodes. Figure 9 presents the
 329 relationships between two indicators of performance (RMSE and the Pearson correlation coefficient), and
 330 the density of nodes (number of nodes per sq. km) for the four methods of interpolation.



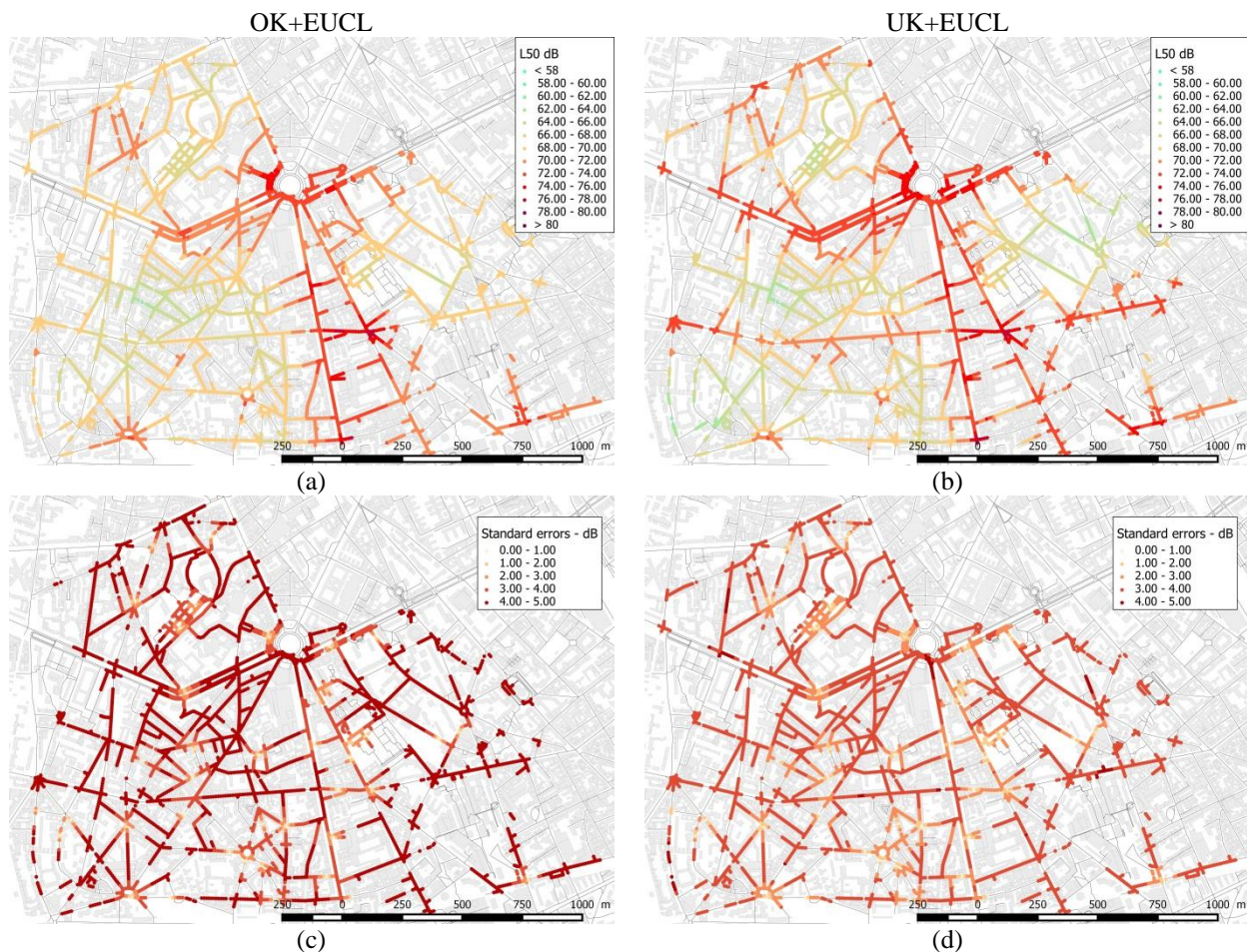
331 Figure 9 (color online) Relationships between the two indicators of quality (RMSE and the Pearson correlation coefficient), and the density of
 332 nodes (number of nodes per sq. km) for the four methods of interpolation. The area corresponds to the standard deviation associated with the 1000
 333 replications. (The red and blue lines are nearly superposed).

334

335 Universal Kriging (UK) considerably increases the quality of the results on both indicators compared
336 to ordinary Kriging (OK). Taking into account the distance along the road network (ROAD) only leads to
337 better results for the ordinary Kriging cases.

338 From a practical point of view, in case only fixed measurement stations are used, 15 observation
339 locations per sq. km is already a large number. In this case, Figure 9 shows that the correlation between
340 interpolated and reference sound levels is between 0.5 and 0.8, and that the RMSE value is between 2.5
341 and 3.5 dB. Figure 10 shows an interpolated sound map, using the UK + EUCL and the OK + EUCL
342 methods, based on one of the random sets of 42 observation locations (15 locations per sq. km). The
343 associated prediction standard error maps are also given, as the Kriging methods give access to this
344 information (Diggle and Ribeiro 2007). Figure 10 shows a good correspondence with the reference sound
345 map (see Figure 7). Nevertheless, the dispersion in Figure 9 shows that the observation locations have an
346 important influence over the performance of the algorithm.

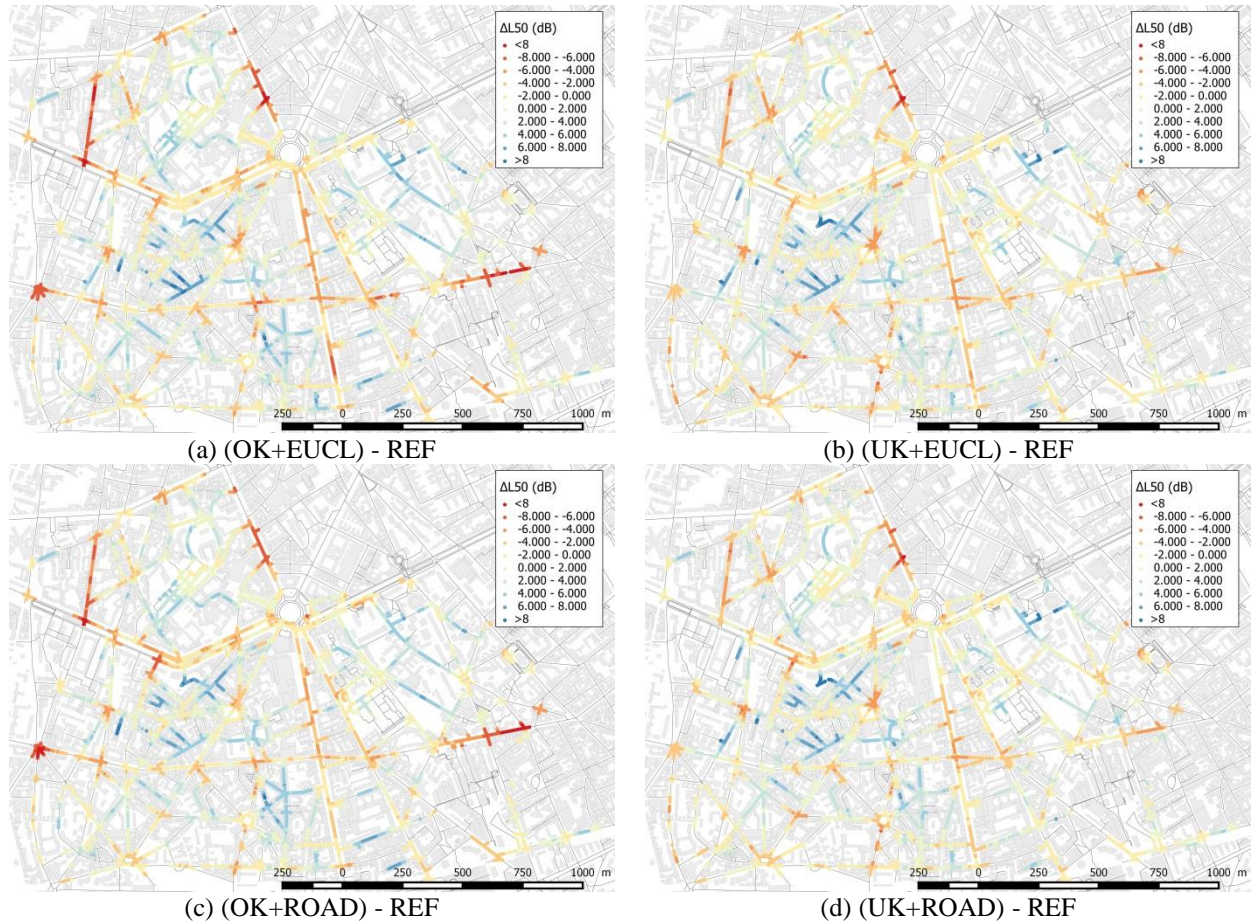
347



348 Figure 10 (color online) Example of an interpolated sound map from 42 observations: Estimated L_{50} values using the (a) OK+EUCL and (b)
349 UK+EUCL Kriging methods, and the associated standard errors maps (c) OK+EUCL and (d) UK+EUCL
350

351 Figure 11 shows the average errors for the 1000 replications (42 observation locations), against the
352 reference map presented in Figure 7. Figure 11 (a-d) shows that all the interpolation strategies

353 underestimate the sound level for large boulevards and overestimate the sound level for quiet places, but
 354 this is a common limitation of interpolation methods. This is less pronounced for universal Kriging
 355 (Figure 11 (b), Figure 11 (d)). In this case, the trend partly corrects this shortcoming, and the errors are
 356 distributed more uniformly over the study area. A comparison between Figure 11(a) and (c) or Figure
 357 11(b) and (d) shows the difference between the use of the Euclidian and the road network distance; no
 358 major influence is observable.



359 Figure 11 (color online) Average errors for 1000 replications (42 observation locations) against the reference map REF for the four interpolation
 360 methods (a) OK+EUCL, (b) UK+EUCL, (c) OK+ROAD and (d) UK+ROAD.
 361

362 Figure 9 shows that the standard deviation of the performance indicators can be important, especially
 363 in case of a small density of measurement locations. In order to identify the principal factors which
 364 influence the performance of the Kriging algorithms, a short statistical analysis has been performed over
 365 geospatial indicators of the spatial distribution of 42 observations. Table 2 presents some of the
 366 parameters which have been calculated for each generated set of observations.

367 The Variance Mean Ratio (VMR), also called index of dispersion, is an indicator of a good dispersion
 368 of the observations over the domain. For this, the domain is divided in 6x6 cells. In each cell, the number
 369 of observations is calculated. The variance mean ratio corresponds to the ratio between (i) the variance of
 370 the number of observations between each cell, and (ii) the mean number of observations for a cell. If
 371 VMR is equal to 1, the observations are randomly dispersed; if $VMR < 1$ the observations are more

372 dispersed than random (*e.g.*, regular distribution); if $VMR > 1$ the observations are more clustered than a
 373 random dispersion.

374 MEAN_ROAD, STD_ROAD, KURT_ROAD, SKEW_ROAD are indicators of the distribution of the
 375 observations over the road categories (integers from 1 to 4, see Figure 2). For example, if MEAN_ROAD
 376 is equal to 1 and STD_ROAD equal to 0, it means that all the observations are located over roads of type 1
 377 (avenue/boulevard).

378 Finally, the CENTER indicator represents the distance between the center of gravity of the observation
 379 locations and the center of gravity of the whole road network within the case study area. If CENTER is
 380 small, the observations are well centered over the domain.

381 Table 2 – Geospatial indicators of the observation locations.

Index	Name	Range
VMR	Variance Mean Ratio	[0.8 – 1.9]
STD_L50	Standard deviation on observed sound levels (L_{50})	[2.8– 6] (dB)
MEAN_ROAD	Average closest road categories	[1.7 – 2.8]
STD_ROAD	Standard deviation on closest road categories	[0.5 – 1.2]
KURT_ROAD	Kurtosis closest road categories	[-0.19 – 1.6]
SKEW_ROAD	Skewness closest road categories	[1.4 – 7.4]
CENTER	Distance between the gravity center of the observations and the gravity center of the whole network	[0.34 – 364] (m)

382 Table 3 presents the Pearson correlation coefficient between the spatial distribution indicators and the
 383 performance indicators. It shows that the STD_L50 is the only geospatial indicator that is well correlated
 384 with the performance indicators, particularly for ordinary Kriging. As it can be expected, this suggests that
 385 the measurement stations should be placed in locations that represent a large distribution of sound levels.
 386 For universal Kriging, this factor is less important because of the correction brought by the trends. The
 387 same analysis was carried out with other performance metrics than the Pearson correlation coefficient, but
 388 this did not bring more information.

389 Table 3 – Pearson correlation coefficient between the geospatial and performance indicators for two Kriging
 390 methods (* $p < 0.05$).

	r (OK+EUCL)	r (UK+EUCL)	RMSE (OK+EUCL)	RMSE (UK+EUCL)
VMR	0.06*		-0.07*	
STD_L50	0.48*	0.27*	-0.55*	-0.21*
MEAN_ROAD	0.07*			-0.04*
STD_ROAD	0.19*	0.07*	-0.19*	-0.10*
KURT_ROAD	-0.10*	-0.06*	0.07*	0.09*
SKEW_ROAD	-0.16*	-0.07*	0.14*	0.11*
CENTER	-0.05*		0.06*	

391 **IV. Discussion**

392 As the study area was the XIIIth district of Paris, the variogram and Kriging parameters were only
 393 adjusted for this district. Even if some of these parameters are comparable with those from previous
 394 studies (A. Can, Dekoninck, and Botteldooren 2014), other replications in other urban contexts will be
 395 necessary to extend the conclusions of this work.

396 On the one hand, the alternative definition of distance along the road network slightly increases the
397 performance of the algorithms, but only for ordinary Kriging methods. On the other hand, the proposed
398 trend based on the distance to the closest roads by category, as defined in Section II.F.4, strongly improves
399 the results for all Kriging methods. These results confirm the observations done in (Morillas et al. 2005;
400 Rey Gozalo, Barrigón Morillas, and Prieto Gajardo 2015; Juan Miguel Barrigón Morillas et al. 2011)
401 which state that using a street categorization method that accounts for street use is particularly appropriate
402 to study the spatial variability of urban sound levels. Thus, to continue to improve the performance of the
403 interpolation methods, efforts should be focused on the trend definition. Nevertheless, a complex trend
404 definition is comparable to the computation of a model-based sound map. Moreover, it might increase the
405 computational cost of the interpolation method considerably, and can possibly introduce new error
406 sources.

407 The errors associated with the observations are not taken into account in our Kriging approach;
408 nevertheless, a follow-up study should investigate the integration of noisy observations with a proper
409 uncertainty associated with each measurement location. Maybe data assimilation methods used in
410 geosciences, *e.g.*, computing the so-called best linear unbiased method (Bouttier et al. 2002; Tilloy et al.
411 2013), could be useful to achieve this task.

412 All the presented results are valid for one homogeneous time period. Time interpolation could be
413 added. This interpolation could be done in pre- or post-processing, relying on previous works dealing with
414 temporal interpolation of urban sound levels (Prieto Gajardo et al. 2016) or directly with a spatio-temporal
415 Kriging.

416 Finally, this study shows the influence of the spread of observation locations over the study area to
417 correctly interpolate sound levels. As it can be expected, the results suggest that fixed measurement
418 stations should be placed to obtain a large distribution of sound levels and so, a large variety of sound
419 environments. Nevertheless, even if it can be estimated, this information is not fully available prior to the
420 installation of the measurement stations. Other indicators, such as the distribution of the measurement
421 stations over the various road categories, or the location of the center of gravity of the measurement
422 stations, which are available a priori, do not show relevant relationship with performance indicators.

423 V. Conclusion

424 By means of a progressive degradation of a reference map of 2.8 km² interpolated from geo-referenced
425 mobile measurements, spatial interpolation methods were compared. The impact of the density of
426 observation points and the performance of four spatial interpolation methods were presented. The four
427 interpolation methods were constructed by combining two algorithms: (i) the Kriging method, either
428 ordinary Kriging or universal Kriging (which consists in adding a linear trend, defined from the distance
429 between each location and its closest road in each category) and (ii) the definition of the distance between
430 locations, either Euclidian or computed from the road network.

431 The main conclusions are:

- 432 • A minimum of 180 1-s measurements are needed to obtain an acceptable level of confidence
433 (1dB) in the L_{50} value calculated at each location within the study area, with an aggregation
434 radius of 25 m.

- 435 • The practical ranges of the variograms computed for the four Kriging methods are between
436 250 and 700 meters.
- 437 • Using the distance along the road network in the Kriging method considerably increases the
438 performance in case of ordinary Kriging, but not in case of universal Kriging.
- 439 • Universal Kriging, which consists of adding a local trend in the ordinary Kriging formulation,
440 is a promising method. Nevertheless, it introduces an additional calculation of the trend that
441 has a pre-processing cost and can in itself be a source of error.
- 442 • Approximately 50 observation locations per sq. km are needed in order to get a correlation
443 coefficient superior to 0.8 and a RMSE value inferior to 2.5 dB between the reference and the
444 interpolated map.

445 In view of the large density of observation locations needed to obtain a sound map with a high
446 accuracy and the strong improvement brought by the trend in the Kriging formulation, further studies
447 should probably focus on fusion or assimilation techniques combining measurements and numerical
448 simulation results.

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