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► **To cite this version:**

Jean Barrère, Gilles Chabriel. BLIND WAVE SEPARATION BY SPATIAL OVERSAMPLING. Proceedings of the 2nd Physics in Signal and Image Processing International Symposium (PSIP'2001), 2001, Marseille, France. hal-01826182

**HAL Id: hal-01826182**

**<https://hal.science/hal-01826182v1>**

Submitted on 29 Jun 2018

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# BLIND WAVE SEPARATION BY SPATIAL OVERSAMPLING

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## ABSTRACT

This paper is a contribution to the problem of the separation of propagating source signals recorded simultaneously by a set of receivers.

We propose to use a small-sized sensor array so that the waves are spatially oversampled. Sensors are assumed to be directional, to have the same complex frequency response and to be differently oriented in space. Under these assumptions, sources are received on each sensor with different attenuations and with different time delays.

When the dimensions of the array are chosen so that time delays are small in comparison with the coherence time of each source, we show that the array outputs can be approximated to a particular model of instantaneous mixtures involving the sources and their first derivative with respect to time. Because sources are statistically dependent to their first derivative, this problem does not appear as a classical Blind Source Separation (BSS) problem. We present then a matched second-order blind identification algorithm in order to estimate this particular mixing system.

The validity of the proposed model and of our algorithm is confirmed by computer simulations in the case of audio sources.

## 1. GENERAL MODEL

It is assumed that a set of  $N$  independent colored signals  $x_1(t), \dots, x_N(t)$  are propagating in an echo-free environment. These signals are recorded on  $M$  sensors without any additive noise (presence of noise will be treated in the full paper). The observation satisfy the equation model below:

$$\begin{aligned} y_1(t) &= x_1(t) + x_2(t) + \dots + x_N(t) \\ y_i(t) &= \sum_{j=1}^{j=N} c_{i,j} x_j(t - \tau_{i,j}), \quad i = 2, \dots, M \end{aligned} \quad (1)$$

where  $\tau_{i,j}$  and  $c_{i,j}$  represent respectively the relative delay and the relative amplitude of source  $x_j(t)$  observed on the  $i^{th}$  sensor versus the first observation  $y_1(t)$ .

We'll show in the full paper that in case of a compact sensor array, delays are sufficiently small when:

$$\tau_{i,j}^2 \ll \frac{1}{2\pi^2\nu_M^2}, \quad \forall i, j,$$

where  $\nu_M$  is the maximum frequency present in the observations.

In this case, an approximation for the observations  $y_i(t)$  ( $i = 2, \dots, M$ ) using an order one Taylor expansion can be considered:

$$\begin{aligned} y_i(t) \approx & c_{i,1}x_1(t) - c_{i,1}\tau_{i,1} \frac{dx_1(t)}{dt} + \\ & c_{i,2}x_2(t) - c_{i,2}\tau_{i,2} \frac{dx_2(t)}{dt} + \\ & \dots + \\ & c_{i,N}x_N(t) - c_{i,N}\tau_{i,N} \frac{dx_N(t)}{dt}. \end{aligned} \quad (2)$$

Let consider the observation vector  $\mathbf{y}(t) = [y_1(t), y_2(t), \dots, y_N(t)]^T$ . Using approximation (2) in (1), the set of equations (1) can be rewritten as:

$$\mathbf{y}(t) \approx \mathbf{M}_1 \mathbf{x}(t) + \mathbf{M}_2 \dot{\mathbf{x}}(t).$$

where,

$$\mathbf{M}_1 = \begin{bmatrix} 1 & \dots & 1 & \dots & 1 \\ c_{2,1} & \dots & c_{2,j} & \dots & c_{2,N} \\ \vdots & & \vdots & & \vdots \\ c_{M,1} & \dots & c_{M,j} & \dots & c_{M,N} \end{bmatrix}, \mathbf{M}_2 = - \begin{bmatrix} 0 & \dots & 0 & \dots & 0 \\ c_{2,1}\tau_{2,1} & \dots & c_{2,j}\tau_{2,j} & \dots & c_{2,N}\tau_{2,N} \\ \vdots & & \vdots & & \vdots \\ c_{M,1}\tau_{M,1} & \dots & c_{M,j}\tau_{M,j} & \dots & c_{M,N}\tau_{M,N} \end{bmatrix},$$

and the components of vector  $\dot{\mathbf{x}}(t)$  are the first derivatives of each source. We can use the following matrix and vector notation:

$$\mathbf{y}(t) \approx [\mathbf{M}_1 \quad \mathbf{M}_2] \begin{bmatrix} \mathbf{x}(t) \\ \dot{\mathbf{x}}(t) \end{bmatrix}. \quad (3)$$

Matrix  $\mathbf{M} = [\mathbf{M}_1 \quad \mathbf{M}_2]$  ( $M \times 2N$ ) is called the instantaneous mixing matrix for small delays.

Equation (3) shows that when the different waves are spatially oversampled ( $\tau_{i,j}^2 \ll \frac{1}{2\pi^2\nu_M^2}$ ,  $\forall i, j$ ) the sensor array delivers a set of instantaneous mixtures involving sources and their first derivatives. For each source signal, original source and its first derivative are dependent signals but uncorrelated at the same time. This last property will be exploited to identify the instantaneous mixing system  $\mathbf{M}$ .

An other important result to be pointed out is: when delays become small, the dimension of the signal sub-space defined by the array outputs tends towards  $2N$ . So, to simplify here the summary, we'll take  $M = 2N$  (twice as much sensors than sources) and we'll assume that, in that case,  $\mathbf{M}$  is a regular matrix (case  $M > 2N$  will be treated in the full paper with the noisy case).

## 2. IDENTIFICATION OF THE INSTANTANEOUS MIXING MATRIX FOR SMALL DELAYS

The BSS problem consists in estimating a separating matrix  $\mathbf{S}$  such as:  $\mathbf{S}\mathbf{M} = \mathbf{D}\mathbf{P}$ , where  $\mathbf{D}$  is a regular diagonal matrix,  $\mathbf{P}$  is a permutation matrix.

The product of  $\mathbf{S}$  with the observations leads to:  $\mathbf{z}(t) = \mathbf{D}\mathbf{P}[\mathbf{x}(t), \dot{\mathbf{x}}(t)]^T$ , representing the sources and their first derivatives except for one permutation and a scaling factor.

Because of the spectra differences of sources, the problem can be solved adapting classical blind identification methods for instantaneous mixtures using second-order statistics of the observations (see Tong's AMUSE [1] [2], SOBI [3], IMISO [5] or [4] ...).

Consider the spatial covariance matrix  $\mathbf{R}_{yy}(\tau) = \mathbb{E}[\mathbf{y}(t)\mathbf{y}(t+\tau)^T]$ . With expression (3),  $\mathbf{R}_{yy}(\tau)$  reads:

$$\begin{aligned} \mathbf{R}_{yy}(\tau) &\approx \mathbf{M} \mathbb{E} \left[ \begin{bmatrix} \mathbf{x}(t) \\ \dot{\mathbf{x}}(t) \end{bmatrix} \begin{bmatrix} \mathbf{x}(t+\tau) \\ \dot{\mathbf{x}}(t+\tau) \end{bmatrix}^T \right] \mathbf{M}^T \\ &\approx \mathbf{M} \begin{bmatrix} \mathbf{R}_{xx}(\tau) & \mathbf{R}_{x\dot{x}}(\tau) \\ \mathbf{R}_{\dot{x}x}(\tau) & \mathbf{R}_{\dot{x}\dot{x}}(\tau) \end{bmatrix} \mathbf{M}^T. \end{aligned} \quad (4)$$

With a lag  $\tau = 0$ , each source  $\mathbf{x}_i(t)$  and its first derivative  $\dot{\mathbf{x}}_i(t)$  are uncorrelated. Matrices  $\mathbf{R}_{x\dot{x}}(0)$  and  $\mathbf{R}_{\dot{x}x}(0)$  are then null matrices. Because sources are uncorrelated, matrices  $\mathbf{R}_{xx}(0)$  and  $\mathbf{R}_{\dot{x}\dot{x}}(0)$  are diagonal matrices. Thus, we can write for spatial covariance  $\mathbf{R}_{yy}(0)$ :

$$\mathbf{R}_{yy}(0) = \mathbf{M}\mathbf{D}_0\mathbf{M}^T, \quad (5)$$

where  $\mathbf{D}_0$  is a diagonal matrix.

Let's linearly filter with the impulse response  $h(t)$  each member of expression (4):

$$\mathbf{R}_{yy}^h(\tau) \approx \mathbf{M} \begin{bmatrix} \mathbf{R}_{xx}^h(\tau) & \mathbf{R}_{x\dot{x}}^h(\tau) \\ \mathbf{R}_{\dot{x}x}^h(\tau) & \mathbf{R}_{\dot{x}\dot{x}}^h(\tau) \end{bmatrix} \mathbf{M}^T,$$

where  $\mathbf{R}_{..}^h(\tau) = (h * \mathbf{R}_{..})(\tau)$ .

The entries of the function matrix  $\mathbf{R}_{x\dot{x}}(\tau) (= \mathbf{R}_{\dot{x}x}(\tau))$  are odd  $\tau$  functions. If the impulse response  $h(t)$  is chosen even, the entries of  $\mathbf{R}_{x\dot{x}}^h(\tau) (= \mathbf{R}_{\dot{x}x}^h(\tau))$  are still odd  $\tau$  functions. Taking  $\tau = 0$  assures these previous matrices to be null.

Matrices  $\mathbf{R}_{xx}^h(0)$  and  $\mathbf{R}_{\dot{x}\dot{x}}^h(0)$  being diagonal, the filtered spatial covariance matrix  $\mathbf{R}_{yy}^h(0)$  can be factorized as:

$$\mathbf{R}_{yy}^h(0) = \mathbf{M}\mathbf{D}_0^h\mathbf{M}^T, \quad (6)$$

where  $\mathbf{D}_0^h = \begin{bmatrix} \mathbf{R}_{xx}^h(0) & \mathbf{R}_{x\dot{x}}^h(0) \\ \mathbf{R}_{\dot{x}x}^h(0) & \mathbf{R}_{\dot{x}\dot{x}}^h(0) \end{bmatrix}$  is a diagonal matrix.

Because matrix  $\mathbf{R}_{yy}(0)$  is regular, we can introduce the matrix:  $\mathbf{R} = [\mathbf{R}_{yy}(0)]^{-1} \mathbf{R}_{yy}^h(0)$ .

Then from (5) and (6), it comes:  $\mathbf{R} = [\mathbf{M}^T]^{-1} [\mathbf{D}_0]^{-1} [\mathbf{D}_0^h] \mathbf{M}^T$ . We can show that  $(\mathbf{M}^T)^{-1}$  can be estimated except for one diagonal matrix and one permutation matrix from the eigenvector matrix of  $\mathbf{R}$ .

### 3. SIMULATIONS

#### Array and source configuration:

Two audio signals propagating in the air are detected by an half spherical sensor array (see fig. 1). In this case of two sources mixed, only four sensors are used; they are plotted in bold on fig. 1. Distance between center of close sensors is 2cm. Sphere radius is 17 cm. Each sensor have the same complex frequency response and the same cosine beam pattern (this for any frequency in the sensor bandwidth). The first audio source is an Beethoven fragment and the second is a swept sine. Each source is sampled at 5.5 KHz and is 6 seconds long (32768 samples). Fig. 2 shows Power Spectral Density (PSD) of the sources.

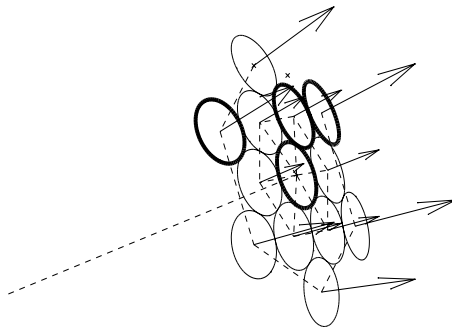


Figure 1: Sensor Array

#### Results:

The performance of our method is measured using the criterion introduced by Shobben and al in [6]. The quality of separation of the  $j^{th}$  separated output is defined as:

$$S_j = 10 \log \left( \frac{\mathbb{E}[(z_{j,x_j})^2]}{\mathbb{E}[(\sum_{i \neq j} z_{j,x_i})^2]} \right),$$

where  $z_{j,x_i}$  is the  $j^{th}$  output when only  $x_i$  is active.

For our numerical experiments the performance measures can be found in the following table:

Estimated Source #1	$S_1 = 39\text{dB}$
Estimated Source #2	$S_2 = 40\text{dB}$

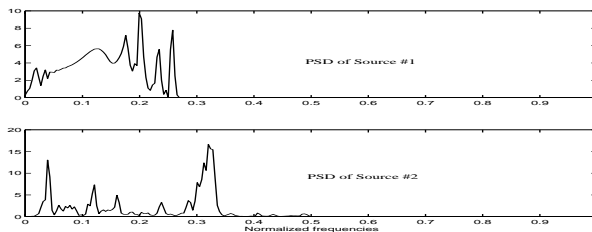


Figure 2: DSP of sources

Fig.3 presents the PSD of the estimated sources which are similar to those of fig. 2.

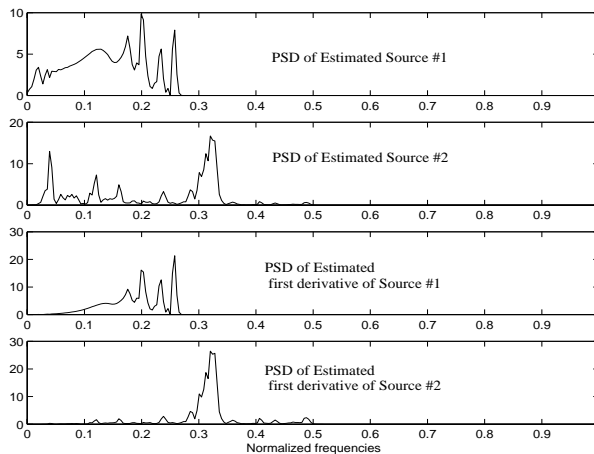


Figure 3: DSP of estimated sources

#### 4. CONCLUSION

A method to separate propagating sources has been presented. This approach needs to record the waves with a compact discrete antenna so that propagation delays become small in comparison to the signal coherence time (spatial oversampling). Let point out that this notion of compactness depends both on the spectra of sources but also on the propagation velocity. For example in sub-marine propagation, the dimensions of antenna can be higher than in aerial propagation.

We showed that in previous conditions, the problem can be seen of a BSS problem of statistically dependent sources. Inversion of the mixing system is obtained with a matched algorithm based on the second-order statistics of waves. Simulation of the extraction of two audio sources in aerial propagation recorded with a feasible discrete sensor array have been provided. Good results obtained prove the practical interest of our approach.

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