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THE ARCHIMEDES PRINCIPLE APPLIED TO SEPARATION OF UNIFORMLY DISTRIBUTED SOURCES

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ABSTRACT

In this work, we are interested in the separation of N source signals recorded simultaneously by N receivers. We present in this work a method based on an analogy between the research of independent axes of an hypercube (geometrical representation of a mixing of uniform sources) and the research of equilibrium states of weighing system submitted to discontinued gravity fields. The method only use one order statistics and is able to treat a large amount of sources. Presently this method is limited to uniformly or symmetrically distributed sources.

1. INTRODUCTION

1.1. Position of the problem

Let's consider N independent sources with uniform probability distribution, simultaneously received on N sensors. The mixing process is characterized by the following equation:

$$\mathbf{y}(t) = \mathbf{M}\mathbf{x}(t), \tag{1}$$

in which $\mathbf{x}(t) = [x_1(t), x_2(t), \dots, x_N(t)]^T$ is the vector of zero-mean statistically independent unknown sources and $\mathbf{y}(t) = [y_1(t), y_2(t), \dots, y_N(t)]^T$ is the observation vector. In (1), \mathbf{M} is the $N \times N$ unknown mixing matrix, assumed to be full column rank.

The identification problem consists in estimating a separating matrix \mathbf{S} such as: $\mathbf{S}\mathbf{M} = \mathbf{D}\mathbf{P}$, where \mathbf{D} is an regular diagonal matrix. \mathbf{P} is a permutation matrix which has one nonzero entry in each row and column.

The product of \mathbf{S} with the observation vector leads to:

$$\mathbf{z}(t) = \mathbf{D}\mathbf{P}\mathbf{x}(t). \tag{2}$$

The vector $\mathbf{z}(t)$ represents the source vector $\mathbf{x}(t)$ except for one permutation and a scaling factor.

We will recall in the main paper how the use of second order statistics permits to reduce the problem to the research of an unitary mixing matrix (whitening of the observations). The Figure 1 represents a two dimensional geometrical illustration of the space sources (a), the mixing space (b), the action of the whitening procedure (c) and the estimated sources in the whitening space (d).

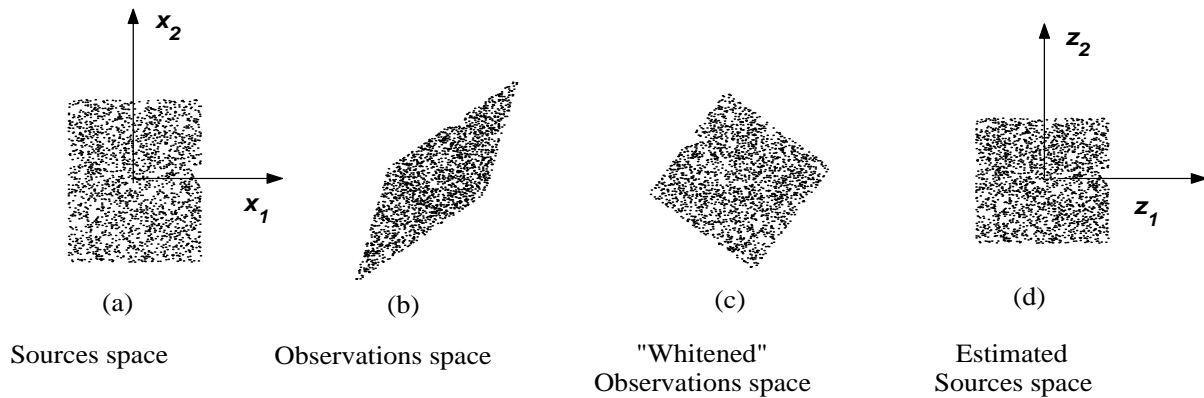


Figure 1: 2D different spaces

So in the following, without any loss of generality, we will only consider the case where \mathbf{M} is an unitary matrix and the sources are power unit. The problem reduces in finding the unitary matrix \mathbf{S} i.e. the unitary transform between (c) and (d) in Figure 1. From a geometrical point of view the problem is to obtain the symmetric axes parallel to the faces of the hypercube of the observations.

1.2. Recall of Static

Let's consider the following ideal two dimensional experiment: an homogeneous square thin plate is immersed in an homogeneous liquid. The plate is rigid and its density is half the density of the liquid. The plate is submitted to two kind of different forces: volume forces (surface force in the two dimensional case) due to the gravity field, and surface forces (line force in the two dimensional case) due to the pressure of liquid acting on the bound of the immersed solid. We denote by \vec{F}_g the resultant of gravity forces, O its point of application and \vec{F}_p the resultant of surface forces, C its point of application. The equilibrium of the plate is obtained when the sum of the resultant is null and when the momentum of forces is null i.e. when \vec{OC} and \vec{F}_g are linearly dependent. The Figure 2 illustrates such experiment.

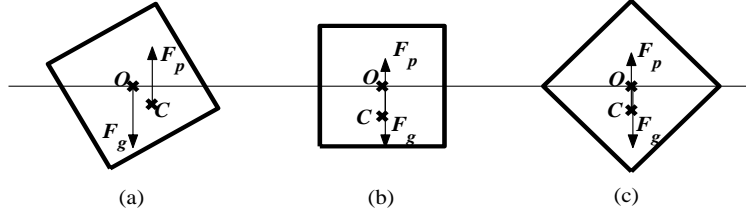


Figure 2: Archimedes Principle

We can consider pressure forces as volume forces acting on an equivalent volume of liquid corresponding to the immersed part of the solid and C is its the center of gravity. This phenomenon is well known as the Archimedes principle. Because of homogeneity, the equilibrium positions give us the axis of symmetry of the plate. For attractive forces, the stable equilibrium state is obtained when the distance $|OC|$ is minimum (see figure (2.c)) unstable when the distance $|OC|$ is maximum (see figure (2.b)). For repulsive forces, the stability states will be inverted. This basic application of static can be easily extend to N dimensional homogeneous systems. Following, we will inspired from it to construct an iterative method extracting the stable state of equilibrium of the N -dimensional hypercube of observations immersed in a discontinuous vector field.

2. ALGORITHM

2.1. Brief description

We describe here one step (from iteration k to iteration $k + 1$) of the iterative method proposed. Let consider the mixing space of N uniformly distributed power unit centred sources as illustrated in Figure 3.a. Each point of this space $\mathbf{y}^k(t) = [y_1^k(t), y_2^k(t), \dots, y_N^k(t)]^T$ is submitted to a discontinuous vector field \mathbf{p} such as:

$$\mathbf{p}(\mathbf{y}^k(t)) = \begin{cases} [1, 0, \dots, 0]^T & \text{if } y_1^k(t) \geq 0 \\ [0, 0, \dots, 0]^T & \text{if } y_1^k(t) < 0 \end{cases} \quad (3)$$

and a continuous vector field \mathbf{g} :

$$\mathbf{g}(\mathbf{y}^k(t)) = [-0.5, 0, \dots, 0]^T \quad (4)$$

Because of uniform distribution of sources, the resultant of two fields are : $\mathbf{F}_p = -\mathbf{F}_g = E\{\mathbf{p}\}$.

The points of application of these resultants are : $C^k = E\{\mathbf{y}^k / y_1^k \geq 0\}$ for the discontinuous field, and $O = E\{\mathbf{y}^k\}$ for the continuous field.

Let's denote \mathbf{u}_1 the vector such as $\mathbf{u}_1 = \frac{\vec{OC}^k}{|OC^k|}$. Using an Gram-Schmidt orthogonalization on \mathbf{u}_1 and a set of $N - 2$ any linearly independent vectors, we construct a set of $N - 1$ ortho-normalized vector \mathbf{u}_i , $i = 2, \dots, N$.

It results a $N \times N$ unitary matrix :

$$\mathbf{U}_1^k = [\mathbf{u}_1, \dots, \mathbf{u}_N].$$

Applying the transform $[\mathbf{U}_1^k]^T$ to the mixing space \mathbf{y}^k we obtain an new balanced mixing space versus the field force in presence as described in Figure 3.b:

$$\mathbf{y}^{k+1} = [\mathbf{U}_1^k]^T \mathbf{y}^k$$

We reiterate the process taking in account the vector fields \mathbf{p} and \mathbf{g} on the new mixing space \mathbf{y}^{k+1} until convergence,

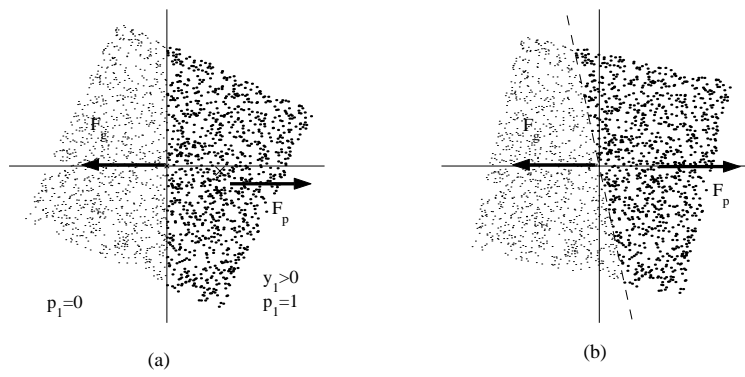


Figure 3: One step

i.e. \mathbf{U}_1 is the an identity matrix. Because of uniformity, after convergence we have separation of one source from the others:

$$\mathbf{y}^n(t) = [z_1(t), y_2^n(t), \dots, y_N^n(t)]^T$$

The procedure can be applied again on the $N - 1$ remaining observations. The separation will be achieve when all sources have been extracted. A proof of convergence will be given for the two-dimensional case in the whole paper.

2.2. Acceleration of convergence

The method can be strongly accelerate if we use in the Gram-Schmidt procedure a set of vector jointly evaluated from N different discontinuous field vectors:

$$\mathbf{p}_j(\mathbf{y}^k(t)) = \begin{cases} 1, \dots, j, \dots, N \text{ index of source} \\ [0, \dots, 1, \dots, 0]^T & \text{if } y_j^k(t) \geq 0 \\ [0, \dots, 0, \dots, 0]^T & \text{if } y_j^k(t) < 0 \end{cases} \quad (5)$$

3. EVALUATION - APPLICATION

The valuation of performance of separation is measured by an index introduced in [3], constructed on the the global matrix $\mathbf{G} = \mathbf{S}\mathbf{M}$, normalized by the number of non-diagonal terms in this matrix:

$$\text{ind}(\mathbf{G}) = \frac{1}{2N(N-1)} \left[\sum_i \left(\sum_j \frac{|G_{i,j}|^2}{\max_l |G_{i,l}|^2} - 1 \right) + \sum_j \left(\sum_i \frac{|G_{i,j}|^2}{\max_l |G_{l,j}|^2} - 1 \right) \right]$$

This non-negative index takes its values between 0 and 1. A small value indicates the proximity of the desired solutions.

The following table gives an idea of the sensibility of the method versus the data number for a mixing of 3 sources. Each line was obtained for a set of 1000 different realizations and mixing matrices.

Data Number	Mean(ind)	Std(ind)	Number of non separated cases
2048	0.00400	0.0030	5
4096	0.00120	0.0010	1
8192	0.00045	0.0034	0

On the Figure 4, we illustrate the robustness of the method for a number of sources equal to 10,20,...,80. The curves are obtained for a data number equal to 32768.

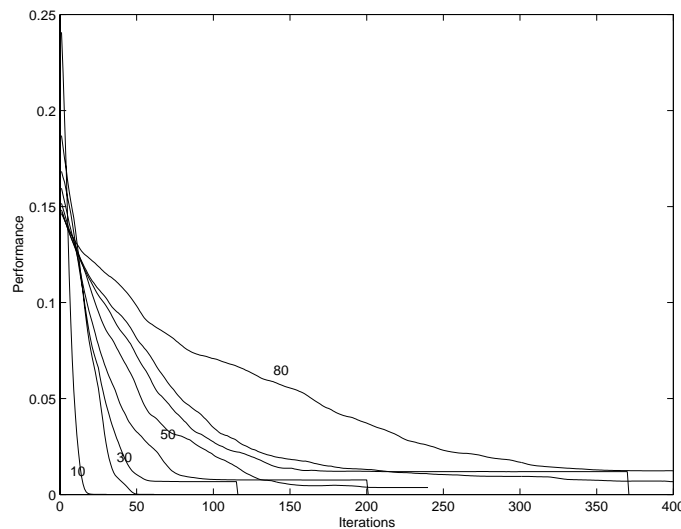


Figure 4: Performances for different source numbers

4. CONCLUSIONS-EXTENSIONS

The method proposed can be considered as an extension of geometric ones, see [1], [2], by the addition of an exterior field vector acting on the space observation. It can be interpreted also as classical second order statistics ones applied to non linear filtered observations. The separation has been presented on uniformly distributed sources. Because the method extracts symmetry axes, it can be extended to symmetrically distributed sources. Of course, many other field can be tested in order to generalize the separating process. In case of n -valuated sources the existence of meta-stable configurations implies some modifications of the algorithm.

REFERENCES

- [1] C.G. Puntonet, A. Prieto, C. Jutten, M. Rodriguez-Alvarez, and J. Ortega. *Separation of sources: a geometry based procedure for reconstruction of n -valued signals*, in Signal Processing, Vol.46 n 3, pp 267-284, 1995.
- [2] C.G. Puntonet, C.A. Mansour, and C. Jutten. *Geometrical algorithms for blind separation of sources*, in Actes du XV^{ème} colloque GRETSI, pp 273-275, Sept. 1995.
- [3] E.Morau and O. Macchi. *High-order contrasts for self-adaptative source separation*, in International Journal of Adaptative Control and Signal Processing, Vol.10, pp 19-46, 1996.